NUI Galway
OÉ Gaillimh

Semester 1 Examinations 2017-2018

Exam Code(s)	4BP, 4BLE, 1MBM, 1CSD
Exam(s)	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Master of Engineering (Biomedical Engineering) Master of Science in Computer Science (Data Analytics)
Module Code(s)	EE445
Module(s)	**Digital Signal Processing**
Paper No.	1
Repeat Paper	No
External Examiner(s)	Prof. B. Foley
Internal Examiner(s)	Prof. G. Ó Laighin Dr. E. Jones
Instructions:	**Answer any three questions from four** All questions carry 20 marks each
Duration	2 hours
No. of Pages	6 pages (including cover page)
Discipline	Electrical & Electronic Engineering
Course Co-ordinator(s)	Dr. E. Jones

Requirements:
MCQ
Handout
Statistical Tables
Graph Paper
Log Graph Paper
Other Material — Standard mathematical tables

Question 1

(a) A digital filter is described by the following difference equation:

$$y(n) = 0.3x(n) - 0.25x(n-1) + 0.5y(n-1) - 0.4y(n-2)$$

Give the transfer function of the system and hence its frequency response. Derive expressions for its magnitude and phase responses. What is the magnitude response of the filter at a frequency equal to one third of the sampling frequency? What is the phase response of the filter at the same frequency? Does the filter have a linear or non-linear phase response? Explain your answer.

[7 marks]

(b) A digital filter has a pair of complex conjugate poles at $z = 0.5e^{\pm j0.7}$, and a pair of complex conjugate zeros at $z = 0.8e^{\pm j2.0}$. Sketch the pole-zero map of the filter. Based on the pole-zero map, and in particular on the locations of the poles and zeros, sketch the magnitude response of the system as a function of both radians and Hz, if the sampling frequency is 2 kHz. Also, using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 500 Hz.

[6 marks]

(c) A discrete-time system has a finite-duration impulse response that consists of the samples {2, -1, 3, -1, 2}, commencing at n = 0. Using time-domain convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {3, 0, -1}, also commencing at n = 0. Indicate in detail the calculations needed to determine $y(3)$.

From only the information given, state whether the filter has a linear or non-linear phase response, and estimate the group delay of the filter.

[7 marks]

Question 2

(a) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{1, 0, 1, -2\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{1, 2, -1, 3\}$, commencing at $n = 0$.

If the input signal $x(n)$ commences at $n = 3$ instead of $n = 0$, what is the impact on the filter output?

[7 marks]

(b) A digital filter has the following transfer function:

$$H(z) = \frac{1 - 0.4z^{-3}}{1 + 0.7z^{-1} - 0.6z^{-2}}$$

Determine the phase response of the system. What is the value of the phase response at a frequency equal to one sixth of the sampling frequency?

[6 marks]

(c) The transfer function of a first-order filter is described by the following equation:

$$H(z) = \frac{0.1}{1 - 0.9z^{-1}}$$

If the sampling rate is 1 kHz, determine the frequency in Hz at which the magnitude response of the filter is equal to -20 dB. What is the phase response of the filter at this frequency?

If the numerator of the filter transfer function is equal to 0.2 instead of 0.1, what effect will this have on the magnitude and phase responses of the filter as a function of frequency?

[7 marks]

Question 3

(a) A sensor in a building management system application requires the removal of mains interference at a frequency of 50 Hz from the sensor signal. Design a notch filter to achieve this objective. The signal is sampled at a frequency of 300 Hz and a notch of width 10 Hz will suffice. Sketch the pole-zero map of the filter and give the difference equation.

[6 marks]

(b) A nonlinear-phase 512-tap FIR digital filter with real coefficients is to be implemented as part of an audio processing system operating at a sampling rate of 96 kHz. Calculate the number of multiplies and additions required to process a 10-second duration of a (real) audio signal, if the filter is implemented using a standard transversal filter structure.

Calculate the possible saving in the number of multiplies required to process the same duration of input signal, if the filter is implemented using fast convolution in the frequency domain with 512-point FFT and Inverse FFT, and with Hamming windowing. You may assume that the coefficients of the window are pre-computed, and that there is overlap of 50% between successive frames. You may ignore any additional overhead associated with overlap-add processing of the FFT output.

[9 marks]

(c) A signal processing application requires spectral analysis of a signal that has a sampling frequency of 30 kHz. The spectral analysis is to be carried out using short-time analysis with a window length of 16 msec, and with a resolution such that the frequency step is no greater than 10 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution, assuming that the FFT algorithm is used for spectral analysis.

[5 marks]

Question 4

(a) A first-order analogue filter is described by the following transfer function:

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where ω_c is the cut-off frequency in radians/s.

Using the bilinear transformation, determine the transfer function of the digital equivalent of this filter, if the desired cut-off frequency is 4 kHz and the sampling frequency is 24 kHz. If pre-warping was not carried out, what would be the actual cut-off frequency of the digital filter?

[7 marks]

(b) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{4}{(s+3)(s+12)}$$

Assuming that the sampling rate is chosen to be eight times the highest pole frequency in the analogue filter, calculate the digital filter coefficients and write down the transfer function.

[7 marks]

(c) Describe the Window method for FIR filter design and outline the conditions necessary for the filter to have linear phase.

A linear phase FIR low-pass filter designed using the Window method operates at a sampling frequency of 8 kHz. If the group delay of the filter is 5 ms, determine the number of coefficients in the filter.

[6 marks]

Table of useful z-Transforms

	Sequence	z-Transform
1. Unit sample	$d(n)$	1
	$d(n\text{-}k)$	z^{-k}
2. Unit step	$u(n)$	$z/(z-1)$
3. Exponential	$a^n\, u(n)$	$z/(z-a)$
4. Sinusoidal	$\sin(\theta_O n)\, u(n)$	$\dfrac{z\sin\theta_0}{z^2-2z\cos\theta_0+1}$
	$\cos(\theta_O n)\, u(n)$	$\dfrac{z^2-z\cos\theta_0}{z^2-2z\cos\theta_0+1}$
5. Unit ramp	$nu(n)$	$\dfrac{z}{(z-1)^2}$
6. Product of ramp and signal	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

7. Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+\dots\dots\dots\dots+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$