Semester 1 Examinations 2015-2016

Exam Code(s)	4BP, 4BLE, 4BSE
Exam(s)	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Fourth Year Energy Systems Engineering – Electrical
Module Code(s)	EE445
Module(s)	**Digital Signal Processing**
Paper No.	1
Repeat Paper	No
External Examiner(s)	Prof. B. Foley
Internal Examiner(s)	Prof. G. Ó Laighin Dr. E. Jones
Instructions:	**Answer any three questions from four** All questions carry 20 marks each
Duration	2 hours
No. of Pages	6 pages (including cover page)
Discipline	Electrical & Electronic Engineering
Course Co-ordinator(s)	Dr. E. Jones

Requirements:

MCQ	
Handout	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	Standard mathematical tables

Question 1

(a) A digital filter is described by the following difference equation:

$$y(n) = 0.3x(n) - 0.75x(n-2) - 0.65y(n-1) + 0.5y(n-2)$$

Give the transfer function of the system and hence its frequency response. Derive expressions for its magnitude and phase responses.

What is the phase response of the filter at a frequency equal to one eighth of the sampling frequency?

[7 marks]

(b) A digital filter has a pair of complex conjugate poles at z = -0.3 ± j0.7, and a pair of complex conjugate zeros at z = 0.7 ± j0.5. Sketch the pole-zero map of the filter. Based on the pole-zero map, and in particular on the locations of the poles and zeros, sketch the magnitude response of the system if the sampling frequency is 2 kHz. Also, using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 500 Hz.

[7 marks]

(c) A discrete-time system has a finite-duration impulse response that consists of the samples {1, 2, -2, 1, -1}, commencing at n = 0. Using time-domain convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {2, 1, -1}, also commencing at n = 0.

Indicate in detail the calculations needed to determine $y(2)$.

[6 marks]

Question 2

(a) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{1, -1, 0, 3\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{1, 3, -1, 2\}$, but commencing at $n = 2$.

[6 marks]

(b) A digital filter has the following transfer function:

$$H(z) = \frac{1 - 0.6z^{-2}}{1 + 0.2z^{-1} - 0.7z^{-2}}$$

Determine the frequency response and hence the phase response of the system. What is the value of the phase response at a frequency equal to one quarter of the sampling frequency?

[4 marks]

(c) The transfer function of a first-order low-pass filter is described by the following equation:

$$H(z) = \frac{0.2}{1 - 0.8z^{-1}}$$

If the sampling rate is 8 kHz, determine the cut off frequency of the filter in Hz (where the cut-off frequency is defined as the "-3 dB frequency").

[6 marks]

(d) A digital low-pass filter has the following difference equation:

$y(n) = ax(n) + by(n - 1)$ $\qquad\qquad$ $0 < b < 1$

Choose a value for a (in terms of b) such that the DC gain of the filter is 0.8.

[4 marks]

Question 3

(a) Using the pole-zero placement method, determine the transfer function of a digital resonator with the following characteristics:

(i) Sampling rate of 20 kHz
(ii) Centre frequency of 1.5 kHz
(iii) Bandwidth of 40 Hz
(iv) DC gain of 0.6

Sketch the pole zero map of the filter and write the difference equation.

[5 marks]

(b) A signal processing application requires spectral analysis of a signal that has a sampling frequency of 32 kHz. The spectral analysis is to be carried out using short-time analysis with a window length of 40 msec, and with a resolution such that the frequency step is no greater than 20 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution, assuming that the FFT algorithm is used for spectral analysis.

[4 marks]

(c) A linear-phase 512-tap FIR digital filter is to be implemented as part of an audio signal processing system at a sampling rate of 96 kHz. Calculate the number of multiplies and additions required to process a 30-second duration of a (real) audio signal, if the filter is implemented using a standard transversal filter structure. Use the fact that the filter is linear phase to reduce the number of multiplies required.

Calculate the saving in the number of multiplies required to process the same duration of input signal, if the filter is implemented using fast convolution in the frequency domain using 512-point FFT and Inverse FFT, with Hamming windowing (assume that the coefficients of the window are pre-computed), and overlap of 50% between frames. You may ignore any additional overhead associated with overlap-add processing of the FFT output.

[7 marks]

(d) Design an oscillator that produces a cosine wave with a frequency of 1 kHz at a sampling rate of 30 kHz. The amplitude of the cosine wave should be 1. Calculate the values of the digital filter coefficients, and the initial conditions for the oscillator, assuming that the cosine wave starts with a phase shift of $\pi/3$.

[4 marks]

Question 4

(a) An environmental monitoring sensor application requires the removal of 50 Hz interference. Design a digital filter to achieve this objective. The signal is sampled at a frequency of 500 Hz and a notch of width 10 Hz will suffice.

[5 marks]

(b) Using the window method, obtain an expression for the impulse response of a linear phase FIR *high-pass* filter with a sampling rate of 8 kHz, a cut-off frequency of 1 kHz, and with a group delay equal to 5 msec.

[7 marks]

(c) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{1}{(s+4)(s+9)}$$

Assuming that the sampling rate is chosen to be eight times the highest pole frequency in the analogue filter, calculate the digital filter coefficients and write down the transfer function.

[8 marks]

Table of useful z-Transforms

		Sequence	z-Transform
1.	Unit sample	$d(n)$	1
		$d(n-k)$	z^{-k}
2.	Unit step	$u(n)$	$z/(z-1)$
3.	Exponential	$a^n\, u(n)$	$z/(z-a)$
4.	Sinusoidal	$\sin(\theta_O n)\, u(n)$	$\dfrac{z\sin\theta_0}{z^2-2z\cos\theta_0+1}$
		$\cos(\theta_O n)\, u(n)$	$\dfrac{z^2-z\cos\theta_0}{z^2-2z\cos\theta_0+1}$
5.	Unit ramp	$nu(n)$	$\dfrac{z}{(z-1)^2}$
6.	Product of ramp and signal	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

7. Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+..........+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$