NUI Galway
OÉ Gaillimh

Semester One Examination 2013-2014

Exam Code(s)	4BP, 4BLE, 4BSE
Exam(s)	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Fourth Year Energy Systems Engineering – Electrical
Module Code(s)	EE445
Module(s)	**Digital Signal Processing**
Paper No.	1
Repeat Paper	No
External Examiner(s)	Prof. G. W. Irwin
Internal Examiner(s)	Prof. G. Ó Laighin Dr. E. Jones
Instructions:	**Answer any three questions from four** All questions carry 20 marks each
Duration	2 hours
No. of Pages	6 pages (including cover page)
Discipline	Electrical & Electronic Engineering
Course Co-ordinator(s)	Dr. E. Jones

Requirements:

MCQ	
Handout	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	Standard mathematical tables

Question 1

(a) A digital filter is described by the following difference equation:

$$y(n) = x(n) - 0.6x(n-1) + 0.4y(n-1) - 0.5y(n-2)$$

State the transfer function of the system, and hence obtain expressions for its magnitude and phase responses.

What is the gain of the system at a frequency equal to one quarter of the sampling frequency?

[9 marks]

(b) A discrete-time system has a finite-duration impulse response that consists of the samples {1, 4, -2}, commencing at $n = 0$. Using time-domain convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {3, -1, 0, 4}, also commencing at $n = 0$.

Indicate in detail the calculations needed to determine $y(3)$.

[6 marks]

(c) A digital filter has a pair of complex conjugate poles at z = -0.2 ± j0.7, and a pair of complex conjugate zeros at z = 0.3 ± j1.1. Sketch the pole-zero map of the filter. Using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 100 Hz, if the sampling frequency is equal to 800 Hz.

[5 marks]

Question 2

(a) The transfer function of a first-order low-pass filter is described by the following equation:

$$H(z) = \frac{0.15}{1 - 0.85z^{-1}}$$

If the sampling rate is 10 kHz, determine the cut off frequency of the filter in Hz (you may assume the cut-off frequency is the "-3 dB frequency").

[5 marks]

(b) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{1, -1, 1\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{1, 2, 3, -1\}$, but commencing at $n = 1$.

[5 marks]

(c) A digital filter has the following transfer function:

$$H(z) = \frac{1 + 3z^{-1}}{1 + 0.4z^{-1} - 0.7z^{-2}}$$

Determine the frequency response, and hence the phase response of the system. What is the value of the phase response at a frequency equal to one-fifth of the sampling frequency?

[5 marks]

(d) Using the pole-zero placement method, determine the transfer function of a digital resonator with the following characteristics:

(i) Sampling rate of 16 kHz
(ii) Centre frequency of 4 kHz
(iii) Bandwidth of 40 Hz
(iv) DC gain of 1

Sketch the pole zero map of the filter.

[5 marks]

Question 3

(a) A healthcare instrumentation application requires the removal of interference at 60 Hz from an ECG signal. Design a digital notch filter to achieve this objective. The signal is sampled at a frequency of 1000 Hz, and a notch of width 25 Hz is required.

[5 marks]

(b) A signal processing application requires spectral analysis of a signal that has a sampling frequency of 16 kHz. The spectral analysis is to be carried out using short-time analysis with a window length of 30 msec, and with a resolution such that the frequency step is no greater than 20 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution, assuming that the FFT algorithm is used for spectral analysis.

[4 marks]

(c) A linear-phase 512-tap FIR digital filter is to be implemented at a sampling rate of 16 kHz using digital hardware in an FPGA. Calculate the number of multiplies and additions required to process a 10-second duration of the (real) input signal, if the filter is implemented using a transversal filter structure. Use the fact that the filter is linear phase to reduce the number of multiplies required.

Also, calculate the saving in the number of multiplies required to process the same duration of input signal, if the filter is implemented using fast convolution in the frequency domain using 512-point FFT and Inverse FFT.

(Assume that windowing with a 512-point Hamming window is used, where the coefficients of the window are pre-computed. Furthermore, you may assume overlap of 50% is used in the frequency-domain filtering approach in determining the number of frames; however, you may ignore any additional overhead associated with overlap-add processing of the FFT output.)

[6 marks]

(d) Design an oscillator that produces a cosine wave with a frequency of 2 kHz, at a sampling rate of 48 kHz. The amplitude of the cosine wave should be 1. Calculate the values of the digital filter coefficients, and the initial conditions for the oscillator, assuming that the cosine wave starts with a phase shift of $\pi/3$. Draw a block diagram of the oscillator.

[5 marks]

Question 4

(a) Using the window method, obtain an expression for the impulse response of a linear phase FIR *high-pass* filter with a sampling rate of 4 kHz, a cut-off frequency of 800 Hz, and with a group delay equal to 5 msec.

[7 marks]

(b) A first-order analogue filter is described by the following transfer function:

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where ω_c is the cut-off frequency in radians/s.

Using the bilinear transformation, determine the transfer function of the digital equivalent of this filter, if the desired cut-off frequency is 1.8 kHz and the sampling frequency is 16 kHz. If pre-warping was not carried out, what would be the actual cut-off frequency of the digital filter?

[7 marks]

(c) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{3}{(s+4)(s+5)}$$

Assuming that the sampling rate is chosen to be ten times the highest pole frequency in the analogue filter, calculate the digital filter coefficients, and write down the transfer function.

[6 marks]

Table of useful z-Transforms

		Sequence	z-Transform
1.	Unit sample	$d(n)$	1
		$d(n-k)$	z^{-k}
2.	Unit step	$u(n)$	$z/(z-1)$
3.	Exponential	$a^n\, u(n)$	$z/(z-a)$
4.	Sinusoidal	$\sin(\theta_O n)\, u(n)$	$\dfrac{z\sin\theta_0}{z^2-2z\cos\theta_0+1}$
		$\cos(\theta_O n)\, u(n)$	$\dfrac{z^2-z\cos\theta_0}{z^2-2z\cos\theta_0+1}$
5.	Unit ramp	$nu(n)$	$\dfrac{z}{(z-1)^2}$
6.	Product of ramp and signal	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

7. Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+..........+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$