EE445 DIGITAL SIGNAL PROCESSING: Assignment I

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1 Exercise 1.

(a). Using the Matlab freqz function, calculate and plot the magnitude response (in dB) of the following discrete-time system:

$$H(z) = \frac{1 + 0.4 \times z^{-1}}{1 - 1.5 \times cos(\frac{\pi}{8}) \times z^{-1} + 0.96 \times z^{-2}}$$

Use 1024 points evenly spaced between 0 and half the sampling frequency. For the plot, the x-axis should be in units of Hz (or kHz); you may assume a sampling rate of 12 kHz.

```
%
% Exercise 1.(a).
% calculate and plot the magnitude response using "freqz" function.
%

clear all;
close all;
clc;

% set up some constants
nsamp = 1024; % number of samples
fs = 12000; % sampling frequency

% specify the filter
b = [1 0.4];
a = [1 -1.5*cos(pi/8) 0.96];

% calculate the magnitude response using "freqz".
% and plot the magnitude response with the x-axis in units of Hz.
```

```
figure;
freqz(b, a, nsamp, fs);
```

Figure1 shows the frequency responses.

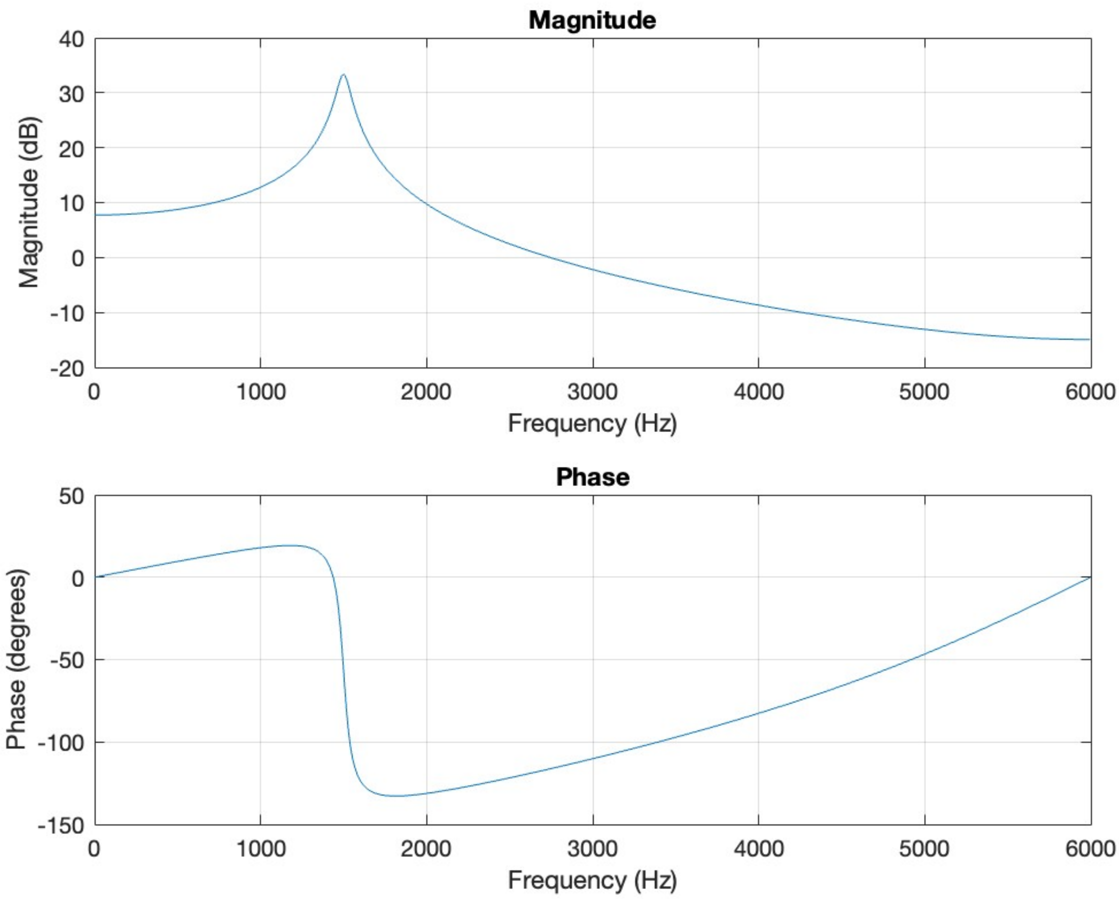


Figure 1: Frequency response of the discrete-time system of exercise 1.

(b). Determine the locations of the poles (in polar form), and calculate the centre frequency of the filter in Hz, assuming a sampling frequency of 12 kHz.

First, we find the location of poles in complex format using Matlab tools1:

$$p_1 = 0.6929 + 0.6927i$$

$$p_2 = 0.6929 - 0.6927i$$

.

```
%
% Exercise 1.(b).
% Determine the locations of the poles (in polar form).
```

```
%

clear all;
close all;
clc;

% transfer function
b = [1 0.4];
a = [1 -1.5*cos(pi/8) 0.96];

% get the roots of the numerator (zeros)
rnum = roots(b);

% get the roots of the denominator (poles)
rden = roots(a);

% get the modulus of poles
rs = abs(rden);

% get the angle of poles
theta = angle(rden);
```

Then, we calculate the modulus and the angles of the respective poles using Matlab1:

$$
\begin{aligned}
r_{p1} &= 0.9798 \\
r_{p2} &= 0.9798 \\
\theta_{p1} &= 0.7853 \\
\theta_{p2} &= -0.7853
\end{aligned}
$$

.

We convert the location of poles to polar form:

$$
\begin{aligned}
p_1 &= r_{p1} \times e^{j \times \theta_{p1}} = 0.9798 \times e^{j \times 0.7853} \\
p_2 &= r_{p2} \times e^{j \times \theta_{p2}} = 0.9798 \times e^{-j \times 0.7853}
\end{aligned}
$$

.

Then, we calculate the centre frequency of the filter by:

$$\theta_0 = 2 \times \pi \times \frac{f_0}{f_s}$$

We obtain the centre frequency:

$$
\begin{aligned}
\theta_{0,p1} &= 2 \times \pi \times \frac{f_{0,p2}}{f_s} = 2 \times \pi \times \frac{f_{0,p1}}{12000} \\
\theta_{0,p2} &= 2 \times \pi \times \frac{f_{0,p2}}{f_s} = 2 \times \pi \times \frac{f_{0,p2}}{12000}
\end{aligned}
$$

2 Exercise 2.

Determine the transfer function, and hence calculate and plot the magnitude response and impulse response, of a second-order filter that has a complex conjugate pole pair with a pole frequency of 3.4 kHz, and pole radius of 0.96. The filter also has a double zero at z = 0 in the z-plane. The sampling frequency is 16 kHz. For the magnitude response, use 2048 points equally spaced between DC and half the sampling frequency. You should ensure that your plots have proper axes.

The filter we want to design with the following specification:

- Pole frequency $= 3400Hz$
- Pole radius $= 0.96$
- Double zeros: $z_1 = z_2 = 0$
- Sampling frequency $= 16000Hz$
- Number of samples $= 2048 \in [0, f_s/2]$

The first step is to calculate the pole frequency (in radians) and the pole radius:

$$\theta_0 = 2 \times \pi \times \frac{f_0}{f_s} = 2 \times \pi \times \frac{3400}{16000} = \frac{17}{40}\pi$$

$$r_p = 0.96$$

.

Then, the coefficients are calculated as follows:

$$b_2 = r_p^2 = 0.96^2 = 0.9216$$

$$b_1 = -2 \times r_p \times \cos\frac{17}{40}\pi \approx -0.4482$$

.

$$1 + b_1 + b_2 = 1 - 0.4482 + 0.9216 = 1.4734$$

Hence, the transfer function can be written as:

$$H(z) = \frac{1.4734}{1 - 0.4482 \times z^{-1} + 0.9216 \times z^{-2}}$$

Let $z = e^{j\times\Theta}$. The frequency response can be given by:

$$\begin{aligned} H(\theta) &= \frac{1.4734}{1 - 0.4482 \times e^{-j\theta} + 0.9216 \times e^{-j2\theta}} &(1)\\ &= \frac{1.4734}{1 - 0.4482\cos\theta + j0.4482\sin\theta + 0.9216\cos 2\theta - j0.9216\sin 2\theta}. &(2)\end{aligned}$$

The magnitude response is obtained by:

$$|H(\theta)| = \frac{\sqrt{1.4734^2}}{\sqrt{(1 - 0.4482\cos\theta + 0.9216\cos 2\theta)^2 + (0.4482\sin\theta - 0.9216\sin 2\theta)^2}}$$

We use Matlab2 to calculate and plot the magnitude response:

```
%
% Exercise 2.
% Calculate and plot magnitude response.
%

clear all;
close all;
clc;

% set up some constants
fs = 16000; % sampling frequency
nsamp = 2048; % number of samples

% transfer function
b = [1];
a = [1 -0.4482 0.9216];

% scale to get a DC gain of 1
gainDC = sum(b)/sum(a);
scalefac = 1/gainDC;
b = b*scalefac;

% get and plot the magnitude response.
figure(1);
freqz(b, a, nsamp, fs);
title("Magnitude response of the filter in exercise 2.");
```

Figure2 shows the frequency responses.

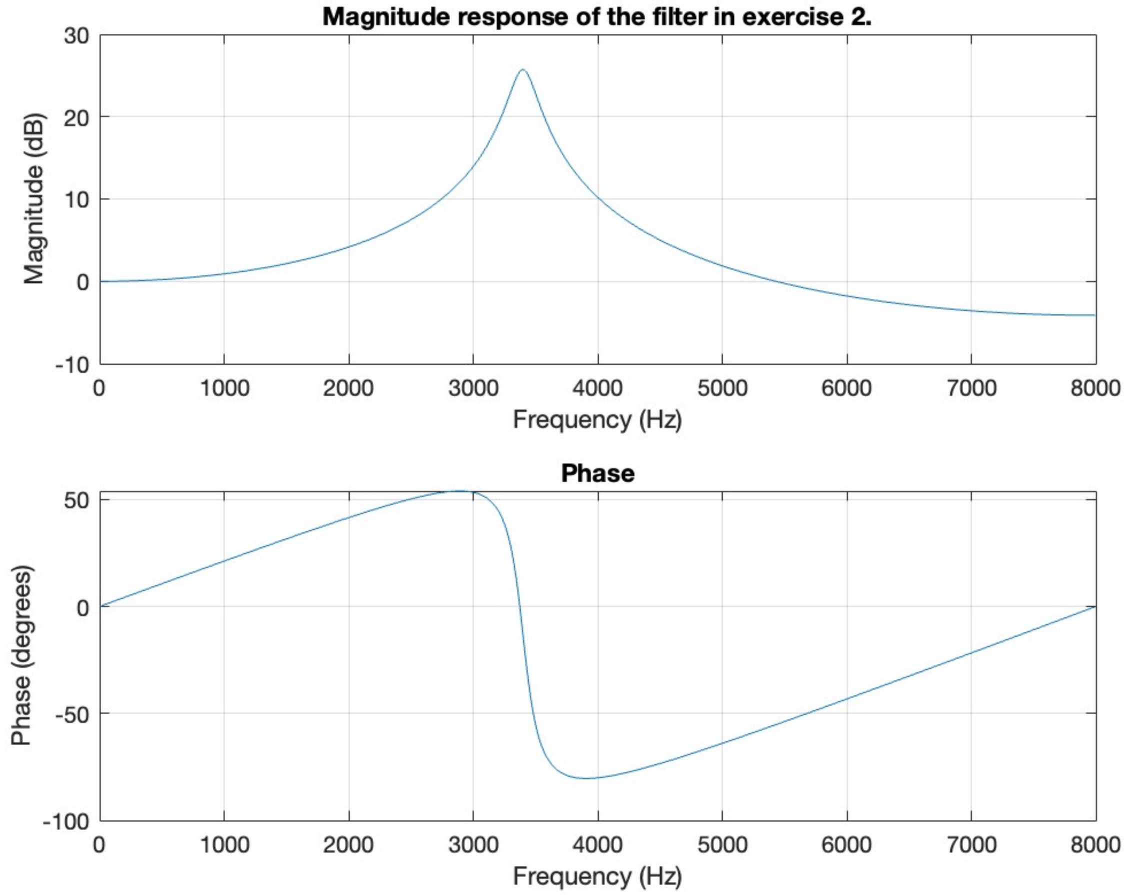


Figure 2: Frequency response of the filter of exercise 2.

The impulse response is obtained by using Matlab2:

```
%
% Exercise 2.
% calculate and plot the impulse response.
%

clear all;
close all;
clc;

% set up some constants
nsamp = 2048; % number of samples

% specify the filter
b = [1];
a = [1 -0.4482 0.9216];

```

```
% set up the inputs
impulse = zeros(nsamp, 1);
impulse(1) = 1;

% use "filter" function
y = filter(b, a, impulse);

% set up sample axis
sample_index = 1:nsamp;

% plot the impulse response
figure(1);
stem(sample_index, y);
grid on;
title("The impulse response for the filter in exercise 2.");
xlabel("Sample number");
ylabel("Amplitude(M)");
```

Figure3 shows the impulse responses.

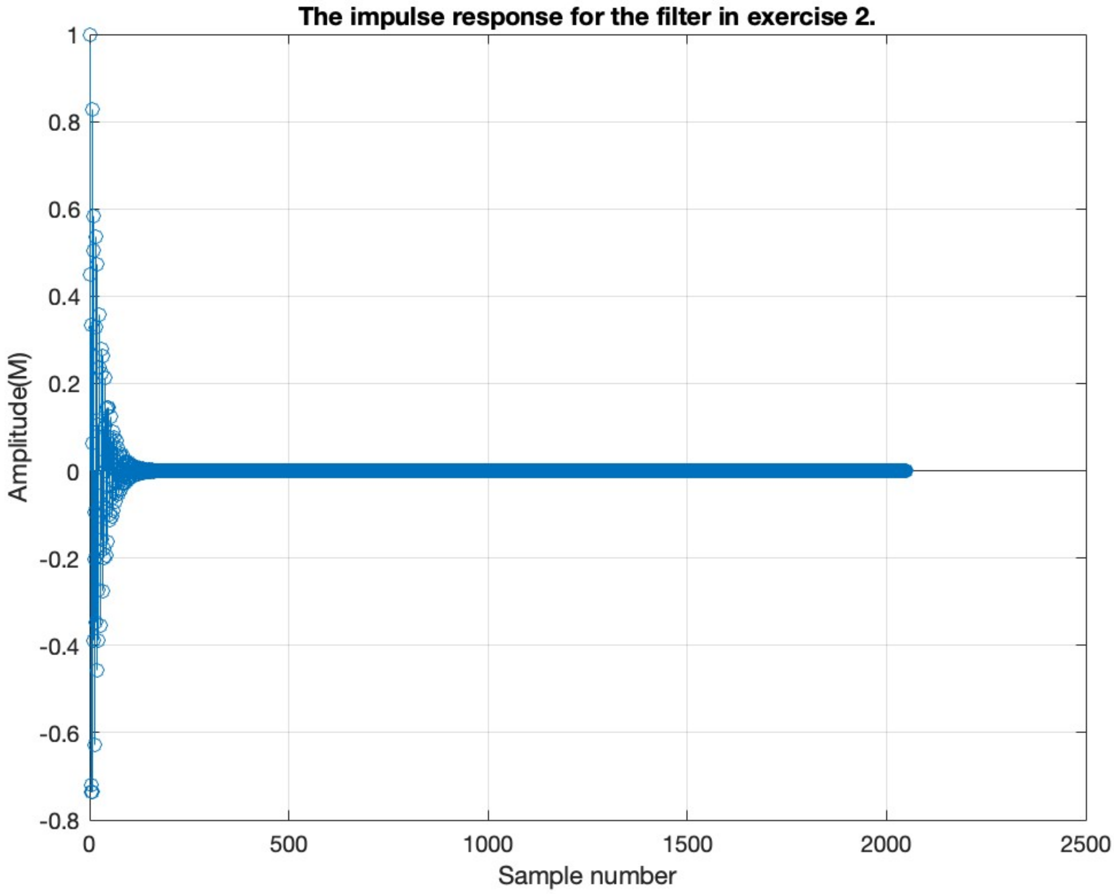


Figure 3: Impulse response of the filter of exercise 2.

3 Exercise 3.

For each of the following difference equations, calculate and plot the magnitude response and the impulse response, and state what type of filter the difference equation represents (i.e. low-pass, band-pass, notch or band-stop etc. – this is not too "exact" so choose the categorization that most suits).

(i). $y(n) + 0.13 \times y(n-1) + 0.52 \times y(n-2) + 0.3 \times y(n-3) = 0.16 \times x(n) - 0.48 \times x(n-1) + 0.48 \times x(n-2) - 0.16 \times x(n-3)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.16 - 0.48 \times z^{-1} + 0.48 \times z^{-2} - 016 \times z^{-3}}{1 + 0.13 \times z^{-1} + 0.52 \times z^{-2} + 0.3 \times z^{-3}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.16 - 0.48 \times e^{-j\theta} + 0.48 \times e^{-j2\theta} - 0.16 \times e^{-j3\theta}}{1 + 0.13 \times e^{-j\theta} + 0.52 \times e^{-j2\theta} + 0.3 \times e^{-j3\theta}} \tag{3}$$

$$= \frac{(0.16 - 0.48\cos\theta + 0.48\cos 2\theta - 0.16\cos 3\theta) + j(0.48\sin\theta - 0.48\sin 2\theta + 0.16\sin 3\theta)}{(1 + 0.13\cos\theta + 0.52\cos 2\theta + 0.3\cos 3\theta) + j(-0.13\sin\theta - 0.52\sin 2\theta - 0.3\sin 3\theta)} \tag{4}$$

The magnitude response is obtained by calculating the magnitude:

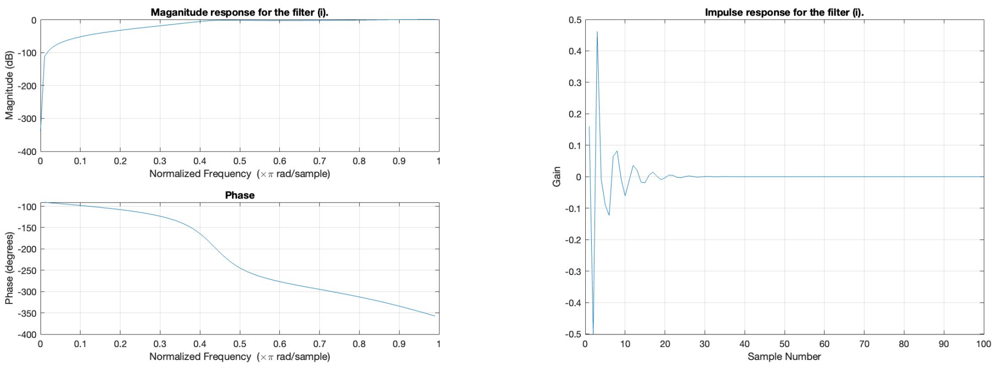
$$|H(\theta)| = \frac{\sqrt{[0.16 - 0.48\cos\theta + 0.48\cos 2\theta - 0.16\cos 3\theta]^2 + [0.48\sin\theta - 0.48\sin 2\theta + 0.16\sin 3\theta]^2}}{\sqrt{[1 + 0.13\cos\theta + 0.52\cos 2\theta + 0.3\cos 3\theta]^2 + [-0.13\sin\theta - 0.52\sin 2\theta - 0.3\sin 3\theta]^2}}$$

The magnitude and impulse responses can be obtained by using Matlab3:

```
%
% Exercise 3.(i).
% Calculate and plot the magnitude and impulse responses for filter
    (i).
%

close all;
clear all;
clc;

% specify the filter
b = [0.16 -0.48 0.48 -0.16];
a = [1 0.13 0.52 0.3];

nsamp = 100; % assume the number of samples

% calculate and plot the magnitude response
figure(1);
freqz(b, a, nsamp);
title("Maganitude response for the filter (i).");

% calculate the impulse response

% set up the impulse input
impulse = zeros(nsamp, 1);
impulse(1) = 1;

% use "filter" function to implement the filter
y = filter(b, a, impulse);

% set up sample axis
```

```
sample_axis = 1:nsamp;

% plot the impulse response
figure(2);
plot(sample_axis, y);
grid on;
title("Impulse response for the filter (i).");
xlabel("Sample Number");
ylabel("Gain");
```

Figure4 shows the magnitude responses and impulse response of the filter (i).



(a) Frequency response of filter (i).

(b) Impulse response of filter (i).

Figure 4: Frequency response and impulse response of filter (i).

The difference equation (i) represents high-pass filter.

(ii). $y(n) = 0.634 \times x(n) - 0.634 \times x(n-2) + 0.268 \times y(n-2)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.634 - 0.634 \times z^{-2}}{1 - 0.268 \times z^{-2}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.634 - 0.634 \times e^{-j2\theta}}{1 - 0.268 \times e^{-j2\theta}} \tag{5}$$

$$= \frac{(0.634 - 0.634 \times \cos 2\theta) + j \times 0.634 \times \sin 2\theta}{(1 - 0.268 \times \cos 2\theta) + j \times 0.268 \times \sin 2\theta} \tag{6}$$

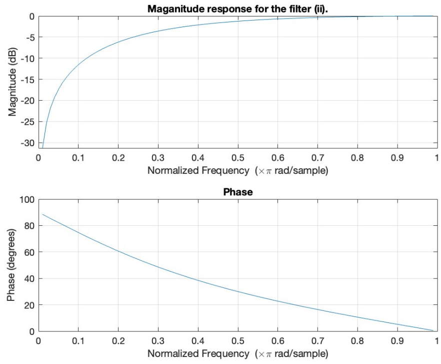
The magnitude response is obtained by calculating the magnitude:

$$|H(\theta)| = \frac{\sqrt{(0.634 - 0.634 \times \cos 2\theta)^2 + (0.634 \times \sin 2\theta)^2}}{\sqrt{(1 - 0.268 \times \cos 2\theta)^2 + (0.268 \times \sin 2\theta)^2}}$$

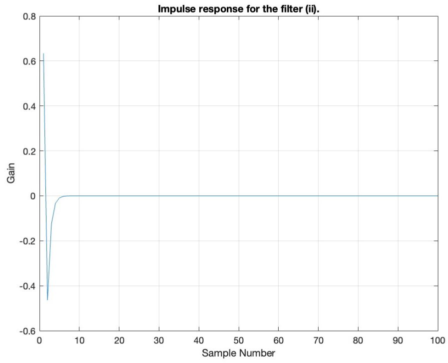
The magnitude and impulse responses can be obtained by using Matlab3:

```
%
% Exercise 3.(ii).
% Calculate and plot the magnitude and impulse responses for filter (ii).
%

close all;
clear all;
clc;

% specify the filter
b = [0.634 -0.634];
a = [1 -0.268];

nsamp = 100; % assume the number of samples

% calculate and plot the magnitude response
figure(1);
freqz(b, a, nsamp);
title("Maganitude response for the filter (ii).");

% calculate the impulse response

% set up the impulse input
impulse = zeros(nsamp, 1);
impulse(1) = 1;

% use "filter" function to implement the filter
y = filter(b, a, impulse);

% set up sample axis
sample_axis = 1:nsamp;

% plot the impulse response
figure(2);
plot(sample_axis, y);
grid on;
title("Impulse response for the filter (ii).");
xlabel("Sample Number");
ylabel("Gain");
```

Figure5 shows the magnitude responses and impulse response of the filter (ii).



(a) Frequency response of filter (ii).



(b) Impulse response of filter (ii).

Figure 5: Frequency response and impulse response of filter (ii).

The difference equation (ii) represents high-pass filter.

(iii). $y(n) + 0.268 \times y(n-2) = 0.634 \times x(n) + 0.634 \times x(n-2)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.634 + 0.634 \times z^{-2}}{1 + 0.268 \times z^{-2}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.634 + 0.634 \times e^{-j2\theta}}{1 + 0.268 \times e^{-j2\theta}} \tag{7}$$

$$= \frac{(0.634 + 0.634 \times \cos 2\theta) - j \times 0.634 \times \sin 2\theta}{(1 + 0.268 \times \cos 2\theta) - j \times 0.268 \times \sin 2\theta} \tag{8}$$

The magnitude response is obtained by calculating the magnitude:

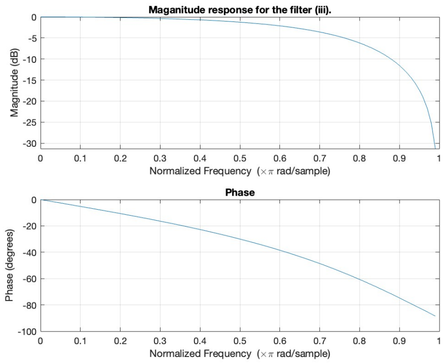
$$|H(\theta)| = \frac{\sqrt{(0.634 + 0.634 \times \cos 2\theta)^2 + (0.634 \times \sin 2\theta)^2}}{\sqrt{(1 + 0.268 \times \cos 2\theta)^2 + (0.268 \times \sin 2\theta)^2}}$$

The magnitude and impulse responses can be obtained by using Matlab3:

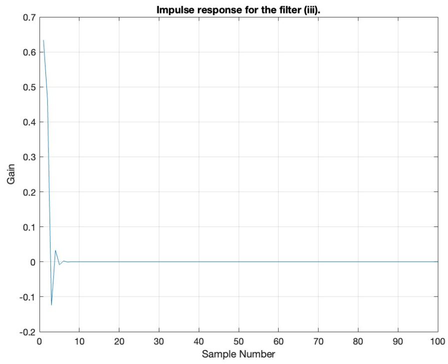
```
%
% Exercise 3.(iii).
% Calculate and plot the magnitude and impulse responses for filter (iii).
%

close all;
```

```
clear all;
clc;

% specify the filter
b = [0.634 0.634];
a = [1 0.268];

nsamp = 100; % assume the number of samples

% calculate and plot the magnitude response
figure(1);
freqz(b, a, nsamp);
title("Maganitude response for the filter (iii).");

% calculate the impulse response

% set up the impulse input
impulse = zeros(nsamp, 1);
impulse(1) = 1;

% use "filter" function to implement the filter
y = filter(b, a, impulse);

% set up sample axis
sample_axis = 1:nsamp;

% plot the impulse response
figure(2);
plot(sample_axis, y);
grid on;
title("Impulse response for the filter (iii).");
xlabel("Sample Number");
ylabel("Gain");
```

Figure6 shows the magnitude responses and impulse response of the filter (iii).



(a) Frequency response of filter (iii).



(b) Impulse response of filter (iii).

Figure 6: Frequency response and impulse response of filter (iii).

The difference equation (iii) represents low-pass filter.

(iv). $10 \times y(n) - 5 \times y(n-1) + y(n-2) = 0.634 \times x(n) - 5 \times x(n-1) + 10 \times x(n-2)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.634 - 5 \times z^{-1} + 10 \times z^{-2}}{10 - 5 \times z^{-1} + z^{-2}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.634 - 5 \times e^{-j\theta} + 10 \times e^{-j2\theta}}{10 - 5 \times e^{-j\theta} + e^{-j2\theta}} \tag{9}$$

$$= \frac{(0.634 - 5 \times \cos\theta + 10 \times \cos 2\theta) + j \times (5 \times \sin\theta - 10 \times \sin 2\theta)}{(10 - 5 \times \cos\theta + \cos 2\theta) + j \times (5 \times \sin\theta - \sin 2\theta)} \tag{10}$$

The magnitude response is obtained by calculating the magnitude:

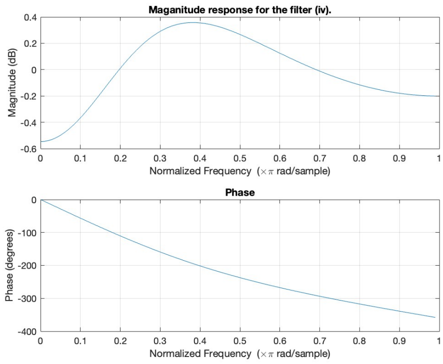
$$|H(\theta)| = \frac{\sqrt{(0.634 - 5 \times \cos\theta + 10 \times \cos 2\theta)^2 + (5 \times \sin\theta - 10 \times \sin 2\theta)^2}}{\sqrt{(10 - 5 \times \cos\theta + \cos 2\theta)^2 + (5 \times \sin\theta - \sin 2\theta)^2}}$$

The magnitude and impulse responses can be obtained by using Matlab3:

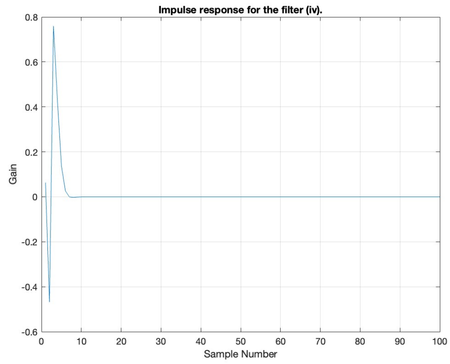
```
%
% Exercise 3.(iv).
% Calculate and plot the magnitude and impulse responses for filter
   (iv).
%

close all;
```

```
clear all;
clc;

% specify the filter
b = [0.634 -5 10];
a = [10 -5 1];

nsamp = 100; % assume the number of samples

% calculate and plot the magnitude response
figure(1);
freqz(b, a, nsamp);
title("Maganitude response for the filter (iv).");

% calculate the impulse response

% set up the impulse input
impulse = zeros(nsamp, 1);
impulse(1) = 1;

% use "filter" function to implement the filter
y = filter(b, a, impulse);

% set up sample axis
sample_axis = 1:nsamp;

% plot the impulse response
figure(2);
plot(sample_axis, y);
grid on;
title("Impulse response for the filter (iv).");
xlabel("Sample Number");
ylabel("Gain");
```

Figure7 shows the magnitude responses and impulse response of the filter (iv).



(a) Frequency response of filter (iv).



(b) Impulse response of filter (iv).

Figure 7: Frequency response and impulse response of filter (iv).

The difference equation (iv) represents band-pass filter.