SEMESTER I EXAMINATIONS 2019-2020

EE445 DIGITAL SIGNAL PROCESSING

SOLUTIONS

Q.1 (a) $y(n) = x(n) + 0.3x(n-1) - 0.4x(n-2) - 0.4y(n-1) + y(n-2)$ 1

Input: $x(n) = u(n)[1 + 0.9^n]$

n	0.9^n	$x(n)$
0	1	2
1	0.9	1.9
2	0.81	1.81
3	0.729	1.729

1

1

n	$\overset{*}{x(n)}$	$x(n-1)$	$x(n-2)$	$\overset{*}{0.3x(n-1)}$	$\overset{*}{-0.4x(n-2)}$
0	2	0	0	0	0
1	1.9	2	0	0.6	0
2	1.81	1.9	2	0.57	−0.8
3	1.729	1.81	1.9	0.543	−0.76

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$y(n-1)$	$\overset{*}{y(n-2)}$	$\overset{*}{-0.4y(n-1)}$	$y(n)$
0	0	0	2
2	0	−0.8	1.7
1.7	2	−0.68	2.9
2.9	1.7	−1.16	2.052

(b) $y(n) = x(n) - 0.4x(n-1) + 0.5y(n-2)$

$$H(z) = \frac{1 - 0.4z^{-1}}{1 - 0.5z^{-2}}$$ 1

$$H(\theta) = \frac{1 - 0.4e^{-j\theta}}{1 - 0.5e^{-j2\theta}}$$

$$= \frac{1 - 0.4\cos\theta + j0.4\sin\theta}{1 - 0.5\cos 2\theta + j0.5\sin 2\theta}$$ 1

$$|H(\theta)| = \frac{[(1-0.4\cos\theta)^2 + (0.4\sin\theta)^2]^{1/2}}{[(1-0.5\cos 2\theta)^2 + (0.5\sin 2\theta)^2]^{1/2}}$$

$$\underline{/H(\theta)} = \tan^{-1}\left[\frac{0.4\sin\theta}{1-0.4\cos\theta}\right] - \tan^{-1}\left[\frac{0.5\sin 2\theta}{1-0.5\cos 2\theta}\right]$$ 1

$$\theta = \frac{2\pi}{5}$$ 1

$$\Rightarrow |H(\theta)| = \frac{[(1-0.4\cos\frac{2\pi}{5})^2 + (0.4\sin\frac{2\pi}{5})^2]^{1/2}}{[(1-0.5\cos\frac{4\pi}{5})^2 + (0.5\sin\frac{4\pi}{5})^2]^{1/2}}$$

$$= \frac{[(0.8764)^2 + (0.3804)^2]^{1/2}}{[(1.4045)^2 + (0.2939)^2]^{1/2}}$$

$$= \frac{[0.9128]^{1/2}}{[2.059]^{1/2}}$$

$$= 0.6658$$ 1

$$\underline{/H(\theta)} = \tan^{-1}\left[\frac{0.3804}{0.8764}\right] - \tan^{-1}\left[\frac{0.2939}{1.4045}\right]$$

$$= 0.4095 - 0.2063 = 0.2032 \text{ rads}$$ 1

(c) Time-domain convolution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

Outputs: $y(n) = \{6, 11, -2, 14, 19, 5, -3\}$ 3

- details of $y(3)$ 1
- cross-check of duration of $y(n)$:
  $N_1 + N_2 - 1 = 3 + 5 - 1 = 7$ 1

Phase response:

filter has non-linear phase response, as impulse response $h(n)$ is not symmetric about its mid-point 1

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Q. 2 (a) $X(z) = 3z^{-2} + z^{-3} - z^{-4} + 2z^{-5} + 3z^{-6}$

$H(z) = 2 + z^{-1} - 3z^{-2}$ 1

$$
\begin{array}{rrrrrrrr}
X(z).H(z): & 6z^{-2} & +2z^{-3} & -2z^{-4} & +4z^{-5} & +6z^{-6} & & \\
 & & 3z^{-3} & +z^{-4} & -z^{-5} & +2z^{-6} & +3z^{-7} & \\
 & & & -9z^{-4} & -3z^{-5} & +3z^{-6} & -6z^{-7} & -9z^{-8} \\
\hline
 & 6z^{-2} & +5z^{-3} & -10z^{-4} & & +11z^{-6} & -3z^{-7} & -9z^{-8}
\end{array}
$$ 2

$h(n) * x(n) = \{6, 5, -10, 0, 11, -3, -9\}$ @ $n = 2$ 1

If $x(n)$ starts at $n = 4$, then $y(n)$ will be the same sample sequence, but delayed by 2 samples 1

(b) $H(z) = \dfrac{a}{1 - 6z^{-1}}$ 1

$H(\theta) = \dfrac{a}{1 - 6e^{-j\theta}}$ 1

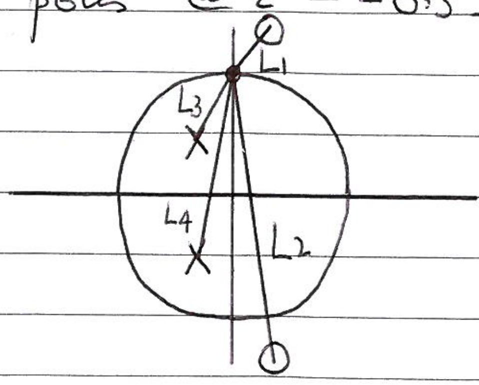
$H(\theta)\big|_{\theta=0} = \dfrac{a}{1-6}$ 1

For DC gain of 0.5, we require $\dfrac{a}{1-6} = 0.5$

$\Rightarrow a = 0.5(1-6)$ 1

(c) zeros @ $z = 0.2 \pm j1.2$

poles @ $z = -0.3 \pm j0.5$



1

$f_x = 100$ Hz

$\Rightarrow \theta_x = 2\pi \dfrac{100}{400}$

$= \dfrac{\pi}{2}$

$= 0 + j$ 1

$$|H(\theta_x)| = \frac{L_1 L_2}{L_3 L_4}$$

$$L_1 = \sqrt{(0.2-0)^2 + (1.2-1)^2} = 0.2828$$
$$L_2 = \sqrt{(0.2-0)^2 + (-1.2-1)^2} = 2.2091$$
$$L_3 = \sqrt{(-0.3-0)^2 + (0.5-1)^2} = 0.5831$$ 2
$$L_4 = \sqrt{(-0.3-0)^2 + (-0.5-1)^2} = 1.5297$$

$$\Rightarrow |H(\theta_x)| = 0.7004$$ 1

(d) Third harmonic $\Rightarrow f_0 = 150\,Hz$

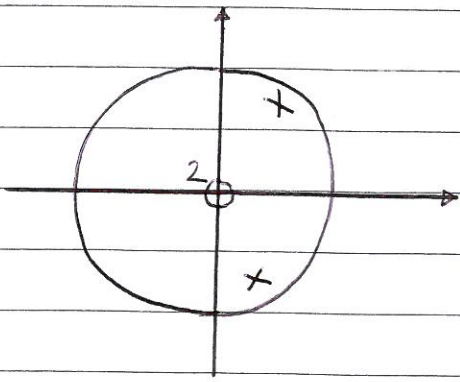
$$\theta_0 = 2\pi \frac{150}{1000} = 0.3\pi$$ 1

$$r \approx 1 - \frac{\Delta f}{f_s}\pi = 1 - \frac{30}{1000}\pi = 0.9057$$ 1

$$b_1 = -2r\cos\theta_0 = 1.0647$$
$$b_2 = r^2 = 0.8203$$ 1

$$H(z) = \frac{1}{1 + 1.0647z^{-1} + 0.8203z^{-2}}$$

$$\text{DC gain} = \frac{1}{1 + 1.0647 + 0.8203} = 0.3466$$ 1



$$y(n) = x(n) - 1.0647y(n-1) - 0.8203y(n-2)$$ 1

1

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Q.3 (a) Windowing
- explanation of window
- trade-off : width of main lobe vs. side lobe suppression — 2
- applications : signal analysis, filter design — 1

Calculation:

$T_{WIN} = 40\text{ ms} \Rightarrow N_{WIN} = 1280$ — 1

$\Delta f = \frac{f_s}{N_{FFT}} \leq 20\text{ Hz}$

$\Rightarrow N_{FFT} \geq \frac{f_s}{20} \geq 1600$ — 1

Next highest power of 2 is 2048

$\Rightarrow$ no. of samples for zero padding $= 2048 - 1280 = 768$ — 1

Computational complexity:

no. of MPY : $2N \log_2 N$

For $N = 2048$, we need 45056 MPY — 1

40 ms frames, with 0% overlap

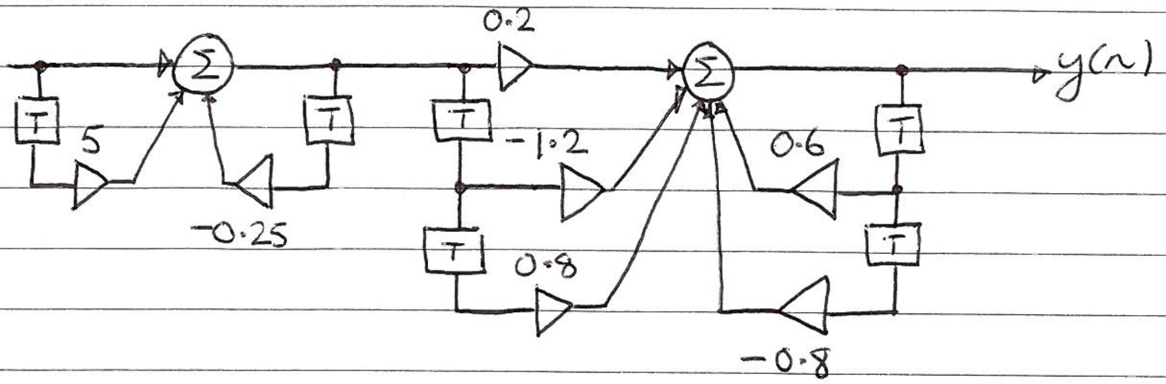
$\Rightarrow$ 25 frames/sec

$\therefore$ we need $25 \times 45056 = 1{,}126{,}400$ MPY/sec — 1

(b) Cascade and parallel decompositions

Points:
- easier to analyse
- re-use H/W or S/W — 3
- finite arithmetic effects

Decomposition:



2

(c) $b_1 = -2\sin\theta_0 \Rightarrow \sin\theta_0 = -\frac{b_1}{2}$ 1

$$\sin(\theta_0 + \Delta\theta_0) = -\frac{(b_1 + \Delta b_1)}{2}$$

$$\Rightarrow \sin\left(\frac{2\pi f_0}{f_s} + \frac{2\pi \Delta f_0}{f_s}\right) = -\frac{b_1}{2} - \frac{\Delta b_1}{2}$$ 1

$$\Rightarrow \sin\left(\frac{2\pi f_0}{f_s}\right)\cos\left(\frac{2\pi \Delta f_0}{f_s}\right) + \cos\left(\frac{2\pi f_0}{f_s}\right)\sin\left(\frac{2\pi \Delta f_0}{f_s}\right) = -\frac{b_1}{2} - \frac{\Delta b_1}{2}$$ 1

if Δf_0 is small, $\cos\left(\frac{2\pi \Delta f_0}{f_s}\right) \approx 1$, $\sin\left(\frac{2\pi \Delta f_0}{f_s}\right) \approx \frac{2\pi \Delta f_0}{f_s}$

Note: $\sin\left(\frac{2\pi f_0}{f_s}\right) = -\frac{b_1}{2}$ 1

$$\Rightarrow \cos\left(\frac{2\pi f_0}{f_s}\right)\frac{2\pi \Delta f_0}{f_s} = -\frac{\Delta b_1}{2}$$ 1

$$\Rightarrow 2\pi \frac{\Delta f_0}{f_s} = \frac{-\frac{\Delta b_1}{2}}{\cos\left(\frac{2\pi f_0}{f_s}\right)}$$ 1

$$\Rightarrow \Delta f_0 = \frac{-f_s \Delta b_1}{4\pi \cos\left(\frac{2\pi f_0}{f_s}\right)}$$ 1

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Q.4 (a) Notch filter

$$\theta_0 = 2\pi \frac{50}{1200} = \frac{\pi}{12}$$

Pole radius : $r \approx 1 - \frac{15}{1200}\pi = 0.9607$ 1

$b_1 = -2r\cos\theta_0 = -1.8559$

$b_2 = r^2 = 0.9229$ 1

Zeroes :

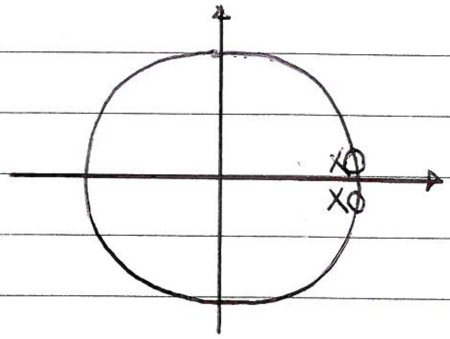
$a_1 = -2r\cos\theta_0 = -1.9318$

$a_2 = r^2 = 1$ 1

$$H(z) = \frac{1 - 1.9318z^{-1} + z^{-2}}{1 - 1.8559z^{-1} + 0.9229z^{-2}}$$ 1

Scaling factor to get DC gain of 1 :

$$K = \frac{1 - 1.8559 + 0.9229}{1 - 1.9318 + 1} = 0.9824$$ 1



$$y(n) = 0.9824[x(n) + 1.9318\,x(n-1) + x(n-2)] + 1.8559\,y(n-1) - 0.9229\,y(n-2)$$ 1

(b) $$H(s) = \frac{W_c}{s + W_c}$$

Desired digital cut-off frequency :

$$\theta_c = 2\pi \frac{2.5}{16} = 0.3125\pi$$ 1

Pre-warp :

$$W_c = \frac{2}{T}\tan\left(\frac{\theta_c}{2}\right)$$

$= 17{,}104.36$ rad/s 1

Bilinear transform

$$s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$ 1

$$H(z) = \frac{\omega_c}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + \omega_c} = \frac{\omega_c(1+z^{-1})}{(2f_s+\omega_c)+(\omega_c-2f_s)z^{-1}}$$ 1

Substitute for f_s and ω_c to get

$$H(z) = \frac{17104.36(1+z^{-1})}{(32000+17104.36)+(17104.36-32000)z^{-1}}$$ 1

$$= \frac{17104.36(1+z^{-1})}{49104.36-14895.64z^{-1}}$$

$$= \frac{0.3483(1+z^{-1})}{1-0.3033z^{-1}}$$ 1

Without pre-warping:

$$\theta_d = 2\tan^{-1}\left(\frac{\omega_c T}{2}\right) = 0.9126 \text{ rad}$$

$$= \frac{2\pi f_d}{f_s}$$

$$f_d = \frac{\theta_d f_s}{2\pi} = 2{,}323.92 \text{ Hz } (< 2.5 \text{ kHz})$$ 1

(c) High pass filter

$$\theta_c = 2\pi\frac{1}{8} = \frac{\pi}{4}$$ 1

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\theta)e^{jn\theta}\,d\theta$$ 1

$$= \delta(n) - \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$ 3

We require group delay of 9 msec

$= 72$ samples @ $f_s = 8$ kHz 1

$\alpha = \frac{N-1}{2} \Rightarrow N = 2\alpha + 1 = 145$ coefficients

$$\therefore h(n) = \delta(n-72) - \frac{1}{(n-72)\pi}\sin\left[\frac{(n-72)\pi}{4}\right]$$ 1

$n = 0, 1, \ldots, 144$

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