



EE445 DIGITAL SIGNAL PROCESSING: Assignment I

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1 Exercise 1.

- (a). Using the Matlab `freqz` function, calculate and plot the magnitude response (in dB) of the following discrete-time system:

$$H(z) = \frac{1 + 0.4 \times z^{-1}}{1 - 1.5 \times \cos\left(\frac{\pi}{8}\right) \times z^{-1} + 0.96 \times z^{-2}}$$

Use 1024 points evenly spaced between 0 and half the sampling frequency. For the plot, the x-axis should be in units of Hz (or kHz); you may assume a sampling rate of 12 kHz.

```
1 %  
2 % Exercise 1.(a).  
3 % calculate and plot the magnitude response using "freqz" function.  
4 %  
5  
6 clear all;  
7 close all;  
8 clc;  
9  
10 % set up some constants  
11 nsamp = 1024; % number of samples  
12 fs = 12000; % sampling frequency  
13  
14 % specify the filter  
15 b = [1 0.4];  
16 a = [1 -1.5*cos(pi/8) 0.96];  
17  
18 % calculate the magnitude response using "freqz".  
19 % and plot the magnitude response with the x-axis in units of Hz.
```

```
20 figure;
21 freqz(b, a, nsamp, fs);
```

Figure 1 shows the frequency responses.

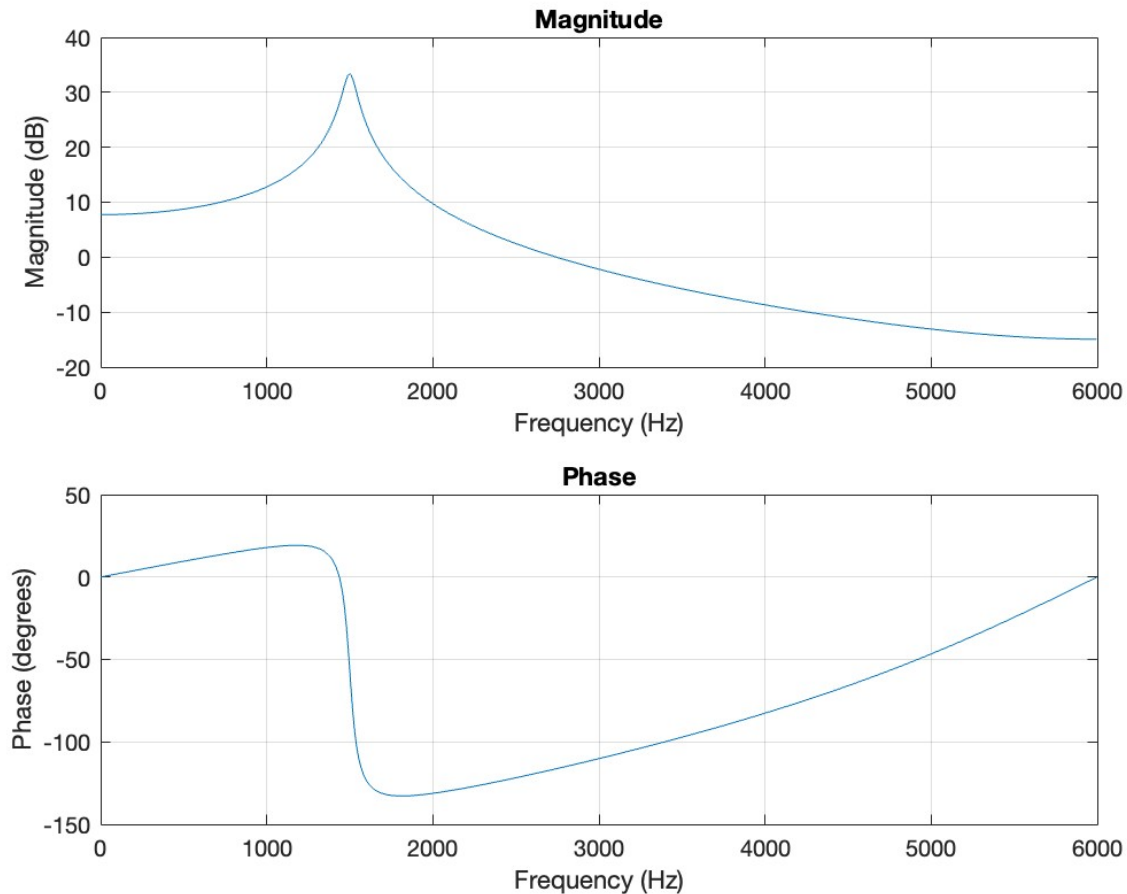


Figure 1: Frequency response of the discrete-time system of exercise 1.

- (b). Determine the locations of the poles (in polar form), and calculate the centre frequency of the filter in Hz, assuming a sampling frequency of 12 kHz.

First, we find the location of poles in a complex format using Matlab tools¹:

$$p_1 = 0.6929 + 0.6927i$$

$$p_2 = 0.6929 - 0.6927i$$

```
1 %
2 % Exercise 1.(b).
3 % Determine the locations of the poles (in polar form),
```



```
4 % and calculate the centre frequency of the filter in Hz,  
5 % assuming a sampling frequency of 12 kHz.  
6 %  
7  
8 clear all;  
9 close all;  
10 clc;  
11  
12 % transfer function  
13 b = [1 0.4];  
14 a = [1 -1.5*cos(pi/8) 0.96];  
15  
16 % get the roots of the numerator (zeros)  
17 rnum = roots(b);  
18  
19 % get the roots of the denominator (poles)  
20 rden = roots(a);  
21  
22 % get the modulus of poles  
23 rs = abs(rden);  
24  
25 % get the angle of poles  
26 theta = angle(rden);  
27  
28 figure;  
29 zplane(b, a);  
30 grid on;
```

Figure 2 shows the pole-zero map for the system.

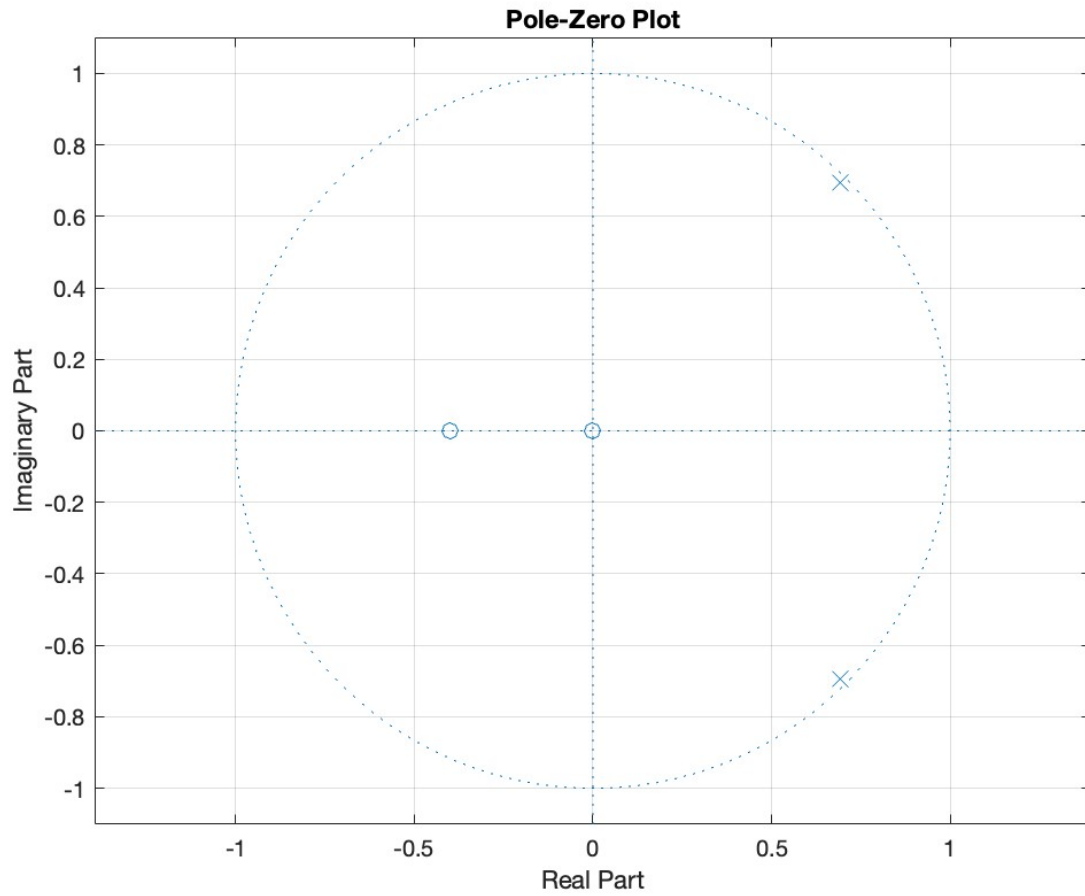


Figure 2: Pole-zero map for system.

Then, we calculate the modulus and the angles of the respective poles using Matlab¹:

$$r_{p1} = 0.9798$$

$$r_{p2} = 0.9798$$

$$\theta_{p1} = 0.7853$$

$$\theta_{p2} = -0.7853$$

.

We convert the location of poles to polar form:

$$p_1 = r_{p1} \times e^{j \times \theta_{p1}} = 0.9798 \times e^{j \times 0.7853}$$

$$p_2 = r_{p2} \times e^{j \times \theta_{p2}} = 0.9798 \times e^{-j \times 0.7853}$$

.

Then, we calculate the centre frequency of the filter by:

$$\theta_0 = 2 \times \pi \times \frac{f_0}{f_s}$$

We obtain the pole frequency:

$$f_{0,p1} = \frac{\theta_{p1}}{2 \times \pi} \times f_s = \frac{0.7853}{2 \times \pi} \times 12000 \approx 1499.8125$$

$$f_{0,p2} = \frac{\theta_{p2}}{2 \times \pi} \times f_s = \frac{-0.7853}{2 \times \pi} \times 12000 \approx -1499.8125$$

We obtain the centre frequency:

$$\theta_{0,p1} = 2 \times \pi \times \frac{f_{0,p1}}{f_s} = 2 \times \pi \times \frac{1499.8125}{12000} = 0.7853$$

$$\theta_{0,p2} = 2 \times \pi \times \frac{f_{0,p2}}{f_s} = 2 \times \pi \times \frac{-1499.8125}{12000} = -0.7853$$

2 Exercise 2.

Determine the transfer function, and hence calculate and plot the magnitude response and impulse response, of a second-order filter that has a complex conjugate pole pair with a pole frequency of 3.4 kHz, and pole radius of 0.96. The filter also has a double zero at $z = 0$ in the z -plane. The sampling frequency is 16 kHz. For the magnitude response, use 2048 points equally spaced between DC and half the sampling frequency. You should ensure that your plots have proper axes.

The filter we want to design with the following specification:

- Pole frequency = $3400Hz$
- Pole radius = 0.96
- Double zeros: $z_1 = z_2 = 0$
- Sampling frequency = $16000Hz$
- Number of samples = $2048 \in [0, f_s/2]$

The first step is to calculate the pole frequency (in radians) and the pole radius:

$$\theta_0 = 2 \times \pi \times \frac{f_0}{f_s} = 2 \times \pi \times \frac{3400}{16000} = \frac{17}{40}\pi$$

$$r_p = 0.96$$

Then, the coefficients are calculated as follows:

$$b_2 = r_p^2 = 0.96^2 = 0.9216$$

$$b_1 = -2 \times r_p \times \cos \frac{17}{40}\pi \approx -0.4482$$

$$1 + b_1 + b_2 = 1 - 0.4482 + 0.9216 = 1.4734$$

Hence, the transfer function can be written as:

$$H(z) = \frac{1.4734}{1 - 0.4482 \times z^{-1} + 0.9216 \times z^{-2}}$$

Let $z = e^{j\omega}$. The frequency response can be given by:

$$H(\theta) = \frac{1.4734}{1 - 0.4482 \times e^{-j\theta} + 0.9216 \times e^{-j2\theta}} \quad (1)$$

$$= \frac{1.4734}{1 - 0.4482 \cos \theta + j0.4482 \sin \theta + 0.9216 \cos 2\theta - j0.9216 \sin 2\theta}. \quad (2)$$

The magnitude response is obtained by:

$$|H(\theta)| = \frac{\sqrt{1.4734^2}}{\sqrt{(1 - 0.4482 \cos \theta + 0.9216 \cos 2\theta)^2 + (0.4482 \sin \theta - 0.9216 \sin 2\theta)^2}}$$

We use Matlab² to calculate and plot the magnitude response:

```
1 %  
2 % Exercise 2.  
3 % Calculate and plot magnitude response.  
4 %  
5  
6 clear all;  
7 close all;  
8 clc;  
9  
10 % set up some constants  
11 fs = 16000; % sampling frequency  
12 nsamp = 2048; % number of samples  
13  
14 % transfer function  
15 b = [1];  
16 a = [1 -0.4482 0.9216];  
17
```

```
18 % scale to get a DC gain of 1
19 gainDC = sum(b)/sum(a);
20 scalefac = 1/gainDC;
21 b = b*scalefac;
22
23 % get and plot the magnitude response.
24 figure(1);
25 freqz(b, a, nsamp, fs);
26 title("Magnitude response of the filter in exercise 2.");
```

Figure 3 shows the frequency responses.

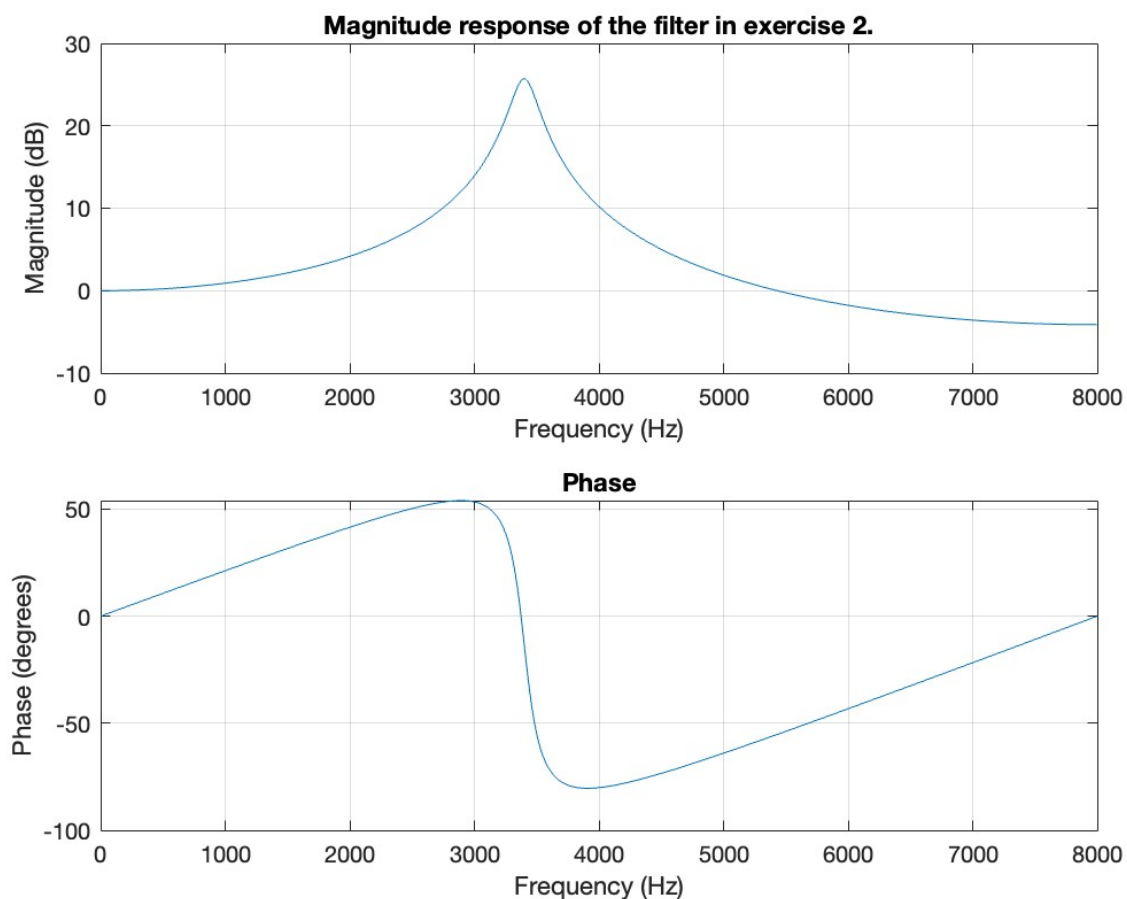


Figure 3: Frequency response of the filter of exercise 2.

The impulse response is obtained by using Matlab2:

```
1 %
2 % Exercise 2.
3 % calculate and plot the impulse response.
4 %
```



```
5
6 clear all;
7 close all;
8 clc;
9
10 % set up some constants
11 nsamp = 2048; % number of samples
12
13 % specify the filter
14 b = [1];
15 a = [1 -0.4482 0.9216];
16
17 % set up the inputs
18 impulse = zeros(nsamp, 1);
19 impulse(1) = 1;
20
21 % use "filter" function
22 y = filter(b, a, impulse);
23
24 % set up sample axis
25 sample_index = 1:nsamp;
26
27 % plot the impulse response
28 figure(1);
29 stem(sample_index, y);
30 grid on;
31 title("The impulse response for the filter in exercise 2.");
32 xlabel("Sample number");
33 ylabel("Amplitude(M)");
```

Figure 4 shows the impulse responses.

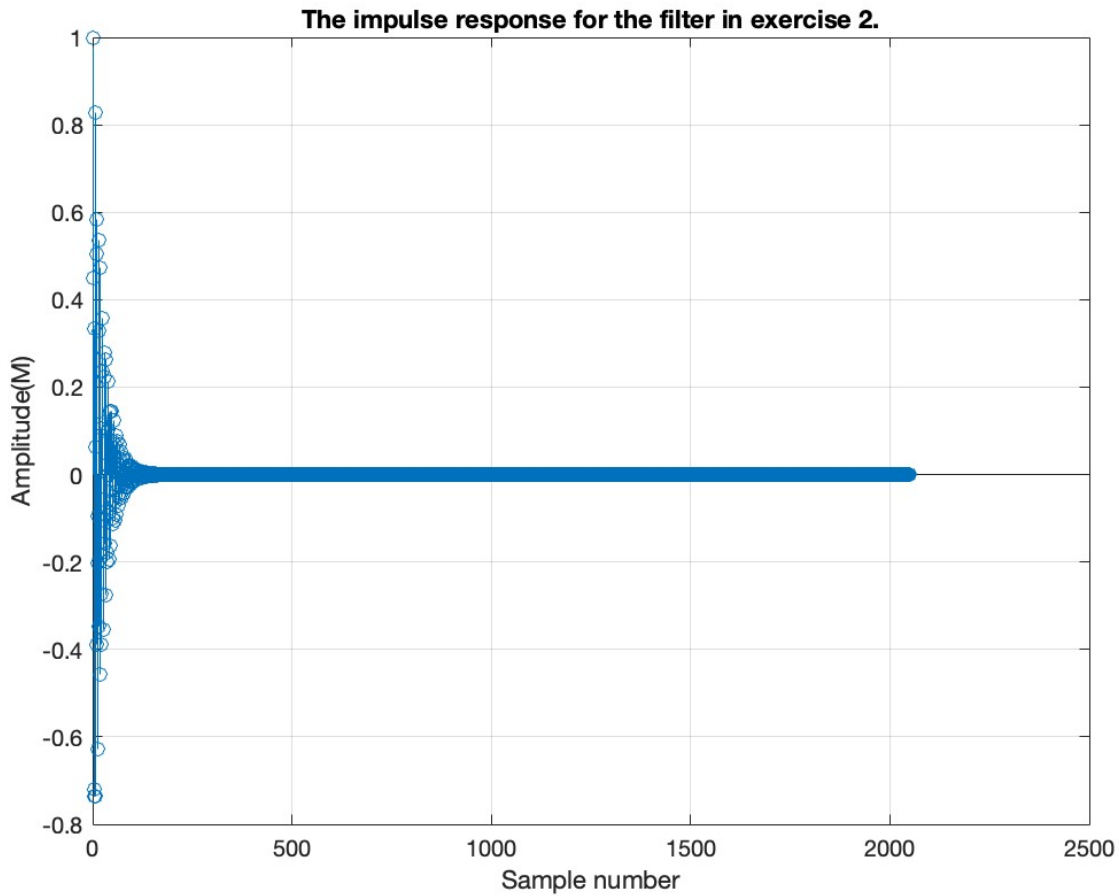


Figure 4: Impulse response of the filter of exercise 2.

3 Exercise 3.

For each of the following difference equations, calculate and plot the magnitude response and the impulse response, and state what type of filter the difference equation represents (i.e. low-pass, band-pass, notch or band-stop etc. – this is not too “exact” so choose the categorization that most suits).

- (i). $y(n) + 0.13 \times y(n-1) + 0.52 \times y(n-2) + 0.3 \times y(n-3) = 0.16 \times x(n) - 0.48 \times x(n-1) + 0.48 \times x(n-2) - 0.16 \times x(n-3)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.16 - 0.48 \times z^{-1} + 0.48 \times z^{-2} - 0.16 \times z^{-3}}{1 + 0.13 \times z^{-1} + 0.52 \times z^{-2} + 0.3 \times z^{-3}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.16 - 0.48 \times e^{-j\theta} + 0.48 \times e^{-j2\theta} - 0.16 \times e^{-j3\theta}}{1 + 0.13 \times e^{-j\theta} + 0.52 \times e^{-j2\theta} + 0.3 \times e^{-j3\theta}} \quad (3)$$
$$= \frac{(0.16 - 0.48 \cos \theta + 0.48 \cos 2\theta - 0.16 \cos 3\theta) + j(0.48 \sin \theta - 0.48 \sin 2\theta + 0.16 \sin 3\theta)}{(1 + 0.13 \cos \theta + 0.52 \cos 2\theta + 0.3 \cos 3\theta) + j(-0.13 \sin \theta - 0.52 \sin 2\theta - 0.3 \sin 3\theta)} \quad (4)$$

The magnitude response is obtained by calculating the magnitude:

$$|H(\theta)| = \frac{\sqrt{[0.16 - 0.48 \cos \theta + 0.48 \cos 2\theta - 0.16 \cos 3\theta]^2 + [0.48 \sin \theta - 0.48 \sin 2\theta + 0.16 \sin 3\theta]^2}}{\sqrt{[1 + 0.13 \cos \theta + 0.52 \cos 2\theta + 0.3 \cos 3\theta]^2 + [-0.13 \sin \theta - 0.52 \sin 2\theta - 0.3 \sin 3\theta]^2}}$$

The magnitude and impulse responses can be obtained by using Matlab³:

```
1 %  
2 % Exercise 3.(i).  
3 % Calculate and plot the magnitude and impulse responses for filter  
4 %   (i).  
5 %  
6 close all;  
7 clear all;  
8 clc;  
9  
10 % specify the filter  
11 b = [0.16 -0.48 0.48 -0.16];  
12 a = [1 0.13 0.52 0.3];  
13  
14 nsamp = 100; % assume the number of samples  
15  
16 % calculate and plot the magnitude response  
17 figure(1);  
18 freqz(b, a, nsamp);  
19 title("Magnitude response for the filter (i).");  
20  
21 % calculate the impulse response  
22  
23 % set up the impulse input  
24 impulse = zeros(nsamp, 1);  
25 impulse(1) = 1;  
26  
27 % use "filter" function to implement the filter  
28 y = filter(b, a, impulse);  
29  
30 % set up sample axis
```

```

31 sample_axis = 1:nsamp;
32
33 % plot the impulse response
34 figure(2);
35 plot(sample_axis, y);
36 grid on;
37 title("Impulse response for the filter (i).");
38 xlabel("Sample Number");
39 ylabel("Gain");

```

Figure 5 shows the frequency responses and impulse response of the filter (i).

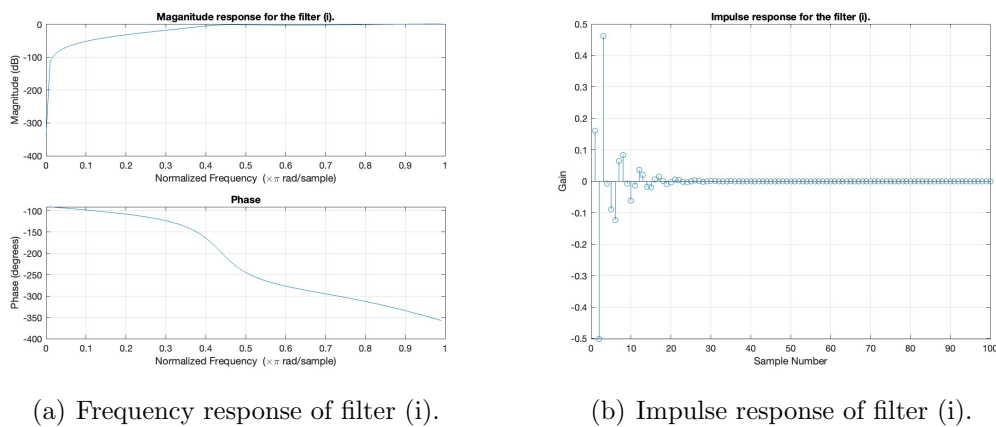


Figure 5: Frequency response and impulse response of filter (i).

The difference equation (i) represents high-pass filter.

(ii). $y(n) = 0.634 \times x(n) - 0.634 \times x(n-2) + 0.268 \times y(n-2)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.634 + 0 \times z^{-1} - 0.634 \times z^{-2}}{1 + 0 \times z^{-1} - 0.268 \times z^{-2}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.634 + 0 \times e^{-j\theta} - 0.634 \times e^{-j2\theta}}{1 + 0 \times e^{-j\theta} - 0.268 \times e^{-j2\theta}} \quad (5)$$

$$= \frac{(0.634 - 0.634 \times \cos 2\theta) + j \times 0.634 \times \sin 2\theta}{(1 - 0.268 \times \cos 2\theta) + j \times 0.268 \times \sin 2\theta} \quad (6)$$

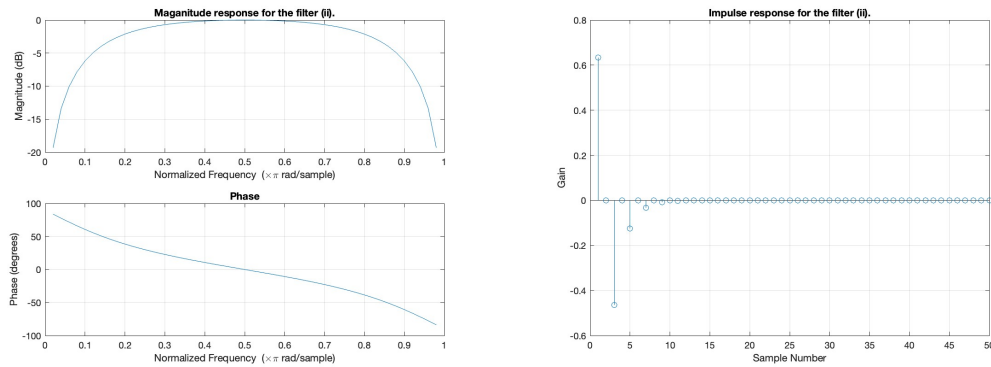
The magnitude response is obtained by calculating the magnitude:

$$|H(\theta)| = \frac{\sqrt{(0.634 - 0.634 \times \cos 2\theta)^2 + (0.634 \times \sin 2\theta)^2}}{\sqrt{(1 - 0.268 \times \cos 2\theta)^2 + (0.268 \times \sin 2\theta)^2}}$$

The magnitude and impulse responses can be obtained by using Matlab³:

```
1 %  
2 % Exercise 3.(ii).  
3 % Calculate and plot the magnitude and impulse responses for filter  
4   → (i).  
5 %  
6 close all;  
7 clear all;  
8 clc;  
9  
10 % specify the filter  
11 b = [0.634 0 -0.634];  
12 a = [1 0 -0.268];  
13  
14 nsamp = 50; % assume the number of samples  
15  
16 % calculate and plot the magnitude response  
17 figure(1);  
18 freqz(b, a, nsamp);  
19 title("Magnitude response for the filter (ii).");  
20  
21 % calculate the impulse response  
22  
23 % set up the impulse input  
24 impulse = zeros(nsamp, 1);  
25 impulse(1) = 1;  
26  
27 % use "filter" function to implement the filter  
28 y = filter(b, a, impulse);  
29  
30 % set up sample axis  
31 sample_axis = 1:nsamp;  
32  
33 % plot the impulse response  
34 figure(2);  
35 stem(sample_axis, y);  
36 grid on;  
37 title("Impulse response for the filter (ii).");  
38 xlabel("Sample Number");  
39 ylabel("Gain");
```

Figure⁶ shows the magnitude response and impulse response of the filter (ii).



(a) Frequency response of filter (ii).

(b) Impulse response of filter (ii).

Figure 6: Frequency response and impulse response of filter (ii).

The difference equation (ii) represents band-pass filter.

(iii). $y(n) + 0.268 \times y(n - 2) = 0.634 \times x(n) + 0.634 \times x(n - 2)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.634 + 0.634 \times z^{-2}}{1 + 0.268 \times z^{-2}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.634 + 0.634 \times e^{-j2\theta}}{1 + 0.268 \times e^{-j2\theta}} \quad (7)$$

$$= \frac{(0.634 + 0.634 \times \cos 2\theta) - j \times 0.634 \times \sin 2\theta}{(1 + 0.268 \times \cos 2\theta) - j \times 0.268 \times \sin 2\theta} \quad (8)$$

The magnitude response is obtained by calculating the magnitude:

$$|H(\theta)| = \frac{\sqrt{(0.634 + 0.634 \times \cos 2\theta)^2 + (0.634 \times \sin 2\theta)^2}}{\sqrt{(1 + 0.268 \times \cos 2\theta)^2 + (0.268 \times \sin 2\theta)^2}}$$

The magnitude and impulse responses can be obtained by using Matlab³:

```

1  %
2  % Exercise 3.(iii).
3  % Calculate and plot the magnitude and impulse responses for filter
   ↪ (iii).
4  %
5
6  close all;
```

```
7 clear all;
8 clc;
9
10 % specify the filter
11 b = [0.634 0 0.634];
12 a = [1 0 0.268];
13
14 nsamp = 100; % assume the number of samples
15
16 % calculate and plot the magnitude response
17 figure(1);
18 freqz(b, a, nsamp);
19 title("Magnitude response for the filter (iii).");
20
21 % calculate the impulse response
22
23 % set up the impulse input
24 impulse = zeros(nsamp, 1);
25 impulse(1) = 1;
26
27 % use "filter" function to implement the filter
28 y = filter(b, a, impulse);
29
30 % set up sample axis
31 sample_axis = 1:nsamp;
32
33 % plot the impulse response
34 figure(2);
35 stem(sample_axis, y);
36 grid on;
37 title("Impulse response for the filter (iii).");
38 xlabel("Sample Number");
39 ylabel("Gain");
```

Figure 7 shows the magnitude response and impulse response of the filter (iii).

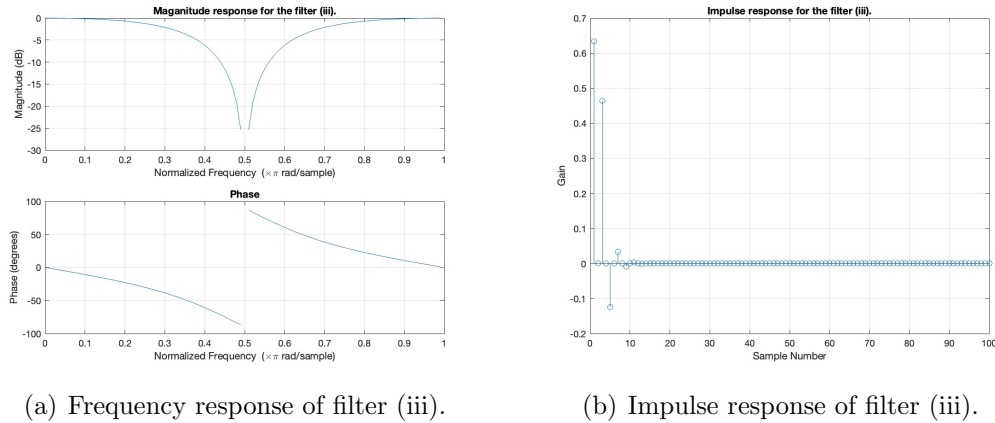


Figure 7: Frequency response and impulse response of filter (iii).

The difference equation (iii) represents band-stop filter.

(iv). $10 \times y(n) - 5 \times y(n-1) + y(n-2) = 0.634 \times x(n) - 5 \times x(n-1) + 10 \times x(n-2)$

Because the impulse response of this filter is of infinite length. Therefore, we will use transfer function to evaluate the frequency response.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.634 - 5 \times z^{-1} + 10 \times z^{-2}}{10 - 5 \times z^{-1} + z^{-2}}$$

Hence, the frequency response is given by:

$$H(\theta) = \frac{0.634 - 5 \times e^{-j\theta} + 10 \times e^{-j2\theta}}{10 - 5 \times e^{-j\theta} + e^{-j2\theta}} \quad (9)$$

$$= \frac{(0.634 - 5 \times \cos \theta + 10 \times \cos 2\theta) + j \times (5 \times \sin \theta - 10 \times \sin 2\theta)}{(10 - 5 \times \cos \theta + \cos 2\theta) + j \times (5 \times \sin \theta - \sin 2\theta)} \quad (10)$$

The magnitude response is obtained by calculating the magnitude:

$$|H(\theta)| = \frac{\sqrt{(0.634 - 5 \times \cos \theta + 10 \times \cos 2\theta)^2 + (5 \times \sin \theta - 10 \times \sin 2\theta)^2}}{\sqrt{(10 - 5 \times \cos \theta + \cos 2\theta)^2 + (5 \times \sin \theta - \sin 2\theta)^2}}$$

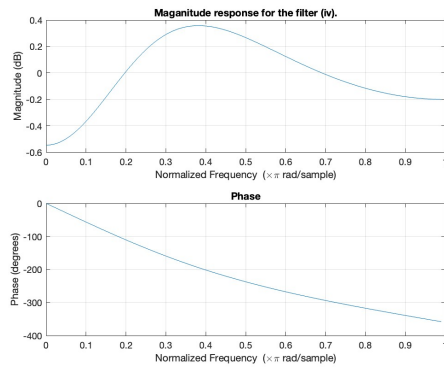
The magnitude and impulse responses can be obtained by using Matlab³:

```

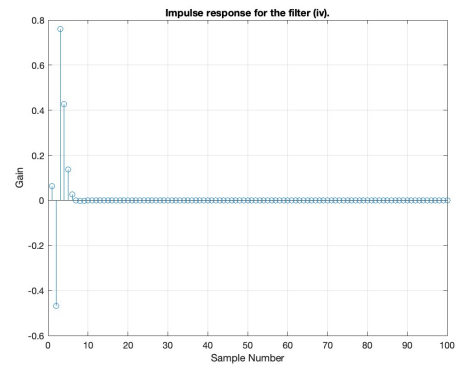
1  %
2  % Exercise 3.(iv).
3  % Calculate and plot the magnitude and impulse responses for filter
   ↪ (iv).
4  %
5
6  close all;
```

```
7 clear all;
8 clc;
9
10 % specify the filter
11 b = [0.634 -5 10];
12 a = [10 -5 1];
13
14 nsamp = 100; % assume the number of samples
15
16 % calculate and plot the magnitude response
17 figure(1);
18 freqz(b, a, nsamp);
19 title("Magnitude response for the filter (iv).");
20
21 % calculate the impulse response
22
23 % set up the impulse input
24 impulse = zeros(nsamp, 1);
25 impulse(1) = 1;
26
27 % use "filter" function to implement the filter
28 y = filter(b, a, impulse);
29
30 % set up sample axis
31 sample_axis = 1:nsamp;
32
33 % plot the impulse response
34 figure(2);
35 plot(sample_axis, y);
36 grid on;
37 title("Impulse response for the filter (iv).");
38 xlabel("Sample Number");
39 ylabel("Gain");
```

Figure 8 shows the magnitude response and impulse response of the filter (iv).



(a) Frequency response of filter (iv).



(b) Impulse response of filter (iv).

Figure 8: Frequency response and impulse response of filter (iv).

The difference equation (iv) represents band-pass filter.