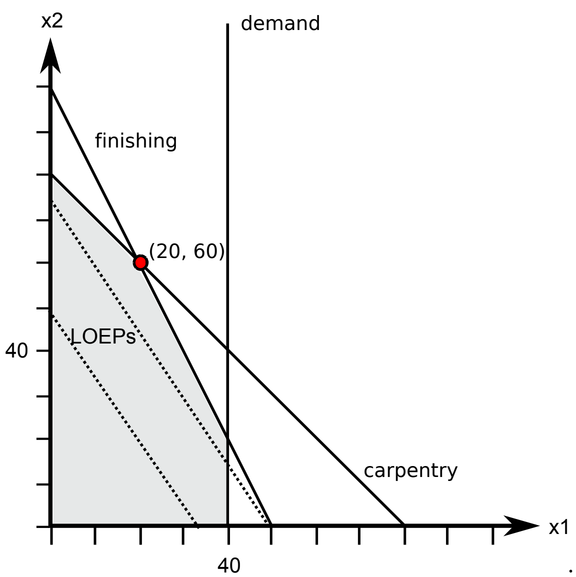
CT5141 Lab Week 2 Solutions

James McDermott

Giapetto solution

- Define decision variables x_1 = number of soldiers, x_2 = number of trains.
- Profit for soldiers is 27 - 10 - 14 = 3.
- Profit for trains is 21 - 9 - 10 = 2.
- Objective: maximise $3x_1 + 2x_2$.
- Constraint on carpentry labour: $x_1 + x_2 \leq 80$
- Constraint on finishing labour: $2x_1 + x_2 \leq 100$
- Constraint on soldier demand: $x_1 \leq 40$.



The optimum is at (20, 60).

Interpretation: Giapetto should manufacture 20 soldiers and 60 trains to achieve a total profit per week of $180.

Here is a nice illustration with an animation of the LOEP, made by Johannes-Lucas Löwe (CT5141 2020-21): https://www.geogebra.org/graphing/xvsyy5q7 – thanks Johannes!

Blend solution

Solution (objective function)

Consider a particular decision variable, e.g. x_{1s}. This represents the quantity of Component 1 used in making Super. The profit associated with each unit of that quantity is the Super selling price less the Component 1 cost price, $23 - 12 = 9$.

Thus we can write the objective as

$$\sum_{i=1,2,3;j=s,p,e} x_{ij}(s_j - c_i)$$

where c_i are the costs per barrel and s_j are the selling prices, so $s_j - c_i$ is the profit associated with the usage of component i in product j.

Solution (constraints)

1. For the $x_{1s} + x_{1p} + x_{1e} \leq 4500$ constraint, the LHS is **the total amount of Component 1 we use**.
2. For the $0.6x_{1p} - 0.4x_{2p} - 0.4x_{3p} \geq 0$ constraint: Premium must contain at least 40% Component 1

$$\begin{aligned}
&\implies x_{1p} \geq 0.4 \times (\text{total premium}) \\
&\implies x_{1p} \geq 0.4(x_{1p} + x_{2p} + x_{3p}) \\
&\implies (1 - 0.4)x_{1p} \geq 0.4(x_{2p} + x_{3p}) \\
&\implies 0.6x_{1p} - 0.4x_{2p} - 0.4x_{3p} \geq 0
\end{aligned}$$

3. For the $x_{1s} + x_{2s} + x_{3s} \geq 3000$ constraint, the LHS is **the total amount of Super we produce**.

Solution (constraints)

$$\begin{aligned}
x_{1s} + x_{1p} + x_{1e} &\leq 4500 && \text{Component 1 availability} \\
x_{2s} + x_{2p} + x_{2e} &\leq 2700 && \text{Component 2 availability} \\
x_{3s} + x_{3p} + x_{3e} &\leq 3500 && \text{Component 3 availability} \\
\\
0.50x_{1s} - 0.50x_{2s} - 0.50x_{3s} &\geq 0 && \text{Super at least 50\% Component 1} \\
0.70x_{2s} - 0.30x_{1s} - 0.30x_{3s} &\leq 0 && \text{Super no more than 30\% Component 2} \\
\\
0.60x_{1p} - 0.40x_{2p} - 0.40x_{3p} &\geq 0 && \text{Premium at least 40\% Component 1} \\
0.75x_{3p} - 0.25x_{1p} - 0.25x_{2p} &\leq 0 && \text{Premium no more than 25\% Component 3} \\
\\
0.40x_{1e} - 0.60x_{2e} - 0.60x_{3e} &\geq 0 && \text{Extra at least 60\% Component 1} \\
0.90x_{2e} - 0.10x_{1e} - 0.10x_{3e} &\geq 0 && \text{Extra at least 10\% Component 2} \\
\\
x_{1s} + x_{2s} + x_{3s} &\geq 3000 && \text{Super contract} \\
x_{1p} + x_{2p} + x_{3p} &\geq 3000 && \text{Premium contract} \\
x_{1e} + x_{2e} + x_{3e} &\geq 3000 && \text{Extra contract} \\
\\
\forall i, j, x_{ij} &\geq 0 && \text{all variables non-negative}
\end{aligned}$$

Advertisement solution

See Topcu and Kabak lecture notes in Bb, p. 21.

Two Mines solution

See Topcu and Kabak lecture notes in Bb, p. 22.