Semester 1 Examinations 2020/2021

Course Instance Code(s):	4BP, 4BLE, 1MECE, 1MAI
Exam(s):	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Master of Engineering (Electronic & Computer Engineering) Master of Science (Computer Science-Artificial Intelligence)
Module Code(s):	EE445
Paper No.:	1
Module(s):	**Digital Signal Processing**
External Examiner(s):	Prof. A. Nandi
Internal Examiner(s):	Prof. G. Ó Laighin Prof. E. Jones
Course Co-ordinators:	Edward Jones (edward.jones@nuigalway.ie)

Remote Examination Instructions:

1. **This Examination, EE445 (Digital Signal Processing), is an Open Book examination.**
2. You may use written materials, including books, notes, and websites.
3. You may communicate with the above Course Co-ordinator by email to ask for clarification of the examination paper, or to ask for advice on an operational problem (e.g., submitting your script).
4. **You may not communicate with any other person in any way.**
5. Write on one side of the page only. Number each page 1/5, 2/5, 3/5 etc.
6. Clearly number each question and part of question on every page.
7. Orient every page in portrait format when scanning.
8. Scan your script and the **signed** cover sheet and submit it as a single PDF, named *Firstname-Lastname-EE445.pdf,* via the designated Blackboard assignment, by the deadline below.
9. Refer to instructions previously provided on scanning and uploading.
10. If, for circumstances outside of your control, you cannot upload via Blackboard on time, email the Course Co-ordinators immediately who will advise you on an alternative means of submission.
11. Check the contents of the uploaded file on Blackboard.
12. Retain your original paper script for future reference.
13. By submitting your script, you are making the following declaration:
    In submitting this work I confirm that it is entirely my own.
    I acknowledge that I may be invited to online interview if there is any concern in relation to the integrity of my exam, and I am aware that any breach will be subject to the University's Procedures for dealing with breaches of Exam Regulations:
    https://www.nuigalway.ie/media/registry/exams/QA230---Procedures-for-Dealing-with-Breaches-of-Examination-Regulations.pdf

Deadline for submission of script: **19:00 on 18th January 2021**

Specific Instructions: Answer 3 questions from 4.
All questions carry equal marks (20 marks).

Duration: 2 hours plus 30 minutes for upload **Pages:** 6 (including cover)

Question 1

(a) A discrete-time system has the following transfer function:

$$H(z) = \frac{0.3 - 0.7z^{-1} + 0.5z^{-2}}{1 + 0.3z^{-1} - 0.4z^{-2}}$$

Calculate the first four samples of the system's response to the following input signal:

$$x(n) = u(n)(-0.8 + 0.5^n)$$

where $u(n)$ is the unit step function. Indicate your calculations in detail using, e.g., a tabular format. State any assumptions you make in the calculation.

[6 marks]

(b) A digital filter is described by the following difference equation:

$$y(n) = 0.4x(n) + 0.6x(n-1) + 0.4x(n-2)$$

Give the transfer function of the system, and hence obtain an expression for the frequency response of the system. From this, give the system magnitude and phase responses and calculate the magnitude and phase responses at a frequency of one quarter of the sampling frequency. What is the group delay of the system at this frequency?

[5 marks]

(c) A discrete-time system has a finite-duration impulse response that consists of the samples {1, 3, -1}, commencing at $n = 0$. Explain how time-domain convolution can be used to calculate the system output, and calculate the response of the system to a finite-duration input signal that consists of the samples {3, 1, 4, 3}, also commencing at $n = 0$. Indicate in detail the calculations needed to determine $y(2)$. From the information provided, state if the filter has a linear or non-linear phase response; justify your answer.

[5 marks]

(d) A continuous-time signal produced by a sensor contains three sinusoidal components with frequencies 150, 300 and 450 Hz. The signal is input to a data acquisition system that operates at a sampling rate of 800 Hz. Calculate the frequencies present in the discrete-time signal after sampling, in units of Hz and radians. Explain any mismatch you observe between the components in the continuous-time and the sampled signal.

[4 marks]

Question 2

(a) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{1, 1, -3, 2\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{3, 1, -1, 2, 3\}$, commencing at $n = 1$.

[5 marks]

(b) A digital filter has a pair of complex conjugate poles at $z = -0.2 \pm j0.6$, and a pair of complex conjugate zeros at $z = 0.8 \pm j1.5$. Sketch the pole-zero map of the filter. Using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 100 Hz, if the sampling frequency is equal to 500 Hz. Show your calculations in detail.

Calculate alternative positions for the zeros given above, such that the filter becomes a minimum-phase filter.

[7 marks]

(c) A microprocessor-based system is used to determine the presence of energy at 100 Hz in the signal produced by an environmental sensor. Design a discrete-time resonator to carry out this function, for a sampling rate of 400 Hz. The bandwidth of the resonator should be 20 Hz. Scale the transfer function such that the DC gain is equal to 1. Sketch the pole zero map of the filter and give the difference equation.

Assume the amplitude of the input signal at 100 Hz is equal to 1.6. Calculate the amplitude of the resonator output at this frequency.

[8 marks]

Question 3

(a) An instrumentation application requires the removal of interference at 50Hz from a sensor signal. Determine the transfer function of a digital notch filter to achieve this objective. The signal is sampled at a frequency of 500 Hz, and a notch of width 25 Hz is required. Sketch the pole-zero map of the filter, and give the difference equation.

[5 marks]

(b) A signal processing application requires spectral analysis of a signal that has a sampling frequency of 20 kHz. The spectral analysis is to be carried out using short-time analysis with a signal window 30 milliseconds long, and with a resolution such that the frequency step is no greater than 10 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution, assuming that the FFT algorithm is used for spectral analysis. Hence, calculate the number of multiples per second needed to analyse 10 seconds of the signal, assuming 50% overlap between successive windows of the signal. State any assumptions made.

Also, calculate the width of the main lobe of the frequency response of the window in radians, if a Hamming window is used for analysis.

[7 marks]

(c) A linear-phase 256-tap FIR digital filter is to be implemented as part of an audio processing system operating at a sampling rate of 48 kHz. Calculate the number of multiplies and additions required to process a 10-second duration of a (real) audio signal, if the filter is implemented using a standard transversal filter structure.

Calculate the possible saving in the number of multiplies required to process the same duration of input signal, if the filter is implemented using fast convolution in the frequency domain, taking other possible computational savings into account. Each frame consists of 10 milliseconds of signal, zero-padded with a further 32 zero-valued samples to permit the use of the FFT. Hamming windowing is also applied. You may assume that the coefficients of the Hamming window are pre-computed, and that there is overlap of 50% between successive frames. You may ignore any additional overhead associated with overlap-add processing of the FFT output.

[8 marks]

Question 4

(a) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{3}{(s+5)(s+4)}$$

Assuming that the sampling rate is chosen to be 10 times the highest pole frequency in the analogue filter, calculate the digital filter coefficients, and write down the transfer function.

Modify the transfer function of the digital filter so that the transfer function has the same number of zeros as poles. Comment on the effect this has on the delay through the filter.

[6 marks]

(b) Using the bilinear transformation, design a first order digital filter with a cut-off frequency of 3 kHz, to operate at a sampling frequency 10 kHz. If pre-warping was not carried out, what would be the cut-off frequency of the digital filter? Comment on your result.

[5 marks]

(c) Using the window method, obtain an expression for the impulse response of a linear phase FIR *low-pass* filter with a sampling rate of 6 kHz, a cut-off frequency of 1 kHz, and with a group delay equal to 13 msec.

[5 marks]

(d) An FIR filter has an impulse response that consists of the following samples:

$$h(n) = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\},\ n = 0..11$$

Implement this filter using a cascade of a comb filter and a first order filter. Give an expression for the values of the zeros and poles of the transfer function. What is the group delay of the filter in samples?

[4 marks]