CT5141 Lab Week 2

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Giapetto problem

The Giapetto problem is a well-known linear programming example. Despite the toy setting, it gets the business ideas across quite well. [From Operations Research: Applications and Algorithms, 4th Edition, by Wayne L. Winston (Thomson, 2004)]

Giapetto's Woodcarving Inc. manufactures two types of wooden toys: soldiers and trains.

- A soldier sells for $27 and uses $10 worth of raw materials. Each soldier that is manufactured increases Giapetto's variable labor and overhead costs by $14.
- A train sells for $21 and uses $9 worth of raw materials. Each train built increases Giapetto's variable labor and overhead costs by $10.
- Each week, Giapetto can obtain all the needed raw material.

The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing.

- A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor.
- A train requires 1 hour of finishing and 1 hour of carpentry labor.
- Each week, only 100 finishing hours and 80 carpentry hours of labor are available.

Concerning demand;

- Demand for trains is unlimited, but at most 40 soldiers are bought each week.

Giapetto wants to maximize weekly profits (revenues minus costs).

1. Formalise this problem.
2. Solve it graphically.
3. Interpret the solution verbally.

Blend problem

Suppose we work for an oil company. We have three components (ingredients), and we produce three products, each a blend of the ingredients. No processing, just blending. We have contracts to produce at least 3,000 barrels of each grade of motor oil per day.

Component	Maximum barrels available/day	Cost/barrel
1	4500	12
2	2700	10
3	3500	14

Grade	Component specification	Selling price/barrel
Super	At least 50% of C1	23
	No more than 30% of C2	
Premium	At least 40% of C1	20
	No more than 25% of C3	
Extra	At least 60% of C1	18
	At least 10% of C2	

Determine the optimal mix of the three components in each grade of motor oil to maximize profit.

We are not asked to solve this problem graphically – there are 9 variables, so we can't!

Decision variables

Define x_{ij} = barrels of component i used in motor oil grade j per day, where i = 1, 2, 3 and j = s (super), p (premium), and e (extra). NB it is the *quantity*, not the *percentage* of component i used in grade j.

Objective

E.g., for x_{1s} our profit is calculated as Super selling price - Component 1 cost price = 23 - 12 = 11. Thus the objective is to maximise profits:

$$11x_{1s} + 13x_{2s} + 9x_{3s} + 8x_{1p} + 10x_{2p} + 6x_{3p} + 6x_{1e} + 8x_{2e} + 4x_{3e}$$

Exercise: write the objective in algebraic form, i.e. using $\sum$ over i, j.

Constraints

The constraints are shown below. **Exercise**: label them, i.e. explain what each constraint means. E.g.:

1. For the $x_{1s} + x_{1p} + x_{1e} \leq 4500$ constraint, what quantity does the left-hand side represent?
2. For the $0.6x_{1p} - 0.4x_{2p} - 0.4x_{3p} \geq 0$ constraint, **derive** the constraint, i.e. show how the verbal problem leads to this constraint.
3. For the $x_{1s} + x_{2s} + x_{3d} \geq 3000$ constraint, what does the left-hand side quantity represent?

$$
\begin{aligned}
x_{1s} + x_{1p} + x_{1e} &\leq 4500 \\
x_{2s} + x_{2p} + x_{2e} &\leq 2700 \\
x_{3s} + x_{3p} + x_{3e} &\leq 3500 \\
\\
0.50x_{1s} - 0.50x_{2s} - 0.50x_{3s} &\geq 0 \\
0.70x_{2s} - 0.30x_{1s} - 0.30x_{3s} &\leq 0 \\
\\
0.60x_{1p} - 0.40x_{2p} - 0.40x_{3p} &\geq 0 \\
0.75x_{3p} - 0.25x_{1p} - 0.25x_{2p} &\leq 0 \\
\\
0.40x_{1e} - 0.60x_{2e} - 0.60x_{3e} &\geq 0 \\
0.90x_{2e} - 0.10x_{1e} - 0.10x_{3e} &\geq 0 \\
\\
x_{1s} + x_{2s} + x_{3s} &\geq 3000 \\
x_{1p} + x_{2p} + x_{3p} &\geq 3000 \\
x_{1e} + x_{2e} + x_{3e} &\geq 3000 \\
\\
\forall i, j, x_{ij} &\geq 0
\end{aligned}
$$

Hint: recall “Rewriting for linearity” in the lecture.

Advertisement problem

(Winston 3.2, p.61)

Dorian makes luxury cars and jeeps for high-income men and women. It wishes to advertise with 1 minute spots in comedy shows and football games. Each comedy spot costs \$50K and is seen by 7M high-income women and 2M high-income men. Each football spot costs \$100K and is seen by 2M high-income women and 12M high-income men. How can Dorian reach 28M high-income women and 24M high-income men at the least cost?

Two Mines problem

The Two Mines Company own two different mines that produce an ore which, after being crushed, is graded into three classes: high, medium and low-grade. The company has contracted to provide a smelting plant with 12 tons of high-grade, 8 tons of medium-grade and 24 tons of low-grade ore per week. The two mines have different operating characteristics as detailed below.

Mine	Cost per day (£’000)	Production (tons/day)
		High/Medium/Low
X	180	6 / 3 / 4
Y	160	1 / 1 / 6

How many days per week should each mine be operated to fulfil the smelting plant contract? (Assume we can use fractions of a day.)