Semester 1 Examinations 2019-2020

Exam Code(s)	4BP, 4BLE, 1MECE, 1MEEE, 1MBM, 1MAI
Exam(s)	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Master of Engineering (Electronic & Computer Engineering) Master of Engineering (Electrical & Electronic Engineering) Master of Engineering (Biomedical Engineering) Master of Science (Computer Science-Artificial Intelligence)
Module Code(s)	EE445
Module(s)	**Digital Signal Processing**
Paper No.	1
Repeat Paper	No
External Examiner(s)	Prof. A. Nandi
Internal Examiner(s)	Prof. G. Ó Laighin Prof. E. Jones
Instructions:	**Answer any three questions from four** All questions carry 20 marks each
Duration	2 hours
No. of Pages	6 pages (including cover page)
Discipline	Electrical & Electronic Engineering
Course Co-ordinator(s)	Prof. E. Jones

Requirements:

MCQ	
Handout	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	Standard mathematical tables

Question 1

(a) Write the difference equation of the discrete-time system shown in Figure Q1, and hence determine the first four samples of the system's response to the following input:

$$x(n) = u(n)(1 + 0.9^n)$$

where $u(n)$ is the unit step function. You may assume zero initial conditions in the filter.

[8 marks]

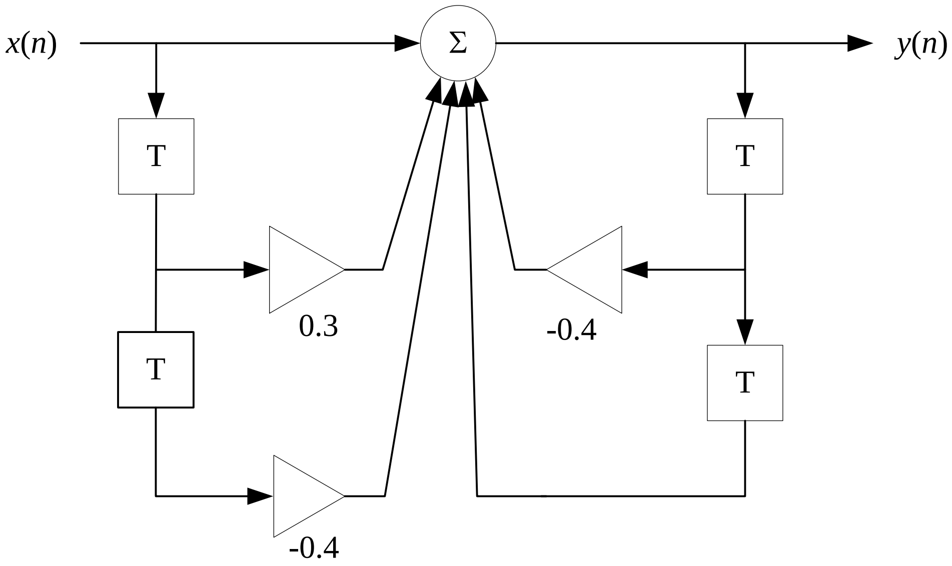

Figure Q1

(b) A digital filter is described by the following difference equation:

$$y(n) = x(n) - 0.4x(n-1) + 0.5y(n-2)$$

Write the transfer function of the system, and hence obtain an expression for the magnitude and phase responses of the system. What is the gain of the system at a frequency equal to one fifth of the sampling frequency? What is the phase response of the system at the same frequency?

[6 marks]

(c) A discrete-time system has a finite-duration impulse response that consists of the samples {2, 3, -1}, commencing at $n = 0$. Using time-domain convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {3, 1, -1, 4, 3}, also commencing at $n = 0$. Indicate in detail the calculations needed to determine $y(3)$. From the information provided, state if the filter has a linear or non-linear phase response; justify your answer.

[6 marks]

Question 2

(a) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{2, 1, -3\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{3, 1, -1, 2, 3\}$, commencing at $n = 2$.

If the input signal $x(n)$ commences at $n = 4$ instead of $n = 2$, what is the impact on the filter output?

[5 marks]

(b) A digital low-pass filter has the following difference equation:

$$y(n) = ax(n) + by(n-1) \qquad 0 < b < 1$$

Choose a value for a (in terms of b) such that the DC gain of the filter is 0.5.

[4 marks]

(c) A digital filter has a pair of complex conjugate poles at $z = -0.3 \pm j0.5$, and a pair of complex conjugate zeros at $z = 0.2 \pm j1.2$. Sketch the pole-zero map of the filter. Using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 100 Hz, if the sampling frequency is equal to 400 Hz.

[5 marks]

(d) A microprocessor-based power quality meter is used to estimate third harmonic distortion in a voltage signal. This requires extracting the third harmonic of a 50 Hz mains voltage waveform using a resonator. Design a discrete-time resonator to carry out this function, for a sampling rate of 1000 Hz. The bandwidth of the resonator should be 30 Hz. Calculate the DC gain of the resonator. Sketch the pole zero map of the filter, and give the difference equation.

[6 marks]

Question 3

(a) Discuss the use of windowing in short-time signal analysis, and outline the different trade-offs to be considered in the choice of window for different applications.

A signal processing application requires spectral analysis of a signal that has a sampling frequency of 32 kHz. The spectral analysis is to be carried out using short-time analysis with a window length of 40 msec, and with a resolution such that the frequency step is no greater than 20 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution, assuming that the FFT algorithm is used for spectral analysis.

Hence, calculate the number of multiples per second needed to analyse the signal, assuming no overlap between successive windows of the signal. State any assumptions made.

[8 marks]

(b) Discuss the advantages of decomposing high-order filters into a cascade or parallel combination of first and second-order sections.

A third-order filter has the following transfer function:

$$H(z) = \frac{(1+5z^{-1})(0.2-1.2z^{-1}+0.8z^{-2})}{(1+0.25z^{-1})(1-0.6z^{-1}+0.8z^{-2})}$$

Draw a block diagram of a cascade implementation of the filter using first and second-order sections, where the second-order section is Direct Form I.

[5 marks]

(c) For an oscillator whose output is $p(n)=\sin(n\theta_0)$, where θ_0 is determined by the coefficient b_1, show that a small change in oscillation frequency Δf_0 is related to a change Δb_1 in the coefficient b_1 by the following expression:

$$\Delta f_0 = \frac{-f_s \Delta b_1}{4\pi \cos\left(2\pi \dfrac{f_0}{f_s}\right)}$$

where f_0 is the frequency of oscillation in Hz, and f_s is the sampling frequency.

[7 marks]

Question 4

(a) An environmental monitoring sensor application requires the removal of 50 Hz interference. Design a digital filter to achieve this objective. The signal is sampled at a frequency of 1200 Hz and a notch of width 15 Hz will suffice. Draw the pole-zero map of the filter and write the difference equation. Scale the filter transfer function such that the DC gain of the filter is equal to 1.

[6 marks]

(b) A first-order analogue filter is described by the following transfer function:

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where ω_c is the cut-off frequency in radians/s.

Using the bilinear transformation, determine the transfer function of the digital equivalent of this filter, if the desired cut-off frequency is 2.5 kHz and the sampling frequency is 16 kHz. If pre-warping was not carried out, what would be the actual cut-off frequency of the digital filter?

[7 marks]

(c) Using the window method, obtain an expression for the impulse response of a linear phase FIR *high-pass* filter with a sampling rate of 8 kHz, a cut-off frequency of 2.5 kHz, and with a group delay equal to 9 msec.

[7 marks]

Table of useful z-Transforms

		Sequence	z-Transform
1.	Unit sample	$d(n)$	1
		$d(n-k)$	z^{-k}
2.	Unit step	$u(n)$	$z/(z-1)$
3.	Exponential	$a^n\, u(n)$	$z/(z-a)$
4.	Sinusoidal	$\sin(\theta_O n)\, u(n)$	$\dfrac{z\sin\theta_0}{z^2-2z\cos\theta_0+1}$
		$\cos(\theta_O n)\, u(n)$	$\dfrac{z^2-z\cos\theta_0}{z^2-2z\cos\theta_0+1}$
5.	Unit ramp	$nu(n)$	$\dfrac{z}{(z-1)^2}$
6.	Product of ramp and signal	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

7. Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+..........+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$