Semester 1 Examinations 2016-2017

Exam Code(s)	4BP, 4BLE, 1MEEE, 1MECE, 1MBM, 1CSD
Exam(s)	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Master of Engineering (Electrical & Electronic Engineering) Master of Engineering (Electronic & Computer Engineering) Master of Engineering (Biomedical Engineering) Master of Science in Computer Science (Data Analytics)
Module Code(s)	EE445
Module(s)	**Digital Signal Processing**
Paper No.	1
Repeat Paper	No
External Examiner(s)	Prof. B. Foley
Internal Examiner(s)	Prof. G. Ó Laighin Dr. E. Jones
Instructions:	**Answer any three questions from four** All questions carry 20 marks each
Duration	2 hours
No. of Pages	6 pages (including cover page)
Discipline	Electrical & Electronic Engineering
Course Co-ordinator(s)	Dr. E. Jones

Requirements:

MCQ	
Handout	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	Standard mathematical tables

Question 1

(a) A digital filter is described by the following difference equation:

$$y(n) = x(n) - 0.5x(n-1) - 0.7y(n-1) - 0.4y(n-2)$$

Give the transfer function of the system and hence its frequency response. Derive expressions for its magnitude and phase responses. What is the DC gain of the filter? What is the phase response of the filter at a frequency equal to one quarter of the sampling frequency?

[6 marks]

(b) A digital filter has a pair of complex conjugate poles at z = -0.1 ± j0.4, and a pair of complex conjugate zeros at z = 0.3 ± j0.6. Sketch the pole-zero map of the filter. Based on the pole-zero map, and in particular on the locations of the poles and zeros, sketch the magnitude response of the system if the sampling frequency is 1 kHz. Also, using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 250 Hz.

[6 marks]

(c) A discrete-time system has a finite-duration impulse response that consists of the samples {1, -2, 3, -2, 1}, commencing at n = 0. Using time-domain convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {2, 2, 1}, also commencing at n = 0. Indicate in detail the calculations needed to determine $y(2)$.

From the information given, state whether the filter has a linear or non-linear phase response. What is the group delay of the filter?

[8 marks]

Question 2

(a) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{1, 0, 0, -3\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{1, -1, 2\}$, commencing at $n = 2$.

[4 marks]

(b) Obtain the inverse z-transforms of the following functions:

(i) $X(z) = 1 + 2z^{-1} - 3z^{-3} - 2z^{-5}$

(ii) $H(z) = \dfrac{1}{z(z-1)(2z-1)}$

[6 marks]

(c) A discrete-time system has impulse response:

$$h(n) = \alpha^{n} u(n)$$

Obtain an expression for the frequency response of the system, and hence calculate the phase response. From the phase response, obtain an expression for the group delay of the system.

Use the fact that

$$\frac{d}{dx}\tan^{-1}\left[\frac{-a\sin x}{1-a\cos x}\right] = -\frac{a^2 - a\cos x}{1 + a^2 - 2a\cos x}$$

[4 marks]

(d) The transfer function of a first-order low-pass filter is described by the following equation:

$$H(z) = \frac{0.1}{1 - 0.9z^{-1}}$$

If the sampling rate is 1 kHz, determine the frequency in Hz at which the gain of the filter is equal to -20 dB.

[6 marks]

Question 3

(a) Using the pole-zero placement method, determine the transfer function of a digital resonator with the following characteristics:

(i) Sampling rate of 10 kHz
(ii) Centre frequency of 2.5 kHz
(iii) Bandwidth of 30 Hz
(iv) DC gain of 0.75

Sketch the pole zero map of the filter and write the difference equation.

[5 marks]

(b) A linear-phase 256-tap FIR digital filter is to be implemented as part of a speech conferencing system at a sampling rate of 32 kHz. Calculate the number of multiplies and additions required to process a 20-second duration of a (real) audio signal, if the filter is implemented using a standard transversal filter structure. Use the fact that the filter is linear phase to reduce the computation required.

Calculate the saving in the number of multiplies required to process the same duration of input signal, if the filter is implemented using fast convolution in the frequency domain with 256-point FFT and Inverse FFT, and with Hamming windowing. You may assume that the coefficients of the window are pre-computed, and that there is overlap of 50% between successive frames. You may ignore any additional overhead associated with overlap-add processing of the FFT output.

Comment on the potential for further savings in the fast convolution approach, on the assumption that the filter has a real impulse response, and the signal being analysed is real.

[10 marks]

(c) A signal processing application requires spectral analysis of a signal that has a sampling frequency of 50 kHz. The spectral analysis is to be carried out using short-time analysis with a window length of 20 msec, and with a resolution such that the frequency step is no greater than 10 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution, assuming that the FFT algorithm is used for spectral analysis.

[5 marks]

Question 4

(a) Show how an FIR filter whose impulse response consists of N samples each of amplitude g, can be implemented efficiently by a cascade of a comb filter and a first-order filter with a single pole at $z = 1$. Explain how this results in a finite impulse response filter, even though it includes a recursive component. Draw the pole-zero map for the resulting structure for N=8.

[6 marks]

(b) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{1}{(s+3)(s+12)}$$

Assuming that the sampling rate is chosen to be ten times the highest pole frequency in the analogue filter, calculate the digital filter coefficients and write down the transfer function.

[6 marks]

(c) Using the window method, derive an expression for the impulse response of a linear phase FIR low-pass filter with the following frequency response:

$$H(\theta) = \begin{cases} e^{-j7\theta}, & |\theta| \le \dfrac{\pi}{3} \\ 0, & \dfrac{\pi}{3} < |\theta| < \pi \end{cases}$$

(hint: consider how the group delay is included in the frequency response above)

[8 marks]

Table of useful z-Transforms

		Sequence	z-Transform
1.	Unit sample	$d(n)$	1
		$d(n\text{-}k)$	z^{-k}
2.	Unit step	$u(n)$	$z/(z-1)$
3.	Exponential	$a^n\, u(n)$	$z/(z-a)$
4.	Sinusoidal	$\sin(\theta_O n)\, u(n)$	$\dfrac{z\sin\theta_0}{z^2-2z\cos\theta_0+1}$
		$\cos(\theta_O n)\, u(n)$	$\dfrac{z^2-z\cos\theta_0}{z^2-2z\cos\theta_0+1}$
5.	Unit ramp	$nu(n)$	$\dfrac{z}{(z-1)^2}$
6.	Product of ramp and signal	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

7. Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+\dots\dots\dots\dots+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$