Semester 1 Examinations 2018-2019

Exam Code(s)	4BP, 4BLE, 1MECE, 1MEEE
Exam(s)	Fourth Year Electronic & Computer Engineering Fourth Year Electrical & Electronic Engineering Master of Engineering (Electronic & Computer Engineering) Master of Engineering (Electrical & Electronic Engineering)
Module Code(s)	EE445
Module(s)	**Digital Signal Processing**
Paper No.	1
Repeat Paper	No
External Examiner(s)	Prof. A. Nandi
Internal Examiner(s)	Prof. G. Ó Laighin Dr. E. Jones
Instructions:	**Answer any three questions from four** All questions carry 20 marks each
Duration	2 hours
No. of Pages	6 pages (including cover page)
Discipline	Electrical & Electronic Engineering
Course Co-ordinator(s)	Dr. E. Jones

Requirements:

MCQ	
Handout	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	Standard mathematical tables

Question 1

(a) A digital filter is described by the following difference equation:

$$y(n) = 0.3x(n) - 0.25x(n-1) + 0.5y(n-1) - 0.4y(n-2)$$

Give the transfer function of the system and hence its frequency response. Derive expressions for its magnitude and phase responses. What is the magnitude response of the filter at a frequency equal to one quarter of the sampling frequency? What is the phase response of the filter at the same frequency?

[7 marks]

(b) A digital filter has a pair of complex conjugate poles at $z = 0.5e^{\pm j0.7}$, and a pair of complex conjugate zeros at $z = 0.8e^{\pm j2.0}$. Sketch the pole-zero map of the filter. Based on the pole-zero map, and in particular on the locations of the poles and zeros, sketch the magnitude response of the system as a function of both radians and Hz, if the sampling frequency is 1 kHz. Also, using only the pole-zero map, calculate the magnitude response of the filter at a frequency of 250 Hz.

[6 marks]

(c) A discrete-time system has a finite-duration impulse response that consists of the samples {1, -1, 3, -1, 1}, commencing at n = 0. Using time-domain convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {3, 2, -1}, also commencing at n = 0. Indicate in detail the calculations needed to determine $y(3)$.

From only the information given, state whether the filter has a linear or non-linear phase response, and determine the group delay of the filter.

[7 marks]

Question 2

(a) A digital filter has an impulse response consisting of the finite duration sequence $h(n) = \{1, 0, 2, -2\}$, commencing at $n = 0$. Using the z-transform convolution property, determine the output of the system in response to the finite duration input signal $x(n) = \{1, 1, -1, 3\}$, commencing at $n = 0$.

If the input signal $x(n)$ commences at $n = 3$ instead of $n = 0$, what is the impact on the filter output?

[5 marks]

(b) A digital filter has the following transfer function:

$$H(z) = \frac{1 - 0.6z^{-2}}{1 + 0.2z^{-1} - 0.7z^{-2}}$$

Determine the frequency response and hence the phase response of the system. What is the value of the phase response at a frequency equal to one quarter of the sampling frequency?

[4 marks]

(c) The transfer function of a first-order low-pass filter is described by the following equation:

$$H(z) = \frac{0.15}{1 - 0.85z^{-1}}$$

If the sampling rate is 10 kHz, determine the cut off frequency of the filter in Hz (you may assume the cut-off frequency is the "-3 dB frequency").

[5 marks]

(d) Using the pole-zero placement method, determine the transfer function of a digital resonator with the following characteristics:

(i) Sampling rate of 16 kHz
(ii) Centre frequency of 4 kHz
(iii) Bandwidth of 40 Hz
(iv) DC gain of 1

Sketch the pole zero map of the filter and write the difference equation.

[6 marks]

Question 3

(a) A healthcare instrumentation application requires the removal of interference at 60 Hz from an ECG signal. Design a digital notch filter to achieve this objective. The signal is sampled at a frequency of 1000 Hz, and a notch of width 25 Hz is required.

[5 marks]

(b) A nonlinear-phase 512-tap FIR digital filter is to be implemented as part of an audio processing system operating at a sampling rate of 96 kHz. Calculate the number of multiplies and additions required to process a 20-second duration of a (real) audio signal, if the filter is implemented using a standard transversal filter structure.

Calculate the possible saving in the number of multiplies required to process the same duration of input signal, if the filter is implemented using fast convolution in the frequency domain with 512-point FFT and Inverse FFT, and with Hamming windowing. You may assume that the coefficients of the window are pre-computed, and that there is overlap of 50% between successive frames. You may ignore any additional overhead associated with overlap-add processing of the FFT output.

[9 marks]

(c) Design an oscillator that produces a cosine wave with a frequency of 2 kHz, at a sampling rate of 48 kHz. The amplitude of the cosine wave should be 1. Calculate the values of the digital filter coefficients, and the initial conditions for the oscillator, assuming that the cosine wave starts with a phase shift of 0 radians. Draw a block diagram of the oscillator.

Modify this oscillator so that a signal with sine phase is generated.

[6 marks]

Question 4

(a) Using the window method, obtain an expression for the impulse response of a linear phase FIR *high-pass* filter with a sampling rate of 4 kHz, a cut-off frequency of 800 Hz, and with a group delay equal to 5 msec.

[7 marks]

(b) A first-order analogue filter is described by the following transfer function:

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where ω_c is the cut-off frequency in radians/s.

Using the bilinear transformation, determine the transfer function of the digital equivalent of this filter, if the desired cut-off frequency is 1.8 kHz and the sampling frequency is 16 kHz. If pre-warping was not carried out, what would be the actual cut-off frequency of the digital filter?

[7 marks]

(c) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{3}{(s+4)(s+5)}$$

Assuming that the sampling rate is chosen to be eight times the highest pole frequency in the analogue filter, calculate the digital filter coefficients, and write down the transfer function.

[6 marks]

Table of useful z-Transforms

		Sequence	z-Transform
1.	Unit sample	$d(n)$	1
		$d(n-k)$	z^{-k}
2.	Unit step	$u(n)$	$z/(z-1)$
3.	Exponential	$a^n\, u(n)$	$z/(z-a)$
4.	Sinusoidal	$\sin(\theta_O n)\, u(n)$	$\dfrac{z\sin\theta_0}{z^2-2z\cos\theta_0+1}$
		$\cos(\theta_O n)\, u(n)$	$\dfrac{z^2-z\cos\theta_0}{z^2-2z\cos\theta_0+1}$
5.	Unit ramp	$nu(n)$	$\dfrac{z}{(z-1)^2}$
6.	Product of ramp and signal	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

7. Sum of Series: $1+z^{-1}+z^{-2}+z^{-3}+..........+z^{-(N-1)}=\dfrac{1-z^{-N}}{1-z^{-1}}$