EE445 DIGITAL SIGNAL PROCESSING: Assignment II

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1 Exercise 1 IIR Filter Design.

A digital low-pass filter has the following specification:

Sampling frequency 12 kHz.

Cutoff frequency 3.5 kHz.

Transition bandwidth 400 Hz.

Stop band attenuation 30 dB.

Passband ripple 0.1 dB.

A Chebyshev Type I filter is to be designed to meet this specification.

```
%
% Exercise 1 - IIR Filter Design
%

close all;
clear all;
clc;

% set up constants
fsamp = 12e3; % sampling frequency
fc = 3.5e3; % cutoff frequency
transition_bandwidth = 400; % transition bandwidth 400Hz
stop_band_attenuation = 30; % stop band attenuation 30dB
passband_ripple = 0.1; % passband ripple 0.1dB

% calculate the desired digital cut off and stop band frequencies.
theta_c = 2*(fc/fsamp); %*pi
```

```
theta_s = 2*((transition_bandwidth+fc)/fsamp); %*pi

% calculate the order of the filter
[n,Wp] = cheb1ord(theta_c,theta_s,passband_ripple,stop_band_attenuation);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Chebyshev IIR filter
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% calculate the actual Chebyshev filter coefficients
[num_poly_c, den_poly_c] = cheby1(n,passband_ripple,Wp);

% calculate the magnitude response of Chebyshev filter
Hc = freqz(num_poly_c, den_poly_c);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Butterworth IIR filter
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% calculate the actual Butterworth filter coefficients
[num_poly_b, den_poly_b] = butter(n,Wp);
% calculate the magnitude response of Butterworth filter
Hb = freqz(num_poly_b, den_poly_b);

% plot
figure;
plot(20*log10(abs(Hc)));
hold on;
plot(20*log10(abs(Hb)), 'r');
grid on;
hold off;

% annotate
xlabel('Frequency (Hz)');
ylabel('Magnitude Response (dB)');
title('Exercise 1 - Magnitude Response of Chebyshev filter and Butterworth
↪ filter');
legend('Chebyshev', 'Butterworth');

fprintf('\n\nFinished ...\n');
```

Figure1 shows the Magnitude Response of Chebyshev filter and Butterworth filter.

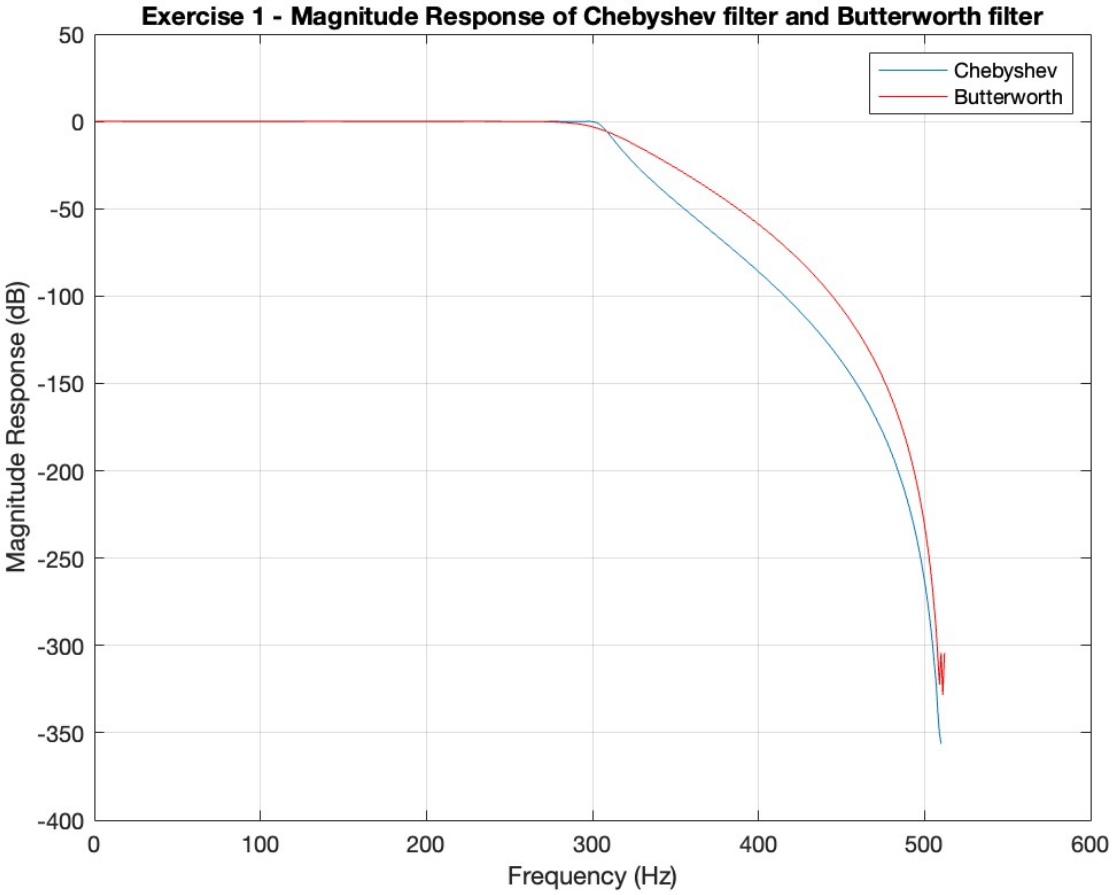


Figure 1: Magnitude Response of Chebyshev filter and Butterworth filter.

Chebyshev and Butterworth filters are designed for different applications. Butterworth filters are used in applications where there is no ripple in the pass band and stop band, while Chebyshev filters are more tolerant of changing magnitude. As Figure 1 shows above, the Chebyshev filter has a sharper transition than the Butterworth filter does in the transition band as there is a cost of ripples in the passband. Figure 1 also shows the distortion of the Butterworth filter in the passband as the Butterworth filter has the flat magnitude frequency response in the passband, whereas our desired filter has a cost of ripple of $0.1dB$ in the pass band.

2 Exercise 2 FIR Filter Design.

A digital low-pass filter has the following specification:

Sampling frequency 10 kHz.

Cutoff frequency 3 kHz.

Transition bandwidth 400 Hz.

Stop band gain 0(desired).

```
%
% FIR Filter Design
%

close all;
clear all;
clc;

% set up constants
fsamp = 10e3; % sampling frequency
fc = 3e3; % cutoff frequency
transition_bandwidth = 400;
stopband_gain = 0;

% calculate the desired digital cut off and stop band frequencies.
theta_c = 2*(fc/fsamp); %*pi
theta_s = 2*((fc+transition_bandwidth)/fsamp); %*pi

% set for Parks-McClellan technique
f = [0, theta_c, theta_s, 1];
a = [1, 1, 0, 0];
% filters of length ranging from 21 to 51 coefficients in increments of 5
for N = 21:51,
    % Parks-McClellan technique
    b_pm = firpm(N, f, a);

    % window method with the default Hamming window
    b_w = fir1(N, theta_c, "low");
%     H = freqz(b_pm);
%     figure(1);
%     plot(20*log10(abs(H)));
%     hold on;
    N = N + 5;
end

% calculate the frequency response for the filter of order 51
H_pm = freqz(b_pm);
H_w = freqz(b_w);

% plot
figure(1);
plot(20*log10(abs(H_pm)));
hold on;
```

```
plot(20*log10(abs(H_w)), 'r');
hold off;

% annotate
xlabel('Frequency (Hz)');
ylabel('Magnitude Response (dB)');
title('Exercise 2 - Magnitude Response the filter of order 51 with Parks-McClellan and Window method');
legend('Parks-McClellan', 'Window');

fprintf('\n\nFinished ...\n');
```

Figure2 shows the Magnitude Response of the filter of order 51 with Parks-McClellan and Window method.

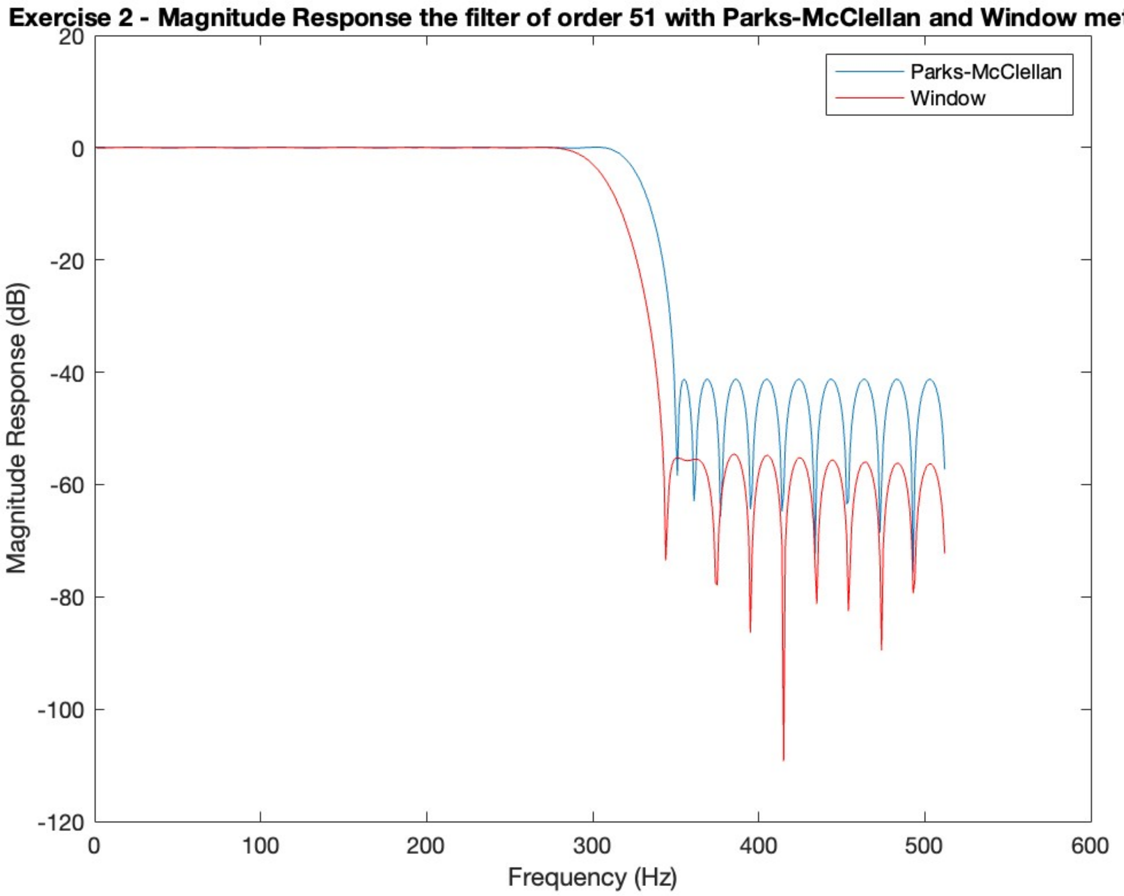
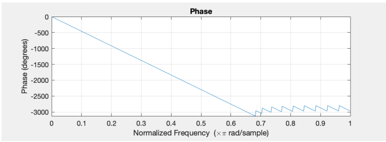
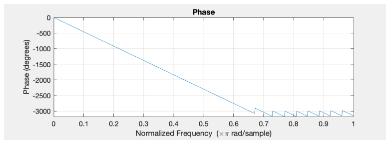


Figure 2: Magnitude Response of the filter of order 51 with Parks-McClellan technique and Window method.

There is no need to plot the phase response as it doesn't give us much more information than we already have, as Figure 3 shows that the phase response of filter of order 51 with Parks-McClellan technique and the Window method are the same.



(a) Phase response of filter of order 51 with Parks-McClellan technique.



(b) Phase response of filter of order 51 with window method.

Figure 3: Phase response of the filter of order 51 with Parks-McClellan technique and Window method.

Figure4 shows the actual transition bandwidth from the magnitude response of $-3dB$ to the the peak of the first ripple in the stop band of the Parks-McClellan technique is $49Hz$ and the Window method is $32Hz$.

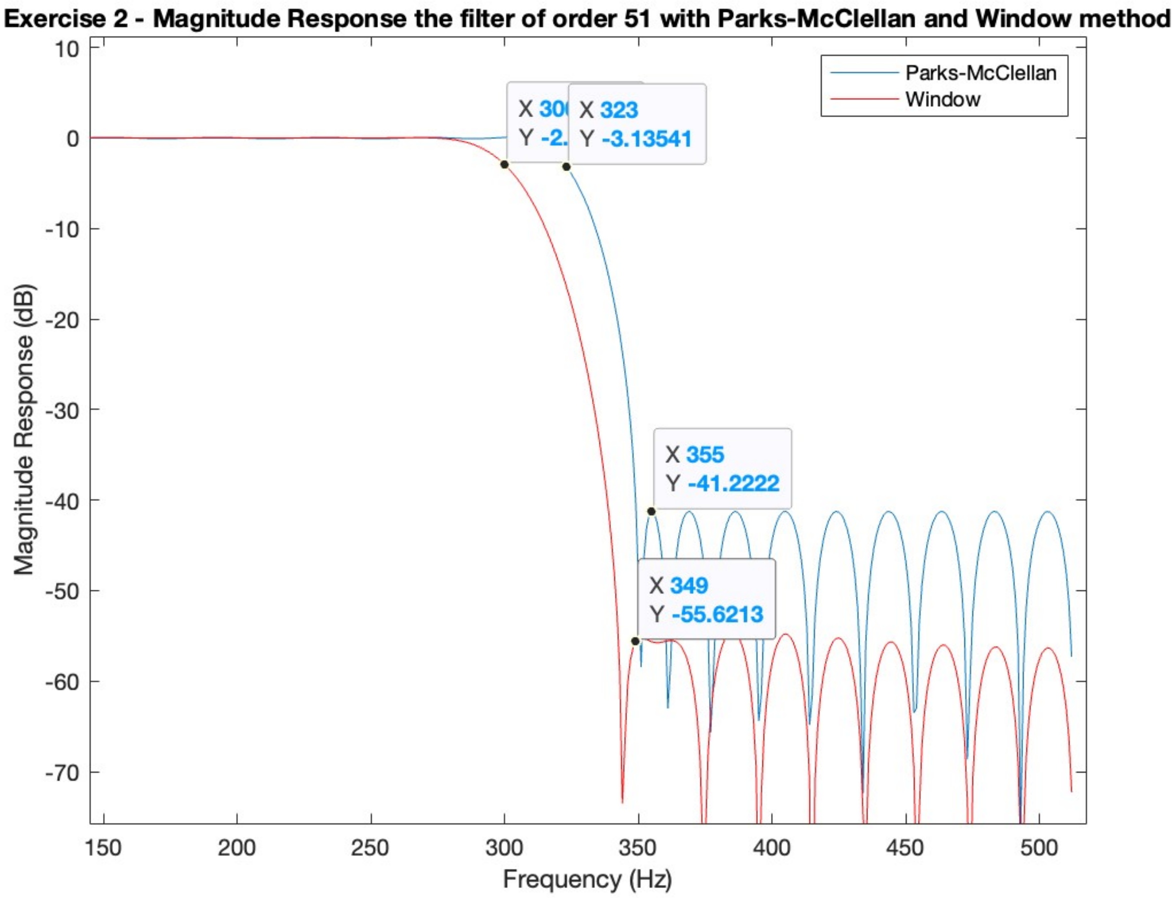


Figure 4: The actual transition bandwidth of the filter designed with Parks-McClellan technique and the filter designed with the Window method.

Filters designed with Parks–McClellan algorithm are optimal because they minimize the maximum error between the actual magnitude response of the filter and the ideal magnitude response of the filter. Besides, it always has a sharp transition band and its magnitude response with the weighted ripple evenly distributed over the passband and stopband [1]. Filters designed with window method can suppress the spectral leakage and smooth the discontinuity at the edges of the FFT slices. As a result, the choice of window function affects the amount of signal and noise that goes inside each filter bank [2].

References

[1] LabVIEW. Designing optimum fir filters using the parks-mcclellan algorithm, 2022. URL https://www.ni.com/docs/en-US/bundle/labview/page/lvanlsconcepts/lvac_design_optimum_fir.html.

[2] Mathuranathan. Window function – figure of merits, 2020. URL https://www.gaussianwaves.com/2020/09/window-function-figure-of-merits/.