NUI Galway
OÉ Gaillimh

Semester I Examinations 2017/2018

Exam Code(s)	1OA1, 1OA9. 1BY1, 1EV1, 2BA1, 2BCS1, 2BME1, 2BCM1, 2BCT1, 2BPT1, 2BS1, 2BFS1, 2EH1, 2MR1, 3BS9
Exam(s)	First Science, Second Arts, Second Science and Engineering, Third Science & Commerce

Module(s)	Statistics/Probability
Module Code(s)	ST237

Paper No	1
Repeat Paper	

External Examiner	Prof. S. Wilson
Internal Examiner	Prof. J. P. Hinde

Instructions:

Answer the 10 Questions in Section A (20 marks)

and

any 4 Questions from Section B (20 marks each).

Duration	2 Hours
No. of Pages	12 Pages
School	School of Mathematics, Statistics and Applied Mathematics

Requirements:

|	
Release to Library:	Yes
Statistical Tables/ Log Tables	Relevant distribution tables are attached to this paper
	The *New Formulae & Tables* are optional.
Other Materials	A calculator is allowed (non-programmable and
	not capable of storing text)

Section A - Compulsory Questions (20 marks)

A maximum of two marks will be awarded for each part.

A1. A lecture room contains 200 students. The professor would like to select a representative sample of 25 students. He considers the following two methods:

(a) take the nearest 25 students from the front rows;

(b) ask each student to toss a coin three times and select the students who get three heads.

Why is method (b) preferred to method (a)? Is there any problem with method (b)?

A2. The following boxplot shows a sample of data with a median at 6.
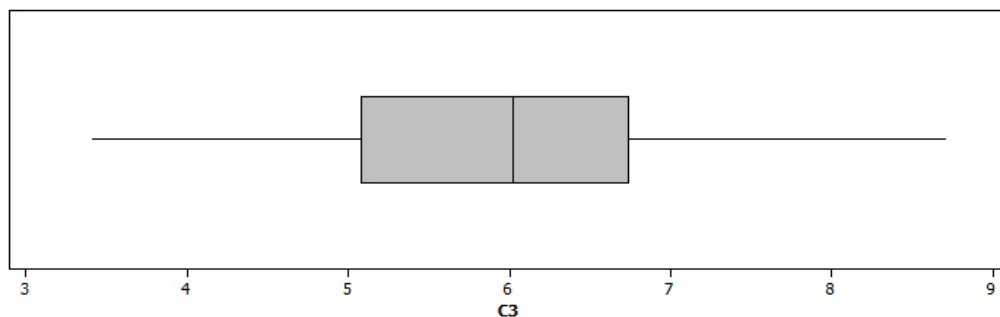
Will the mean of the data be

(a) less than the median;

(b) approximately equal to the median;

(c) larger than the median?

A3. The 2016 monthly recorded rainfall figures (mm) at Mace Head, Co. Galway were:

$$162.7 \quad 130.7 \quad 69.1 \quad 55.9 \quad 53.4 \quad 81.6$$
$$63.4 \quad 73.8 \quad 154.4 \quad 65.3 \quad 58.8 \quad 77.6$$

Calculate the median monthly rainfall.

A4. What is the relationship between **standard deviation** and **variance**?
Why is it usually preferable to quote the standard deviation as a measure of spread?

A5. An inclusive lunch deal has a choice of 5 different sandwiches, 3 chocolate bars, and 4 drink options. How many different lunch combinations are there?

A6. Two events A and B are **mutually exclusive** and have probabilities $P(A) = 0.3$ and $P(B) = 0.5$. Find $P(A \mid B)$.

A7. If two events A and B are **independent** and if $P(A) = 0.5$ and $P(B) = 0.2$, find $P(A \cup B)$.

A8. In a single roll of a pair of fair dice what is the probability of getting a **double**?

A9. A random variable X has the following probability distribution

x	0	1	2	3	4
$P(X = x)$	0.4	0.25	0	0.1	0.25

Find $\mathrm{E}\left[X\right]$, the expected (mean) value of X.

A10. For a random variable X with variance 9, find the standard deviation of $4X + 5$.

Section B — Answer 4 Questions

B1. (a) The following data are the number of vessels registered for an Alaskan fishery over a 23 year period:

143	148	195	181	189	175	213	227	178	136
100	89	88	129	158	169	195	179	194	247
164	246	309							

    i) Construct a stem-and-leaf plot using intervals for the stems of width 10. (4)

    ii) Find the five number summary. (5)

    iii) Draw a boxplot for these data (you do not need to use graph paper for this, but you may do so if you wish). (3)

    iv) Calculate the sample mean and sample standard deviation. (4)

(b) The following boxplot shows cell division times (hours) for samples of three different species of yeast. What can you say about the cell division times for the three species?

Why is the box for species B narrower than those for the other two species? (4)
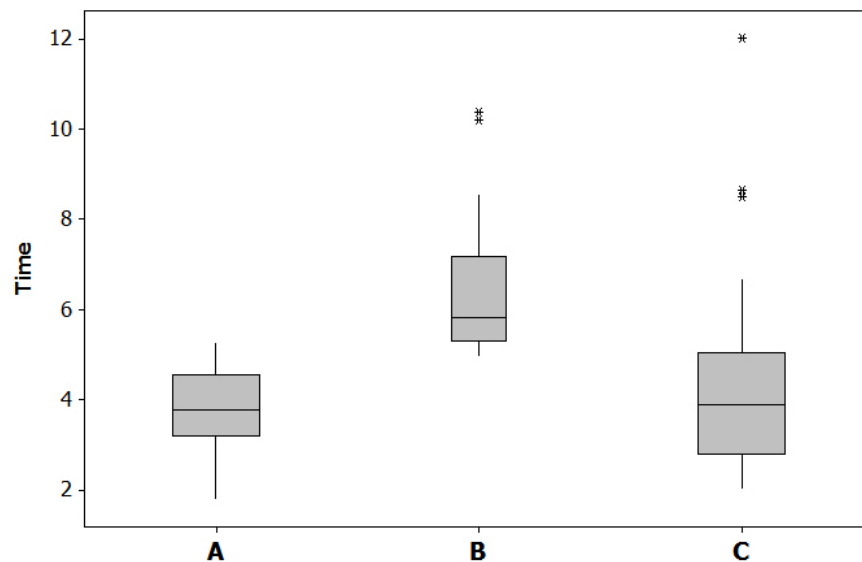
B2. (a) i) How many different ways are there of writing down (in a sequence) the letters in the title of the 1969 Grateful Dead album (3)

AOXOMOXOA

ii) What is the probability that a random rearrangement of these letters has all of the vowels (A's and O's) at the beginning and the consonants (X's and M) at the end? (2)

(b) A box of party surprise chocolates contains 20 identical looking sweets, however 2 of them contain a hidden chilli pepper. Guests take turns to pick a chocolate at random and eat it. What is the probability that

i) the first guest to take a chocolate gets a hot surprise? (1)

ii) the second guest also gets a chilli, given that the first got one? (1)

iii) the second guest gets a chilli, irrespective of what happened to the first guest? (2)

Is it better to go first or last in this "game"? (1)

(c) In the over-75 Irish population there are 4 times as many women as men and 40% of the population have high blood pressure. For males the (*conditional*) probability of having high blood pressure is 0.5.

i) Find the joint probability of being male and having high blood pressure. (2)

ii) Hence, complete a table of joint and marginal probabilities as begun below. (2)

Blood Pressure

		High	OK	Total
Gender	Male			
	Female			
	Total	0.4		1.0

iii) Is high blood pressure independent of gender? (1)

(d) Suppose that 4% of the population has a particular disease. There is a test on a blood sample which gives one of two results, positive or negative. It is found that 90% of the people having the disease produce a positive result. However, 5% of the people without the disease will also produce a positive result. For a person randomly chosen from the population,

i) what is the probability that they will test positive? (3)

ii) given that the person's test was positive, what is the probability that they have the disease? (2)

B3. (a) A fair coin (equal probabilities of a head and a tail) is tossed 10 times. What is the probability of obtaining

    i) equal numbers of head and tails (i.e. 5 of each)? (2)

    ii) at least two heads? (2)

If the coin is tossed a further 10 times, what is the overall probability of getting equal numbers of heads and tails in **each** set of 10 tosses? (2)

Is this the same as the probability of getting equal numbers of heads and tails in the **combined** set of 20 tosses? Explain. (2)

Note: relevant tables of the binomial distribution are given at the back of this paper.

(b) The number of seismic events in Christchurch, New Zealand is Poisson distributed with a mean of 1 per day. Find the probability that

    i) there are no events on a given day; (2)

    ii) there are at least two events on a given day; (2)

    iii) there are no events in a particular week. (2)

(c) The daily number of visits to the library by a population of college students is normally distributed with a mean of 1600 and a standard deviation of 200.

    i) What is the probability that the number of visits on a random day is

      - more than 2000? (2)

      - between 1300 and 1800? (2)

    ii) What is the number of visits that is exceeded on 10% of days? (2)

B4. The joint probability mass function of two random variables X and Y is given by the following table:

		Y		
		1	2	3
	-1	0.1	0.2	0.1
X	0	0	0.2	0
	1	0.1	0.2	0.1

(a) Find the marginal probability functions for X and Y. (3)

(b) What is the conditional probability that $Y = 1$ given that $X = 1$? (2)

(c) Calculate the mean and variance of each of X and Y. (6)

(d) Calculate the covariance between X and Y. (3)

(e) Find the correlation between X and Y. (2)

(f) Are X and Y independent? Justify your answer. (2)

(g) What is $P(X + Y \leq 3)$? (2)

B5. (a) A certain US airline, believing that 10% of passengers fail to show up for flights, overbooks (i.e. sells more tickets than there are seats). Suppose that a plane will hold 188 passengers and the airline sells 200 seats.

    i) What is an appropriate binomial probability model for the number of passengers who will show up? (3)

    ii) Use a normal approximation to the binomial distribution to calculate the probability that the airline will have enough seats on the plane. (4)

    iii) What are the two assumptions of the binomial model? Are they reasonable here? (3)

(b) The normal distribution plays an important role in statistics. Write a very brief comment/answer to each of the following aspects:

    i) the relationship between the mean and the median of the normal distribution; (2)

    ii) the $68 - 95 - 99.7$ rule; (3)

    iii) the use of normal probability plots, with reference to the two examples below; (3)
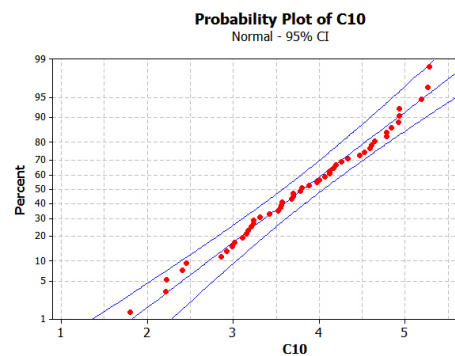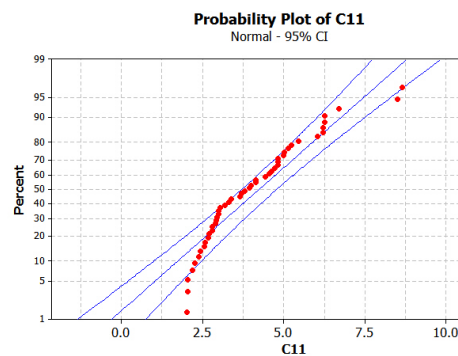
    iv) the Central Limit Theorem. (2)

Formulæ

- Sample Mean

$$\bar{x} = \frac{\sum\limits_{i=1}^{n} x_i}{n}$$

- Sample Standard Deviation

$$s = \sqrt{\frac{\sum\limits_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum\limits_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}}$$

- Binomial Distribution

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, \ldots, n$$

$$\mathsf{E}\left[X\right] = np \qquad \mathsf{Var}\left(X\right) = np(1-p)$$

- Poisson Distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0, 1, \ldots$$

$$\mathsf{E}\left[X\right] = \lambda \qquad \mathsf{Var}\left(X\right) = \lambda$$

- Variances, Covariances and Correlation

$$\mathsf{Var}\left(X\right) = \mathsf{E}\left[X^2\right] - (\mathsf{E}\left[X\right])^2$$

$$\mathsf{cov}\left[X, Y\right] = \mathsf{E}\left[XY\right] - \mathsf{E}\left[X\right]\mathsf{E}\left[Y\right]$$

$$\mathsf{Correlation}(X, Y) = \frac{\mathsf{cov}\left[X, Y\right]}{\sqrt{\mathsf{Var}\left(X\right)\mathsf{Var}\left(Y\right)}}$$

Table C Binomial probabilities (p.4 of 6)

Entry is $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

| n	k	.10	.15	.20	.25	.30	.35	.40	.45	.50
9	0	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7			.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8				.0001	.0004	.0013	.0035	.0083	.0176
	9						.0001	.0003	.0008	.0020
10	0	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7		.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8			.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9					.0001	.0005	.0016	.0042	.0098
	10							.0001	.0003	.0010
12	0	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7		.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8		.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9			.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10					.0002	.0008	.0025	.0068	.0161
	11						.0001	.0003	.0010	.0029
	12								.0001	.0002
15	0	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	
	1	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005
	2	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032
	3	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139
	4	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417
	5	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916
	6	.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527
	7	.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964
	8		.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964
	9		.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527
	10			.0001	.0007	.0030	.0096	.0245	.0515	.0916
	11				.0001	.0006	.0024	.0074	.0191	.0417
	12					.0001	.0004	.0016	.0052	.0139
	13						.0001	.0003	.0010	.0032
	14								.0001	.0005
	15									
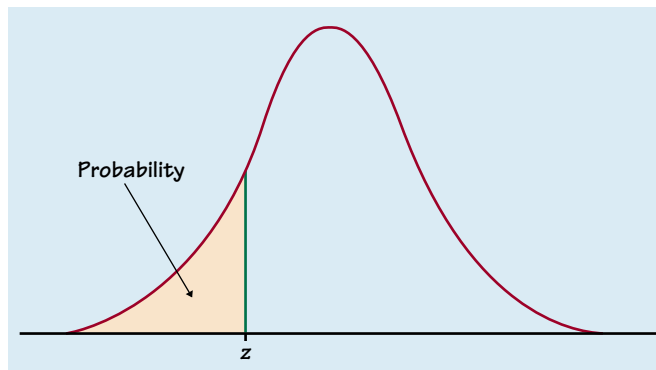
Table entry for z is the probability lying below z.

TABLE A   Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z.

TABLE A   Standard normal probabilities (*Continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998