



Semester I Examinations 2016/2017

Exam Code(s)	1OA1, 1BY1, 1EV1, 2BA1, 2BME1, 2BCS1, 2BCW1 2BCM1, 2BCT1, 2BCW1, 2BPT1, 2BS1, 2EH1, 2MR1, 3BS9
Exam(s)	First Science, Second Arts, Second Science, Second Engineering, Third Science
Module(s)	Statistics/Probability
Module Code(s)	ST237
Paper No	1
Repeat Paper	
External Examiner	Prof. S. Wilson
Internal Examiner	Prof. J. Hinde; C. Dooley

Instructions:

**Answer the 10 Questions in Section A (20 marks)
and
any 4 Questions from Section B (20 marks each).**

Duration	2 Hours
No. of Pages	11 Pages
School	School of Mathematics, Statistics and Applied Mathematics

Requirements:

Release to Library:	Yes
Statistical / Log Tables	Relevant distribution tables are attached to this paper. The <i>New Formulae & Tables</i> are optional.
Other Materials Allowed	A calculator is allowed (non-programmable and not capable of storing text)

Section A - Compulsory Questions (20 marks)

A maximum of two marks will be awarded for each part.

- A1.** A doctor is involved in a trial comparing a new medication to the current standard medication. Before deciding which medication a patient receives the doctor examines the patient. Do you have any concerns with the treatment allocation for this trial? If so, explain these concerns.
- A2.** Which of the following measure of the spread is **least affected** by outliers?
(A) Standard deviation
(B) Range
(C) Inter-quartile range.
- A3.** The test scores of a class were as follows

98 81 83 98 91 92 96 74 82 79 82 62 60 89 63

Calculate the median test score for this class.

- A4.** What is the relationship between **standard deviation** and **variance**?
Why is it usually preferable to quote the standard deviation as a measure of spread?
- A5.** A companies uniform has 4 different choices for tops, 3 for bottoms and 2 hats. How many possible uniform combinations are there?
- A6.** If two events A and B are **mutually exclusive** and if $P(A) = 0.35$ and $P(B) = 0.45$, find $P(\overline{A} \cup \overline{B})$.
- A7.** Two events A and B have probabilities $P(A) = 0.8$ and $P(B) = 0.5$. If $P(A|B) = 0.4$, find $P(A \cap B)$.
- A8.** In two rolls of a fair die, what is the probability of getting at **least** one two?
- A9.** A random variable X has the following probability distribution

x	-2	0	2	4	6
$P(X = x)$	0.30	0.20	0.1	0.25	0.15

Find $E[X]$, the expected (mean) value of X .

- A10.** For a random variable X with standard deviation 4, find the variance of $3X + 8$.

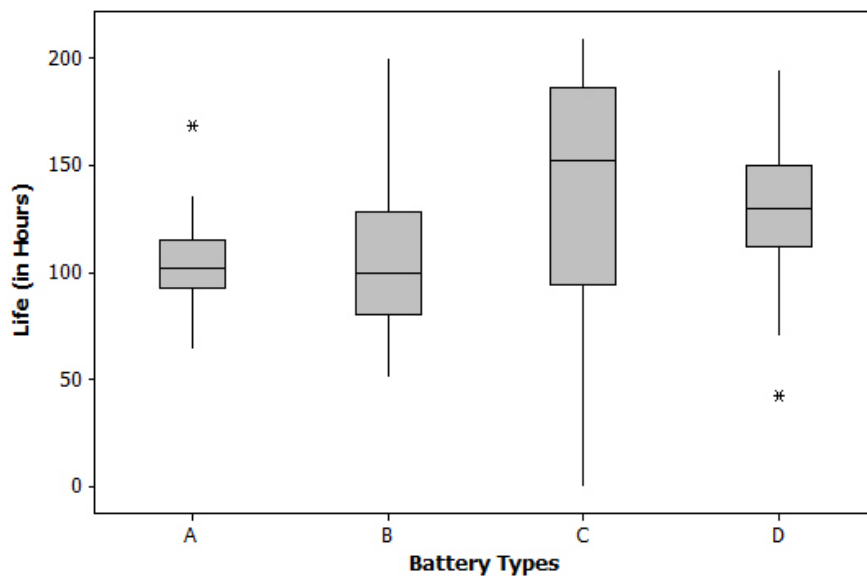
Section B — Answer 4 Questions

B1. (a) The following data are the 19 scores a business received on a review website :

68 74 67 71 72
71 64 76 72 51
75 65 59 68 59
62 69 77 72

- i) Construct a stem-and-leaf plot using intervals for the stems of width 5. (4)
- ii) Find the five number summary. (5)
- iii) Draw a boxplot for these data (you do not need to use graph paper for this, but you may do so if you wish). (3)
- iv) Calculate the sample mean and sample standard deviation. (4)

- (b) The following boxplot shows battery life (in hours) for samples from 4 different manufacturers. What can you say about the performances of the different types of battery? (4)



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- B2.** (a) i) How many different ways are there of writing down (in a distinguishable sequence) the letters in **TOMORROW** (3)
- ii) What is the probability that a random rearrangement of these letters contains the string 'TOM'? (2)
- (b) In dealing cards from a well-shuffled pack, what is the probability that
- i) the first card is a diamond? (1)
- ii) the second card is a diamond given the first card was a diamond? (2)
- iii) the second card is a diamond? (2)
- (c) In the population of over-65s in Ireland, approximately 48% are male. 40% of all over-65s are believed to have high cholesterol as women.
- i) For males the (conditional) probability of having high cholesterol is 0.375. Find the joint probability of being male and having high cholesterol. (2)
- ii) Hence, complete a table of joint and marginal probabilities as begun below. (2)

		High Cholesterol		Total
		Yes	No	
Gender	Male			0.48
	Female			
	Total	0.4		1.0

- iii) Is having high cholesterol independent of gender? (1)
- (d) An insurance company writes policies for a large number of newly-licensed drivers each year. Suppose 70% of these are low-risk drivers and 30% are high risk. The company has no way to know which group any individual driver falls in when it writes the policies. 5% of the low-risk drivers will have an at-fault accident in the next year, but 20% of the high-risk drivers will have such an accident.
- If a driver has an at-fault accident in the next year, what is the probability that he or she is high-risk? (5)

B3. (a) A coffee shop believes that 60% of its coffee orders are takeaway.

- i) If 10 customers enter the store. What is the probability
 - exactly four customers order their coffee for takeaway?
 - exactly six customers order their coffee for takeaway?
 - no more than eight customers order their coffee for takeaway?(3)
- ii) What is the probability that exactly 7 customers have their coffee in the shop? (2)
- iii) What is the probability that you have to wait until the 4th customer before a customer orders their coffee takeaway? (2)

Note: relevant tables of the binomial distribution are given at the back of this paper.

(b) The number of customers that enter the coffeshop is Poisson distributed with a mean of 10 per hour. Find the probability of the following events.

- i) there are no customers in a given hour; (2)
- ii) there are at least two customers in a given hour; (2)
- iii) there are no customers in half an hour, assuming the rate is constant over the hour. (3)

(c) Daily sales in a the coffee shop are normally distributed with a mean of €300 and a standard deviation of €20.

- i) What is the probability that on a random day the sales are
 - more than €280? (2)
 - between €260 and €320? (2)
- ii) What is the amount that is exceeded on 5% of days? (2)

B4. The joint probability mass function of two random variables X and Y is given by the following table:

		Y		
		0	2	4
X	-2	$1/20$	$3/20$	$7/20$
	0	0	$1/20$	$3/20$
	2	$2/20$	$2/20$	$1/20$

- (a) Find the marginal probability functions for X and Y . (3)
- (b) What is the conditional probability that $Y = 2$ given that $X = 2$? (2)
- (c) Calculate the mean and variance of each of X and Y . (6)
- (d) Calculate the covariance between X and Y . (3)
- (e) Find the correlation between X and Y . (2)
- (f) Are X and Y independent? Justify your answer. (2)
- (g) What is $P(X + Y > 3)$? (2)

B5. (a) Political advisors are interested in whether the electorate is happy with the government or not. They believe the probability a member of the public is satisfied with the current government is 0.45. They question 180 people.

i) What is an appropriate binomial probability model for the number of people who will be satisfied with the current government. (3)

ii) Use a normal approximation to the binomial distribution to calculate the probability that more than 70 people will be satisfied with the current government. (4)

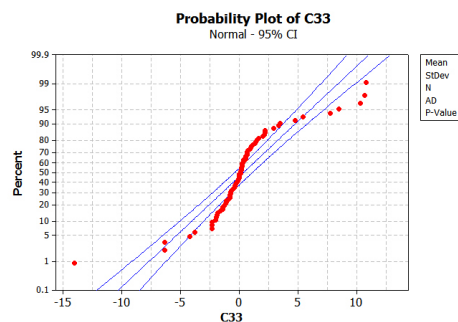
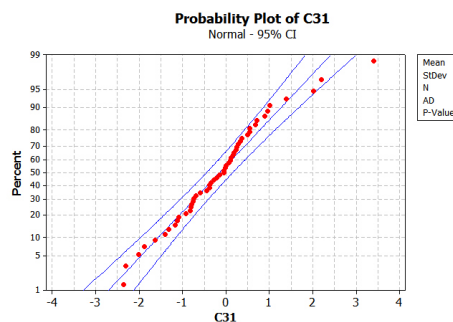
iii) What are the two assumptions of the binomial model? Are they reasonable here? (3)

(b) The normal distribution plays an important role in statistics. Write a very brief comment/answer to each of the following aspects:

i) the relationship between the mean and the median of the normal distribution; (2)

ii) the 68 – 95 – 99.7 rule; (3)

iii) the use of normal probability plots, with reference to the two examples below; (3)



iv) the Central Limit Theorem. (2)

Formulæ

- Sample Mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

- Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n-1}}$$

- Binomial Distribution

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, \dots, n$$

$$E[X] = np \quad \text{Var}(X) = np(1-p)$$

- Poisson Distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0, 1, \dots$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda$$

- Variances, Covariances and Correlation

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

$$\text{cov}[X, Y] = E[XY] - E[X]E[Y]$$

$$\text{Correlation}(X, Y) = \frac{\text{cov}[X, Y]}{\sqrt{\text{Var}(X) \text{Var}(Y)}}$$

Table C Binomial probabilities (p.4 of 6)

		Entry is $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$								
		p								
n	k	.10	.15	.20	.25	.30	.35	.40	.45	.50
9	0	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7			.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8				.0001	.0004	.0013	.0035	.0083	.0176
	9						.0001	.0003	.0008	.0020
10	0	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7		.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8			.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9					.0001	.0005	.0016	.0042	.0098
	10							.0001	.0003	.0010
12	0	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7		.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8		.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9			.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10					.0002	.0008	.0025	.0068	.0161
	11						.0001	.0003	.0010	.0029
	12								.0001	.0002
15	0	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	
	1	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005
	2	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032
	3	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139
	4	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417
	5	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916
	6	.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527
	7	.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964
	8		.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964
	9		.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527
	10			.0001	.0007	.0030	.0096	.0245	.0515	.0916
	11				.0001	.0006	.0024	.0074	.0191	.0417
	12					.0001	.0004	.0016	.0052	.0139
	13						.0001	.0003	.0010	.0032
	14								.0001	.0005
	15									

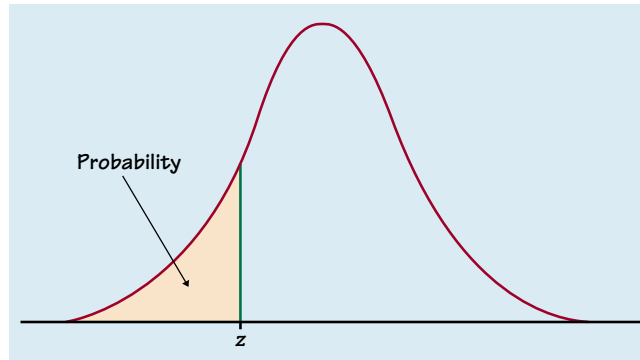


Table entry for z is the probability lying below z .

TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

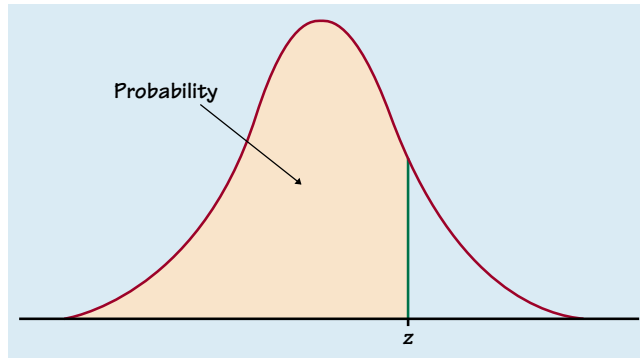


Table entry for z is the probability lying below z .

TABLE A Standard normal probabilities (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998