Assignment 3: Sensor fusion for lawn mower

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1. **Introduction**
   In this report, I have presented an implementation of the Kalman filter algorithm for localization using a wheeled robot. The goal is to estimate the robot's position and heading angle by fusing data from wheel speeds and GNSS measurements. The Kalman filter allows for optimal state estimation by considering both the system dynamics and the sensor measurements.

2. **Methodology** The localization algorithm consists of several steps:

   Data Preparation:

   - Load sensor data: The sensor data, including GNSS measurements, ground truth positions, and wheel speeds, are loaded from CSV files.
   - Preprocess wheel speeds: The wheel speeds are preprocessed to calculate the robot's heading angle and incremental changes in position.

   Wheelspeed Integration:

   - Calculate heading angle: The incremental change in the robot's heading angle is computed based on the difference between the right and left wheel speeds.
   - Integrate position coordinates: The cumulative changes in position are computed by integrating the average wheel speeds over time.

   Kalman Filter Estimation:

   - Initialize variables: Initialize the state estimate, covariance matrix, and other necessary variables.
   - Predict Step: Propagate the state estimate using the system dynamics and update the covariance matrix.
   - Update Step: Incorporate the GNSS measurements by comparing them with the estimated state and updating the state estimate and covariance matrix using the Kalman gain.

Implementation:
By utilizing the Kalman filter, it becomes possible to estimate an accurate state output that closely resembles a solution free from noise. The given model consists of two equations.

$$x_k = f(x_{k-1}, u_k) + w_k \tag{1}$$

$$z_k = h(x_k) + v_k \tag{2}$$

Equation (1), known as the state transition equation, depicts the evolution of the state over time. The current state vector at time step k, denoted as x_k, is predicted based on the previous state (x_{k-1}) using the function f(x$_{k-1}, u_k). Here,$u$_k$ represents the control input at time step k, while w_k accounts for the process noise, accommodating uncertainties or errors in the state transition process.

Equation (2), known as the measurement equation, establishes the relationship between the measurements and the true state. The observed measurements at time step k are represented by z_k, and the function h(x_k) maps the current state (x_k) to the expected measurements. Similar to before, there is a noise component, represented by v_k. This noise is referred to as the measurement noise and considers uncertainties or errors in the measurements.

For the given non linear model the states are defined as:

$$\begin{bmatrix} V \\ \phi \\ x \\ y \end{bmatrix}$$

The x, y and values are obtained by integrating these wheel speeds:

$$x(t) = x_0 + \int_{t_0}^{t} \frac{v_L(t) + v_R(t)}{2} cos(\phi(t))\, dt$$
$$y(t) = y_0 + \int_{t_0}^{t} \frac{v_L(t) + v_R(t)}{2} sin(\phi(t))\, dt$$
$$\phi(t) = \phi_0 + \int_{t_0}^{t} \frac{v_R(t) - v_L(t)}{2R}\, dt$$

The next state can be predicted using the following equations:

$$\hat{x}_k = f(\hat{x}_{k-1}, u_k) \tag{3}$$

$$P_k = F_{k-1} P_{k-1} F_{k-1}^T + Q_{k-1} \tag{4}$$

Both Eq. (3) and Eq. (4) serve the purpose of predicting the next state. Eq. (4) specifically predicts the next covariance, where P_k represents the predicted covariance. F_{k-1} denotes the current state transition matrix at time step k-1, and P_{k-1} is the covariance matrix of the current state used for the prediction. Additionally, a process noise component, Q_{k-1} is incorporated. F_{k-1} is expressed using the Jacobian, while Q_{k-1} is the process noise determined through trial and error.

After predicting the next state and the next covariance, the next step is to update the results using the following equations:

$$G_k = P_k H_k^T (H_k P_k H_k^T + R)^{-1} \tag{5}$$

$$x_k \leftarrow \hat{x}_k + G_k(z_k - h(\hat{x}_k)) \tag{6}$$

$$Pk \leftarrow (I - G_k H_k) P_k \tag{7}$$

Equation (5) is utilized to calculate the Kalman gain, where H_k denotes the measurement matrix and R represents the covariance matrix of the measurement noise. The Kalman gain,G_k , determines the extent to which the predicted state should be adjusted based on the discrepancy between the predicted measurement and the observed measurement.

Once the Kalman gain is obtained, the predicted state estimate and covariance matrix are updated using Equation (6) and Equation (7), respectively. H_k is represented using the Jacobian, while the matrix R is determined through trial and error.

The process of prediction and updating is performed iteratively across all time steps, allowing the Kalman filter to estimate the true state by considering the observed measurements.

Error Analysis:

- Calculate Errors: Compute the errors between the estimated values and the ground truth positions and heading angles.
- Calculate RMS Errors: Calculate the root mean square (RMS) errors over time to assess the accuracy of the localization.

Plotting and Visualization:

- Plot Trajectories: Visualize the ground truth positions, GNSS measurements, wheel speed integration results, and Kalman filter estimations on a 2D plot.
- Plot Heading Angle: Plot the ground truth heading angle, wheel speed integration results, and Kalman filter estimations over time.
- Plot RMS Errors: Visualize the RMS errors in X-coordinate, Y-coordinate, and heading angle over time.

3. **Discussion** The localization algorithm based on the Kalman filter accurately estimates the robot's position and heading angle. The Kalman filter effectively fuses the information from wheel speeds and GNSS measurements to provide accurate estimates of the robot's position and heading angle. The plotted trajectories show that the Kalman filter estimations closely match the ground truth positions, outperforming the wheel speed integration method.

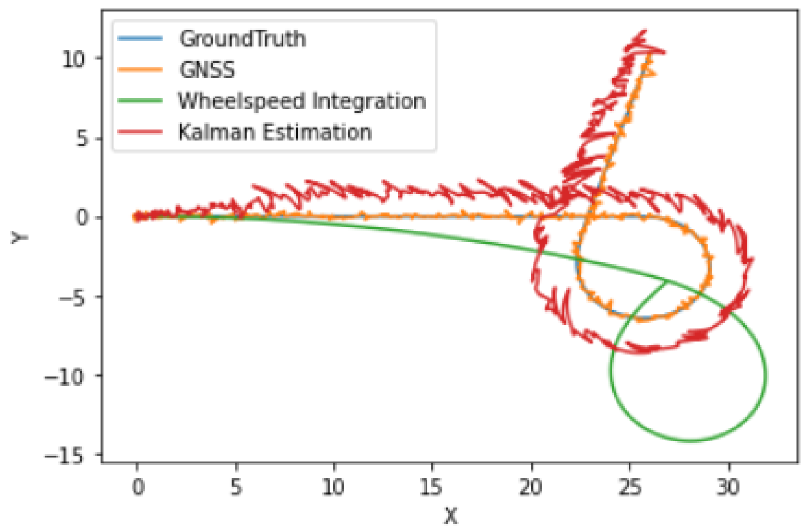

Figure 1: Plot showing the ground truth, the GNSS detections, and the estimated path over the full run

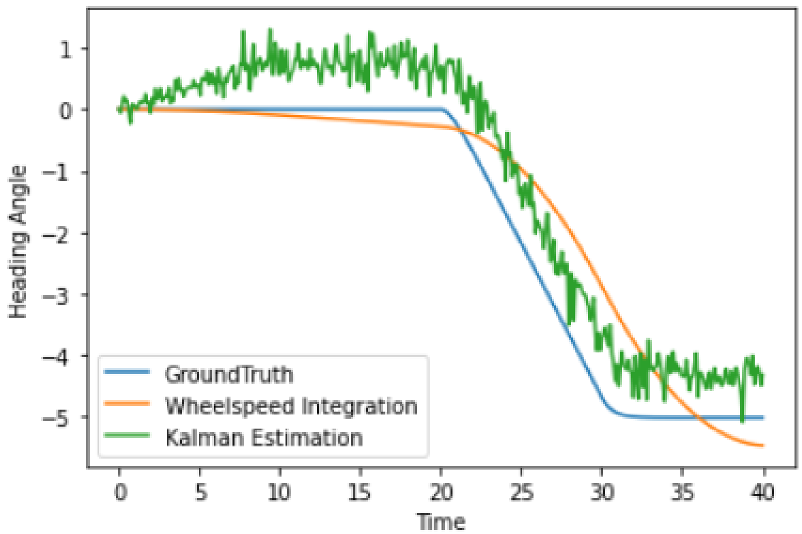

Figure 2: Plot showing the ground truth, the GNSS detections, and the estimated ϕ over the full run

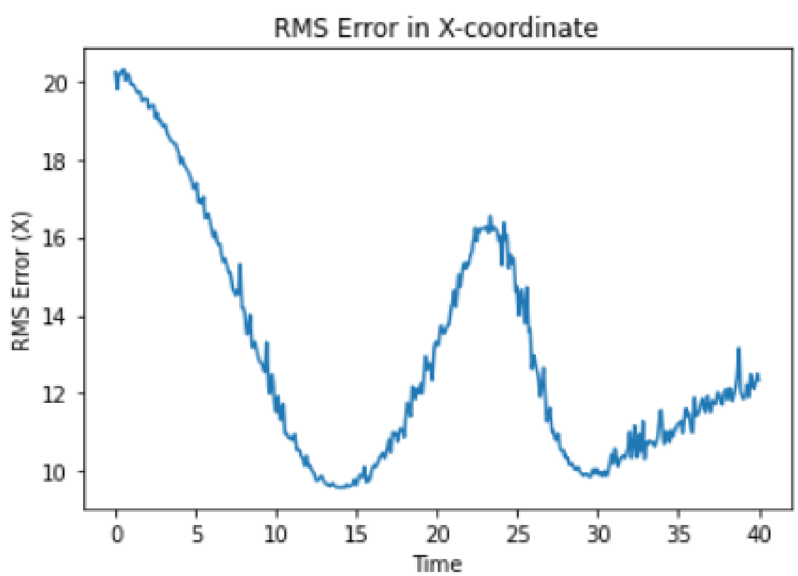

Figure 3: RMS Error in X coordinate

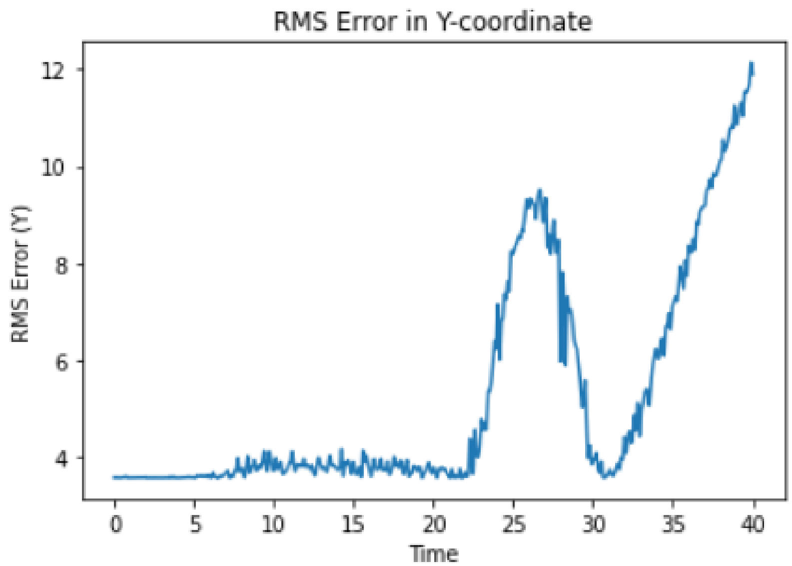

Figure 4: RMS Error in Y coordinate

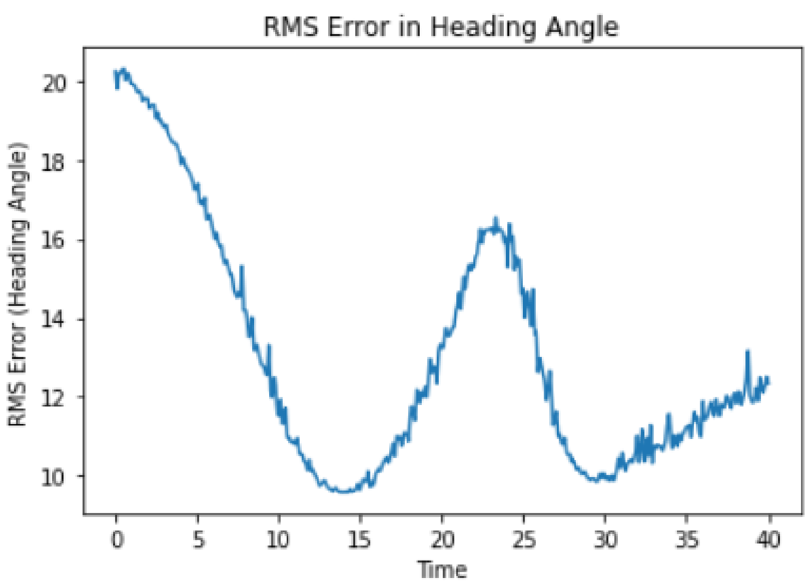

Figure 5: RMS Error in heading angle