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Constraint programming and applied optimization (EEN025)

Assignment 1

Group 10

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We hereby declare that we have both actively participated in solving every exercise. All solutions are entirely our own work, without having taken part in other solutions.

Task 1

Given data:

Plants	Mirafiori	Tychy	Melfi	Cassino
Model	Panda	500	Musa	Giulia
Tax (%)**	30	15	20	30
Price (thousands Kr)	106	136	150	427
Material Cost (%)*	57	60	55	45
Salary (Kr/month)	20000	11000	20000	26000
Manhour (h)	40	45	38	100
Min-Prod.	120000	100000	80000	15000

It is needed to schedule the production for the coming year such that the net income for the company is maximized.

Let x, y, z, and w represent the cars - Panda, 500, Musa, and Giulia produced in Mirafori, Tychy, Melfi, and Cassino.

We have been given the selling price of the cars, however, to calculate the cost price of the car we need to use the material cost, salary, and man-hour information from the table.

The cost price is calculated as follows:

$$\text{Cost Price} = \text{Material Cost} + \text{Man-hour Cost}$$

where,

$$\text{Material Cost} = \frac{\text{Selling Price} \cdot \text{Material Cost}(\%)}{100}$$

and,

$$\text{Man-hour Cost} = \frac{\text{Salary (kr/month)} \cdot \text{Man-hour (h)}}{160}$$

Then we calculate the net income:

$$\text{Profit} = \text{Selling Price} - \text{Cost Price}$$

$$\text{Net Income} = \text{Profit} \cdot (1 - \text{Tax Rate}(\%))$$

Using the given information the constraints are:

Constraint 1:

$$x + z \leq 300,000$$

Constraint 2:

$$x \geq 120,000$$

Constraint 3:

$$y \geq 100,000$$

Constraint 4:

$$z \geq 80,000$$

Constraint 5:

$$w \geq 15,000$$

Constraint 6:

$$\text{Cost Price(Panda)}\, x + \text{Cost Price(Tychy)}\, y + \text{Cost Price(Melfi)}\, z + \text{Cost Price(Cassino)}\, w \leq 40,000,000,000$$

Objective Function:

$$\max(\text{Net Income(Panda)}\, x + \text{Net Income(Tychy)}\, y + \text{Net Income(Melfi)}\, z + \text{Net Income(Cassino)}\, w)$$

Results obtained:

In order to achieve the maximum revenue:
120000 units of *Panda* should be produced in *Mirafori*
100000 units of *500* should be produced in *Tychy*
80001 units of *Musa* should be produced in *Melfi*
80135 units of *Giulia* should be produced in *Cassino*

Task 2

Given data:

	Panda			500			Musa			Giulia		
	Dl	Price	Dm	Dl	Price	Dm	Dl	Price	Dm	Dl	Price	Dm
Poland	800	86k	75k	/	92k	20k	12k	100k	10k	/	/	/
Italy	/	106k	35k	1000	136k	40k	/	150k	80k	/	427k	8k
US	3500	150k	40k	2800	170k	50k	/	/	/	5000	550k	3k
Sweden	2000	112k	2k	1600	150k	5k	2200	170k	1k	2500	500k	1k

Let q_{nv} represent the country (nation) n and the quantity needed of car (vehicle) v in that country.

therefore,

$q_{11} \geq 75k$, $q_{12} \geq 20k$, $q_{13} \geq 10k$, $q_{14} = 0$;
$q_{21} \geq 35k$, $q_{22} \geq 40k$, $q_{23} \geq 80k$, $q_{24} \geq 8k$;
$q_{31} \geq 40k$, $q_{32} \geq 50k$, $q_{33} = 0$, $q_{34} \geq 3k$;
$q_{41} \geq 2k$, $q_{42} \geq 5k$, $q_{43} \geq 1k$, $q_{44} \geq 1k$;

Since we have a constraint on the cost of production, therefore,

Constraint:

$$\sum_{n=1}^{4}\sum_{v=1}^{4}(C_{nv}) \leq 4 \times 10^6 \text{ k sek}$$

such that,

$$Cv = [Mv + \frac{Pv}{160} \cdot Hv + Dv]$$

where,
C_v = Cost of vehicle v to the company
M_v = Material cost of vehicle v
P_v= Salary (Payment) to employee to produce vehicle v
H_v= Man-hour required to produce vehicle v
D_{nv} = Delivery cost of vehicle v in country (nation) n

Objective Function:

$$\max\left(Z = \sum_{n=1}^{4}\sum_{v=1}^{4}(S_{nv} - C_v)\cdot(1 - T_v - E_{nv})\right)$$

such that,

$$Cv = [Mv + \frac{Pv}{160} \cdot Hv + Dv]$$

where,
S_{nv} = Selling price of vehicle v in nation n
C_v= Cost price of vehicle v
T_v = Tax on vehicle v
E_{nv} = Export Tax on vehicle v in nation n

Results obtained:

Table 1:

Variable	Quantity (in thousands)	Description
q_{11}	75	75000 cars should be produced of model *Panda* for *Poland*
q_{12}	30	30000 cars should be produced of model *500* for *Poland*
q_{13}	10	10000 cars should be produced of model *Musa* for *Poland*
q_{14}	0	
q_{21}	35	35000 cars should be produced of model *Panda* for *Italy*
q_{22}	40	40000 cars should be produced of model *500* for *Italy*
q_{23}	80	80000 cars should be produced of model *Musa* for *Italy*
q_{24}	8	8000 cars should be produced of model *Giulia* for *Italy*
q_{31}	40	40000 cars should be produced of model *Panda* for *US*
q_{32}	50	50000 cars should be produced of model *500* for *US*
q_{33}	0	
q_{34}	3	3000 cars should be produced of model *Giulia* for *US*
q_{41}	2	2000 cars should be produced of model *Panda* for *Sweden*
q_{42}	5	5000 cars should be produced of model *500* for *Sweden*
q_{43}	1	1000 cars should be produced of model *Musa* for *Sweden*
q_{44}	1	1000 cars should be produced of model *Giulia* for *Sweden*

Task 3

Given data:

Line	Capacity (hours)	Production Rate (units/hour)		
		Component 1	Component 2	Component 3
1	100	10	15	5
2	150	15	10	5
3	80	20	5	10
4	200	10	15	20

Let,

a, b, c represent the number of hours needed to make components 1, 2, and 3 respectively on line 1

d, e, f represent the number of hours needed to make components 1, 2, and 3 respectively on line 2

u, v, w represent the number of hours needed to make components 1, 2, and 3 respectively on line 3

x, y, z represent the number of hours needed to make components 1, 2, and 3 respectively on line 4

It is needed to maximize the production of foam, therefore we need to have all three components in equal quantity.

Objective Function:

$$\max(10a + 15b + 5c + 15d + 10e + 5f + 20u + 5v + 10w + 10x + 15y + 20z)$$

subject to the constraints:

Constraint 1:

$$a + b + c \leq 100$$

Constraint 2:

$$d + e + f \leq 150$$

Constraint 3:

$$u + v + w \leq 80$$

Constraint 4:

$$x + y + z \leq 200$$

Constraint 5:

$$10a + 15d + 20u + 10x \geq 0$$

Constraint 6:

$$15b + 10e + 5v + 15y \geq 0$$

Constraint 7:

$$5c + 5f + 10w + 20z \geq 0$$

Constraint 8:

$$10a + 15d + 20u + 10x - 15b - 10e - 5v - 15y = 0$$

Constraint 9:

$$10a + 15d + 20u + 10x - 5c - 5f - 10w - 20z = 0$$

Results obtained:

Table 2:

Variable	Duration (hours)
a	0
b	100
c	0
d	88
e	61
f	0
u	80
v	0
w	0
x	0
y	54
z	146

The result of the objective function on maximization is 8760 which denotes the max. quantity of foam that should be produced.

The line 1 should run for 100 hours and produce only component 2,
The line 2 should run for 88 and 61 hours to produce components 1 and 2,
The line 3 should run for 80 hours and produce only component 1,
The line 4 should run for 54 and 146 hours to produce components 2 and 3.

Task 4

Let,

x_i represents a binary decision variable such that:

$$x_i = \begin{cases} 1, & \text{if center } i \text{ is opened,} \\ 0, & \text{otherwise,} \end{cases}$$

W denotes the revenue from the previous year,
K signifies the maximum allowable number of centers,
I represents the set of potential locations for new centers,
F_i stands for the fixed cost associated with the opening of center i,
C_i denotes the variable cost linked to center i,
L_i represents the minimum size requirement for center i,
U_i indicates the maximum size capacity for center i,
R_i signifies the expected revenue generated by center i,
which is contingent on its size.
s_i denote the size of center i.

Constraints

Constraint 1

The size of center i is restricted by whether the center is opened or not, which means, The size of center i is greater than zero if the center i is open.

$$L_i \cdot x_i \leq s_i \leq U_i \cdot x_i, \quad \forall i \in I$$

Constraint 2

The total number of open centers should not exceed the maximum allowed:

$$\sum_{i=1}^{I} x_i \leq K$$

Constraint 3

The cost of opening the centers should not exceed W:

$$\sum_{i=1}^{I} (F_i \cdot x_i) + \sum_{i=1}^{I} (C_i \cdot s_i \cdot x_i) \leq W$$

Objective Function

The objective is to maximize revenue:

$$\max \left(\sum_{i=1}^{I} (R_i \cdot s_i \cdot x_i) \right)$$

Task 5

Let x_1, x_2, x_3, x_4 and x_5 represent the percentage of total capital invested in funds A, B, C, D, and E.

Table 3:

Name	Duration 2	Revenue(%)	Revenue(billion SEK)	Risk
A	9	4.5	$R_a = [1 + \frac{(0.045 \times 9)}{12}] \times 1 \times 10^9 \times x1$	2
B	15	5.4	$R_b = [1 + \frac{(0.054 \times 15)}{12}] \times 1 \times 10^9 \times x2$	3
C	4	5.1	$R_c = [1 + \frac{(0.051 \times 4)}{12}] \times 1 \times 10^9 \times x3$	1
D	3	4.4	$R_d = [1 + \frac{(0.044 \times 3)}{12}] \times 1 \times 10^9 \times x4$	4
E	2	6.1	$R_e = [1 + \frac{(0.061 \times 2)}{12}] \times 1 \times 10^9 \times x5$	5

The aim is to maximize the revenue using the objective function:

Objective Function:

$$\max(R_a \cdot x1 + R_b \cdot x2 + R_c \cdot x3 \cdot C + R_d \cdot x4 \cdot D + R_e \cdot x5 \cdot E)$$

where, the values R_a, R_b, R_c, R_d, R_e are as shown in Table (3)

The objective function is subjected to the following constraints:

Constraint 1:

$$x_2 + x_3 + x_4 \geq 0.4$$

Constraint 2:

$$2x_1 + 3x_2 + x_3 + 4x_4 + 5x_5 \leq 7.5$$

Constraint 3:

$$9x_1 + 15x_2 + 4x_3 + 3x_4 + 2x_5 \leq 300$$

Constraint 4:

$$C + D \leq 1$$

Constraint 5:

$$1.0 + (x1 - \frac{1.0}{1000.0}) \geq 1.0 \cdot E$$

Constraint 6:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

Results obtained:

Table 4:

Variable	Value
x_1	1.0
x_2	0.0
x_3	1.0
x_4	0.0
x_5	0.0
C	1.0
D	0.0
E	0.0