Volatility Shocks in Networks

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Abstract

Existing studies on uncertainty shocks focus on economy-wide shocks that affect all sectors symmetrically and simultaneously. However, as highlighted by the recent COVID-19 pandemic, a rise in uncertainty can exhibit substantial heterogeneity across sectors, with some experiencing significantly larger increases than others. In this paper, I study how these sector-specific volatility shocks propagate and shape aggregate outcomes. First, using industry-level data, I estimate sectoral TFP processes allowing for stochastic volatility. Sectoral TFP displays nontrivial fluctuations in volatility even after controlling for economy-wide variations. During recessions, the number of sectors with heightened volatility rises sharply. Second, I simulate the impact of sector-specific volatility shocks in a calibrated multi-sector New Keynesian model with input-output networks. I find that sectoral volatility shocks generate contractions in aggregate economic activity. The *network precautionary pricing multiplier* emerges as a key transmission mechanism: input-output networks amplify and propagate firms' motive to preemptively raise prices in response to heightened uncertainty.

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1 Introduction

Business analysts and policymakers have emphasized fluctuations in uncertainty as a significant contributor to aggregate fluctuations. In most studies, fluctuations in uncertainty are treated as economy-wide shocks—changes in aggregate risk, firm-level volatility, or policy uncertainty that affect all sectors symmetrically and simultaneously. In reality, however, a significant rise in uncertainty often appears to be concentrated in several specific sectors. For example, during the Great Recession, a substantial increase in risk seems to have originated from financial industries. The COVID-19 pandemic has significantly raised uncertainty surrounding the tourism and recreation industries due to lockdowns and people altering their behavior. Motivated by these observations, in this paper, I relax the conventional assumption of economy-wide uncertainty shocks and study the aggregate implications of the sector-specific volatility shocks using industry-level data and a multi-sector business cycle model.

Sector-Specific Volatility Shocks. In the first part of the paper, I construct a data set for quarterly U.S. sectoral total-factor productivity (TFP) and use it to estimate parameters and historical realizations of aggregate and sector-specific TFP volatility shocks. The focus on sectoral TFP volatility is motivated by prior research, such as Acemoglu et al. (2012), which shows that first-moment sector-specific TFP shocks can account for significant aggregate fluctuations in a network economy. I interpret an unexpected rise in volatility in these TFP innovations as a TFP uncertainty shock that represents an increase in risk. Fluctuations in TFP volatility are allowed to be sector-specific, meaning that a change in uncertainty in one sector could be distinct from changes in uncertainty in other sectors or at the economy-wide level.

Based on the estimated sectoral TFP volatility process, I provide four key facts about sector-specific volatility shocks. First, I find that sector-specific TFP, which controls for economy-wide TFP variations, exhibits sizable fluctuations in volatility. Second, sectoral uncertainty is countercyclical at the macro level: the number of sectors experiencing increased TFP volatility surges during recessions. For example, in 2009:Q1 (the Great Recession) and 2020:Q2 (the COVID-19 recession), 22 and 30 out of 66 sectors, respectively, experienced innovations in TFP volatility exceeding one standard deviation. This is significantly higher than during expansions, where on average only two sectors per quarter experience such volatility shocks. Third, despite the overall countercyclicality, the identity of the sectors experiencing spikes in TFP volatility displays substantial heterogeneity. Moreover, the specific sectors affected map onto the historical narratives of recessions. For instance, sectors such as "Federal Reserve banks, credit intermediation, and related activities" and "Management of companies and enterprises" were hit with the largest innovation to TFP volatility during the Great Recession. In contrast, during the COVID-19 recession, sectors like "Air transportation" and "Amusements, gambling, and recreation industries" saw the most significant increases in volatility. Fourth, sectoral uncertainty is stagflationary at the local level. Specifically, I estimate local projections (Jordá (2005)) for a panel of sectors and show that innovations to sectoral TFP volatility reduce the sector's own gross output and raise the price level.

Network Precautionary Pricing Multiplier. Motivated by the empirical findings, I consider a multi-sector New Keynesian model similar to Bouakez et al. (2009) and Pasten et al. (2020) to study the macroeconomic implications of sector-specific TFP volatility shocks. The model features nominal rigidities and input–output networks, both of which are critical ingredients. First, previous research, such as Fernández-Villaverde et al. (2015) and Basu and Bundick (2017), has found that nominal rigidities propagate and amplify the impact of uncertainty shocks. Thus, it is of interest to examine whether a similar transmission mechanism occurs in the case of sector-specific volatility shocks as well. Second, it is important to consider input–output networks because they may transmit the effects of sectoral volatility shocks across sectors.

I first theoretically analyze the model to gain analytical insights. In the model, nominal rigidities play a crucial role: an increase in uncertainty, such as from sectoral TFP volatility shocks, induces firms to price their goods higher than they would in the absence of an increase in uncertainty. The intuition behind this nominal pricing bias follows from the firm's self-insurance behavior (Fernández-Villaverde et al. (2015)). When uncertainty rises, firms hedge against the risk of pricing their goods too low relative to marginal cost by raising prices. This occurs because the profit loss from setting prices too high (and selling fewer) is smaller than the loss from setting prices too low (and selling more). Consequently, sectoral volatility shocks exhibit stagflationary effects, leading to higher prices and, under sufficiently accommodative monetary policy, lower aggregate GDP. I explicitly characterize the equilibrium elasticities of sectoral volatility shocks on sectoral prices and aggregate GDP. These elasticities depend on the convolution of nominal rigidity, sectoral sizes, steady-state sectoral volatility, and the positions of shocked sectors within the input-output network. I identify a novel theoretical mechanism: through input-output linkages, the *network precautionary pricing multiplier* amplifies and propagates firms' motive to preemptively raise prices in response to heightened uncertainty.

Quantitative Evaluation. To quantitatively assess the aggregate impact of sectoral volatility shocks, I consider a 66-sectors version of the model and calibrate it to U.S. sectoral and macro facts. To solve the model, I use the risk-adjusted log-linearization method as in Jermann (1998), Uhlig (2010), Dew-Becker (2012), and Malkhozov (2014). The advantage of this method is that it captures the effects of sectoral volatility shocks through the risk-adjustment terms while preserving tractability. As emphasized in Bianchi et al. (2023), the method has two additional advantages. First, it allows for an explicit decomposition of the propagation mechanism of volatility shocks into several distinct endogenous risk wedges. Second, the method produces a good approximation to the model solution.

I analyze the impulse response functions (IRFs) to aggregate and sector-specific TFP volatility shocks. Both types of shocks lead to a contraction in aggregate GDP, consumption, investment, and hours worked, accompanied by an increase in inflation and nominal interest rates. The contraction in economic activity resulting from sector-specific volatility shocks is comparable in magnitude to that caused by policy uncertainty shocks simulated in structural models (Mumtaz and Zanetti (2013), Born and Pfeifer (2014), Fernández-Villaverde et al. (2015)). Notably, the response to sector-specific TFP volatility shocks is significantly larger than the response to aggregate volatility shocks, with the cumulative GDP impact being five times greater for the former. By isolating endogenous risk wedges, I show that the nominal pricing bias channel identified in the analytical model is the primary driver of the IRF to sector-specific TFP volatility shocks. Furthermore, the network precautionary pricing multiplier is crucial in amplifying this channel: without input-output linkages, the IRF to sector-specific volatility shocks becomes negligible. The input-output linkages also play a key role in determining the impact of sector identity on the aggregate effects of sectoral volatility shocks. For example, under the baseline model with input-output linkages, the top five most responsive sectors to sector-specific shocks are "Securities, commodity contracts, and investments", "Oil and gas extraction", "Computer and electronic products", "Housing", "Other real estate". When input-output linkages are removed, the ranking changes dramatically, with sectors such as "Insurance carriers and related activities", "Apparel and leather and allied products", "Motor vehicle and parts dealers", "Ambulatory health care services", entering the top five, leaving only "Securities, commodity contracts, and investments" common across both cases.

Literature. This paper is related to two strands of the literature. First, the paper contributes to the expansive literature on uncertainty shocks, such as Bloom (2009), Fernández-Villaverde et al. (2011), Fernández-Villaverde et al. (2015), Bachmann and Bayer (2013), Born and Pfeifer (2014), Christiano et al. (2014), Basu and Bundick (2017), and Bloom et al. (2018). While these studies focus on economy-wide volatility, the empirical results in this paper show that sector-specific spikes in uncertainty occur regularly and can be substantial. Thus, I consider a network economy, which allows me to study these sector-specific volatility shocks and their propagation rather than assuming a single, aggregate source of risk.

Second, this paper belongs to the fast-growing research agenda that studies how microeconomic shocks drive aggregate fluctuations, such as Gabaix (2011), Foerster et al. (2011), and Acemoglu et al. (2012). In particular, since this paper studies how price stickiness critically affects the macroeconomic effects of sectoral volatility shocks, it is closely related to Pasten et al. (2024) and Bouakez et al. (2023), which examine the transmission of sector-specific shocks in the presence of nominal rigidities and input-output linkages. The paper is also related to Baqaee and Farhi (2019), who explore how the aggregate implications of sectoral productivity shocks in an efficient, non-linear environment depend on micro-level details. I focus on a particular form of non-linearity, namely, sectoral volatility shocks, in an inefficient economy. Two recent papers examine how uncertainty matters for macroeconomic outcome in a network economy. Kopytov et al. (2024) consider the impact of an aggregate change in uncertainty in a real economy with endogenous network formation. Nikolakoudis (2025) studies how time-varying uncertainty affects the transmission of first-moment sectoral shocks under incomplete information. In contrast to these two papers, my paper analyzes the impact of a sector-specific change in uncertainty in a complete information environment with nominal frictions.

Outline. The rest of the paper is organized as follows. Section 2 estimates sectoral TFP processes with stochastic volatility and explores their key properties. Section 3 introduces the quantitative model and Section 4 analyzes a simplified version to derive analytical insights. Section 5 describes the solution methodology and calibration strategy. The main results are presented in Section 6, and Section 7 concludes.

2 Sector-Specific Volatility Shocks: Measurement and Facts

In this section, I construct a data set for quarterly U.S. sectoral productivity and use it to estimate the parameters of the stochastic volatility processes and historical realizations of sectoral volatility shocks. Based on these estimation results, I then provide four stylized facts regarding sectoral volatility shocks.

In the baseline empirical analysis, the sample period is 2006:Q2 to 2022:Q1,[1] which encompasses two recessions: the Great Recession (2008:Q1–2009:Q2) and the COVID-19 recession (2020:Q1–2020:Q2). I consider 66 sectors for both the empirical analysis and the calibration exercise below, roughly corresponding to the 3-digit NAICS level (excluding government industries). Tables A.1 in the Appendix list these 66 sectors.

2.1 Measuring Sector-Specific Volatility Shocks

Sectoral TFP Data. The two main challenges in measuring time-varying TFP volatility at the industry level are: (i) the measurement of sectoral productivity needs to take into account the contribution of intermediate inputs to sectoral output and (ii) the data should be measured at least in a quarterly frequency to mitigate the time-aggregation bias (Bloom (2009)). The integrated industry-level production account (KLEMS) by the BEA provides industry-level productivity series that controls for intermediate inputs but is measured in an annual frequency so it is not suitable for my purpose. Instead, the approach I take in this paper is to leverage the optimality conditions implied by the firms' cost minimization problem to control for the impact of intermediate inputs.

First, to measure sectoral TFP, I consider the following production function:

$$y_{i,t} = e^{z_{i,t}}(k_{i,t-1})^{\alpha_i^k}(h_{i,t})^{\alpha_i^h}(\widetilde{m}_{i,t})^{1-\alpha_i^k-\alpha_i^h},$$

where $y_{i,t}$ is the gross output of sector i at period t, $z_{i,t}$ is the TFP level, $k_{i,t-1}$ is the capital stock, $h_{i,t}$ is hours worked, and $\widetilde{m}_{i,t}$ is the combined intermediate inputs,

$$\widetilde{m}_{i,t} = \left[\sum_{i=1}^{n} a_{ij}^{\frac{1}{\varepsilon_m}}(m_{ij,t})^{\frac{\varepsilon_m-1}{\varepsilon_m}}\right]^{\frac{\varepsilon_m}{\varepsilon_m-1}}, \tag{1}$$

where $m_{ij,t}$ is sector j goods used as intermediate inputs and $\varepsilon_m > 0$ controls the elasticity of

[1] The starting point is dictated by the availability of quarterly aggregate hours worked data (for all employees) at the sectoral level.

substitution. The production function exhibits constant returns to scale:

$$\sum_{j=1}^{n} a_{ij} = 1, \tag{2}$$

where $\alpha_i^k > 0$, $\alpha_i^h > 0$, $a_{ij} \geq 0$ for all j. TFP is then measured using the standard accounting method:

$$z_{i,t} = \ln y_{i,t} - \alpha_i^k \ln k_{i,t-1} - \alpha_i^h \ln h_{i,t} - (1 - \alpha_i^k - \alpha_i^h) \ln \widetilde{m}_{i,t}.$$

To measure $y_{i,t}$, I use the quarterly sectoral real gross output series by the BEA. I measure sectoral capital stock using the chain-type quantity indexes for net stock of private fixed assets by industry, provided by the BEA. Since the series are available only at an anuual frequency, I interpolate them into a quarterly frequency by appplying cubic spline interpolation.[2] I measure sectoral hours worked, $h_{i,t}$, using the industry-level quarterly aggregate hours worked series of all employees, provided by the BLS.

I estimate the values of factor shares $\{\alpha_i^k\}_{i=1}^n$, $\{\alpha_i^h\}_{i=1}^n$, and the input-output matrix $\{a_{ij}\}_{i,j=1}^n$ from the "use tables" of the input-output accounts constructed by the BEA. The use table shows the uses of commodities by sectors as intermediate inputs and their final uses. It also shows the sector's value added components, which is the labor income (compensation to employees) and capital income (gross operating surplus). The sum of the value of intermediate inputs and value added, given by a column sum of the table, is the industry's gross output. Thus, to compute the intermediate shares a_{ij}, I compute the value of payments from sector i to sector j divided by sector i's gross output. To obtain α_i^k and α_i^h, I compute the ratios of capital and labor income to gross output, respectively. Finally, I take the averages of these objects ($\{\alpha_i^k\}_{i=1}^n$, $\{\alpha_i^h\}_{i=1}^n$, and $\{a_{ij}\}_{i,j=1}^n$) calculated year by year from the use table from 2006 to 2020.

Finally, I explain how I measure the sectoral use of intermediate inputs, $\widetilde{m}_{i,t}$. BEA provides data for quarterly nominal expenditures on total intermediate inputs for each sector, $\sum_{j=1}^n P_{j,t} m_{ij,t}$, where $P_{j,t}$ is the nominal price for goods j. I first convert them into real terms by dividing them by the GDP deflator P_t so now I have total real expenditures on inputs, $\sum_{j=1}^n p_{j,t} m_{ij,t}$, where $p_{j,t} \equiv P_{j,t}/P_t$ is the real price for goods j. Next, the optimality condition for the firms' cost minimization problem given the CES aggregator for intermediate inputs (1) imply

$$\sum_{j=1}^{n} p_{j,t} m_{ij,t} = p_{i,t}^m \widetilde{m}_{i,t},$$

where

$$p_{i,t}^m \equiv \left[\sum_{j=1}^{n} a_{ij} p_{j,t}^{1-\varepsilon_m} \right]^{\frac{1}{1-\varepsilon_m}}, \tag{3}$$

[2] The advantage of cubic spline interpolation compared to linear interpolation is that the former produces smoother and better-behaved series relative to the latter.

which can be computed given $\{a_{ij}\}_{i,j=1}^{n}$ and the data on $p_{j,t}$. I set $\varepsilon_m = 0.1$ following the estimation result by Atalay (2017). Then we can compute the intemediate input use as

$$\ln \widetilde{m}_{i,t} = \ln \left(\sum_{j=1}^{n} p_{j,t} m_{ij,t} \right) - \ln p_{i,t}^{m}.$$

Stochastic Volatility Process. Following the approach by Fernández-Villaverde et al. (2011), Born and Pfeifer (2014), and Fernández-Villaverde et al. (2015), I model uncertainty shocks as an innovation to an AR(1) stochastic volatility process. I assume that sectoral TFP can be decomposed into an aggregate, economy-wide component $\bar{z}_t$ and a sector-specific component $u_{i,t}$:

$$z_{i,t} = \bar{z}_t + u_{i,t}, \tag{4}$$

where the aggregate component $\bar{z}_t$ is the weighted cross-sectional average of sectoral productivity, $\bar{z}_t \equiv \sum_{i=1}^{n} \left(\frac{GDP_{i,t}}{GDP_t} \right) z_{i,t}$, where $GDP_{i,t}$ is sectoral GDP (i.e. sectoral value-added) and GDP_t is aggergate GDP. $\bar{z}_t$ and sector-specific TFP component $u_{i,t}$ follow independent AR(1) processes with stochastic volatility:

$$\bar{z}_t = a + \tau_1 t + \tau_2 t^2 + \rho \bar{z}_{t-1} + e^{\sigma_{t-1}} \varepsilon_t, \quad \varepsilon_t \sim N(0,1), \tag{5}$$

$$u_{i,t} = a_i + \tau_{1,i} t + \tau_{2,i} t^2 + \rho_i u_{i,t-1} + e^{\sigma_{i,t-1}} \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(0,1), \quad i = 1, \ldots, n, \tag{6}$$

where a and a_i control the unconditional mean of the TFP process and τ_1, τ_2 and $\tau_{1,i}, \tau_{2,i}$ control the linear-quadratic trends. I allow for sector-specific trend terms in order to capture industry-level differences in long-term TFP growth. Following Bloom et al. (2018), the timing in (5) and (6) reflects the assumption that firms know in advance the volatility of the shocks next period. In turn, the (log of) standard deviations σ_t and $\sigma_{i,t}$ follow AR(1) processes with

$$\sigma_t = (1 - \rho_\sigma)\sigma + \rho_\sigma \sigma_{t-1} + \eta \varepsilon_{\sigma,t}, \quad \varepsilon_{\sigma,t} \sim N(0,1), \tag{7}$$

$$\sigma_{i,t} = (1 - \rho_{\sigma_i})\sigma_i + \rho_{\sigma_i} \sigma_{i,t-1} + \eta_i \varepsilon_{\sigma_i,t}, \quad \varepsilon_{\sigma_i,t} \sim N(0,1), \quad i = 1, \ldots, n, \tag{8}$$

where σ and $\sigma_i, i = 1, \ldots, n$, are the unconditional means of the standard deviations of the TFP level shocks. $\eta \varepsilon_{\sigma,t}$ is the aggregate TFP volatility shock and $\eta_i \varepsilon_{\sigma_i,t}$ are the sectoral TFP volatility shocks, where η and $\eta_i, i = 1, \ldots, n$, are the standard deviations of the TFP volatility shocks. I do not impose contemporaneous correlations between a first-moment shock and a volatility shock (between ε_t and $\varepsilon_{\sigma,t}$ or between $\varepsilon_{i,t}$ and $\varepsilon_{\sigma_i,t}$). This assumption allows me to study the independent effects of sectoral volatility shocks $\varepsilon_{\sigma_i,t}$ transparently without the added impact of first-moment shocks through correlations. Since I study model dynamics under decision rules that are conditionally linear in sectoral volatility (as I describe below in Section 5.1), the combined effect of volatility shocks across several sectors can be easily simulated by simultaneously feeding volatility

Table 1: Summary of posterior means

A. Aggregate TFP	ρ	σ	ρ_σ	η
	0.51	-6.56	0.63	0.50
B. Sector-specific TFP	ρ_i	σ_i	ρ_{σ_i}	η_i
Cross-sectional median	0.81	-5.27	0.62	0.46
(10th percentile, 90th percentile)	(0.68, 0.90)	(-5.95, -4.64)	(0.49, 0.77)	(0.43, 0.52)

Notes: The Table reports the posterior mean estimates of first- and second-moments aggregate and sector-specific TFP shocks. For sector-specific TFP shocks, I report the cross-sectional median of posterior means and, in parentheses, 10th and 90th percentiles of posterior means.

shocks of different sectors into the decision rules.

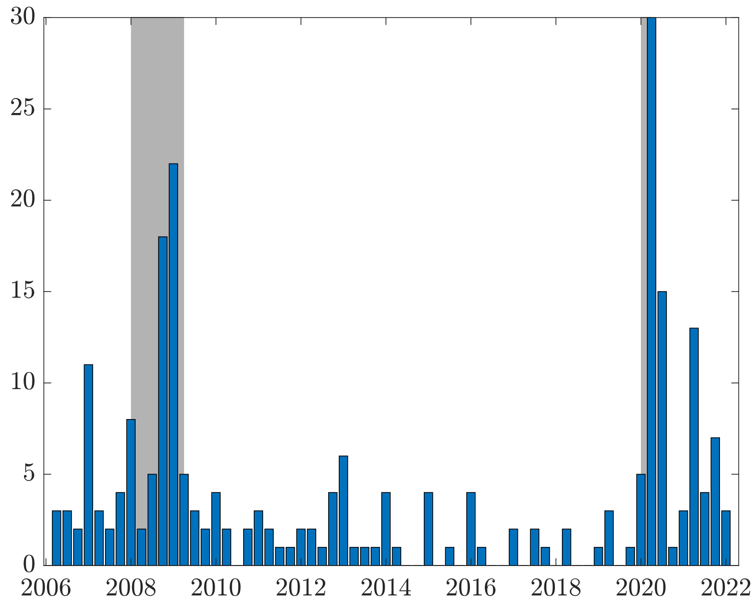
I estimate the first- and second-moments macro TFP processes ((5) and (7)), first- and second-moments sector-specific TFP processes ((6) and (8)) separately for each sector using a Bayesian Markov-Chain-Monte-Carlo approach. I set flat priors (uniform distribution with loose bounds) for a, a_i, τ_1, τ_2, $\tau_{1,i}$, $\tau_{2,i}$, σ, and σ_i. I use a Beta distribution with mean 0.6 and 0.2 for the priors for ρ, ρ_i, ρ_σ, and ρ_{σ_i}. The priors for η and η_i are set to a Gamma distribution with mean 0.5 and standard deviation 0.1. Because of the nonlinearity induced by stochastic volatility, the likelihood of each posterior draw from a random-walk Metropolis-Hastings is evaluated using the particle filter as in Fernández-Villaverde et al. (2011), Born and Pfeifer (2014), and Fernández-Villaverde et al. (2015).

2.2 Facts about Sector-Specific Volatility Shocks

Using the estimation results, I identify four key facts about sectoral volatility shocks, underscoring their relevance for macroeconomic dynamics.

Table 1 provides the summary of posterior means. The key observation is that, compared to the aggregate TFP, sector-specific TFP displays a higher average standard deviation, greater persistence, and similar stochastic volatility. The persistence parameter ρ of aggregate TFP is 0.51, which is substantially lower than the cross-sectional median of the persistence parameter ρ_i of sector-specific TFP of 0.81. The mean standard deviation σ of aggregate TFP is -6.56 while the cross-sectional median of the mean standard deviations σ_i of sector-specific TFP is -5.27. In other words, the standard deviation of sector-specific TFP is about $(\exp(-5.27)/\exp(-6.56) \approx)$ 3.6 times larger than that of the aggregate TFP. Next, consider the stochastic volatility of both aggregate and sector-specific TFP. A two-standard-deviations aggregate TFP uncertainty shock—assuming the volatility is at its steady state—raises the standard deviation of the innovation from $100\exp(-6.56) = 0.14\%$ to $100\exp(-6.56 + 2 \times 0.5) = 0.38\%$. For sector-specific TFP, I use the cross-sectional median parameters as a representative case; under those values, a two-standard-deviations shock—again assuming steady-state volatility—raises the innovation's standard deviation from $100\exp(-5.27) = 0.51\%$ to $100\exp(-5.27 + 2 \times 0.46) = 1.29\%$. Moreover, since the sector-specific TFP process is quite

Figure 1: Number of sectors with a greater-than-one-standard-deviation innovation in volatility



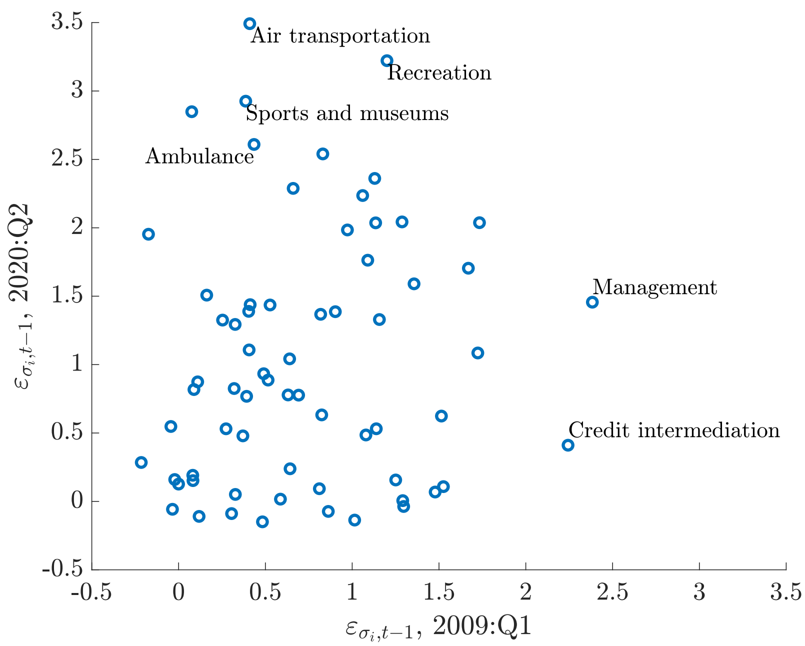
Notes: The figure reports, for each quarter, how many sectors experience a positive innovation to sector-specific TFP volatility $\varepsilon_{\sigma_i,t}$ that exceeds one standard deviation. The shaded areas are the NBER recession dates.

persistent, at $\rho_i = 0.81$ in the median case, even a transitory spike in volatility has lasting effects on sector-specific TFP

Next, I examine the time-series properties of sectoral volatility shocks. First, I measure $\sigma_{i,t}$ using a particle smoother, conditional on parameters set at their posterior means. I then compute the implied innovations, $\varepsilon_{\sigma_i,t}$, by substituting the median smoothed volatility into (8). Figure 1 plots, for each quarter, the number of sectors experiencing a greater-than-one-standard-deviation innovation in TFP volatility. Notably, the count spikes sharply in both the Great Recession and the COVID-19 recession. For instance, in 2009:Q1 and 2020:Q2, 22 and 30 (out of 66) sectors, respectively, had a greater-than-one-standard-deviation innovation—far above the average of two sectors per quarter observed during expansions.

Examining which sectors experienced volatility shocks highlights how distinct these two recessions were. To illustrate, Figure 2 plots the innovation $\varepsilon_{\sigma_i,t-1}$ to sector-specific TFP volatility in 2009:Q1 against the innovation in 2020:Q2. The correlation is weakly positive, at 0.10. In 2009:Q1, "Federal Reserve banks, credit intermediation, and related activities" and "Management of companies and enterprises" underwent the largest increases in volatility, whereas in 2020:Q2 a greater-than-three-standard-deviations spike occurred in "Air transportation" and "Amusements, gambling, and recreation industries." Other sectors registering a substantial jump in volatility include "Ambulatory health care services" and "Performing arts, spectator sports, museums, and related

Figure 2: Innovations to sectoral volatilities in 2009:Q1 and 2020:Q2



Notes: The figure shows the scatterplot of innovations $\varepsilon_{\sigma_i,t-1}$ to sector-specific TFP volatilities in 2009:Q1 against those in 2020:Q2. Each circle represents one of the 66 sectors. The sectors labeled are as follows (from top to bottom): "Air transportation", "Amusements, gambling, and recreation industries", "Performing arts, spectator sports, museums, and related activities", "Ambulatory health care services", "Management of companies and enterprises", "Federal Reserve banks, credit intermediation, and related activities".

activities."

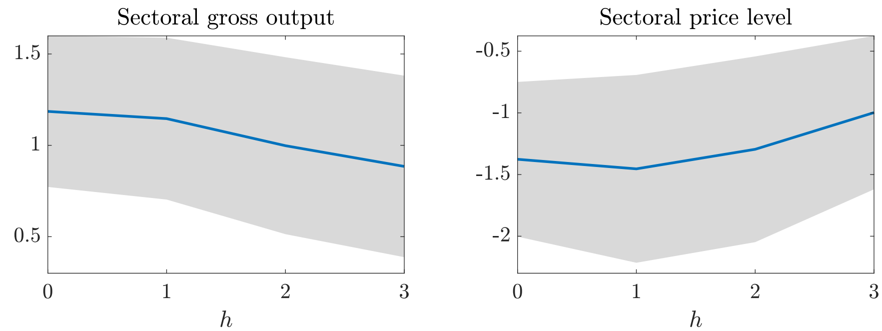
Finally, to examine the economic impact of sector-specific TFP and volatility shocks, I estimate the following fixed-effect panel version of a local projection by Jordá (2005):

$$x_{i,t+h} = \alpha_i + \gamma_t + \beta_1^{(h)} \varepsilon_{i,t} + \beta_2^{(h)} \varepsilon_{\sigma_i,t} + \nu_{i,t+h}, \quad h = 0, \ldots, H$$

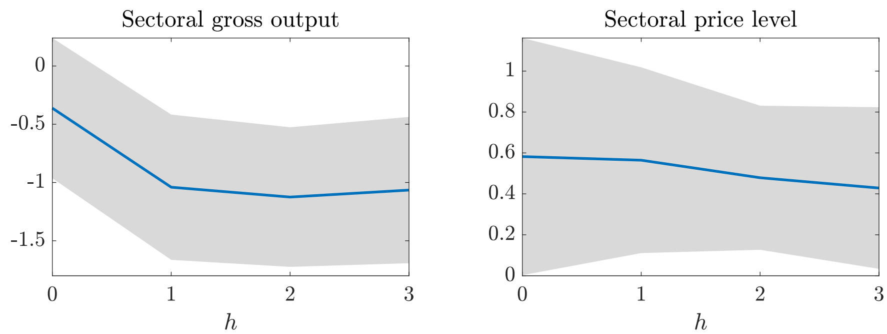
where $x_{i,t}$ is the variable of interest α_i is a sector fixed effect, γ_t is a time fixed effect, and $\{\beta_1^{(h)}\}_{h=0}^H$ and $\{\beta_2^{(h)}\}_{h=0}^H$ are the coefficients of interest. $\varepsilon_{z_i,t}$ and $\varepsilon_{\sigma_i,t}$ are the sector-specific TFP and volatility shocks, respectively, extracted from the particle smoother. Intuitively, the idea of this panel local projection is to exploit cross-sectional variations in the shock realizations to infer the impact of sectoral volatility shocks: in any given quarter, some sectors face larger volatility shocks than others. I compute the impulse responses for $H = 3$-quarter horizons and estimate the responses of sectoral gross output $y_{i,t}$ and sectoral real price level $p_{i,t}$. Figure 3 reports the estimated responses to a one-standard-deviation first-moment sector-specific TFP shock (a positive $\varepsilon_{i,t}$) and a one-standard-deviation sector-specific volatility shock (a positive $\varepsilon_{\sigma_i,t}$) in the top and bottom panels, respectively. In the top panel, a positive innovation to sector-specific TFP raises the sec-

Figure 3: Local projection impulse responses

(a) A one-standard-deviation positive innovation to sector-specific TFP



(b) A one-standard-deviation positive innovation to sector-specific volatility



Notes: The top panel shows the responses of sectoral variables to a one-standard-deviation positive innovation in their own TFP ($\varepsilon_{i,t} > 0$). The bottom panel shows the responses of sectoral variables to a one-standard-deviation positive innovation in their own TFP volatility ($\varepsilon_{\sigma_i,t} > 0$). Units are in percent. The shaded areas are the 90% confidence intervals. Standard errors are clustered by sector.

tor's own gross output by 1.2 percent and lowers its price level by 1.4 percent on impact, at the point estimate. The bottom panel shows that a sector-specific volatility shock has stagflationary consequences for the affected sector. Two quarters after a positive innovation to volatility, sectoral output declines by 1.1 percent, while the price level rises by 0.5 percent at the point estimate.

3 Model

Motivated by the empirical results in the previous section, I study a multi-sector New Keynesian model, extending Bouakez et al. (2009) and Pasten et al. (2020) to allow for sector-specific TFP volatility shocks. The model features nominal rigidities and input-output networks, both of which are critical ingredients. First, previous work, such as Fernández-Villaverde et al. (2015) and Basu and Bundick (2017), has found that nominal rigidities propagate and magnify the impact of uncertainty shocks. Thus, it is of interest to examine whether a similar transmission mechanism takes place in the case of sector-specific volatility shocks as well. Second, it is important to consider input-output networks because they could propagate the effects of sectoral volatility shocks across sectors through linkages.

3.1 Household

A representative household maximizes utility

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} C_t^{1-\sigma} - \left[\sum_{i=1}^{n} h_{i,t}^{\frac{1+\nu}{\nu}} \right]^{\frac{\nu}{1+\nu}} \right],$$

where C_t is aggregate consumption, β is the discount factor, and σ is the coefficient of relative risk aversion. To capture the notion of imperfect labor mobility across sectors in a parsimonious manner, as in Bouakez et al. (2009), total hours worked are aggregated according to a CES function of hours $h_{i,t}$ worked at each sector, where ν is the elasticity of substitution of labor across sectors. The household's budget constraint is

$$P_t C_t + \sum_{i=1}^{n} P_{i,t}^{q} x_{i,t} + B_t = \sum_{i=1}^{n} W_{i,t} h_{i,t} + \sum_{i=1}^{n} R_{i,t}^{k} k_{i,t-1} + R_{t-1} B_{t-1} + D_t,$$

where P_t is the nominal price of an aggregate consumption unit, $P_{i,t}^{q}$ is the nominal price of sector i investment, $x_{i,t}$ is investment in sector i capital, and B_t is the nominal bond holding. $W_{i,t}$ is the nominal wage rate in sector i, $R_{i,t}^{k}$ is the nominal capital rental rate in sector i, $k_{i,t}$ is the capital stock in sector i, R_t is the nominal interest rate, and D_t is the nominal profit from all intermediate firms.

The household aggregates consumption from each sector according to

$$C_t = \left[\sum_{i=1}^{n} \omega_i^{\frac{1}{\varepsilon_c}} c_{i,t}^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \right]^{\frac{\varepsilon_c}{\varepsilon_c - 1}}, \qquad \sum_{i=1}^{n} \omega_i = 1,$$

where ε_c controls the elasticity of substitution of consumption goods across sectors, yielding the demand function

$$c_{i,t} = \omega_i \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon_c} C_t,$$

where $P_{i,t}$ is the nominal price of sector i goods. The aggregate price level is given by

$$P_t = \left[\sum_{i=1}^{n} \omega_i P_{i,t}^{1-\varepsilon_c} \right]^{\frac{1}{1-\varepsilon_c}}.$$

Capital accumulation in each sector i is subject to an investment adjustment cost:

$$k_{i,t} = (1-\delta)k_{i,t-1} + \left\{1 - \frac{\kappa}{2}\left(\frac{x_{i,t}}{x_{i,t-1}} - 1\right)^2\right\}x_{i,t},$$

where δ is the depreciation rate and κ is a parameter that controls the size of the investment adjustment cost. Each sector i produces investment $x_{i,t}$ using inputs $q_{ij,t}$ from other sectors j, via

$$x_{i,t} = \left[\sum_{i=1}^{n} b_{ij}^{\frac{1}{\varepsilon_q}} q_{ij,t}^{\frac{\varepsilon_q - 1}{\varepsilon_q}}\right]^{\frac{\varepsilon_q}{\varepsilon_q - 1}}, \qquad \sum_{j=1}^{n} b_{ij} = 1,$$

where ε_q controls the elasticity of substitution across inputs. The nominal price of sector i investment is given by

$$P_{i,t}^q = \left[\sum_{i=1}^{n} b_{ij} P_{j,t}^{1-\varepsilon_q}\right]^{\frac{1}{1-\varepsilon_q}}.$$

A capital use matrix B describes each sector's contribution to other sectors' capital-goods production:

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & & & \\ & & \ddots & \\ b_{n1} & & & b_{nn} \end{pmatrix}.$$

3.2 Production

In each sector i, final goods $y_{i,t}$ are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods $y_{i,t}(l)$ for $l \in [0,1]$ via

$$y_{i,t} = \left[\int_0^1 (y_{i,t}(l))^{\frac{\theta-1}{\theta}} dl\right]^{\frac{\theta}{\theta-1}},$$

where θ controls the price elasticity of demand for each intermediate good. The demand function for good l is then

$$y_{i,t}(l) = \left(\frac{P_{i,t}(l)}{P_{i,t}}\right)^{-\theta} y_{i,t}, \tag{9}$$

and

$$P_{i,t} = \left[\int_0^1 P_{i,t}(l)^{1-\theta} dl\right]^{\frac{1}{1-\theta}}.$$

Intermediate goods in sector i are produced by monopolistically competitive firms with technology

$$y_{i,t}(l) = e^{z_{i,t}}(k_{i,t}(l))^{\alpha_i^k}(h_{i,t}(l))^{\alpha_i^h}(\widetilde{m}_{i,t}(l))^{1-\alpha_i^k-\alpha_i^h},$$

where

$$\widetilde{m}_{i,t}(l) = \left[\sum_{i=1}^{n} a_{ij}^{\frac{1}{\varepsilon_m}} \left(m_{ij,t}(l) \right)^{\frac{\varepsilon_m - 1}{\varepsilon_m}} \right]^{\frac{\varepsilon_m}{\varepsilon_m - 1}},$$

where $m_{ij,t}$ is the amount of materials produced by sector j used as inputs by sector i. The production function exhibits constant returns to scale as in (2). The input-output matrix A captures the network structure of the economy:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ & & \ddots & \\ a_{n1} & & & a_{nn} \end{pmatrix}.$$

Sectoral TFP includes an economy-wide component and a sector-specific component, each evolving according to AR(1) processes with stochastic volatility, as in (4) and (5)–(8).

The optimality input choices satisfy

$$r_{i,t}^{k} = mc_{i,t}\alpha_i^k \frac{y_{i,t}(l)}{k_{i,t}(l)}, \quad w_{i,t} = mc_{i,t}\alpha_i^h \frac{y_{i,t}(l)}{h_{i,t}(l)}, \quad p_{i,t}^{m} = mc_{i,t}(1 - \alpha_i^k - \alpha_i^h) \frac{y_{i,t}(l)}{\widetilde{m}_{i,t}(l)},$$

where $w_{i,t} \equiv W_{i,t}/P_t$, $r_{i,t}^k \equiv R_{i,t}^k/P_t$, and $p_{i,t}^m$ is defined in (3). The real marginal cost $mc_{i,t}$ for sector i is

$$mc_{i,t} = \frac{1}{e^{z_{i,t}}} \left(\frac{1}{\alpha_i^k} \right)^{\alpha_i^k} \left(\frac{1}{\alpha_i^h} \right)^{\alpha_i^h} \left(\frac{1}{1 - \alpha_i^k - \alpha_i^h} \right)^{1 - \alpha_i^k - \alpha_i^h} (r_{i,t}^k)^{\alpha_i^k} (w_{i,t})^{\alpha_i^h} (p_{i,t}^m)^{1 - \alpha_i^k - \alpha_i^h}.$$

Intermediate firms face a Calvo pricing friction: in each period, $1 - \xi_i$ fraction of firms reset their prices, while the rest index to steady-state inflation Π. A resetting firm chooses $P_{i,t}^*$ to maximize a sum of present discounted values of real profits:

$$\max_{P_{i,t}(l)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_i)^s \left\{ \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{\Pi^s P_{i,t}(l)}{P_{t+s}} - mc_{i,t+s} \right] y_{i,t+s}(l) \right\},$$

subject to the demand (9). The resulting first-order condition is

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_i)^s \left\{ \frac{\lambda_{t+s}}{\lambda_t} \left[(1 - \theta) \left(\frac{\Pi^s P_{i,t}^*}{P_{i,t+s}} \right) \left(\frac{P_{i,t+s}}{P_{t+s}} \right) + \theta mc_{i,t+s} \right] \left(\frac{1}{P_{i,t}^*} \right) \left(\frac{\Pi^s P_{i,t}^*}{P_{i,t+s}} \right)^{-\theta} y_{i,t+s} \right\} = 0.$$

Market clearing in sector i is

$$y_{i,t} = c_{i,t} + \sum_{j=1}^{n} m_{ji,t} + \sum_{j=1}^{n} q_{ji,t}.$$

Sectoral GDP is defined as

$$GDP_{i,t} = p_{i,t}y_{i,t} - \sum_{j=1}^{n} p_{j,t}m_{ij,t}, \tag{10}$$

where $p_{i,t} \equiv \frac{P_{i,t}}{P_t}$.

3.3 Aggregation and Monetary Policy

I aggregate intermediate firms' output by defining

$$\widetilde{y}_{i,t} \equiv \int_0^1 y_{i,t}(l)dl,$$

where $\widetilde{y}_{i,t}$ is related to $y_{i,t}$ via the relationship

$$\widetilde{y}_{i,t} = \Delta_{i,t}y_{i,t},$$

where $\Delta_{i,t} \equiv \int_0^1 \left(\frac{p_{i,t}(l)}{p_{i,t}}\right)^{-\theta} dl$ is a measure of price dispersion that follows the law of motion

$$\Delta_{i,t} = (1-\xi_i)\left(\frac{p_{i,t}^*}{p_{i,t}}\right)^{-\theta} + \xi_i\left(\frac{\Pi}{\pi_{i,t}}\right)^{-\theta}.$$

Aggregate GDP is

$$Y_t = C_t + X_t,$$

where $X_t \equiv \sum_{i=1}^{n}\sum_{j=1}^{n} p_{j,t}q_{ij,t}$. The central bank follows a monetary policy rule given by

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left\{\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}\right\}^{1-\rho_R}, \tag{11}$$

where $\Pi_t \equiv P_t/P_{t-1}$, ρ_R is the smoothing parameter, and ϕ_Π and ϕ_Y are the response coefficients to inflation and output, respectively.

4 Analytical Insights

In this Section, I analyze a simplified version of the model presented above. It is similar to that in Pasten et al. (2024), except that I now consider sectoral volatility shocks. I use the simplified model to achieve two objectives. First, I discuss the intuitions for how sectoral TFP volatility shocks propagate through the input-output network. Second, I illustrate how the risk-adjusted log-linearization method is used to capture the effects of sectoral volatility shocks. I make the following simplifications:

(i) The household has a log consumption utility ($\sigma = 1$) and sectoral labor supplies are perfect substitutes ($\nu \to \infty$). These assumptions simplify the household labor supply and imply that real wages are identical across sectors and track real consumption.

(ii) Capital does not enter into the production function ($\alpha_i^k = 0, \forall i$). This implies that the consumption preference share, ω_i, corresponds to the steady-state GDP share, and that total consumption equals total GDP ($C_t = Y_t$).

(iii) The Calvo pricing friction is replaced with a following simple informational friction: all firms adjust their prices every period but $\widetilde{\xi}_i$ fraction of firms has to set their prices based on the $t-1$ information set.

(iv) Instead of the Taylor rule (11), monetary policy now targets steady-state nominal GDP:

$$P_t Y_t = \overline{Y}.$$

Since nominal consumption will be kept constant, this assumption, together with assumption (i), shuts down the bond Euler equation channel of consumption fluctuations.

(v) Sectoral TFP is driven entirely by the sector-specific component (thus there is no economy-wide TFP shock) and shocks are i.i.d. ($\rho_i = 0, \rho_{\sigma_i} = 0, \forall i$). I abstract from sectoral heterogeneity in the standard deviations of TFP volatility shocks by normalizing them to one ($\eta_{z_i} = 1, \forall i$), which eliminates mechanical cross-sectoral differences in the effects of volatility shocks due to the size differences of the volatility innovations. However, I still allow for heterogeneity in each sector's steady-state TFP volatility.

Consider the firms' pricing problem. The optimal price $P_{i,t}^f$ for the $(1 - \widetilde{\xi}_i)$ fraction of firms in sector i who adjust their prices based on current information set ("flexible" firms) is given by

$$P_{i,t}^f = \left(\frac{\theta}{\theta - 1}\right) MC_{i,t},$$

where $MC_{i,t} \equiv P_t mc_{i,t}$ is the nominal marginal cost for sector i. The optimal price $P_{i,t}^s$ for the $\widetilde{\xi}_i$ fraction of firms in sector i who adjust their prices based on $t-1$ information set ("sticky" firms) has to satisfy the first-order condition

$$\mathbb{E}_{t-1}\left[\frac{P_{i,t}^s}{P_t} - \left(\frac{\theta}{\theta - 1}\right)\frac{MC_{i,t}}{P_t}\right] = 0. \tag{12}$$

To approximate the condition (12), I use the risk-adjusted log-linearization method of Jermann (1998), Uhlig (2010), Dew-Becker (2012), Malkhozov (2014), and others. The advantage of the method is that the resulting decision rules become conditionally linear in volatility. More precisely, the risk-adjusted log-linearization method utilizes the fact that log-linearized variables follow a normal distribution. In turn, this implies that variables in levels follow a log-normal distribution. Thus, the method risk-adjusts all the expectational variables as these variables are log-normal. Denoting variables expressed in terms of log-deviations from their non-stochastic steady states as *hats*, (12) becomes

$$\widehat{P}^s_{i,t} = \mathbb{E}_{t-1}[\widehat{MC}_{i,t}] + 0.5\mathbb{VAR}_{t-1}[\widehat{MC}_{i,t}] - \mathbb{COV}_{t-1}[\widehat{MC}_{i,t}, \widehat{P}_t], \tag{13}$$

where the last two terms are the risk-adjustments. All else equal, an increase in uncertainty about the marginal cost ($\mathbb{VAR}_{t-1}[\widehat{MC}_{i,t}]$) raises the optimal price but higher co-variance between the marginal cost and the aggregate price level ($\mathbb{COV}_{t-1}[\widehat{MC}_{i,t}, \widehat{P}_t]$) lowers the optimal price. Intuitively, when uncertainty increases, firms self-insure against the possibility of pricing their goods too low relative to the marginal cost by raising their prices: the profit loss due to pricing their goods too high and selling less is lower than that due to pricing too low and selling more. Since the firms' objective is to maximize the expected *real* profit, they temper their upward pricing bias when there is high co-movement between the nominal marginal cost and the aggregate price level. The nominal price of good i then evolves according to

$$\widehat{P}_{i,t} = (1 - \widetilde{\xi}_i)\widehat{P}^f_{i,t} + \widetilde{\xi}_i \widehat{P}^s_{i,t},$$

and the equilibrium nominal marginal cost can be expressed as (details in Appendix)

$$\widehat{MC}_{i,t} = -z_{i,t} + \sum_{j=1}^{n} a_{ij}\widehat{P}_{j,t}. \tag{14}$$

Hence, the nominal marginal cost can be expressed as a linear combination of sectoral TFP and nominal prices of intermediate inputs.

To compute the equilibrium law of motion, I proceed in two steps. In the first step, I solve for the policy rules without the risk adjustment, that is, assuming $\widehat{P}^s_{i,t} = \mathbb{E}_{t-1}[\widehat{MC}_{i,t}]$. In the second step, I perform the risk adjustment (13) by computing the variance and co-variance of $\widehat{MC}_{i,t}$ and $\widehat{P}_t$ based on the policy rules I obtained from the first step and solve for the equilibrium.

4.1 Economy without Input-Output Linkages

First, consider an economy without input-output linkages ($a_{ij} = 0, \forall i, j$). In the Appendix, I show that the sectoral nominal price levels $\widehat{\boldsymbol{P}}_t \equiv [\widehat{P}_{1,t}, \ldots, \widehat{P}_{n,t}]'$ and gross output $\widehat{\boldsymbol{y}}_t \equiv [\widehat{y}_{1,t}, \ldots, \widehat{y}_{n,t}]'$ satisfy

$$\widehat{\boldsymbol{P}}_t = -(I - \Xi)\boldsymbol{z}_t + \Xi[I - 2[\iota\boldsymbol{\omega}'(I - \Xi)]']\Sigma_\sigma \boldsymbol{\sigma}_t, \tag{15}$$

$$\widehat{\boldsymbol{y}}_t = -\iota\boldsymbol{\omega}'\widehat{\boldsymbol{P}}_t, \tag{16}$$

where I is an identity matrix and Ξ and Σ_σ are diagonal matrices with $[\widetilde{\xi}_1, \ldots, \widetilde{\xi}_n]'$ and $[e^{2\sigma_{z_1}}, \ldots, e^{2\sigma_{z_n}}]'$ as entries, respectively.[3] $\boldsymbol{\omega} \equiv [\omega_1, \ldots, \omega_n]'$, and ι is an $n \times 1$ vector of 1's. $\boldsymbol{z}_t$ and $\boldsymbol{\sigma}_t$ are vec-

[3] Note that in an economy without linkages sectoral gross output equals sectoral GDP so that $\widehat{\boldsymbol{y}}_t = \widehat{\boldsymbol{GDP}}_t$ where $\widehat{\boldsymbol{GDP}}_t \equiv [\widehat{GDP}_{1,t}, \ldots, \widehat{GDP}_{n,t}]'$.

tors of sector-specific TFP level and volatility shocks so that $\boldsymbol{z}_t = [z_{1,t}, \ldots, z_{n,t}]'$ and $\boldsymbol{\sigma}_t = [\sigma_{z_1,t} - \sigma_{z_1}, \ldots, \sigma_{z_n,t} - \sigma_{z_n}]'$. In turn, the aggregate nominal price level $\widehat{P}_t$ and output $\widehat{Y}_t$ are

$$\widehat{P}_t = \boldsymbol{\omega}' \widehat{\boldsymbol{P}}_t, \tag{17}$$

$$\widehat{Y}_t = -\boldsymbol{\omega}' \widehat{\boldsymbol{P}}_t. \tag{18}$$

To understand the solution, consider sector i's nominal price using (15):

$$\widehat{P}_{i,t} = -(1-\widetilde{\xi}_i) z_{i,t} + \widetilde{\xi}_i [1 - 2\omega_i (1-\widetilde{\xi}_i)] e^{2\sigma_i} (\sigma_{i,t-1} - \sigma_i), \tag{19}$$

which can be aggregated using (18) to obtain total GDP:

$$\widehat{Y}_t = \sum_{i=1}^{n} \omega_i (1-\widetilde{\xi}_i) z_{i,t} - \sum_{i=1}^{n} \omega_i \widetilde{\xi}_i [1 - 2\omega_i (1-\widetilde{\xi}_i)] e^{2\sigma_i} (\sigma_{i,t-1} - \sigma_i). \tag{20}$$

First, the sectoral price $\widehat{P}_{i,t}$ is responsive only to shocks within its own sector ($z_{i,t}$ and $\sigma_{i,t}$). This is because, without input-output networks, sector i marginal cost (14) only depends on sector i TFP. Second, an increase in TFP level $z_{i,t}$ lowers sectoral price $\widehat{P}_{i,t}$ and, through (17) and (18), lowers aggregate price $\widehat{P}_t$ and raises GDP $\widehat{Y}_t$. In contrast, as long as $\omega_i < 1/[2(1-\widetilde{\xi}_i)]$,[4] an increase in TFP volatility $\sigma_{i,t-1}$ raises sectoral price $\widehat{P}_{i,t}$ and, through (17) and (18), raises aggregate price $\widehat{P}_t$ and lowers GDP $\widehat{Y}_t$. Third, shocks originating in sectors with higher steady-state GDP shares ω_i have a larger impact at the aggregate level. Fourth, as described in Pasten et al. (2024), price rigidity $\widetilde{\xi}_i$ dampens the response of GDP to sectoral TFP level shocks. In comparison, everything else equal, sectors with higher price rigidity (higher $\widetilde{\xi}_i$) generate more aggregate volatility in GDP and price in response to TFP volatility shocks. This is because, as described in (13), the sticky firms' optimal price is affected by the increase in risk, unlike the static pricing problem of flexible firms. Fifth, sectors with higher steady-state TFP volatility (higher σ_i) generate more aggregate volatility in GDP and price in response to TFP volatility shocks, all else being equal.

4.2 Economy with Input-Output Linkages: The Network Precautionary Pricing Multiplier

We now introduce input-output linkages. The sectoral nominal price levels become

$$\begin{aligned} \widehat{\boldsymbol{P}}_t = &-L(I-\Xi)\boldsymbol{z}_t \\ &+ L\Xi \left\{ \left[I + \widetilde{A}L\,(I-\Xi)\right] \circ \left[I + \widetilde{A}L\,(I-\Xi)\right] - 2\left[I + \widetilde{A}L\,(I-\Xi)\right] \circ \left[\iota\boldsymbol{\omega}' L\,(I-\Xi)\right] \right\} \Sigma_\sigma \boldsymbol{\sigma}_t, \end{aligned} \tag{21}$$

[4] Intuitively, this condition ensures that the steady-state GDP share of sector i is not "too large" so that the co-variance of sector i nominal marginal cost and the aggregate price index is sufficiently small.

where $\circ$ denotes the element-wise product (Hadamard product) of two matrices. Define $\widetilde{A} \equiv (\iota - \alpha^h)A$, with $\alpha^h \equiv [\alpha_1^h, \ldots, \alpha_n^h]'$, so that $L \equiv [I - (I - \Xi)\widetilde{A}]^{-1}$ is the *effective* Leontief inverse matrix. Since $L = \sum_{k=0}^{\infty}[(I - \Xi)\widetilde{A}]^k$, the (i,j)-th element of the effective Leontief inverse matrix measures the sector j's direct and indirect (i.e. intermediated through other sectors) importance as an input supplier to sector i, weighted by the degree of nominal rigidities.[5] As shown in Pasten et al. (2024), the effective Leontief inverse matrix controls the propagation of sectoral TFP shocks $\boldsymbol{z}_t$ in an economy with nominal rigidites and input-output networks. The aggregate nominal price level $\widehat{P}_t$ and GDP $\widehat{Y}_t$ are given by (17) and (18).

Denoting (i,j)-th element of the effective Leontief inverse matrix as l_{ij}, from (21) sector i's nominal price is

$$\begin{aligned} \widehat{P}_{i,t} = &- \sum_{j=1}^{n} l_{ij}(1 - \widetilde{\xi}_j) z_{j,t} \\ &+ \sum_{j=1}^{n} l_{ij}\widetilde{\xi}_j \left[\sum_{h=1}^{n} \zeta_{jh} e^{2\sigma_h} (\sigma_{h,t-1} - \sigma_h) \right], \end{aligned} \tag{22}$$

where

$$\zeta_{jh} \equiv \begin{cases} \underbrace{\left[1 + \underbrace{(1-\alpha_j^h)\sum_{k=1}^{n} a_{jk} l_{kj}(1-\widetilde{\xi}_j)}_{\text{Network multiplier}} \right]^2}_{\text{Effect through } \mathbb{VAR}_{t-1}(\widehat{MC}_{j,t})} - 2 \underbrace{\left[1 + \underbrace{(1-\alpha_j^h)\sum_{k=1}^{n} a_{jk} l_{kj}(1-\widetilde{\xi}_j)}_{\text{Network multiplier}} \right] \varphi_j}_{\text{Effect through } \mathbb{COV}_{t-1}(\widehat{MC}_{j,t}, \widehat{P}_t)}, & \text{if } j = h \\ \underbrace{\left[\underbrace{(1-\alpha_j^h)\sum_{k=1}^{n} a_{jk} l_{kh}(1-\widetilde{\xi}_h)}_{\text{Network multiplier}} \right]^2}_{\text{Effect through } \mathbb{VAR}_{t-1}(\widehat{MC}_{j,t})} - 2 \underbrace{\left[\underbrace{(1-\alpha_j^h)\sum_{k=1}^{n} a_{jk} l_{kh}(1-\widetilde{\xi}_h)}_{\text{Network multiplier}} \right] \varphi_h}_{\text{Effect through } \mathbb{COV}_{t-1}(\widehat{MC}_{j,t}, \widehat{P}_t)}, & \text{otherwise} \end{cases} \tag{23}$$

and

$$\varphi_i \equiv (1 - \widetilde{\xi}_i) \sum_{j=1}^{n} \omega_j l_{ji}. \tag{24}$$

The first line of the right-hand-side of (22) contains the elasticity, given by $l_{ij}(1 - \widetilde{\xi}_j)$, of sector i price $P_{i,t}$ to sectoral TFP shocks $z_{j,t}$. The elasticity is captured by l_{ij} — the shocked sector's direct and indirect importance as an input supplier to sector i, convoluted with price rigidities — and the shocked sector's degree of price flexibility $(1 - \widetilde{\xi}_j)$.

[5] When prices are fully flexible (i.e. $\widetilde{\xi}_i = 0, \forall i$), we have $\Xi = 0$ so the effective Leontief inverse matrix collapses to the standard Leontief inverse matrix $[I - \widetilde{A}]^{-1}$ described in Acemoglu et al. (2012). For an anologous discussion of the Leontief inverse matrix in an flexible price environment, see Carvalho and Tahbaz-Salehi (2019).

The second line of (22) contains the elasticity, given by $\sum_{j=1}^{n} l_{ij}\widetilde{\xi}_j\zeta_{jh}e^{2\sigma_h}$, of $P_{i,t}$ to sectoral TFP volatility shocks originating in industry h, $\sigma_{h,t-1}$. First, as in the case of an economy without input-output linkages, the elasticity is proportional to the steady-state TFP volatility of sector h, $e^{2\sigma_h}$. Second, the elasticity is affected by ζ_{jh}, which is the responsiveness of optimal price $\widehat{P}^s_{j,t}$ for sticky firms in sector j to a TFP volatility shock in sector h, and the share of sticky firms $\widetilde{\xi}_j$. Third, l_{ij} captures how a sector j sticky firms' pricing, through input-output linkages, affects $P_{i,t}$. The elasticity of $P_{i,t}$ to $\sigma_{h,t-1}$ is then obtained by summing $l_{ij}\widetilde{\xi}_j\zeta_{jh}e^{2\sigma_h}$ across all sectors $j = 1, \ldots, n$.

As described in (23), the responsiveness of the optimal sticky price to a TFP volatility shock, ζ_{jh}, can be decomposed into two components: the effect through $\mathbb{VAR}_{t-1}(\widehat{MC}_{j,t})$, which is positively related to the elasticity, and the effect through $\mathbb{COV}_{t-1}(\widehat{MC}_{j,t}, \widehat{P}_t)$, which is negatively related. To understand the implication of input-output linkages, consider the effect through $\mathbb{VAR}_{t-1}(\widehat{MC}_{j,t})$ in the case of $j = h$. An increase in sector j TFP volatility raises $\mathbb{VAR}_{t-1}(\widehat{MC}_{j,t})$ through two channels. The first is the direct channel, which is equal to 1. As we can see from (14), a one-unit increase of the volatility of $z_{j,t}$ directly raises the conditional variance of $\widehat{MC}_{j,t}$ by one unit. The second channel is the *network precautionary pricing multiplier*, which comes from the fact that sector j's marginal cost depends on sectoral prices of other sectors in (14). Intuitively, the network channel captures sector j's direct and indirect influence as an input supplier to its own and other sectors, say sector k, summarized by l_{kj}. In turn, sector j uses those goods k as intermediate inputs so it must be weighted by a_{jk}. The network precautionary pricing multiplier amplifies sectoral TFP volatility shocks, provided that the magnification effect on $\mathbb{VAR}_{t-1}(\widehat{MC}_{j,t})$ is not dominated by the impact on $\mathbb{COV}_{t-1}(\widehat{MC}_{j,t}, \widehat{P}_t)$. Next, consider ζ_{jh} in the case of $j \neq h$. The structure of the elasticity is similar to that on its own volatility shock ($j = h$), except that there is no direct channel in the variance effect and everything operates through the network channel. Finally, if there are no input-output linkages, $L = I$ and $A = 0$ so (22) reduces to (19).

We can use (18) on (22) to obtain total GDP:

$$\widehat{Y}_t = \sum_{i=1}^{n} \varphi_i z_{i,t} - \sum_{i=1}^{n} \chi_i e^{2\sigma_i}(\sigma_{i,t} - \sigma_i), \tag{25}$$

where

$$\chi_i \equiv \sum_{j=1}^{n}\sum_{h=1}^{n} \omega_j l_{jh}\widetilde{\xi}_h\zeta_{hi}. \tag{26}$$

Finally, note again that if there are no input-output linkages, (25) reduces to (20).

To summarize, the analytical model shows that the transmission mechanism of sector-specific TFP volatility shocks is qualitatively distinct from that of first-moment sector-specific TFP shocks. An increase in sector-specific TFP volatility is stagflationary and its aggregate impact depend on the convolution of several sectoral characteristics, such as the degree of price rigidity, the steady-state TFP volatility, and the impacted sector's position in the production network. The nominal pricing bias and input-output linkages give rise to the network precautionary pricing multiplier:

production networks amplify and propagate firms' motive to preemptively raise prices in response to heightened uncertainty.

5 Solution and Calibration

I now return to the quantitative model to provide a general description of the risk-adjusted log-linearization method to solve the model and explain the calibration to match U.S. sectoral facts.

5.1 Risk-adjusted Log-linearization

This model is large, comprising 66 sectors, so it requires a solution method that handles second-moment shocks efficiently. I therefore adopt a risk-adjusted log-linearization approach, which captures the impact of volatility shocks while keeping the problem computationally tractable. In this regard, my paper relates closely to Bianchi et al. (2023), who incorporate macro uncertainty shocks in a single-sector New Keynesian model using similar risk-adjustment techniques. I extend this framework to a multi-sector environment, allowing me to explore how sector-specific volatility shocks propagate through input–output linkages.

Expectational equations and endogenous risk wedges. As a first step, I list the model's expectational equations alongside their risk-adjusted log-linear forms. Following Bianchi et al. (2023), this decomposition shows how volatility shocks transmit through endogenous risk wedges, which arise from the second moments of endogenous variables. In total, the quantitative model includes four distinct risk wedges. First, consider the consumption Euler equation by the representative household:

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right].$$

In the risk-adjusted log-linearization form,

$$\widehat{\lambda}_t = \widehat{R}_t + \mathbb{E}_t \widehat{\lambda}_{t+1} - \mathbb{E}_t \widehat{\Pi}_{t+1} + \underbrace{\frac{1}{2}\mathbb{VAR}_t(\widehat{\lambda}_{t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{\Pi}_{t+1}) - \mathbb{COV}_t(\widehat{\lambda}_{t+1}, \widehat{\Pi}_{t+1})}_{\text{Precautionary savings}}, \tag{27}$$

where the risk-adjustment term captures the precautionary savings channel as in Basu and Bundick (2017).

Next, consider the capital Euler equation for each sector i:

$$\mu_{i,t} = \beta \mathbb{E}_t \left[\lambda_{t+1} mc_{i,t+1} \alpha_i^k \frac{y_{i,t+1}}{k_{i,t}} + \mu_{i,t+1}(1-\delta) \right],$$

where $\mu_{i,t}$ is the Lagrangian multiplier for the sector i capital accumulation equation. In the risk-adjusted log-linearized form,

$$\widehat{\mu}_{i,t} = \{1-\beta(1-\delta)\}\left[\mathbb{E}_t\widehat{\lambda}_{t+1} + \mathbb{E}_t\widehat{mc}_{i,t+1} + \mathbb{E}_t\widehat{y}_{i,t+1} - \widehat{k}_{i,t}\right] + \beta(1-\delta)\mathbb{E}_t\mu_{i,t+1}$$
$$+ \underbrace{\{1-\beta(1-\delta)\}\left[\frac{1}{2}\mathbb{VAR}_t(\widehat{\lambda}_{t+1}+\widehat{mc}_{i,t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{y}_{i,t+1}) + \mathbb{COV}_t(\widehat{\lambda}_{t+1}+\widehat{mc}_{i,t+1}, \widehat{y}_{i,t+1})\right] + \frac{1}{2}\beta(1-\delta)\mathbb{VAR}_t(\widehat{\mu}_{i,t+1})}_{\text{Capital return risk}}, \tag{28}$$

where the risk-adjustment term captures the fact that the future return on capital is uncertain.

Next, consider the optimality condition for investment in each sector i:

$$\lambda_t p^q_{i,t} = \mu_{i,t}\left[1 - \frac{\kappa}{2}\left(\frac{x_{i,t}}{x_{i,t-1}}-1\right)^2 - \kappa\left(\frac{x_{i,t}}{x_{i,t-1}}-1\right)\frac{x_{i,t}}{x_{i,t-1}}\right]$$
$$+ \beta\kappa\mathbb{E}_t\left[\mu_{i,t+1}\left(\frac{x_{i,t+1}}{x_{i,t}}-1\right)\left(\frac{x_{i,t+1}}{x_{i,t}}\right)^2\right],$$

which can be log-linearized as

$$\widehat{\lambda}_t + \widehat{p}^q_{i,t} = \widehat{\mu}_{i,t} - \kappa\Delta\widehat{x}_{i,t} + \beta\kappa\mathbb{E}_t\Delta\widehat{x}_{i,t+1} + \underbrace{\beta\kappa\left[\frac{5}{2}\mathbb{VAR}_t(\Delta\widehat{x}_{i,t+1}) + \mathbb{COV}_t(\widehat{\mu}_{i,t+1}, \Delta\widehat{x}_{i,t+1})\right]}_{\text{Investment adjustment cost}}, \tag{29}$$

where the risk-adjustment term captures the fact that the impact of current investment on a future adjustment cost is uncertain.

Finally, consider sticky prices. The optimal reset price $p^*_{i,t}$ for retailers in sector i satisfies

$$p^*_{i,t} = \left(\frac{\theta}{\theta-1}\right)\frac{P^n_{i,t}}{P^d_{i,t}},$$

where

$$P^n_{i,t} \equiv \lambda_t mc_{i,t} y_{i,t} + \xi_i\beta\mathbb{E}_t\left[\left(\frac{\pi_{i,t+1}}{\Pi}\right)^{\theta} P^n_{i,t+1}\right],$$
$$P^d_{i,t} \equiv \lambda_t y_{i,t} + \xi_i\beta\mathbb{E}_t\left[\left(\frac{\pi_{i,t+1}}{\Pi}\right)^{\theta-1}\left(\frac{\pi_{i,t+1}}{\Pi_{t+1}}\right) P^d_{i,t+1}\right].$$

$P^n_{i,t}$ and $P^d_{i,t}$ can be risk-adjusted as

$$\widehat{P}^n_{i,t} = (1-\xi_i\beta)\left[\widehat{\lambda}_t + \widehat{mc}_{i,t} + \widehat{y}_{i,t}\right] + \xi_i\beta\left[\theta\mathbb{E}_t\widehat{\pi}_{i,t+1} + \mathbb{E}_t\widehat{P}^n_{i,t+1}\right]$$
$$+ \underbrace{\xi_i\beta\left[\frac{\theta^2}{2}\mathbb{VAR}_t(\widehat{\pi}_{i,t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{P}^n_{i,t+1}) + \theta\mathbb{COV}_t(\widehat{\pi}_{i,t+1}, \widehat{P}^n_{i,t+1})\right]}_{\text{Nominal pricing bias}}, \tag{30}$$

and

$$\widehat{P}^d_{i,t} = (1-\xi_i\beta)\left[\widehat{\lambda}_t + \widehat{y}_{i,t}\right] + \xi_i\beta\left[\theta\mathbb{E}_t\widehat{\pi}_{i,t+1} - \mathbb{E}_t\widehat{\Pi}_{t+1} + \mathbb{E}_t\widehat{P}^d_{i,t+1}\right]$$
$$+\,\xi_i\beta\underbrace{\left[\frac{\theta^2}{2}\mathbb{VAR}_t(\widehat{\pi}_{i,t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{P}^d_{i,t+1} - \widehat{\Pi}_{t+1}) + \theta\mathbb{COV}_t(\widehat{\pi}_{i,t+1}, \widehat{P}^d_{i,t+1} - \widehat{\Pi}_{t+1})\right]}_{\text{Nominal pricing bias}}. \tag{31}$$

The risk-adjustment term, or the nominal pricing bias, is related to the precautionary price setting motive in Fernández-Villaverde et al. (2015). It is also related to the optimal price for the sticky firms (13) in the analytical model above. The nominal pricing bias arises because, when firms set current prices, the future economic environment—and thus their future profits—remains uncertain.

Conditionally linear decision rules. In its risk-adjusted log-linearized form, the model can be written as the following linear rational-expectations (RE) system (Sims, 2002):

$$\Gamma_0\widehat{\boldsymbol{y}}_t = \Gamma_1\widehat{\boldsymbol{y}}_{t-1} + \Psi\Sigma_{t-1}\boldsymbol{\varepsilon}_t + \Omega\boldsymbol{\zeta}_t + C(\Sigma_{t-1}), \tag{32}$$

where $\widehat{\boldsymbol{y}}_t$ is a vector of endogenous variables $\boldsymbol{y}_t$ in terms of its log deviations from the non-stochastic steady states, $\boldsymbol{\varepsilon}_t$ is a vector of first-moment shocks (the economy-wide TFP shock and sector-specific TFP shocks), and $\boldsymbol{\zeta}_t$ is a vector of expectation errors. Γ_0, Γ_1, Ψ, and Ω are coefficient matrices, Σ_{t-1} is a matrix that collects the standard deviations of first-moment shocks, and $C(\Sigma_{t-1})$ is a vector that collects the risk-adjustment terms for relevant equations, which is a function of Σ_{t-1} to reflect the fact that the risk wedges depend on the stochastic volatility. In turn, the log-deviation of Σ_t from the nonstochastic steady state evolves according to

$$\widehat{\Sigma}_t = P\widehat{\Sigma}_{t-1} + Q\boldsymbol{e}_t,$$

where P and Q are coefficient matrices and $\boldsymbol{e}_t$ is a vector that collects innovations to volatility.

The goal is to derive a decision rule that is conditionally linear in volatility:

$$\widehat{\boldsymbol{y}}_t = \overline{J} + J\widehat{\Sigma}_{t-1} + R\widehat{\boldsymbol{y}}_{t-1} + S\Sigma_{t-1}\boldsymbol{\varepsilon}_t, \tag{33}$$

where $\overline{J}$ is a vector of constant terms that reflects the risk-adjustments $C(\Sigma_{t-1})$ in (32) evaluated at the steady-state volatility Σ, and J captures how the additional terms due to risk adjustments $C(\Sigma_{t-1})$ respond to changes in volatility. R and S are coefficient matrices. The decision rule (33) makes clear that volatility shocks affect the endogenous variables through two channels. First, mechanically through the term $S\Sigma_t\boldsymbol{\varepsilon}_t$, the shock realization becomes more dispersed. Second, as can be seen in the term $J\widehat{\Sigma}_{t-1}$, changes in volatility affect endogenous variables because they alter risk adjustments. This second channel is the primary focus.

To solve for (33), I proceed in two steps. First, I leverage the fact that the coefficient matrices R and S are invariant to risk adjustments and compute R and S using a standard solution algorithm (e.g. Sims (2002)) from the RE system (32) without taking into account the risk adjustments $C(\Sigma_{t-1})$. Second, I use S to characterize variances and co-variances that appear in the risk-adjustment terms to obtain $\overline{J}$ and J.[6]

5.2 Calibration

I set the discount factor β at 0.998, depreciation rate δ at 0.025, and the risk-aversion parameter σ at 1. Following Horvath (2000), I choose $\nu = 1$ for the elasticity of substitution of labor across sectors. Following the analysis of long-run sectoral price changes and consumption expenditure shares by Herrendorf et al. (2013), I set $\varepsilon_c = 1$ for the elasticity of substitution of consumption expenditure across sectors. I set $\varepsilon_m = 0.1$ following the estimation result by Atalay (2017). By symmetry, I also set the elasticity of substitution of intermediate inputs for investment to $\varepsilon_q = 0.1$. The investment adjustment cost is set to $\kappa = 0.5$. Following Fernández-Villaverde et al. (2015), I choose $\theta = 21$, implying a steady-state markup of 5%. To calibrate sectoral price stickiness ξ_i, I use the industry-level frequency of price adjustments from Pasten et al. (2020), which is based on BLS confidential microdata. For the Taylor rule I set $\rho_R = 0.5$, $\phi_\Pi = 2$, and $\phi_Y = 0.05$.

As explained in Section 2, I estimate the values of factor shares $\{\alpha_i^k\}_{i=1}^n$, $\{\alpha_i^h\}_{i=1}^n$, the input-output matrix A and the preference weights $\{\omega_i\}_{i=1}^n$ from the "use tables" of the input-output accounts constructed by the BEA. I use the 1997 capital flows table provided by the BEA to estimate the parameters (b_{ij}'s) for the capital goods production. The capital flow table describes the distribution of new structures, equipment and software produced by individual sectors to using industries. Hence, to obtain the investment ratio b_{ij}, I use the table to calculate the share of the value of commodities purchased by sector i from sector j in total investment made by sector i. For the TFP level and volatility processes, I take the estimates from the particle filter in Section 2.

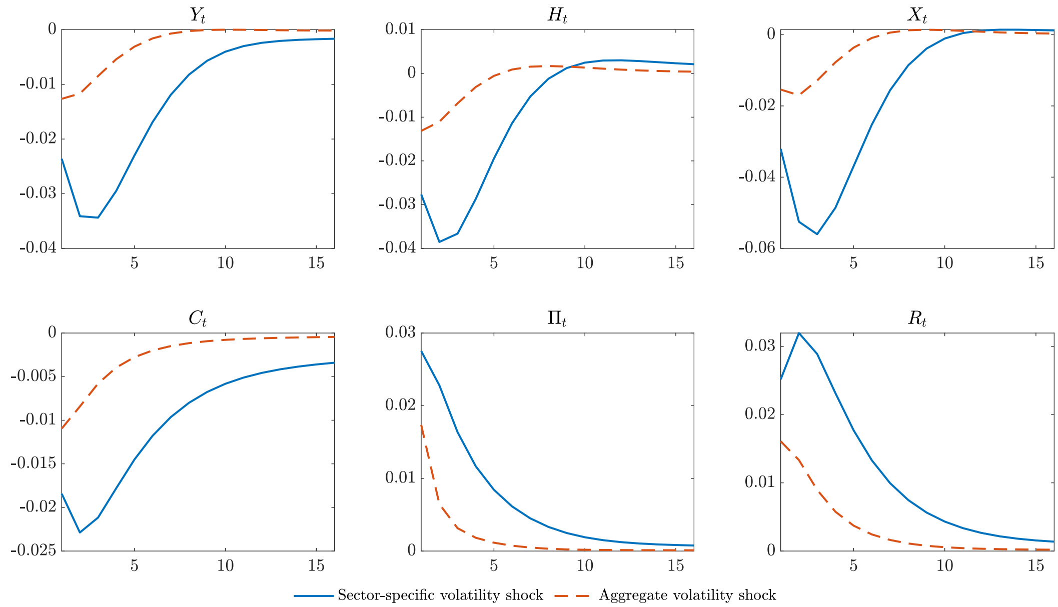
6 Results

I first analyze the impulse response functions (IRFs) of macro variables to an aggregate TFP volatility shock and to sector-specific TFP volatility shocks. To capture the cross-sectoral comovement of sector-specific TFP volatility innovations reported in Figure 1 and to give sector-specific volatility shocks a fair comparison relative to the aggregate volatility shock, I consider a single factor model on sectoral volatility innovations $\varepsilon_{\sigma_i,t}$:

$$\varepsilon_{\sigma_i,t} = \zeta_i F_t + \sigma_{v_i} v_{i,t}, \quad v_{i,t} \sim N(0,1), \quad i = 1, \ldots, n, \tag{34}$$

[6] To see how this works, consider $\mathbb{VAR}_t(\widehat{\boldsymbol{y}}_{t+1})$. Evaluated at the nonstochastic steady state, this vector of one-step-ahead variances is given by $(\mathbb{VAR}_t(\widehat{\boldsymbol{y}}_{t+1}))_{ss} = S\Sigma\Sigma' S'$. Similarly, the log deviation of $\mathbb{VAR}_t(\widehat{\boldsymbol{y}}_{t+1})$ from its nonstochastic steady state is given by $\widehat{\mathbb{VAR}_t(\widehat{\boldsymbol{y}}_{t+1})} = 2S\Sigma\widehat{\Sigma}_t\Sigma' S'$. These relationships allow me to characterize how the risk-adjustment terms evolve in response to volatility shocks.

Figure 4: Impulse responses to sector-specific and aggregate volatility shocks



Notes: The figure reports the IRFs to a two-standard-deviations shock to the common factor for sector-specific TFP volatility innovations (labeled "Sector-specific volatility shock") and to a two-standard-deviations shock to the aggregate TFP volatility. The unit is in percents. Inflation and the nominal interest rate are annualized.

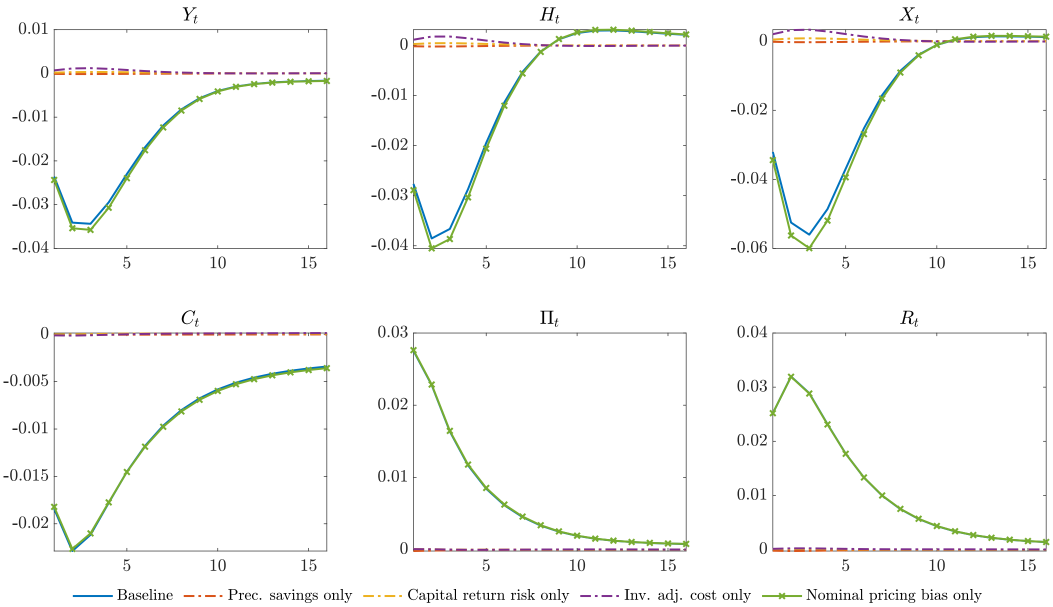
where ζ_i is the factor loading, F_t is the common factor, and $v_{i,t}$ is the idiosyncratic shock. In turn, F_t follows

$$F_t = \rho_F F_{t-1} + \sigma_F \varepsilon_{F,t}, \quad \varepsilon_{F,t} \sim N(0,1).$$

I estimate $\rho_F = 0.44$ and the R^2 for (34) is 0.20. As I show in the Appendix, the estimated factor F_t shows two spikes, one during the Great Recession and another one during the COVID-19 recession, consistent with the patterns observed in Figure 1. Figure 4 reports the IRF to a two-standard-deviation shock (a positive $\varepsilon_{F,t}$) to the common factor in sector-specific volatility.[7] Output, consumption, investment, and hours worked all drop after an increase in sectoral TFP volatility, while inflation and the nominal interest rate rise. The drop in economic activity is similar in magnitude to that in response to a policy uncertainty shock simulated on a structural model (Mumtaz and Zanetti (2013), Born and Pfeifer (2014), Fernández-Villaverde et al. (2015)). For comparison I also plot the IRF of a two-standard-deviation shock to the aggregate TFP volatility. While the IRF is qualitatively similar to that of the sector-specific TFP volatility shock, the magnitude is much smaller. Indeed the impact and cumulative IRFs for output are -0.01 percent and -0.04 percent, respectively, for the aggregate TFP volatility, while they are -0.02 percent and -0.20 percent, respectively, for the sector-specific TFP volatility.

[7] As is common in the uncertainty shock literature (e.g. Fernández-Villaverde et al. (2015)), I consider two-standard-deviation shocks to volatility.

Figure 5: Impulse responses to sector-specific volatility shocks: Endogenous risk wedges

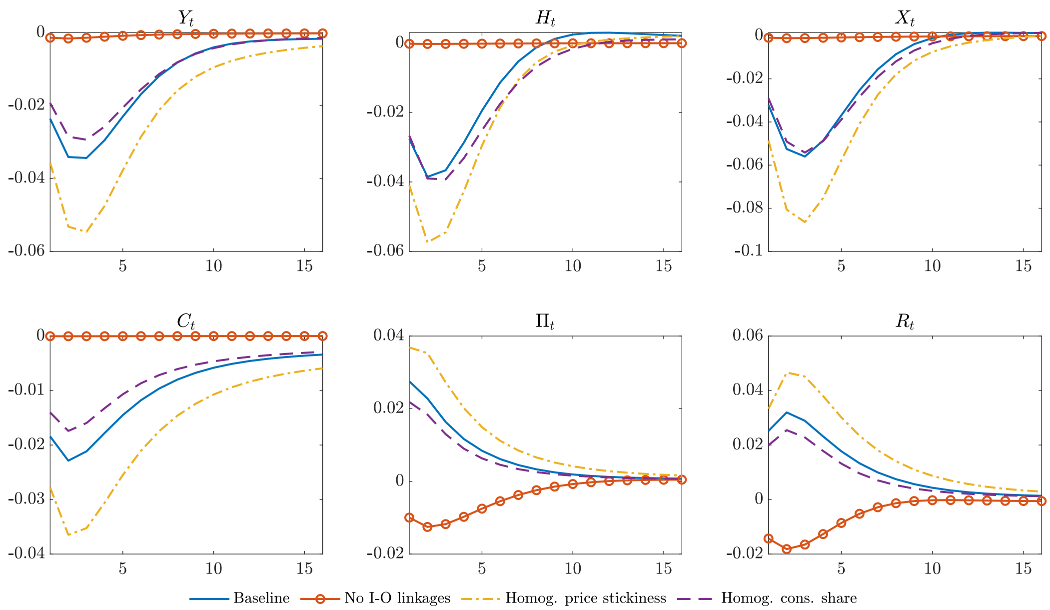


Notes: The figure reports the IRFs to a two-standard-deviations shock to the common factor for sector-specific TFP volatility innovations. Each line represents an IRF where all four endogenous risk wedges are activated ("Baseline"), only the precautionary savings wedge is activated ("Prec. savings only"), only the capital return risk wedge is activated ("Capital return risk only"), only the investment adjustment cost wedge is activated ("Inv. adj. cost only"), and only the nominal pricing bias wedge is activated ("Nominal pricing bias only"). The unit is in percents. Inflation and the nominal interest rate are annualized.

What is the mechanism driving the drop in economic activity and the rise in nominal variables to sector-specific volatility shocks? To understand this, in Figure 5 I compute counterfactual IRFs where I activate only one of the endogenous risk wedges identified in Section 5.1: precautionary savings (27), capital return risk (28), investment adjustment cost (29), and nominal pricing bias ((30) and (31)). Figure 5 shows that when only the nominal pricing bias is activated, the IRF is very similar to that of the baseline IRF where all four risk wedges are activated. In contrast, when only one of the other three risk wedges is activated, the IRFs show little movement. Thus, consistent with the theoretical analysis in Section 4, the nominal pricing bias channel is the dominant driver of the aggregate IRF to sector-specific volatility shocks.

The macroeconomic impact of sector-specific TFP volatility shocks depends on the input-output linkages and the distribution of sectoral characteristics such as price stickiness and preference shares along these linkages (see for instance (25) and (26)). To quantify the importance of these elements, in Figure 6 I report the IRFs to sector-specific volatility shocks when I perturb some features of the model. First, consider the case where I turn off the input-output network by setting $\sum_{j=1} a_{ij} = 0$ for all sector i while setting $B = I$. In this case, the IRF becomes negligible, un-

Figure 6: Counterfactual impulse responses to sector-specific volatility shocks



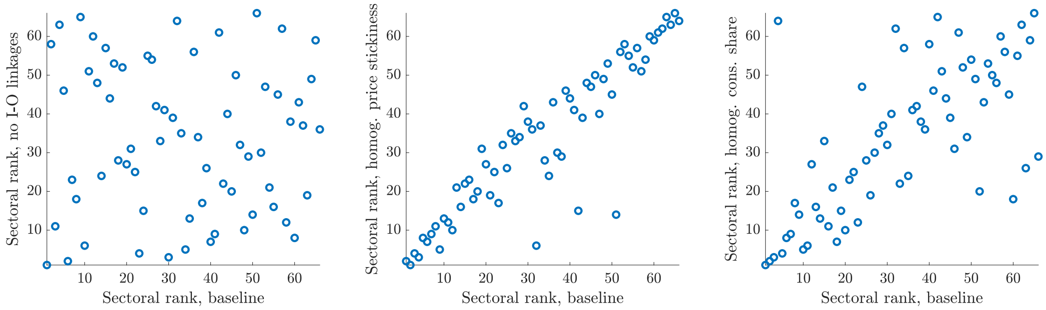
Notes: The figure reports the IRFs to a two-standard-deviations shock to the common factor for sector-specific TFP volatility innovations. Each line represents an IRF of the baseline model ("Baseline"), the model without input-output linkages ("No I-O linkages"), the model with homogenous price stickiness ("Homog. price stickiness"), and the model with homogeneous consumption preference share ("Homog. cons. share"). The unit is in percents. Inflation and the nominal interest rate are annualized.

derscoring the importance of the network precautionary pricing multiplier.[8] Next, consider the scenario where sectoral price stickiness is assumed to be homogeneous across all sectors, set equal to the average price stickiness calculated across all sectors. Interestingly, the homogenous price stickiness case generates a larger aggregate response relative to the baseline scenario; indeed, the cumulative IRFs to aggregate GDP is -0.35 percent so they are about twice as large compared to the baseline IRF. Third, in the case where the preference shares are assumed to be homogeneous across sectors, the IRF is similar to the baseline case.

Finally, I examine the role of sector identity in shaping the effects of volatility shocks and how this identity effect is influenced by the model's features. Specifically, I consider a two-standard-deviations sector-specific volatility shock for each sector and calculate the cumulative aggregate output response. I then rank the sectors based on their responsiveness, from the most to the least responsive. In other words, volatility shocks have a larger macroeconomic impact when they occur in the more responsive sectors. In Figure 7, I visualize how the sectoral rankings change as I vary the model specification. First, shutting down the input-output linkages alters the sectoral ranking dramatically. For instance, the top five most responsive sectors in the baseline case are "Securities, commodity contracts, and investments", "Oil and gas extraction", "Computer and electronic

[8] A similar result holds when $\sum_{j=1} a_{ij} = 0$ is imposed for all sectors i, but B is retained at its original calibration.

Figure 7: Ranking of sectors



Notes: The figure shows how the ranking of sectors, sorted based on the cumulative aggregate output responses to sector-specific TFP volatility shocks, change as I vary the model specification. The left panel compares the baseline model and the model without input-output linkages, the middle panel compares the baseline model and the model with homogeneous price stickiness, and the right panel compares the baseline model and the model with homogeneous consumption preference share.

products", "Housing", "Other real estate". When the linkages are turned off, the top five most responsive sectors become "Securities, commodity contracts, and investments", "Insurance carriers and related activities", "Apparel and leather and allied products", "Motor vehicle and parts dealers", "Ambulatory health care services", so only one out of five sectors remains in the top five ("Securities, commodity contracts, and investments"). In contrast, imposing homogeneous price stickiness or preference shares has smaller effects on the sectoral rankings. Indeed, both of these specifications result in four out of top five sectors in the baseline case remaining in the top five.

7 Conclusion

Recent macroeconomic experience indicates that a rise in uncertainty can exhibit substantial heterogeneity across sectors, with some experiencing significantly larger increases than others. Despite this observation, the role of time-varying sectoral uncertainty in driving aggregate fluctuations has been largely overlooked to date. Using both empirical analysis and a multi-sector New Keynesian model, I show that sector-specific volatility shocks can considerably affect aggregate outcomes. The interaction of nominal rigidities and input-output linkages gives rise to the network precautionary pricing multiplier that amplifies and propagates the sectoral uncertainty shocks throughout the economy. These findings highlight the need for policymakers to consider sector-specific uncertainty shocks when designing stabilization policies.

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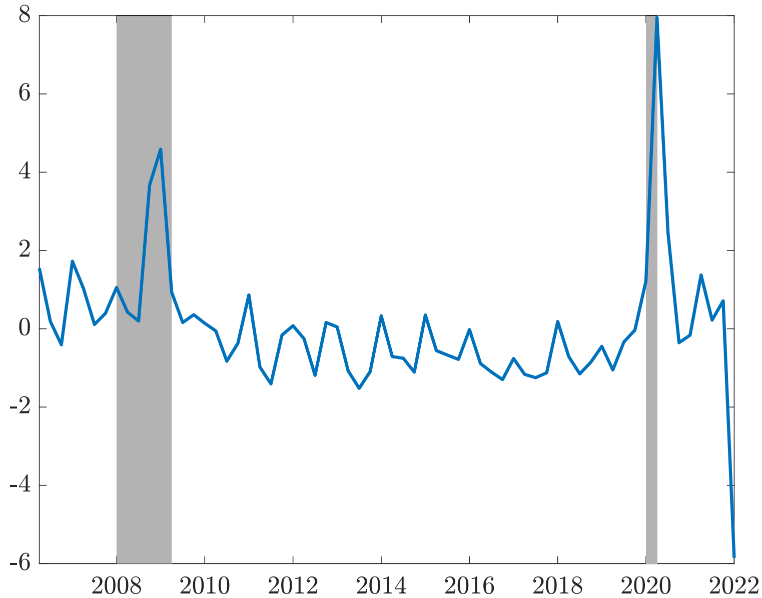
Appendices

A Additional Tables and Figures

Table A.1: List of sectors

1. Farms	34. Water transportation
2. Forestry, fishing, and related activities	35. Truck transportation
3. Oil and gas extraction	36. Transit and ground passenger transportation
4. Mining, except oil and gas	37. Pipeline transportation
5. Support activities for mining	38. Other transportation and support activities
6. Utilities	39. Warehousing and storage
7. Construction	40. Publishing industries, except internet (includes software)
8. Wood products	41. Motion picture and sound recording industries
9. Nonmetallic mineral products	42. Broadcasting and telecommunications
10. Primary metals	43. Data processing, internet publishing, and other information services
11. Fabricated metal products	44. Federal Reserve banks, credit intermediation, and related activities
12. Machinery	45. Securities, commodity contracts, and investments
13. Computer and electronic products	46. Insurance carriers and related activities
14. Electrical equipment, appliances, and components	47. Funds, trusts, and other financial vehicles
15. Motor vehicles, bodies and trailers, and parts	48. Housing
16. Other transportation equipment	49. Other real estate
17. Furniture and related products	50. Rental and leasing services and lessors of intangible assets
18. Miscellaneous manufacturing	51. Legal services
19. Food and beverage and tobacco products	52. Computer systems design and related services
20. Textile mills and textile product mills	53. Miscellaneous professional, scientific, and technical services
21. Apparel and leather and allied products	54. Management of companies and enterprises
22. Paper products	55. Administrative and support services
23. Printing and related support activities	56. Waste management and remediation services
24. Petroleum and coal products	57. Educational services
25. Chemical products	58. Ambulatory health care services
26. Plastics and rubber products	59. Hospitals
27. Wholesale trade	60. Nursing and residential care facilities
28. Motor vehicle and parts dealers	61. Social assistance
29. Food and beverage stores	62. Performing arts, spectator sports, museums, and related activities
30. General merchandise stores	63. Amusements, gambling, and recreation industries
31. Other retail	64. Accommodation
32. Air transportation	65. Food services and drinking places
33. Rail transportation	66. Other services, except government

Figure A.1: Time series of the common factor F_t



Notes: The figure reports the time series of the common factor F_t of sector-specific TFP volatility innovations, given by (34) in the main text. The shaded areas are the NBER recession dates.

B Details on the Simplified Model

B.1 Equilibrium Conditions

1. Marginal utility of consumption:

$$\lambda_t = C_t^{-1} \tag{B.1}$$

2. Labor supply condition:

$$\lambda_t w_{i,t} = 1, \quad i = 1, \ldots, n \tag{B.2}$$

3. Bond Euler equation:

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right] \tag{B.3}$$

4. Consumption demand:

$$c_{i,t} = \omega_i \left(p_{i,t}\right)^{-\varepsilon_c} C_t, \quad i = 1, \ldots, n \tag{B.4}$$

5. Sectoral price index:

$$1 = \left[\sum_{i=1}^{n} \omega_i p_{i,t}^{1-\varepsilon_c} \right]^{\frac{1}{1-\varepsilon_c}} \tag{B.5}$$

6. Real marginal cost:

$$mc_{i,t} = \frac{1}{e^{z_{i,t}}} \left(\frac{1}{\alpha_i^h} \right)^{\alpha_i^h} \left(\frac{1}{1-\alpha_i^h} \right)^{1-\alpha_i^h} (w_{i,t})^{\alpha_i^h} (p_{i,t}^m)^{1-\alpha_i^h}, \quad i = 1, \ldots, n \tag{B.6}$$

7. Labor demand:

$$w_{i,t} = mc_{i,t} \alpha_i^h \frac{y_{i,t}(l)}{h_{i,t}(l)}, \quad i = 1, \ldots, n \tag{B.7}$$

8. Intermediate inputs demand (1):

$$p_{i,t}^m = mc_{i,t}(1-\alpha_i^h) \frac{y_{i,t}(l)}{\widetilde{m}_{i,t}(l)}, \quad i, j = 1, \ldots, n \tag{B.8}$$

9. Intermediate inputs demand (2):

$$p_{i,t}^m = \left[\sum_{j=1}^{n} a_{ij} p_{j,t}^{1-\varepsilon_m} \right]^{\frac{1}{1-\varepsilon_m}}, \quad i = 1, \ldots, n \tag{B.9}$$

10. Intermediate inputs demand (3):

$$m_{ij,t} = a_{ij} \left(\frac{p_{j,t}}{p_{i,t}^m} \right)^{-\varepsilon_m} \widetilde{m}_{i,t}, \quad i, j = 1, \ldots, n \tag{B.10}$$

11. Sectoral output and input:
$$\widetilde{y}_{i,t} = \Delta_{i,t} y_{i,t}, \quad i = 1, \ldots, n, \tag{B.11}$$
where $\widetilde{y}_{i,t} = \int_0^1 y_{i,t}(l)dl$ and $\Delta_{i,t} \equiv \int_0^1 \left(\frac{P_{i,t}(l)}{P_{i,t}}\right)^{-\theta} dl$.

12. Law of motion for price dispersion:
$$\Delta_{i,t} = (1-\widetilde{\xi}_i)\left(\frac{P^f_{i,t}}{P_{i,t}}\right)^{-\theta} + \widetilde{\xi}_i \left(\frac{P^s_{i,t}}{P_{i,t}}\right)^{-\theta}, \quad i = 1, \ldots, n \tag{B.12}$$

13. Sectoral resource constraint:
$$y_{i,t} = c_{i,t} + \sum_{j=1}^{n} m_{ji,t}, \quad i = 1, \ldots, n \tag{B.13}$$

14. Sectoral GDP:
$$GDP_{i,t} = p_{i,t} y_{i,t} - p^m_{i,t} \widetilde{m}_{i,t}, \quad i = 1, \ldots, n \tag{B.14}$$

15. Nominal marginal cost:
$$MC_{i,t} = P_t mc_{i,t}, \quad i = 1, \ldots, n \tag{B.15}$$

16. Flexible firms pricing:
$$P^f_{i,t} = \left(\frac{\theta}{\theta - 1}\right) MC_{i,t}, \quad i = 1, \ldots, n \tag{B.16}$$

17. Sticky firms pricing:
$$\mathbb{E}_{t-1}\left[\frac{P^s_{i,t}}{P_t} - \left(\frac{\theta}{\theta-1}\right)\frac{MC_{i,t}}{P_t}\right] = 0, \quad i = 1, \ldots, n \tag{B.17}$$

18. Nominal sectoral price:
$$P_{i,t} = \left[(1-\widetilde{\xi}_i)(P^f_{i,t})^{1-\theta} + \widetilde{\xi}_i (P^s_{i,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}, \quad i = 1, \ldots, n \tag{B.18}$$

19. Real sectoral price:
$$p_{i,t} = \frac{P_{i,t}}{P_t}, \quad i = 1, \ldots, n \tag{B.19}$$

20. Sectoral inflation:
$$\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}, \quad i = 1, \ldots, n \tag{B.20}$$

21. Aggregate inflation:
$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{B.21}$$

22. Aggregate resource constraint:

$$C_t = Y_t \tag{B.22}$$

23. Monetary policy:

$$P_t Y_t = \overline{Y} \tag{B.23}$$

B.2 Risk-adjusted Log-linearized Equilibrium Conditions

1. Marginal utility of consumption:

$$\widehat{\lambda}_t = -\widehat{C}_t \tag{B.24}$$

2. Labor supply condition:

$$\widehat{\lambda}_t + \widehat{w}_{i,t} = 0, \quad i = 1, \dots, n \tag{B.25}$$

3. Bond Euler equation:

$$\widehat{\lambda}_t = \widehat{R}_t + \mathbb{E}_t \widehat{\lambda}_{t+1} - \mathbb{E}_t \widehat{\Pi}_{t+1} + \frac{1}{2} \mathbb{VAR}_t(\widehat{\lambda}_{t+1}) + \frac{1}{2} \mathbb{VAR}_t(\widehat{\Pi}_{t+1}) - \mathbb{COV}_t(\widehat{\lambda}_{t+1}, \widehat{\Pi}_{t+1}) \tag{B.26}$$

4. Consumption demand:

$$\varepsilon_c \widehat{p}_{i,t} + \widehat{c}_{i,t} = \widehat{C}_t, \quad i = 1, \dots, n \tag{B.27}$$

5. Sectoral price index:

$$0 = \sum_{i=1}^{n} \omega_i \widehat{p}_{i,t} \tag{B.28}$$

6. Real marginal cost:

$$\widehat{mc}_{i,t} = -z_{i,t} + \alpha_i^h \widehat{w}_{i,t} + (1 - \alpha_i^h) \widehat{p}^m_{i,t}, \quad i = 1, \dots, n \tag{B.29}$$

7. Labor demand:

$$\widehat{w}_{i,t} = \widehat{mc}_{i,t} + \widehat{y}_{i,t}(l) - \widehat{h}_{i,t}(l), \quad i = 1, \dots, n \tag{B.30}$$

8. Intermediate inputs demand (1):

$$\widehat{p}^m_{i,t} = \widehat{mc}_{i,t} + \widehat{y}_{i,t}(l) - \widehat{\widehat{m}}_{i,t}(l), \quad i, j = 1, \dots, n \tag{B.31}$$

9. Intermediate inputs demand (2):

$$\widehat{p}^m_{i,t} = \sum_{j=1}^{n} a_{ij} \widehat{p}_{j,t}, \quad i = 1, \dots, n \tag{B.32}$$

10. Intermediate inputs demand (3):

$$\widehat{m}_{ij,t} = -\varepsilon_m \left(p_{j,t} - p^m_{i,t}\right) + \widehat{\widetilde{m}}_{i,t}, \quad i,j = 1,\ldots,n \tag{B.33}$$

11. Sectoral output and input:

$$\widehat{\widetilde{y}}_{i,t} = \widehat{\Delta}_{i,t} + \widehat{y}_{i,t}, \quad i = 1,\ldots,n \tag{B.34}$$

12. Law of motion for price dispersion:

$$\widehat{\Delta}_{i,t} = -\theta(1-\widetilde{\xi}_i)\left(\widehat{P}^f_{i,t} - \widehat{P}_{i,t}\right) - \theta\widetilde{\xi}_i\left(\widehat{P}^s_{i,t} - \widehat{P}_{i,t}\right), \quad i = 1,\ldots,n \tag{B.35}$$

13. Sectoral resource constraint:

$$\widehat{y}_{i,t} = \frac{c_i}{y_i}\widehat{c}_{i,t} + \sum_{j=1}^{n}\frac{m_{ji}}{y_i}\widehat{m}_{ji,t}, \quad i = 1,\ldots,n \tag{B.36}$$

14. Sectoral GDP:

$$\widehat{GDP}_{i,t} = \frac{p_i y_i}{GDP_i}\left(\widehat{p}_{i,t} + \widehat{y}_{i,t}\right) - \frac{p^m_i \widetilde{m}_i}{GDP_i}\left(\widehat{p}^m_{i,t} + \widehat{\widetilde{m}}_{i,t}\right), \quad i = 1,\ldots,n \tag{B.37}$$

15. Nominal marginal cost:

$$\widehat{MC}_{i,t} = \widehat{P}_t + \widehat{mc}_{i,t}, \quad i = 1,\ldots,n \tag{B.38}$$

16. Flexible firms pricing:

$$\widehat{P}^f_{i,t} = \widehat{MC}_{i,t}, \quad i = 1,\ldots,n \tag{B.39}$$

17. Sticky firms pricing:

$$\widehat{P}^s_{i,t} = \mathbb{E}_{t-1}[\widehat{MC}_{i,t}] + \frac{1}{2}\mathbb{VAR}_{t-1}[\widehat{MC}_{i,t}] - \mathbb{COV}_{t-1}[\widehat{MC}_{i,t}, \widehat{P}_t], \quad i = 1,\ldots,n \tag{B.40}$$

18. Nominal sectoral price:

$$\widehat{P}_{i,t} = (1-\widetilde{\xi}_i)\widehat{P}^f_{i,t} + \widetilde{\xi}_i\widehat{P}^s_{i,t}, \quad i = 1,\ldots,n \tag{B.41}$$

19. Real sectoral price:

$$\widehat{p}_{i,t} = \widehat{P}_{i,t} - \widehat{P}_t, \quad i = 1,\ldots,n \tag{B.42}$$

20. Sectoral inflation:

$$\widehat{\pi}_{i,t} = \widehat{P}_{i,t} - \widehat{P}_{i,t-1}, \quad i = 1,\ldots,n \tag{B.43}$$

21. Aggregate inflation:

$$\widehat{\Pi}_t = \widehat{P}_t - \widehat{P}_{t-1} \tag{B.44}$$

22. Aggregate resource constraint:
$$\widehat{C}_t = \widehat{Y}_t \tag{B.45}$$

23. Monetary policy:
$$\widehat{P}_t + \widehat{Y}_t = 0 \tag{B.46}$$

B.3 Equilibrium Law of Motion

Using (B.24), (B.44), (B.45), and (B.46) and substituting into (B.26), the Euler equation becomes $\widehat{R}_t = 0$, which gives the equilibrium nominal interest rate that supports (B.46).

From (B.24), (B.25), (B.45), and (B.46), substitute $\widehat{w}_{i,t} = \widehat{C}_t = -\widehat{P}_t$ into (B.29) and (B.32):

$$\widehat{mc}_{i,t} = -z_{i,t} - \alpha_i^h \widehat{P}_t + (1-\alpha_i^h)\sum_{j=1}^{n} a_{ij}\widehat{p}_{j,t},$$

which using (B.38) and (B.42), becomes

$$\widehat{MC}_{i,t} = -z_{i,t} + (1-\alpha_i^h)\sum_{j=1}^{n} a_{ij}\widehat{P}_{j,t}. \tag{B.47}$$

Denote $\widehat{\boldsymbol{MC}}_t \equiv \left[\widehat{MC}_{1,t}, \ldots, \widehat{MC}_{n,t}\right]'$ then (B.47) is written as

$$\widehat{\boldsymbol{MC}}_t = -\boldsymbol{z}_t + \widetilde{A}\widehat{\boldsymbol{P}}_t, \tag{B.48}$$

where $\boldsymbol{z}_t \equiv [z_{1,t}, \ldots, z_{n,t}]'$, $\widehat{\boldsymbol{P}}_t \equiv [\widehat{P}_{1,t}, \ldots, \widehat{P}_{n,t}]'$, and $\widetilde{A} \equiv (\iota - \alpha^h)A$, where $\alpha^h \equiv [\alpha_1^h, \ldots, \alpha_n^h]'$ and ι is a $n \times 1$ vector of 1's.

As a first step, I solve for the policy rules without risk adjustment. Using $\mathbb{E}_{t-1}\left[\widehat{\boldsymbol{MC}}_t\right] = 0$,

$$\begin{aligned}\widehat{\boldsymbol{P}}_t &= \Xi\mathbb{E}_{t-1}\left[\widehat{\boldsymbol{MC}}_t\right] + (I-\Xi)\,\widehat{\boldsymbol{MC}}_t \\ &= (I-\Xi)\left[-\boldsymbol{z}_t + \widetilde{A}\widehat{\boldsymbol{P}}_t\right],\end{aligned} \tag{B.49}$$

so we have
$$\widehat{\boldsymbol{P}}_t = -L\,(I-\Xi)\,\boldsymbol{z}_t. \tag{B.50}$$

where $L \equiv \left[I - (I-\Xi)\,\widetilde{A}\right]^{-1}$. Substitute (B.50) into (B.48) to obtain

$$\widehat{\boldsymbol{MC}}_t = -\left[I + (\iota - \alpha^h)AL\,(I-\Xi)\right]\boldsymbol{z}_t. \tag{B.51}$$

Next, consider policy rules with risk adjustment. Note we have

$$\widehat{\mathbb{VAR}_{t-1}\left[\widehat{\boldsymbol{MC}}_t\right]} = 2\left[I + \widetilde{A}L\left(I - \Xi\right)\right] \circ \left[I + \widetilde{A}L\left(I - \Xi\right)\right]\Sigma_\sigma \boldsymbol{\sigma}_t, \tag{B.52}$$

$$\widehat{\mathbb{COV}_{t-1}\left[\widehat{\boldsymbol{MC}}_t, \widehat{P}_t\right]} = 2\left[I + \widetilde{A}L\left(I - \Xi\right)\right] \circ \left[\iota\boldsymbol{\omega}' L\left(I - \Xi\right)\right]\Sigma_\sigma \boldsymbol{\sigma}_t, \tag{B.53}$$

where $\circ$ denotes the element-wise product (Hadamard product) of two matrices and we denote

$$\widehat{\mathbb{VAR}_{t-1}\left[\widehat{\boldsymbol{MC}}_t\right]} \equiv \left[\widehat{\mathbb{VAR}_{t-1}\left[\widehat{MC}_{1,t}\right]}, \ldots, \widehat{\mathbb{VAR}_{t-1}\left[\widehat{MC}_{n,t}\right]}\right]', \tag{B.54}$$

$$\widehat{\mathbb{COV}_{t-1}\left[\widehat{\boldsymbol{MC}}_t, \widehat{P}_t\right]} \equiv \left[\widehat{\mathbb{COV}_{t-1}\left[\widehat{MC}_{1,t}, \widehat{P}_t\right]}, \ldots, \widehat{\mathbb{COV}_{t-1}\left[\widehat{MC}_{n,t}, \widehat{P}_t\right]}\right]'. \tag{B.55}$$

Note that $\widehat{\mathbb{VAR}_{t-1}\left[\widehat{MC}_{i,t}\right]}$ and $\widehat{\mathbb{COV}_{t-1}\left[\widehat{MC}_{i,t}, \widehat{\boldsymbol{P}}_t\right]}$ are log-deviations of $\mathbb{VAR}_{t-1}\left[\widehat{MC}_{i,t}\right]$ and $\mathbb{COV}_{t-1}\left[\widehat{MC}_{i,t}, \widehat{\boldsymbol{P}}_t\right]$ from their steady states, respectively and ι is an $n \times 1$ vector of 1's.

Substitute (B.38), (B.39), (B.40), (B.54), and (B.55) into (B.41):

$$\begin{aligned}\widehat{\boldsymbol{P}}_t =& \Xi\left\{\left[I + \widetilde{A}L\left(I - \Xi\right)\right] \circ \left[I + \widetilde{A}L\left(I - \Xi\right)\right] - 2\left[I + \widetilde{A}L\left(I - \Xi\right)\right] \circ \left[\iota\boldsymbol{\omega}' L\left(I - \Xi\right)\right]\right\}\Sigma_\sigma \boldsymbol{\sigma}_t \\ &+ \left(I - \Xi\right)\left[-\boldsymbol{z}_t + \widetilde{A}\widehat{\boldsymbol{P}}_t\right].\end{aligned}$$

Solving for $\widehat{\boldsymbol{P}}_t$ gives us (21).

C Details on the Quantitative Model

C.1 Equilibrium Conditions

1. Marginal utility of consumption:

$$\lambda_t = C_t^{-\sigma} \tag{C.56}$$

2. Labor supply condition:

$$\lambda_t w_{i,t} = H_t^{-\frac{1}{\nu}} h_{i,t}^{\frac{1}{\nu}}, \quad i = 1, \ldots, n \tag{C.57}$$

3. Total hours worked:

$$H_t = \left[\sum_{i=1}^{n} h_{i,t}^{\frac{1+\nu}{\nu}}\right]^{\frac{\nu}{1+\nu}} \tag{C.58}$$

4. Bond Euler equation:

$$\lambda_t = \beta \mathbb{E}_t\left[\lambda_{t+1}\frac{R_t}{\Pi_{t+1}}\right] \tag{C.59}$$

5. Capital Euler equation:

$$\mu_{i,t} = \beta \mathbb{E}_t \left[\lambda_{t+1} mc_{i,t+1} \alpha_i^k \frac{y_{i,t+1}}{k_{i,t}} + \mu_{i,t+1}(1-\delta) \right], \quad i = 1, \ldots, n \tag{C.60}$$

6. Investment optimality condition:

$$\begin{aligned} \lambda_t p_{i,t}^q = \mu_{i,t} & \left\{ 1 - \frac{\kappa}{2} \left(\frac{x_{i,t}}{x_{i,t-1}} - 1 \right)^2 - \kappa \left(\frac{x_{i,t}}{x_{i,t-1}} - 1 \right) \frac{x_{i,t}}{x_{i,t-1}} \right\} \\ & + \beta \kappa \mathbb{E}_t \left\{ \mu_{i,t+1} \left(\frac{x_{i,t+1}}{x_{i,t}} - 1 \right) \left(\frac{x_{i,t+1}}{x_{i,t}} \right)^2 \right\}, \quad i, j = 1, \ldots, n \end{aligned} \tag{C.61}$$

7. Sectoral investment growth:

$$\Delta x_{i,t} = \frac{x_{i,t}}{x_{i,t-1}}, \quad i = 1, \ldots, n \tag{C.62}$$

8. Capital accumulation equation:

$$k_{i,t} = (1-\delta) k_{i,t-1} + \left\{ 1 - \frac{\kappa}{2} \left(\frac{x_{i,t}}{x_{i,t-1}} - 1 \right)^2 \right\} x_{i,t}, \quad i = 1, \ldots, n \tag{C.63}$$

9. Consumption demand:

$$c_{i,t} = \omega_i \left(p_{i,t} \right)^{-\varepsilon_c} C_t, \quad i = 1, \ldots, n \tag{C.64}$$

10. Sectoral price index:

$$1 = \left[\sum_{i=1}^{n} \omega_i p_{i,t}^{1-\varepsilon_c} \right]^{\frac{1}{1-\varepsilon_c}} \tag{C.65}$$

11. Production function:

$$y_{i,t}(l) = e^{z_{i,t}} (k_{i,t}(l))^{\alpha_i^k} (h_{i,t}(l))^{\alpha_i^h} (\widetilde{m}_{i,t}(l))^{1-\alpha_i^k-\alpha_i^h}, \quad i = 1, \ldots, n \tag{C.66}$$

12. Labor demand:

$$w_{i,t} = mc_{i,t} \alpha_i^h \frac{y_{i,t}(l)}{h_{i,t}(l)}, \quad i = 1, \ldots, n \tag{C.67}$$

13. Intermediate inputs demand (1):

$$p_{i,t}^m = mc_{i,t} (1 - \alpha_i^k - \alpha_i^h) \frac{y_{i,t}(l)}{\widetilde{m}_{i,t}(l)}, \quad i, j = 1, \ldots, n \tag{C.68}$$

14. Intermediate inputs demand (2):

$$p^m_{i,t} = \left[\sum_{j=1}^{n} a_{ij} p_{j,t}^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}}, \quad i = 1, \ldots, n \tag{C.69}$$

15. Intermediate inputs demand (3):

$$m_{ij,t} = a_{ij} \left(\frac{p_{j,t}}{p^m_{i,t}}\right)^{-\varepsilon_m} \widetilde{m}_{i,t}, \quad i, j = 1, \ldots, n \tag{C.70}$$

16. Investment inputs demand (1):

$$p^q_{i,t} = \left[\sum_{j=1}^{n} b_{ij} p_{j,t}^{1-\varepsilon_q}\right]^{\frac{1}{1-\varepsilon_q}}, \quad i = 1, \ldots, n \tag{C.71}$$

17. Investment inputs demand (2):

$$q_{ij,t} = b_{ij} \left(\frac{p_{j,t}}{p^q_{i,t}}\right)^{-\varepsilon_q} x_{i,t}, \quad i, j = 1, \ldots, n \tag{C.72}$$

18. Sectoral output and input:

$$\widetilde{y}_{i,t} = \Delta_{i,t} y_{i,t}, \quad i = 1, \ldots, n, \tag{C.73}$$

where $\widetilde{y}_{i,t} = \int_0^1 y_{i,t}(l) dl$ and $\Delta_{i,t} \equiv \int_0^1 \left(\frac{P_{i,t}(l)}{P_{i,t}}\right)^{-\theta} dl$.

19. Law of motion for price dispersion:

$$\Delta_{i,t} = (1 - \xi_i) \left(\frac{p^*_{i,t}}{p_{i,t}}\right)^{-\theta} + \xi_i \left(\frac{\Pi}{\pi_{i,t}}\right)^{-\theta}, \quad i = 1, \ldots, n \tag{C.74}$$

20. Sectoral resource constraint:

$$y_{i,t} = c_{i,t} + \sum_{j=1}^{n} m_{ji,t} + \sum_{j=1}^{n} q_{ji,t}, \quad i = 1, \ldots, n \tag{C.75}$$

21. Sectoral GDP:

$$GDP_{i,t} = p_{i,t} y_{i,t} - p^m_{i,t} \widetilde{m}_{i,t}, \quad i = 1, \ldots, n \tag{C.76}$$

22. Optimal reset price (I):

$$p^*_{i,t} = \left(\frac{\theta}{\theta - 1}\right) \frac{P^n_{i,t}}{P^d_{i,t}}, \quad i = 1, \ldots, n \tag{C.77}$$

23. Optimal reset price (II):

$$P_{i,t}^{n} = \lambda_t mc_{i,t} y_{i,t} + \xi_i \beta \mathbb{E}_t \left[\left(\frac{\pi_{i,t+1}}{\Pi} \right)^{\theta} P_{i,t+1}^{n} \right], \quad i = 1, \ldots, n \tag{C.78}$$

24. Optimal reset price (III):

$$P_{i,t}^{d} = \lambda_t y_{i,t} + \xi_i \beta \mathbb{E}_t \left[\left(\frac{\pi_{i,t+1}}{\Pi} \right)^{\theta-1} \left(\frac{\pi_{i,t+1}}{\Pi_{t+1}} \right) P_{i,t+1}^{d} \right], \quad i = 1, \ldots, n \tag{C.79}$$

25. Law of motion for sectoral price level:

$$1 = (1 - \xi_i) \left(\frac{p_{i,t}^{*}}{p_{i,t}} \right)^{1-\theta} + \xi_i \left(\frac{\Pi}{\pi_{i,t}} \right)^{1-\theta}, \quad i = 1, \ldots, n \tag{C.80}$$

26. Sectoral inflation:

$$\pi_{i,t} = \frac{p_{i,t}}{p_{i,t-1}} \Pi_t, \quad i = 1, \ldots, n \tag{C.81}$$

27. Aggregate resource constraint:

$$C_t + X_t = Y_t \tag{C.82}$$

28. Total investment:

$$X_t = \sum_{i=1}^{n} p_{i,t}^{q} x_{i,t} \tag{C.83}$$

29. Monetary policy:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left\{ \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} \left(\frac{Y_t}{Y} \right)^{\phi_Y} \right\}^{1-\rho_R} \tag{C.84}$$

C.2 Solving for the Steady State

To solve for the steady state, I first normalize sectoral prices $p_i = 1, i, \ldots, n$. This in turn implies $p_i^m = p_i^q = 1$ from (C.69) and (C.71). In addition, I normalize to scale of the economy by setting the steady-state aggregate consumption to $C = 1$. This immediately implies, from (C.56), $\ln \lambda = 0$. Finally, I guess the steady-state sectoral hours as $\widetilde{h}_i, i, \ldots, n$. Given the guess of the sectoral hours, the guessed aggregate hours is given by $\widetilde{H} = \left[\sum_{i=1}^{n} \widetilde{h}_i^{\frac{1+\nu}{\nu}} \right]^{\frac{\nu}{1+\nu}}$ using (C.58).

Evaluating (C.61) at the steady state, we have

$$\underset{n\times 1}{\mathbf{1}} \ln \lambda = \ln \boldsymbol{\mu} = \underset{n\times 1}{\mathbf{0}}, \tag{C.85}$$

where $\boldsymbol{\mu} \equiv [\mu_1, \ldots, \mu_n]'$. From (C.64), we have

$$c_i = \omega_i,$$

where in a vector form can be written as

$$\boldsymbol{c} = \boldsymbol{\omega}. \tag{C.86}$$

Evaluating (C.77), (C.78), and (C.79) at the steady states, the real marginal cost can be expressed as

$$mc_i = \frac{\theta - 1}{\theta}. \tag{C.87}$$

Substituting (C.87) into (C.60), we have

$$\frac{1-\beta(1-\delta)}{\beta\left(\frac{\theta-1}{\theta}\right)}\mu_i = \lambda\alpha_i^k \frac{y_i}{k_i},$$

which taking logs and re-arranging and using (C.85), gives the expression for sectoral capital in a vector form:

$$\ln \boldsymbol{k} = \ln \boldsymbol{\alpha^k} + \ln \boldsymbol{y} - \underset{n\times 1}{\mathbf{1}} \ln\left[\frac{1-\beta(1-\delta)}{\beta\left(\frac{\theta-1}{\theta}\right)}\right], \tag{C.88}$$

where $\boldsymbol{k} = [k_1, \ldots, k_n]'$, $\boldsymbol{\alpha^k} = [\alpha_1^k, \ldots, \alpha_n^k]'$, and $\boldsymbol{y} = [y_1, \ldots, y_n]'$.

Evaluating (C.57) and (C.67) at the steady state and also using (C.87), the sectoral hours in a vector form, given the guessed $\widetilde{h}_i$ and $\widetilde{H}$ can be expressed as

$$\ln \boldsymbol{h} = \underset{n\times 1}{\mathbf{1}} \ln\left(\frac{1-\theta}{\theta}\right) + \ln \boldsymbol{\alpha^h} + \ln \boldsymbol{y} + \frac{1}{\nu}\underset{n\times 1}{\mathbf{1}} \ln \widetilde{H} - \frac{1}{\nu}\ln \widetilde{\boldsymbol{h}}, \tag{C.89}$$

where $\boldsymbol{h} = [h_1, \ldots, h_n]'$, $\boldsymbol{\alpha^h} = [\alpha_1^h, \ldots, \alpha_n^h]'$, and $\widetilde{\boldsymbol{h}} = [\widetilde{h}_1, \ldots, \widetilde{h}_n]'$.

Evaluating (C.68) at the steady state and also using (C.87), the intermediate input demand can be expressed as

$$\ln \widetilde{\boldsymbol{m}} = \underset{n\times 1}{\mathbf{1}} \ln\left(\frac{\theta-1}{\theta}\right) + \ln\left(\underset{n\times 1}{\mathbf{1}} - \boldsymbol{\alpha^k} - \boldsymbol{\alpha^h}\right) + \ln \boldsymbol{y}, \tag{C.90}$$

where $\widetilde{\boldsymbol{m}} = [\widetilde{m}_1, \ldots, \widetilde{m}_n]'$.

Evaluating (C.66) at the steady state and taking logs, we can express the production function in a vector form:

$$\ln \boldsymbol{y} = \boldsymbol{z} + \alpha_d^k \ln \boldsymbol{k} + \alpha_d^h \ln \boldsymbol{h} + (\boldsymbol{I} - \alpha_d^k - \alpha_d^h)\ln \widetilde{\boldsymbol{m}}, \tag{C.91}$$

where $\boldsymbol{z} = [z_1, \ldots, z_n]'$, $\alpha_d^k = diag(\boldsymbol{\alpha^k})$, and $\alpha_d^h = diag(\boldsymbol{\alpha^h})$.

Substitute (C.88), (C.89), and (C.90) into (C.91) and rearranging,

$$\begin{aligned} &\underbrace{\left(\boldsymbol{I} - \alpha_d^k - \alpha_d^h - (\boldsymbol{I} - \alpha_d^k - \alpha_d^h)\right)}_{=\mathbf{0}} \ln \boldsymbol{y} = \boldsymbol{z} + \mathbf{1}\ln\left(\frac{1-\theta}{\theta}\right) \\ &+ \underbrace{\left[\alpha_d^k\left(\ln \boldsymbol{\alpha^k} - \mathbf{1}\ln\left[\frac{1-\beta(1-\delta)}{\beta}\right]\right) + \alpha_d^h\left(\ln \boldsymbol{\alpha^h} + \frac{1}{\nu}\mathbf{1}\ln \widetilde{H} - \frac{1}{\nu}\ln \widetilde{\boldsymbol{h}}\right) + (\boldsymbol{I} - \alpha_d^k - \alpha_d^h)\ln(\mathbf{1} - \boldsymbol{\alpha^k} - \boldsymbol{\alpha^h})\right]}_{\equiv(\mathrm{A})}. \end{aligned} \tag{C.92}$$

Thus we can express sectoral productivity as a function of guessed $\widetilde{\boldsymbol{h}}$:

$$\boldsymbol{z} = -\left(\mathbf{1}\ln\left(\frac{1-\theta}{\theta}\right) + (A)\right).$$

Evaluating (C.75) at the steady state and substituting in (C.68), (C.70), and (C.72) we have

$$\boldsymbol{c} + M_{my}\boldsymbol{y} + B'\boldsymbol{x} = \boldsymbol{y}, \tag{C.93}$$

where $\boldsymbol{x} = [x_1, \ldots, x_n]'$ and

$$M_{my} = \begin{bmatrix} \left(\frac{\theta-1}{\theta}\right)(1-\alpha_1^k-\alpha_1^h)a_{11} & \ldots & \left(\frac{\theta-1}{\theta}\right)(1-\alpha_n^k-\alpha_n^h)a_{n1} \\ \vdots & \ddots & \vdots \\ \left(\frac{\theta-1}{\theta}\right)(1-\alpha_1^k-\alpha_1^h)a_{1n} & \ldots & \left(\frac{\theta-1}{\theta}\right)(1-\alpha_n^k-\alpha_n^h)a_{nn} \end{bmatrix}.$$

Take an exponential on (C.88):

$$k_i = \underbrace{\exp\left(\ln\alpha_i^k - \ln\left[\frac{1-\beta(1-\delta)}{\beta\left(\frac{\theta-1}{\theta}\right)}\right]\right)}_{\equiv \boldsymbol{K}_i} y_i,$$

and using (C.63),

$$\boldsymbol{x} = \delta\boldsymbol{k} = \delta M_k \boldsymbol{y},$$

where $M_k = diag\left([\boldsymbol{K}_1, \ldots, \boldsymbol{K}_n]'\right)$. Substitute this into (C.93) to obtain y as a function of parameters and guessed $\widetilde{h}$:

$$\boldsymbol{y} = \left(\boldsymbol{I} - M_{my} - \delta B' M_k\right)^{-1}\boldsymbol{c}.$$

Finally, substitute in $\ln\boldsymbol{y}$ into the right hand side of (C.89) to obtain sectoral hours as a function of parameters and guessed $\widetilde{\boldsymbol{h}}$. Denoting the new sectoral hours as $\boldsymbol{h}'$, stop if $\boldsymbol{h}' \approx \widetilde{\boldsymbol{h}}$. Otherwise set $\widetilde{\boldsymbol{h}} = \boldsymbol{h}'$ and repeat until convergence.

C.3 Risk-adjusted Log-linearized Equilibrium Conditions

1. Marginal utility of consumption:

$$\widehat{\lambda}_t = -\sigma\widehat{C}_t \tag{C.94}$$

2. Labor supply condition:

$$\widehat{\lambda}_t + \widehat{w}_{i,t} = -\frac{1}{\nu}\widehat{H}_t + \frac{1}{\nu}\widehat{h}_{i,t}, \quad i = 1, \ldots, n \tag{C.95}$$

3. Total hours worked:

$$\widehat{H}_t = \sum_{i=1}^{n}\left(\frac{h_i}{H}\right)^{\frac{1+\nu}{\nu}}\widehat{h}_{i,t} \tag{C.96}$$

4. Bond Euler equation:

$$\widehat{\lambda}_t = \widehat{R}_t + \mathbb{E}_t[\widehat{\lambda}_{t+1}] - \mathbb{E}_t[\widehat{\Pi}_{t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{\lambda}_{t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{\pi}_{t+1}] - \mathbb{COV}_t[\widehat{\lambda}_{t+1}, \widehat{\Pi}_{t+1}] \quad \text{(C.97)}$$

5. Capital Euler equation:

$$\begin{aligned}\widehat{\mu}_{i,t} &= \{1-\beta(1-\delta)\}\left[\mathbb{E}_t[\widehat{\lambda}_{t+1}] + \mathbb{E}_t[\widehat{mc}_{i,t+1}] + \mathbb{E}_t[\widehat{y}_{i,t+1}] - \widehat{k}_{i,t}\right] + \beta(1-\delta)\mathbb{E}_t[\mu_{i,t+1}] \\ &+ \{1-\beta(1-\delta)\}\left[\frac{1}{2}\mathbb{VAR}_t[\widehat{\lambda}_{t+1} + \widehat{mc}_{i,t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{y}_{i,t+1}] + \mathbb{COV}_t[\widehat{\lambda}_{t+1} + \widehat{mc}_{i,t+1}, \widehat{y}_{i,t+1}]\right] \\ &+ \frac{1}{2}\beta(1-\delta)\mathbb{VAR}_t[\widehat{\mu}_{i,t+1}], \quad i = 1,\ldots,n\end{aligned} \quad \text{(C.98)}$$

6. Investment optimality condition:

$$\begin{aligned}\widehat{\lambda}_t + \widehat{p}^q_{i,t} &= \widehat{\mu}_{i,t} - \kappa\Delta\widehat{x}_{i,t} \\ &+ \beta\kappa\mathbb{E}_t[\Delta\widehat{x}_{i,t+1}] + \beta\kappa\left[\frac{5}{2}\mathbb{VAR}_t[\Delta\widehat{x}_{i,t+1}] + \mathbb{COV}_t(\widehat{\mu}_{i,t+1}, \Delta\widehat{x}_{i,t+1})\right], \quad i,j = 1,\ldots,n\end{aligned} \quad \text{(C.99)}$$

7. Sectoral investment growth:

$$\Delta\widehat{x}_{i,t} = \widehat{x}_{i,t} - \widehat{x}_{i,t-1}, \quad i = 1,\ldots,n \quad \text{(C.100)}$$

8. Capital accumulation equation:

$$\widehat{k}_{i,t} = (1-\delta)\widehat{k}_{i,t-1} + \delta\widehat{x}_{i,t}, \quad i = 1,\ldots,n \quad \text{(C.101)}$$

9. Consumption demand:

$$\varepsilon_c\widehat{p}_{i,t} + \widehat{c}_{i,t} = \widehat{C}_t, \quad i = 1,\ldots,n \quad \text{(C.102)}$$

10. Sectoral price index:

$$0 = \sum_{i=1}^{n}\omega_i\widehat{p}_{i,t}, \quad \text{(C.103)}$$

11. Production function:

$$\widehat{y}_{i,t}(l) = z_{i,t} + \alpha_i^k\widehat{k}_{i,t}(l) + \alpha_i^h\widehat{h}_{i,t}(l) + (1-\alpha_i^k-\alpha_i^h)\widehat{\widetilde{m}}_{i,t}(l), \quad i = 1,\ldots,n \quad \text{(C.104)}$$

12. Labor demand:

$$\widehat{w}_{i,t} = \widehat{mc}_{i,t} + \widehat{y}_{i,t}(l) - \widehat{h}_{i,t}(l), \quad i = 1,\ldots,n \quad \text{(C.105)}$$

13. Intermediate inputs demand (I):

$$\widehat{p}_{i,t}^{m} = \widehat{mc}_{i,t} + \widehat{y}_{i,t}(l) - \widehat{\widetilde{m}}_{i,t}(l), \quad i,j = 1,\ldots,n \tag{C.106}$$

14. Intermediate inputs demand (II):

$$\widehat{p}_{i,t}^{m} = \sum_{j=1}^{n} a_{ij}\widehat{p}_{j,t}, \quad i = 1,\ldots,n \tag{C.107}$$

15. Intermediate inputs demand (III):

$$\widehat{m}_{ij,t} = -\varepsilon_m \left(\widehat{p}_{j,t} - \widehat{p}_{i,t}^{m}\right) + \widehat{\widetilde{m}}_{i,t}, \quad i,j = 1,\ldots,n \tag{C.108}$$

16. Investment inputs demand (I):

$$\widehat{p}_{i,t}^{q} = \sum_{j=1}^{n} b_{ij}\widehat{p}_{j,t}, \quad i = 1,\ldots,n \tag{C.109}$$

17. Investment inputs demand (II):

$$\widehat{q}_{ij,t} = -\varepsilon_q \left(\widehat{p}_{j,t} - \widehat{p}_{i,t}^{q}\right) + \widehat{x}_{i,t}, \quad i,j = 1,\ldots,n \tag{C.110}$$

18. Sectoral output and input:

$$\widehat{\widetilde{y}}_{i,t} = \widehat{\Delta}_{i,t} + \widehat{y}_{i,t}, \quad i = 1,\ldots,n \tag{C.111}$$

19. Law of motion for price dispersion:

$$\widehat{\Delta}_{i,t} = \theta(1-\xi_i)\left(\widehat{p}_{i,t} - \widehat{p}_{i,t}^{*}\right) + \theta\xi_i\widehat{\pi}_{i,t}, \quad i = 1,\ldots,n \tag{C.112}$$

20. Sectoral resource constraint:

$$\widehat{y}_{i,t} = \frac{c_i}{y_i}\widehat{c}_{i,t} + \sum_{j=1}^{n}\frac{m_{ji}}{y_i}\widehat{m}_{ji,t} + \sum_{j=1}^{n}\frac{q_{ji}}{y_i}\widehat{q}_{ji,t}, \quad i = 1,\ldots,n \tag{C.113}$$

21. Sectoral GDP:

$$\widehat{GDP}_{i,t} = \frac{p_i y_i}{GDP_i}\left(\widehat{p}_{i,t} + \widehat{y}_{i,t}\right) - \frac{p_i^m \widetilde{m}_i}{GDP_i}\left(\widehat{p}_{i,t}^{m} + \widehat{\widetilde{m}}_{i,t}\right), \quad i = 1,\ldots,n \tag{C.114}$$

22. Optimal reset price (I):

$$\widehat{p}_{i,t}^{*} = \widehat{P}_{i,t}^{n} - \widehat{P}_{i,t}^{d}, \quad i = 1,\ldots,n \tag{C.115}$$

23. Optimal reset price (II):

$$\widehat{P}^n_{i,t} = (1-\xi_i\beta)(\widehat{\lambda}_t + \widehat{mc}_{i,t} + \widehat{y}_{i,t}) \\ + \xi_i\beta\left\{\theta\mathbb{E}_t\widehat{\pi}_{i,t+1} + \mathbb{E}_t\widehat{P}^n_{i,t+1} + \frac{\theta^2}{2}\mathbb{VAR}_t(\widehat{\pi}_{i,t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{P}^n_{i,t+1}) + \theta\mathbb{COV}_t(\widehat{\pi}_{i,t+1}, \widehat{P}^n_{i,t+1})\right\}, \quad i = 1,\dots,n \tag{C.116}$$

24. Optimal reset price (III):

$$\widehat{P}^d_{i,t} = (1-\xi_i\beta)(\widehat{\lambda}_t + \widehat{y}_{i,t}) \\ + \xi_i\beta\left\{\theta\mathbb{E}_t\widehat{\pi}_{i,t+1} - \mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{P}^d_{i,t+1} + \frac{\theta^2}{2}\mathbb{VAR}_t(\widehat{\pi}_{i,t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{P}^d_{i,t+1} - \widehat{\pi}_{t+1}) + \theta\mathbb{COV}_t(\widehat{\pi}_{i,t+1}, \widehat{P}^d_{i,t+1} - \tag{C.117}$$

25. Law of motion for sectoral price level:

$$0 = (1-\xi_i)\left(\widehat{p}^*_{i,t} - \widehat{p}_{i,t}\right) - \xi_i\widehat{\pi}_{i,t}, \quad i = 1,\dots,n \tag{C.118}$$

26. Sectoral inflation:

$$\widehat{\pi}_{i,t} = \widehat{p}_{i,t} - \widehat{p}_{i,t-1} + \widehat{\Pi}_t, \quad i = 1,\dots,n \tag{C.119}$$

27. Aggregate resource constraint:

$$C\widehat{C}_t + X\widehat{X}_t = Y\widehat{Y}_t \tag{C.120}$$

28. Total investment:

$$X\widehat{X}_t = \sum_{i=1}^{n} p^q_i x_i \left(\widehat{p}^q_{i,t} + \widehat{x}_{i,t}\right) \tag{C.121}$$

29. Monetary policy:

$$\widehat{R}_t = \rho_R\widehat{R}_{t-1} + (1-\rho_R)\left(\phi_\Pi\widehat{\Pi}_t + \phi_Y\widehat{Y}_t\right) \tag{C.122}$$

C.4 Risk-adjusted Log-linearized Equilibrium Conditions in a Vector Form

1. Marginal utility of consumption:

$$\widehat{\lambda}_t = -\sigma\widehat{C}_t \tag{C.123}$$

2. Labor supply condition:

$$\mathbf{1}\widehat{\lambda}_t + \widehat{\boldsymbol{w}}_t = -\frac{1}{\nu}\mathbf{1}\widehat{H}_t + \frac{1}{\nu}\widehat{\boldsymbol{h}}_t \tag{C.124}$$

3. Total hours worked:

$$\widehat{H}_t = V_h\widehat{\boldsymbol{h}}_t, \tag{C.125}$$

where $V_h = \left[\left(\frac{h_1}{H}\right)^{\frac{1+\nu}{\nu}}, \dots, \left(\frac{h_n}{H}\right)^{\frac{1+\nu}{\nu}}\right]'$.

4. Bond Euler equation:

$$\widehat{\lambda}_t = \widehat{R}_t + \mathbb{E}_t[\widehat{\lambda}_{t+1}] - \mathbb{E}_t[\widehat{\Pi}_{t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{\lambda}_{t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{\pi}_{t+1}] - \mathbb{COV}_t[\widehat{\lambda}_{t+1}, \widehat{\Pi}_{t+1}] \quad \text{(C.126)}$$

5. Capital Euler equation:

$$\begin{aligned}\widehat{\boldsymbol{\mu}}_t &= \{1-\beta(1-\delta)\}\left[\mathbf{1}\mathbb{E}_t[\widehat{\lambda}_{t+1}] + \mathbb{E}_t[\widehat{\boldsymbol{mc}}_{t+1}] + \mathbb{E}_t[\widehat{\boldsymbol{y}}_{t+1}] - \widehat{\boldsymbol{k}}_t\right] + \beta(1-\delta)\mathbb{E}_t[\widehat{\boldsymbol{\mu}}_{t+1}] \\ &+ \{1-\beta(1-\delta)\}\left[\frac{1}{2}\mathbb{VAR}_t[\mathbf{1}\widehat{\lambda}_{t+1} + \widehat{\boldsymbol{mc}}_{t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{\boldsymbol{y}}_{t+1}] + \mathbb{COV}_t[\mathbf{1}\widehat{\lambda}_{t+1} + \widehat{\boldsymbol{mc}}_{t+1}, \widehat{\boldsymbol{y}}_{t+1}]\right] \\ &+ \frac{1}{2}\beta(1-\delta)\mathbb{VAR}_t[\widehat{\boldsymbol{\mu}}_{t+1}]\end{aligned} \quad \text{(C.127)}$$

6. Investment optimality condition:

$$\begin{aligned}\mathbf{1}\widehat{\lambda}_t + \widehat{\boldsymbol{p}}_t^q &= \widehat{\boldsymbol{\mu}}_t - \kappa\Delta\widehat{\boldsymbol{x}}_t \\ &\quad + \beta\kappa\mathbb{E}_t[\Delta\widehat{\boldsymbol{x}}_{t+1}] + \beta\kappa\left[\frac{5}{2}\mathbb{VAR}_t[\Delta\widehat{\boldsymbol{x}}_{t+1}] + \mathbb{COV}_t(\widehat{\boldsymbol{\mu}}_{t+1}, \Delta\widehat{\boldsymbol{x}}_{t+1})\right]\end{aligned} \quad \text{(C.128)}$$

7. Sectoral investment growth:

$$\Delta\widehat{\boldsymbol{x}}_t = \boldsymbol{x}_t - \boldsymbol{x}_{t-1} \quad \text{(C.129)}$$

8. Capital accumulation equation:

$$\widehat{\boldsymbol{k}}_t = (1-\delta)\widehat{\boldsymbol{k}}_{t-1} + \delta\widehat{\boldsymbol{x}}_t \quad \text{(C.130)}$$

9. Consumption demand:

$$\varepsilon_c M_{cp}\widehat{\boldsymbol{p}}_t + \widehat{\widetilde{\boldsymbol{c}}}_t = \underset{n_c\times 1}{\mathbf{1}}\widehat{C}_t, \quad \text{(C.131)}$$

where M_{cp} is a matrix where we remove, from $\underset{n\times n}{\mathbf{I}}$, rows corresponding to zero elements in $\boldsymbol{\omega} \equiv [\omega_1, \ldots, \omega_n]'$, $\widehat{\widetilde{\boldsymbol{c}}}_t$ is a vector that removes $\widehat{c}_{i,t}$ corresponding to zero elements in ω, and n_c is the length of $\widehat{\widetilde{\boldsymbol{c}}}_t$.

10. Sectoral price index:

$$0 = \boldsymbol{\omega}'\widehat{\boldsymbol{p}}_t \quad \text{(C.132)}$$

11. Production function:

$$\widehat{\boldsymbol{y}}_t = \boldsymbol{z}_t + \alpha_d^k\widehat{\boldsymbol{k}}_{t-1} + \alpha_d^h\widehat{\boldsymbol{h}}_{i,t} + (I - \alpha_d^k - \alpha_d^h)\widehat{\widetilde{\boldsymbol{m}}}_t \quad \text{(C.133)}$$

12. Labor demand:

$$\widehat{\boldsymbol{w}}_t = \widehat{\boldsymbol{mc}}_t + \widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{h}}_t \quad \text{(C.134)}$$

13. Intermediate inputs demand (I):

$$\widehat{\boldsymbol{p}}_t^m = \widehat{\boldsymbol{mc}}_t + \widehat{\boldsymbol{y}}_t - \widehat{\widetilde{\boldsymbol{m}}}_t \tag{C.135}$$

14. Intermediate inputs demand (II):

$$\widehat{\boldsymbol{p}}_t^m = A\widehat{\boldsymbol{p}}_t \tag{C.136}$$

15. Intermediate inputs demand (III):

$$\widehat{\boldsymbol{m}}_t = \varepsilon_m M_1^m \widehat{\boldsymbol{p}}_t^m - \varepsilon_m M_2^m \widehat{\boldsymbol{p}}_t + M_1^m \widehat{\widetilde{\boldsymbol{m}}}_t \tag{C.137}$$

where $\widehat{\boldsymbol{m}}_t$ is a $(\sum_{i=1}^n n_{m_i} \times 1)$ vector that collects the log-deviations of $m_{ij,t}$ from all non-zero elements of steady-state m_{ij}:

$$\widehat{\boldsymbol{m}}_t \equiv [\widehat{m}_{11,t}, \dots, \widehat{m}_{1n,t}, \dots, \widehat{m}_{n1,t}, \dots, \widehat{m}_{nn,t}]',$$

where n_{m_i} is the number of non-zero elements in the steady-state intermediate inputs use by sector i: $[m_{i1}, \dots, m_{in}]'$. I define matrices M_1^m as

$$\underset{\sum_{i=1}^n n_{m_i} \times n}{M_1^m} \equiv \begin{bmatrix} \underset{n_{m_1} \times 1}{\mathbf{1}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \underset{n_{m_n} \times 1}{\mathbf{1}} \end{bmatrix},$$

and M_2^m is a $(\sum_{i=1}^n n_{m_i} \times n)$ matrix where we remove rows corresponding to zero elements in $vec(A')$ from a matrix $\underset{n \times 1}{\mathbf{1}} \otimes \underset{n \times n}{\mathbf{I}}$.

16. Investment inputs demand (I):

$$\widehat{\boldsymbol{p}}_t^q = B\widehat{\boldsymbol{p}}_t \tag{C.138}$$

17. Investment inputs demand (II):

$$\widehat{\boldsymbol{q}}_t = \varepsilon_q M_1^q \widehat{\boldsymbol{p}}_t^q - \varepsilon_q M_2^q \widehat{\boldsymbol{p}}_t + M_1^q \widehat{\boldsymbol{x}}_t \tag{C.139}$$

where $\widehat{\boldsymbol{q}}_t$ is a $(\sum_{i=1}^n n_{q_i} \times 1)$ vector that collects the log-deviations of $q_{ij,t}$ from all non-zero elements of steady-state q_{ij}:

$$\widehat{\boldsymbol{q}}_t \equiv [\widehat{q}_{11,t}, \dots, \widehat{q}_{1n,t}, \dots, \widehat{q}_{n1,t}, \dots, \widehat{q}_{nn,t}]',$$

where n_{q_i} is the number of non-zero elements in the steady-state intermediate inputs use by sector i: $[q_{i1}, \ldots, q_{in}]'$. I define matrices M_1^q as

$$\underset{\sum_{i=1}^n n_{q_i} \times n}{M_1^q} \equiv \begin{bmatrix} \underset{n_{q_1}\times 1}{\mathbf{1}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \underset{n_{q_n}\times 1}{\mathbf{1}} \end{bmatrix},$$

and M_2^q is a $(\sum_{i=1}^n n_{q_i} \times n)$ matrix where we remove rows corresponding to zero elements in $vec(B')$ from a matrix $\underset{n\times 1}{\mathbf{1}} \otimes \underset{n\times n}{\mathbf{I}}$.

18. Sectoral output and input:

$$\widehat{\widehat{\boldsymbol{y}}}_t = \widehat{\boldsymbol{\Delta}}_t + \widehat{\boldsymbol{y}}_t \tag{C.140}$$

19. Law of motion for price dispersion:

$$\widehat{\boldsymbol{\Delta}}_t = \theta(I - diag(\boldsymbol{\xi}))\left(\widehat{\boldsymbol{p}}_t - \widehat{\boldsymbol{p}}_t^*\right) + \theta diag(\boldsymbol{\xi})\widehat{\boldsymbol{\pi}}_t \tag{C.141}$$

20. Sectoral resource constraint:

$$S_y \widehat{\boldsymbol{y}}_t = S_c \widehat{\widehat{\boldsymbol{c}}}_t + S_m \widehat{\boldsymbol{m}}_t + S_q \widehat{\boldsymbol{q}}_t,$$

where $S_y = diag(\boldsymbol{y})$, S_c is a matrix where we remove zero columns from $diag(\boldsymbol{c})$, and S_m is a matrix constructed from the vector m such that

$$\underset{n\times\sum_{i=1}^n n_{m_i}}{S_m} = [S_{m_1}, \cdots, S_{m_n}],$$

where

$$\underset{n\times n_{m_i}}{S_{m_i}} = \begin{bmatrix} m_{i1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & m_{in} \end{bmatrix},$$

and S_q is a matrix constructed from the vector q such that

$$\underset{n\times\sum_{i=1}^n n_{q_i}}{S_q} = [S_{q_1}, \cdots, S_{q_n}],$$

where

$$\underset{n\times n_{q_i}}{S_{q_i}} = \begin{bmatrix} q_{i1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & q_{in} \end{bmatrix}.$$

We can eliminate $\widehat{\boldsymbol{m}}_t$ and $\widehat{\boldsymbol{q}}_t$ by substituting in (C.136) and (C.138):

$$\begin{aligned}S_y\widehat{\boldsymbol{y}}_t &= S_c\widehat{\widetilde{\boldsymbol{c}}}_t + S_m\left[\varepsilon_m M_1^m\widehat{\boldsymbol{p}}_t^m - \varepsilon_m M_2^m\widehat{\boldsymbol{p}}_t + M_1^m\widehat{\widetilde{\boldsymbol{m}}}_t\right] + S_q\left[\varepsilon_q M_1^q\widehat{\boldsymbol{p}}_t^q - \varepsilon_q M_2^q\widehat{\boldsymbol{p}}_t + M_1^q\widehat{\boldsymbol{x}}_t\right]\\ &= S_c\widehat{\widetilde{\boldsymbol{c}}}_t + \varepsilon_m S_m M_1^m\widehat{\boldsymbol{p}}_t^m + \varepsilon_q S_q M_1^q\widehat{\boldsymbol{p}}_t^q - \left[\varepsilon_m S_m M_2^m + \varepsilon_q S_q M_2^q\right]\widehat{\boldsymbol{p}}_t + S_m M_1^m\widehat{\widetilde{\boldsymbol{m}}}_t + S_q M_1^q\widehat{\boldsymbol{x}}_t\end{aligned} \tag{C.142}$$

21. Sectoral GDP:

$$diag(\boldsymbol{GDP})\widehat{\boldsymbol{GDP}}_t = diag(\boldsymbol{y})\left(\widehat{\boldsymbol{p}}_t + \widehat{\boldsymbol{y}}_t\right) - diag(\widetilde{\boldsymbol{m}})\left(\widehat{\boldsymbol{p}}_t^m + \widehat{\widetilde{\boldsymbol{m}}}_t\right) \tag{C.143}$$

22. Optimal reset price (I):

$$\widehat{\boldsymbol{p}}_t^* = \widehat{\boldsymbol{P}}_t^n - \widehat{\boldsymbol{P}}_t^d \tag{C.144}$$

23. Optimal reset price (II):

$$\begin{aligned}\widehat{\boldsymbol{P}}_t^n &= (I - \beta diag(\boldsymbol{\xi}))(\boldsymbol{1}\widehat{\lambda}_t + \widehat{\boldsymbol{mc}}_t + \widehat{\boldsymbol{y}}_t)\\ &\quad + \beta diag(\boldsymbol{\xi})\left\{\theta\mathbb{E}_t[\widehat{\boldsymbol{\pi}}_{t+1}] + \mathbb{E}_t[\widehat{\boldsymbol{P}}_{t+1}^n] + \frac{\theta^2}{2}\mathbb{VAR}_t(\widehat{\boldsymbol{\pi}}_{t+1}) + \frac{1}{2}\mathbb{VAR}_t(\widehat{\boldsymbol{P}}_{t+1}^n) + \theta\mathbb{COV}_t(\widehat{\boldsymbol{\pi}}_{t+1}, \widehat{\boldsymbol{P}}_{t+1}^n)\right\}\end{aligned} \tag{C.145}$$

24. Optimal reset price (III):

$$\begin{aligned}\widehat{\boldsymbol{P}}_t^d &= (I - \beta diag(\boldsymbol{\xi}))(\boldsymbol{1}\widehat{\lambda}_t + \widehat{\boldsymbol{y}}_t)\\ &\quad + \beta diag(\boldsymbol{\xi})\left\{\theta\mathbb{E}_t[\widehat{\boldsymbol{\pi}}_{t+1}] - \mathbb{E}_t[\boldsymbol{1}\widehat{\pi}_{t+1}] + \mathbb{E}_t[\widehat{\boldsymbol{P}}_{t+1}^d] + \frac{\theta^2}{2}\mathbb{VAR}_t[\widehat{\boldsymbol{\pi}}_{t+1}] + \frac{1}{2}\mathbb{VAR}_t[\widehat{\boldsymbol{P}}_{t+1}^d - \boldsymbol{1}\widehat{\pi}_{t+1}]\right.\\ &\quad \left. + \theta\mathbb{COV}_t[\widehat{\boldsymbol{\pi}}_{t+1}, \widehat{\boldsymbol{P}}_{t+1}^d - \boldsymbol{1}\widehat{\pi}_{t+1}]\right\}\end{aligned} \tag{C.146}$$

25. Law of motion for sectoral price level:

$$\boldsymbol{0} = (I - diag(\boldsymbol{\xi}))\left(\widehat{\boldsymbol{p}}_t^* - \widehat{\boldsymbol{p}}_t\right) - diag(\boldsymbol{\xi})\widehat{\boldsymbol{\pi}}_t \tag{C.147}$$

26. Sectoral inflation:

$$\widehat{\boldsymbol{\pi}}_t = \widehat{\boldsymbol{p}}_t - \widehat{\boldsymbol{p}}_{t-1} + \boldsymbol{1}\widehat{\pi}_t \tag{C.148}$$

27. Aggregate resource constraint:

$$C\widehat{C}_t + X\widehat{X}_t = Y\widehat{Y}_t \tag{C.149}$$

28. Total investment:

$$X\widehat{X}_t = \boldsymbol{x}'\left(\widehat{\boldsymbol{p}}_t^q + \widehat{\boldsymbol{x}}_t\right) \tag{C.150}$$

29. Monetary policy:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R) \left(\phi_\Pi \widehat{\Pi}_t + \phi_Y \widehat{Y}_t \right) \tag{C.151}$$