The Relational Model

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Recap

- The relational model is based on the concept of a relation or table.
- Two example relations:

Teams

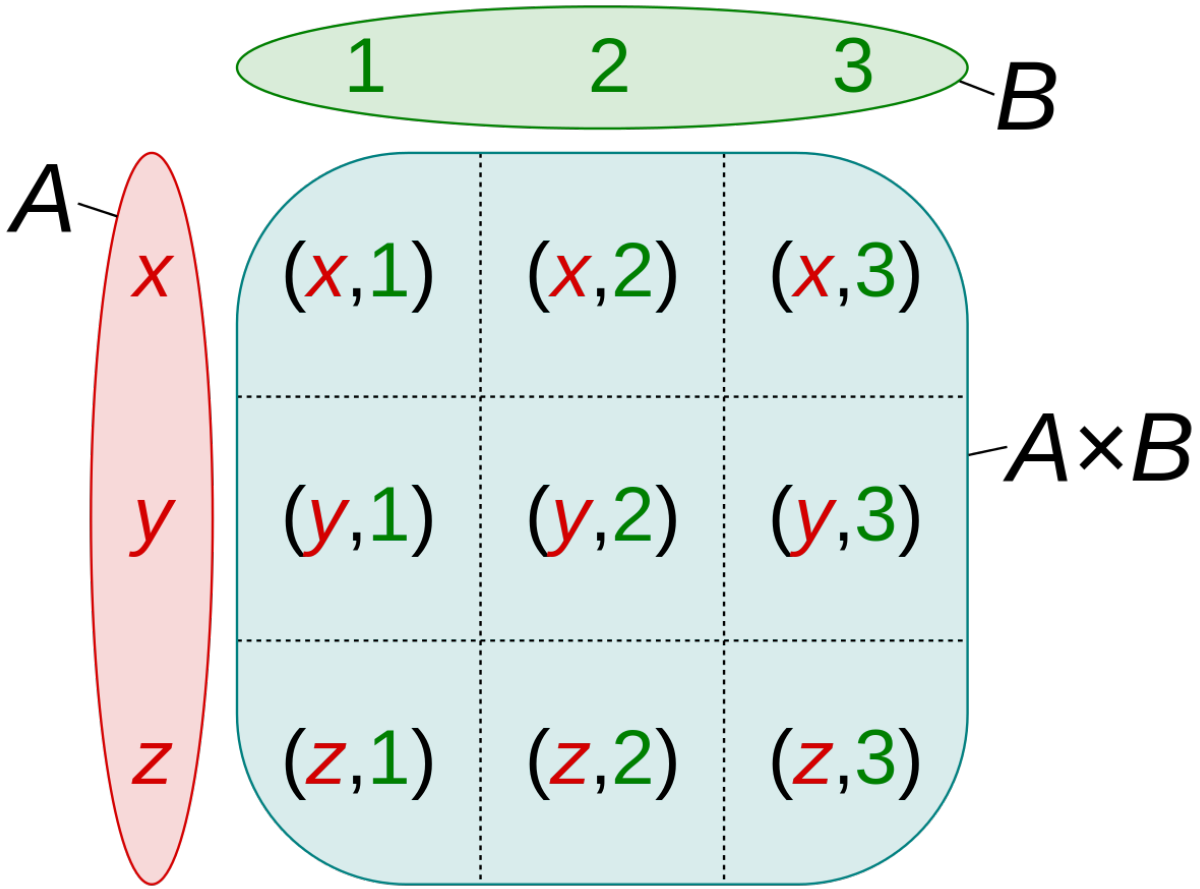
Name	Home Field	Coach
Rangers	Runnymede CI	Tarvo Sinervo
Ducks	Humber Public	Tracy Zheng
Choppers	High Park	Ammar Jalali

Games

Home team	Away team	Home goals	Away goals
Rangers	Ducks	3	0
Ducks	Choppers	1	1
Rangers	Choppers	4	2
Choppers	Ducks	0	5

Relations in Math: The Cartesian Product

Cartesian product of two domains **A** and **B**



<x, 1>
<y, 1>
<z, 1>
<x, 2>
<y, 2>
<z, 2>
<x, 3>
<y, 3>
<z, 3>

Relations in Math

- A domain is a set of values.
- Suppose $\mathbf{D_1, D_2, \ldots D_n}$ are domains.
  - The Cartesian product

    $D_1 \mathbf{x} D_2 \mathbf{x} \ldots \mathbf{x} D_n$

  - this is the set of all **tuples** $\mathbf{<d_1, d_2, \ldots, d_n>}$ such that $d_1 \in D_1,\ d_2 \in D_2,\ \ldots,\ d_n \in D_n$.
  - I.e., every combination of a value from D_1, a value from D_2 etc.
- A (mathematical) **relation** on $D_1, D_2, \ldots D_n$ is a **subset** of the Cartesian product.

- Example of a mathematical relation:
  - Let **A** = {p, q, r, s}, **B** = {1, 2, 3} and **C** = {100, 200}.
  - **R** = {<q, 2, 100>, <s, 3, 200>, <p, 1, 200>} is a **relation** on **A, B, C**.
    - Clearly it is a subset of the Cartesian Product
- Turns out that our **database tables are relations** too.
- Example:

  {<Rangers, Ducks, 3, 0>, <Ducks, Choppers, 1, 1>,

  <Rangers, Choppers, 4, 2>, <Choppers, Ducks, 0, 5>}

Relation schemas vs instances

- **Schema**: definition of the structure of the relation. Example:

  Teams have 3 attributes: name, home field, coach.
  No two teams can have the same name.

- Notation for expressing a relation's schema

  Teams(Name, HomeField, Coach)

- **Instance**: particular data in the relation.
- Instances change *constantly*; schemas *rarely*.
- Conventional databases store the **current version** of the data. Databases that record the history are called ***temporal*** databases.

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Terminology

Teams

Name	Home Field	Coach
Rangers	Runnymede CI	Tarvo Sinervo
Ducks	Humber Public	Tracy Zheng
Choppers	High Park	Ammar Jalali
Crullers	WTCS	Anna Liu

- relation (table)
- attribute (column)
- tuple (row)
- arity of a relation: number of attributes
- cardinality of a relation: number of tuples

Relations are sets

- A relation is a ***set*** of tuples, which means:
  - there can be **no duplicate tuples**
  - **order** of the tuples **doesn't matter**
- In another model, relations are ***bags*** — a generalization of sets that allows duplicates.
  - Commercial DBMSs use this bag model.
  - But for now, we will stick with relations as sets.

Let's focus on…
Database schemas and instances

- Database schema: a set of relation schemas
- Database instance: a set of relation instances

A schema for our league data

Teams(name, homefield, coach)
Games(hometeam, awayteam, homegoals, awaygoals)

- Can there be >1 team with the same name?
  - Didn't happen in our dataset, but could it?
  - That's up to the league, not us!
- Suppose the league wants to allow
  - multiple teams with same name
  - multiple teams with same home field
- The **schema allows it**: nothing says otherwise.
- But what if the league wants to **disallow:**
  - multiple teams with the same name *and* home field?

Constraining the data

- More *formally*: ∄ tuples t_1 and t_2 such that
  $(t_1.\text{name} = t_2.\text{name}) \wedge$
  $(t_1.\text{homefield} = t_2.\text{homefield})$
- *Implication*: If we know the values for {name, homefield}, we can look up ***exactly* one team** in the relation.
- I.e., attributes {name, homefield} **uniquely identify** tuples in this relation.
- We say that {name, homefield} is a "**superkey**" for relation Teams.

Superkeys

- Informally:
  A **superkey** is a set of one or more attributes whose combined values are unique.
    - I.e., no two tuples can have the same values on all of these attributes.
- *Formally*:
  If attributes $a_1, a_2, \ldots, a_n$ form a superkey for relation R, ∄ tuples t_1 and t_2 such that

$$(t_1.a_1 = t_2.a_1) \wedge (t_1.a_2 = t_2.a_2) \wedge \ldots \wedge (t_1.a_n = t_2.a_n)$$

Example

- **Course**(dept, number, name, breadth)
  - One tuple might be
    <“csc”, “343”, “Introduction to Databases”, True>
- Suppose our knowledge of the domain tells us that no two tuples can have the same value for dept and number.
- This means that {dept, number} is a superkey.
- This is a constraint on what can go in the relation.
- Create an instance of Course that violates it.
- Does every relation have a superkey?

Keeping it minimal

- If {dept, number} is a superkey, then so is {dept, number, name}.
  - This follows from the definition.
- But we are more interested in a *minimal* set of attributes with the superkey property.
  - Minimal in the sense that no attributes can be removed from the superkey without making it no longer a superkey.

Keys

- key: a minimal superkey.
- In the schema, by convention we often underline a key.
- Aside: The term “superkey” is related to the term “superset”.
  - A superkey is a superset of some key. (Not necessarily a proper superset.)
- Can a relation have more than one key?

Coincidence vs key

- If a set of attributes is a key for a relation:
  - It does not mean merely that there are no duplicates in a particular instance of the relation
  - It means that **in principle** there *cannot* be any.
  - Only a *domain expert* can determine that.
- Often we invent an attribute to ensure all tuples will be unique.
  This predates databases.
  E.g., SIN, ISBN number.

- A key defines a kind of **integrity constraint.**

“Primary key”??

- Some of you have heard the term **primary key**.
- This is specific to SQL and we’ll learn about it later.

Example: Movies schema handout, Q1-6

References between relations

- Relations often refer to each other.
- Example:
  In the Roles relation, the tuple about Han Solo needs to say he is played by Ford.
- Rather than repeat information already in the Artists table, we store Ford's key.
- If aID is a key for Artists, does that mean a particular aID can appear only once in Roles??

Foreign keys

- The referring attribute is called a foreign key because it refers to an attribute that is a key in another table.
- This gives us a way to refer to a single tuple in that relation.
- A foreign key may need to have several attributes.

Declaring foreign keys

- A bit of notation: R[A]
  - R is a relation and
    A is a list of attributes in R.
  - R[A] is the set of all tuples from R,
    but with only the attributes in list A.
- We declare foreign key constraints this way:
  $R_1[X] \subseteq R_2[Y]$
  - X and Y may be lists of attributes, of same arity
  - Y must be a key in R_2
- Example: Roles[mID] ⊆ Movies[mID]

Example: Movies schema handout, Q7-9

Referential integrity constraints

- These $R_1[X] \subseteq R_2[Y]$ relationships are a kind of referential integrity constraint or inclusion dependency.
- Not all referential integrity constraints are foreign key constraints.
- For example, we could say

  $$\text{Artists}[\text{aID}] \subseteq \text{Roles}[\text{aID}]$$

  Here we are not referring to a unique tuple.
- $R_1[X] \subseteq R_2[Y]$ is a foreign key constraint iff Y is a key for relation R2.

Designing a schema

- Mapping from the real world to a relational schema is surprisingly challenging and interesting.
- There are always many possible schemas.
- Two important goals:
  - Represent the data well.
    For example, avoid constraints that prevent expressing things that occur in the domain.
  - Avoid redundancy.
- Later, we'll learn some elegant theory that provides sound principles for good design.

What's next

- First we'll learn how to write queries in relational algebra.
  - Relational algebra is the foundation for SQL.
  - Other important concepts, like query optimization, are defined in terms of RA.
- Later, we'll learn how to use SQL to
  - define a database's structure,
  - put data in it, and
  - write queries on it.