University of Toronto
CSC343 Summer 2019

In-class Exercises: Projection and Minimal Basis

1. Suppose we have these FDs: : $S = \{ABE \to CF, \quad DF \to BD, \quad C \to DF, \quad E \to A, \quad AF \to B\}$

   Project the FDs onto: $L = CDEF$

Attributes to take all subsets X of: C D E F	Closure of the subset X^+	Functional dependencies inferred

Solution:

C	D	E	F	closure	FDs
✓				$C^+ = CDFBD$	$C \to DF$
	✓			$D^+ = D$	nothing
		✓		$E^+ = EA$	nothing
			✓	$F^+ = F$	nothing
✓	✓			$CD^+ = CDFB$	nothing, since $CD \to DF$ is weaker than $C \to DF$ which we have already
✓		✓		$CE^+ = CEDFAB$	nothing, since $CE \to DF$ is weaker than $C \to DF$ which we have already
✓			✓	$CF^+ = CFDB$	nothing, since $CF \to D$ is weaker than $C \to DF$ which we have already
	✓	✓		$DE^+ = DEA$	nothing
	✓		✓	$DF^+ = DFB$	nothing
		✓	✓	$EF^+ = EFABCD$	$EF \to CD$
✓	✓	✓		$CDE^+ = CDEF$	nothing, since $CDE \to F$ is weaker than $C \to DF$ which we have already
✓	✓		✓	$CDF^+ = CDFB$	nothing
✓		✓	✓	since EF is a key, supersets of EF can only yield FDs that are weaker than ones we have.	
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Final answer: The projection of S onto L is $C \to DF$, $EF \to CD$.

2. Find a minimal basis for this set of FDs: $S = \{ABF \to G, \quad BC \to H, \quad BCH \to EG, \quad BE \to GH\}$.

Solution:

Step 1: Split the RHSs to get our initial set of FDs, $S1$:

(a) $ABF \to G$
(b) $BC \to H$
(c) $BCH \to E$
(d) $BCH \to G$
(e) $BE \to G$
(f) $BE \to H$.

Step 2: For each FD, try to reduce the LHS:

(a) $A^+ = A, B^+ = B, F^+ = F$. In fact, no singleton LHS yields anything. $AB^+ = AB, AF^+ = AF$, and $BF^+ = BF$, so none of them yields G either. We cannot reduce the LHS of this FD.
(b) Since this FD has only two attributes on the LHS, and no singleton LHS yields anything, we cannot reduce the LHS of this FD.
(c) Since no singleton LHS yields anything, we need only consider LHSs with two or more attributes. We only have three to begin with, so that leaves LHSs with two attributes. $BC^+ = BCHEG$. So we can reduce the LHS of this FD, yielding the new FD: $BC \to E$.
(d) By the same argument, we can reduce this FD to: $BC \to G$.
(e) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.
(f) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.

Our new set of FDs, let's call it $S2$, is

(a) $ABF \to G$
(b) $BC \to H$
(c) $BC \to E$
(d) $BC \to G$
(e) $BE \to G$
(f) $BE \to H$.

Step 3: Try to eliminate each FD.

(a) $ABF^+_{S2-(a)} = ABF$. We need this FD.
(b) $BC^+_{S2-(b)} = BCEG\underline{H}$. We can remove this FD.
(c) $BC^+_{S2-\{(b),(c)\}} = BCG$. We need this FD.
(d) $BC^+_{S2-\{(b),(d)\}} = BCE\underline{G}H$. We can remove this FD.
(e) $BE^+_{S2-\{(b),(d),(e)\}} = BEH$. We need this FD.
(f) $BE^+_{S2-\{(b),(d),(f)\}} = BEG$. We need this FD.

Our final set of FDs is:

(a) $ABF \to G$
(b) $BC \to E$
(c) $BE \to G$
(d) $BE \to H$.