

# Relational Algebra Exercises

These are some sample solutions of the the exercises we did in class. Please read the explanations that accompany some of the solutions. **Important:** There are other good answers to each of these queries, and we encourage you to think about other ways to write them.

## Schema

Note: “breadth” is a boolean indicating whether or not a course satisfies the breadth requirement for degrees in the Faculty of Arts and Science.

Student(sID, surName, firstName, campus, email, cgpa)

Course(dept, cNum, name, breadth)

Offering(oID, dept, cNum, term, instructor)

Took(sID, oID, grade)

Offering[dept, cNum]  $\subseteq$  Course[dept, cNum]

Took[sID]  $\subseteq$  Student[sID]

Took[oID]  $\subseteq$  Offering[oID]

## Queries

Write a query for each of the following:

1. Student number of all students who have taken csc343.

**Answer:**

$\Pi_{sID} \sigma_{dept="csc" \wedge cNum=343} (Took \bowtie Offering)$

2. Student number of all students who have taken csc343 and earned an A+ in it.

**Answer:**

$Good343(sID) := \Pi_{sID} \sigma_{dept="csc" \wedge cNum=343 \wedge grade \geq 90} (Took \bowtie Offering)$

3. The names of all such students.

**Answer:**

Here we reuse relation Good343 from the previous question.

$\Pi_{surName, firstName} (Good343 \bowtie Student)$

4. The names of all students who have passed a breadth course with Professor Picky.

**Answer:**

$$PickyBreadth(oID) := \Pi_{oID} \sigma_{breadth=true \wedge instructor="Picky"} (Course \bowtie Offering)$$

$$Passers(sID) := \Pi_{sID} \sigma_{grade \geq 50} (PickyBreadth \bowtie Took)$$

$$Answer(surName, firstName) := \Pi_{surName, firstName} (Passers \bowtie Student)$$

5. sID of all students who have earned some grade over 80 and some grade below 50.

**Answer:**

$$(\Pi_{sID} \sigma_{grade > 80} Took) \cap (\Pi_{sID} \sigma_{grade < 50} Took)$$

6. Terms when Cook and Pitassi were both teaching something.

**Answer:**

$$(\Pi_{term} \sigma_{instructor="Cook"} Offering) \cap (\Pi_{term} \sigma_{instructor="Pitassi"} Offering)$$

7. Terms when either of them was teaching csc463.

**Answer:**

$$\Pi_{term} (\sigma_{dept="csc" \wedge cNum=463 \wedge (instructor="Cook" \vee instructor="Pitassi")} Offering)$$

8. sID of students who have earned a grade of 85 or more, or who have passed a course taught by Atwood.

**Answer:**

$$HaveHighGrade(sID) = \Pi_{sID} \sigma_{grade \geq 85} Took$$

$$PassedAtwood(sID) = \Pi_{sID} \sigma_{instructor="Atwood" \wedge grade \geq 50} (Took \bowtie Offering)$$

$$Answer(sID) := HaveHighGrade \cup PassedAtwood$$

9. Terms when csc369 was not offered.

**Answer:**

$$(\Pi_{term} Offering) - (\Pi_{term} \sigma_{dept="csc" \wedge cNum=369} Offering)$$

10. Department and course number of courses that have never been offered.

**Answer:**

$$(\Pi_{dept,cNum} Course) - (\Pi_{dept,cNum} Offering)$$

11. SIDs and surnames of all pairs of students who've taken a course together.

**Answer:**

$$Pairs(sID1, sID2) := \Pi_{T1.sID, T2.sID} \sigma_{T1.sID < T2.sID \wedge T1.oID = T2.oID} [(\rho_{T1} Took) \times (\rho_{T2} Took)]$$

$$OneName(sID1, sID2, name1) := \Pi_{sID1, sID2, surName} \sigma_{sID1 = sID} (Pairs \times Student)$$

$$Answer(sID1, sID2, name1, name2) :=$$

$$\Pi_{sID1, sID2, name1, surName} \sigma_{sID2 = sID} (OneName \times Student)$$

The strategy used here is to first identify the entities that should be in the answer by their keys, and then later add in the other required attributes. If many additional attributes are required in the answer, this is cleaner. Because you end up working with fewer attributes through the hard parts of the query, it also avoids accidentally doing an inappropriate natural join – one that would force attributes to match that we don't actually need to match. Here, I brought in one surname at a time. Rather than getting the surnames in two steps we could have done it all at once. To do so, we'd have to bring in two copies of Student, and therefore would need to do some renaming.

When there aren't many other attributes needed in the final answer, it may be simpler to keep them all along. Here's that solution:

– Who took what, and their surname.

$$TookName(sID, oID, name) :=$$

$$\Pi_{sID, oID, surName} (Took \bowtie Student)$$

$$Answer(sID1, sID2, name1, name2) :=$$

$$\Pi_{T1.sID, T2.sID, T1.name, T2.name} \sigma_{T1.sID < T2.sID \wedge T1.oID = T2.oID} [(\rho_{T1} TookName) \times (\rho_{T2} TookName)]$$

12. sID of student(s) with the highest grade in csc343, in term 20099.

**Answer:**

This works except for one wrinkle discussed below:

$$Takers(sID, grade) := \Pi_{sID, grade} [(\sigma_{dept="csc" \wedge cNum=343 \wedge term=20099} Offering) \bowtie Took]$$

$$NotTop(sID) := \Pi_{T1.sID} \sigma_{T1.grade < T2.grade} [(\rho_{T1} Takers) \times (\rho_{T2} Takers)]$$

$$Answer(sID) := (\Pi_{sID} Takers) - NotTop$$

Notice that the schema doesn't disallow having two different offerings of csc343 in 20099, and this makes sense in our domain. For instance, we have a day and an evening section of csc343 this term. The schema also does not prevent a student from being in the Took relation for both of them, even though this probably doesn't make sense in our domain. If such a student were to get the top mark for one and a non-top mark for the other, they would appear in NotTop and therefore not in Answer. To prevent that, we can keep the oID until the very end:

$$Takers(sID, oID, grade) := \Pi_{sID, oID, grade} (\sigma_{dept="csc" \wedge cNum=343 \wedge term=20099} Offering) \bowtie Took$$

$$NotTop(sID, oID, grade) := \Pi_{T1.sID, T1.oID, T1.grade} \sigma_{T1.grade < T2.grade} [(\rho_{T1} Takers) \times (\rho_{T2} Takers)]$$

$$Answer(sID) := \Pi_{sID} (Takers - NotTop)$$

Notice that the final projection onto sID has moved. Make sure you know why. These small matters

often make the difference between a correct and an incorrect or even syntically ill-formed query.

13. sID of students who have a grade of 100 at least twice.

**Answer:**

$$AtLeastTwice(sID) :=$$

$$\Pi_{T1.sID} \sigma_{T1.oID \neq T2.oID \wedge T1.sID = T2.sID \wedge T1.grade = 100 \wedge T2.grade = 100} [(\rho_{T1} Took) \times (\rho_{T2} Took)]$$

14. sID of students who have a grade of 100 exactly twice.

**Answer:**

$$AtLeastThrice(sID) :=$$

$$\Pi_{T1.sID}$$

$$\sigma_{T1.oID < T2.oID < T3.oID \wedge T1.sID = T2.sID = T3.sID \wedge T1.grade = T2.grade = T3.grade = 100}$$

$$[(\rho_{T1} Took) \times (\rho_{T2} Took) \times (\rho_{T3} Took)]$$

$$ExactlyTwice(sID) := atLeastTwice - AtLeastThrice$$

Notice that, since  $\neq$  is not transitive, we can't string together  $T1.oID \neq T2.oID \neq T3.oID$  and still enforce that  $T1.oID \neq T3.oID$ . We could compare the three oIDs all three ways, but this use of " $<$ " works too and is more concise.

15. sID of students who have a grade of 100 at most twice.

**Answer:**

$$(\Pi_{sID} Student) - AtLeastThrice$$

16. Department and cNum of all courses that have been taught in every term when csc448 was taught.

**Answer:**

$$448Terms(term) := \Pi_{term} (\sigma_{dept = "csc" \wedge cNum = 448} Offering)$$

$$CourseTerms(dept, cNum, term) := \Pi_{dept, cNum, term} Offering$$

$$ShouldHaveBeen(dept, cNum, term) := \Pi_{dept, cNum} CourseTerms \times 448Terms$$

$$WereNotAlways(dept, cNum, term) := ShouldHaveBeen - CourseTerms$$

$$Answer(dept, cNum) := (\Pi_{dept, cNum} CourseTerms) - (\Pi_{dept, cNum} WereNotAlways)$$

17. Name of all students who have taken, at some point, every course Gries has taught (but not necessarily taken them from Gries).

**Answer:**

This one is very similar to question 16. Try it on your own. It is great practise for the "every" strategy.

## Integrity Constraints

Use the notation

$$\langle \text{relational algebra expression} \rangle = \emptyset$$

to write an integrity constraint for each of the following.

1. Courses at the 400-level cannot count for breadth.

**Answer:**

$$\sigma_{400 \leq cNum < 500 \wedge breadth} Course = \emptyset$$

2. CSC490 can only be offered at the same time as CSC454.

**Answer:**

$$490Terms(term) := \Pi_{term}(\sigma_{dept='CSC' \wedge cNum=490} Offering)$$

$$454Terms(term) := \Pi_{term}(\sigma_{dept='CSC' \wedge cNum=454} Offering)$$

$$490Terms - 454Terms = \emptyset$$