MIP AND BIP OPTIMIZATION

HOPE ALBERS

1. Tool Selection

The problem given is that a new homeowner, Jill, needs a specific set of tools. As is the case with most optimization problems she wants to get all the tools for the lowest price. The challenge here is that there are multiple options for the sets of tools that all have different prices and tools. Posted below is an image of the sets of tools and how they are grouped and priced.

	Set																													
Item	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Small, Straight Screwdriver	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
Medium, Strainght Screwdriver	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
Large, Straight Screwdriver	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
Small, Philip Screwdriver	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
Medium Philip Screwdriver	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
Large Philip Screwdriver	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
8-inch Crescent Wrench	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
12-inch Crescent Wrench	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
Needle Nose Pliers	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
Wire Cutters	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
Small Adjustable Pliers	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1
Large Adjustable Pliers	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1
Screwdriver Bit Set	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
Drill Bit Set	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
Cordless Drill	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
Cordless Circular Saw	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0
Chisel Set	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
Framing Hammer	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
Finish Hammer	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
Cost	5	5	5	5	5	5	21	25	15	17	27	32	14	17	89	95	52	30	37	29	50	30	55	39	200	135	85	82	275	155

(1) The type of problem here is bin packing. Jill wants to find the lowest amount of money for a group of tools that need to be purchased. This was picked over a knapsack problem since the objective would have to be how many tools can Jill buy without exceeding a given price.

(2) Decision Variables: s_i: Set values 0 - 29

Constraints: Where s_i is the set number and c_i is the value from the coefficient matrix

$$\sum_{i=0}^{29} s_i c_i = 1 \text{ for j = 0...18}$$

Objective Function: Sum of tools multiplied by their cost where t_i is the tool and co_j is the cost

$$\text{Minimize} \sum_{i=0}^{18} \sum_{j=0}^{29} t_i co_j$$

(3) The optimal purchase consists of the tool sets 6, 7, 12-19 and 27 to get the lowest possible amount of \$491.

2. Glass Plant

A glass company, Perfectly Clear (PC), is considering purchasing its own silica mines and shipping silica via railroad to its 15 plants. Total startup costs to restart a plant are $40 million per plant, not counting the lost sales during the downtime. The body of the table below shows the transportation cost per ton from each potential mine site to each of PCs 15 plants. The bottom row shows the acquisition cost of each mine. The right-most column shows the number of tons of silica used by each plant. Each plant uses the same number of tons of silica each year. PCs goal is to minimize the upfront cost of purchasing the mines plus the first year of transportation costs by determining which mines to buy and which plants should be supplied by which of the mines.

		Potential Mine Sites															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Annual Tons of Silica Used
Float Glass Plant	0	24.92	30.36	34.90	39.30	31.78	28.89	44.49	34.99	30.40	30.79	33.31	29.42	34.62	36.18	41.88	153,300
	1	35.57	21.70	36.79	43.38	31.42	40.79	30.96	29.92	29.62	25.74	41.77	32.04	40.69	43.05	30.46	153,300
	2	44.57	22.94	23.97	40.35	26.31	27.98	35.00	38.77	29.88	25.47	38.02	33.53	41.75	27.88	37.74	153,300
	3	36.38	36.78	38.07	26.37	29.46	32.75	26.71	30.67	30.15	30.96	39.77	37.08	30.97	37.20	29.04	153,300
	4	43.06	39.52	28.24	39.64	21.02	32.27	39.11	26.87	39.30	36.78	34.29	27.80	40.69	39.36	26.62	153,300
	5	40.00	40.78	30.37	29.06	36.83	23.30	27.66	34.10	25.21	35.97	33.24	43.08	34.48	25.04	27.67	153,300
	6	42.78	40.26	30.13	30.07	38.74	25.42	26.52	22.90	30.33	31.91	31.21	41.51	30.11	27.16	42.24	153,300
	7	28.51	37.04	33.64	40.15	31.57	31.21	38.05	20.55	40.37	37.04	39.44	40.03	41.89	40.68	28.46	153,300
	8	32.68	41.09	31.26	31.12	36.60	42.88	28.38	33.22	25.05	29.22	33.83	36.00	38.53	34.74	37.84	153,300
	9	32.83	31.23	39.85	29.23	21.06	35.46	44.64	37.50	31.33	21.34	42.79	41.56	37.13	42.00	31.39	153,300
	10	39.44	22.16	39.22	34.29	37.82	33.98	43.78	26.06	44.11	26.37	26.82	40.44	34.71	42.37	42.86	153,300
	11	34.41	39.82	27.01	33.52	28.79	29.19	42.17	28.96	39.16	39.48	39.45	26.71	24.97	35.39	41.33	153,300
	12	27.51	35.59	26.86	45.97	32.64	34.21	45.88	36.42	32.76	28.71	36.66	36.66	23.18	26.14	26.42	153,300
	13	29.01	40.13	28.84	43.70	32.75	29.23	33.11	32.37	25.10	22.92	34.16	27.38	26.28	24.67	39.63	153,300
	14	26.07	28.96	33.83	36.53	32.95	28.25	32.21	36.24	29.18	22.37	30.78	26.84	36.74	30.18	26.33	153,300
	Purchase Cost	9,355,730	4,652,163	9,908,390	8,379,730	6,946,479	4,760,629	4,880,341	9,804,748	6,100,783	7,224,557	7,756,098	4,015,446	10,580,870	8,088,620	10,866,700	

(1) Decision Variables: pm_ij: All the values in the transportation cost per ton matrix.

Constraints: Where m_j is the cost of the mine and b_i is a binary value. The constraints are in order of non negativity, row summation that does not exceed 153,300 and the last being if the mine is purchased or not.

$$\sum_{i=0}^{14}\sum_{j=0}^{14} pm_{ij} \geq 0$$

$$\sum_{j=0}^{14} pm_{ij} = 153,300 \text{ for i} = 0...14$$

$$pm_{ij} \leq m_j \times b_j$$

Objective Function: The sum of transportation cost represented as tc_{ij} multiplied by the decision variables which is then added to the sum of the starting cost of opening a mine represented by pc_j.

$$\text{Minimize} \sum_{i=0}^{14}\sum_{j=0}^{14} tc_{ij}pm_{ij} + \sum_{j=0}^{14} pc_j b_j$$

(2) The lowest transportation cost for the first year is $73823261.8197.

(3) Mines 5 and 9 should be bought to minimize the cost for the year.

(4) The list of mines that serve plants are as follows: mine 5 to plant 1, mine 9 to plant 1, mine 9 to plant 2, mine 9 to plant 3, mine 5 to plant 4, mine 5 to plant 5, mine 5 to plant 6, mine 5 to plant 7, mine 9 to plant 8, mine 9 to plant 9, mine 9 to plant 10, mine 5 to plant 11, mine 9 to plant 12, mine 9 to plant 13, mine 9 to plant 14.

(5) Buying mines 5 and 9 are optimal because of the combination of upfront fees as well as transportation costs.