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Question 1

Let M be a matrix

$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Matrix M is non-singular if and only if

- ○ $ad + bc \neq 0$
- ○ $ad - bc \neq 0$

Singular values tell you how much the matrix stretches space along orthogonal directions.

A matrix is non-singular (invertible) if it's determinant is **not zero**.

$$det(M) = ad - bc$$ for 2by2 matrix

Question 2

Given the matrices

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 & 0 \\ -2 & 7 \end{bmatrix}$

Find the matrix

$(AB)^T$

- ○ $\begin{bmatrix} 0 & 4 \\ 21 & 28 \end{bmatrix}$
- ○ $\begin{bmatrix} 6 & 12 \\ 18 & 22 \end{bmatrix}$
- ○ $\begin{bmatrix} 6 & 18 \\ 12 & 22 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1\cdot6+3\cdot-2 & 1\cdot0+3\cdot7 \\ 2\cdot6+4\cdot-2 & 2\cdot0+4\cdot7 \end{bmatrix} = \begin{bmatrix} 0 & 21 \\ 4 & 28 \end{bmatrix} = AB$$

$$(AB)^T = \begin{bmatrix} 0 & 4 \\ 21 & 28 \end{bmatrix}$$

Matrix Multiplication is not communicative (order matters)

$$AB \neq BA$$

Question 3

Use elementary row operations to find the RREF for a counting matrix

$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

- ○ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -8 \\ 0 & -6 & -12 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
- ✓ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow[R_3:R_3-7R_1]{R_2:R_2-4R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \xrightarrow{R_2:\frac{R_2}{-3}}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix} \xrightarrow[R_3:R_3+6R_2]{R_1:R_1-2R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 4 — 2 pts

The general solution for the linear homogeneous system depends only on the free variables and there is a solution for any choice of these variables. The vector coefficients of each one of the free variables are called the fundamental solutions.

If coefficient matrix A has dimensions 9x27, is the statement that the linear system $Ax=0$ must have at least 18 fundamental solutions correct?

- ✓ True
- ○ False

- Pivot columns = linearly independent column
- Pivot variables = dependent (leading) variable

A homogeneous system of linear equations is a system where all constant terms (from the right side of the equation) are zero. This matrix could have 9 (or less) pivot columns (aka the leading or dependent variable). Assuming that we have 9 pivot columns then that leaves us with 18 non-pivot columns (aka free variables). Non-pivot columns can lead to fundamental solutions.

Even if we don't have 9 pivot columns the answer is still true since the question is saying that the system of linear equations must have **at least** 18 fundamental solutions.

Question 5

Find the rank of A

$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$

- ○ 2
- ✓ 3
- ○ 1

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix} \xrightarrow[R_3:R_3-R_1]{R_2:R_2-R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{bmatrix} \xrightarrow{R_2:\frac{R_2}{2}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 6 & 24 \end{bmatrix} \xrightarrow[R_3:R_3-6R_2]{R_1:R_1-2R_2}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 6 \end{bmatrix} \xrightarrow{R_3:\frac{R_3}{6}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_2:R_2-3R_3]{R_1:R_1+3R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 6

Please choose the right answer for the statement below:

Suppose V1,V2,V3,V4,V5 are five vectors in the space R^3

Then number of redundant vectors....

- ○ must be two
- ○ can be any number from 0 to 3
- ✓ can be any number from two to five

Since this matrix has 5 factors in a subspace of R^3, the rank (aks the number of pivot columns or lineary independent vectors) can be at most 3. This means that at least 2 vectors must be redundant (aka dependent variables)

Question 7

Which of the following are the eigenvectors of the matrix

$M = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$?

More than one answer can be correct and zero or more options can be correct

- [x] $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- [x] $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$
- [x] $\begin{bmatrix} -2 \\ 6 \end{bmatrix}$
- [x] $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$
- [ ] $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

Step 1: Solve for eigenvalues

$$\begin{bmatrix} 2-\lambda & 1 \\ 3 & 0-\lambda \end{bmatrix} = A - \lambda I$$

$$\det(A-\lambda I) = (2-\lambda)(-\lambda) - 3\cdot 1 = -2\lambda + \lambda^2 - 3$$

$$= \lambda^2 - 2\lambda - 3 = (\lambda-3)(\lambda+1)$$

$$\lambda = 3 \qquad \lambda = -1$$

Step 2: Find eigenvector for each eigenvalue

$\lambda = 3$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 3 & -3 & 0 \end{array}\right] \xrightarrow{R_2 : R_2 + 3R_1} \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 : -R_1} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x - y = 0 \qquad \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x = y$$

$\lambda = -1$

$$\left[\begin{array}{cc|c} 3 & 1 & 0 \\ 3 & 1 & 0 \end{array}\right]$$

$$3x + y = 0$$
$$3x = -y$$
$$x = -\frac{y}{3}$$

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{3}y \\ y \end{bmatrix} = y\begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix} = -3\begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Question 8

Suppose you have an mxn matrix A and its Singular Value Decomposition (SVD): $A = U\Sigma V^T$

Pick all of the following that are true:

- [x] V has n rows
- [ ] Σ has m columns
- [x] V has n columns
- [ ] U has n columns
- [ ] Σ has n rows
- [ ] U has n rows

Question 9

2 / 2 pts

In descent methods the particular choice of search direction does not matter so much, true or false?

- ( ) True
- (•) False

Correct!

Question 10

2 / 2 pts

Which of the following statements are true, check all that apply

- [x] Given an input $x \in R^n$, PCA compresses it to lower dimensions $z \in R^k$, where $k < n$
- [ ] PCA can be used only to reduce dimensionality of data
- [x] If the input features are on the very different scales it is a good idea to perform feature scaling before applying PCA