Find the range of the matrix below:

The set of all vectors $A\vec{v}$ such that $\vec{v} \in \mathbb{R}^4$

$$\operatorname{ran}\left(\begin{array}{c} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \\ \begin{array}{cccc} x & y & z & w \end{array} \end{array}\right) = \{A\vec{v} \mid \vec{v} \in R^4\} = \left\{ \begin{pmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \middle| \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \right\}.$$

Pivots

$\vec{v}$ = Denotes a vector. The arrow indicates that this isn't a scalar

$\vec{v} \in R^4$ = Means the vector $\vec{v}$ is in a 4 dimensional real space. It means the vector has 4 components/variables.

Vertical line "I" = means "such that"

$$= \left\{ x\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + y\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + z\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + w\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \middle| x, y, z, w \in \mathbb{R} \right\}.$$

The notation of v (with a arrow) is not used in this expression since we are unable to connect the vector v (with a arrow) to the individual scalars (x, y, z, w)

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 : R_2 - R_1} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_3 : R_3 - R_2} \begin{array}{c} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccc} x & y & z & w \end{array} \end{array}$$

Pivots

A pivot is defined as:

1. A row with a leading one
2. That one is the first nonzero entry in the row (reading left to right)
3. It is used to eliminate entries above and below it (has zeros above and below it)

$$range(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

- The range (or column space) of matrix A is the set of all possible output vectors $A \cdot \vec{v}$
- The range of matrix A lives in the co domain of the matrix which is based on how many rows the matrix has

Domain: R^4

Range: A subspace of R^3 Specifically a 2 dimensional subspace for this example since there were only 2 pivot columns. So the range is a 2 dimensional subspace of R^3 (a plane in a 3D space)

Question 1

If each component of a vector in R^3 is doubled, the L2 norm of that vector is doubled

- [x] True
- [ ] False

- The L2 norm (aka the Euclidean norm) of a vector is its length or distance from the orgin
- R^3 is the entire 3 dimensional space
- Doubling each component of a vector is a linear transformation. More specifically, a scalar multiplication transformation

The transformation described is:

- A linear transformation, because it satisfies:
  - Additivity: $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$
  - Homogeneity: $T(c\,\vec{v}) = cT(\vec{v})$
- A dilation (scaling) which means it stretches vectors away from the origin

Question 2

Let matrix T be

$$T = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix}$$

and let vectors u and v be

$$u = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

For each of the vectors determine whether the vector is in the null space of matrix T: $Null(T)$

Please select the correct answer below:

- [ ] Both vectors u and v are in the Null(T)
- [ ] Vector u is in the $Null(T)$ and vector v is not
- [x] Vector u is not in the Null(T) and vector v is
- [ ] None of the vectors are in the $Null(T)$

- The null space is when you multiply a matrix by a vector and the output is a zero vector

$$T\vec{x} = \vec{0}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

2 x 3   3 x 1   2 x 1

$$-3 + 2 + 1 = 0$$

$$-9 + 6 + 4 = 1$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2 x 3   3 x 1   2 x 1

$$-2 + 2 + 0 = 0$$

$$-6 + 6 + 0 = 0$$

Question 3

Suppose that $v1=(2,1,0,3)$, $v2=(3,-1,5,2)$, $v3=(-1,0,2,1)$ are column vectors temporarily lying down

Which of the following vectors are in span of $(v1,v2,v3)$?

(There are multiple correct answers)

- ✗ $(2,3,-7,3)$
- ✗ $(0,0,0,0)$
- ☐ $(1,1,1,1)$

The question is asking whether each vector (listed in the answer choices) can be written as a linear combination of V_1, V_2, V_3

If this is true, then the vectors are in the span of V_1, V_2, V_3

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{array}\right] \xrightarrow{R_1: \frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & -\frac{1}{2} & 1 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{array}\right] \xrightarrow[R_4: R_4 - 3R_1]{R_2: R_2 - R_1} \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & -\frac{1}{2} & 1 \\ 0 & -\frac{5}{2} & \frac{1}{2} & 2 \\ 0 & 5 & 2 & -7 \\ 0 & -\frac{5}{2} & \frac{5}{2} & 0 \end{array}\right] \xrightarrow{R_2: -\frac{2}{5}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & -\frac{1}{2} & 1 \\ 0 & 1 & -\frac{1}{5} & -\frac{4}{5} \\ 0 & 5 & 2 & -7 \\ 0 & -\frac{5}{2} & \frac{5}{2} & 0 \end{array}\right] \xrightarrow[\substack{R_3: R_3 - 5R_2 \\ R_4: R_4 + \frac{5}{2}R_2}]{R_1: R_1 - \frac{3}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{5} & \frac{11}{5} \\ 0 & 1 & -\frac{1}{5} & -\frac{4}{5} \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 2 & -2 \end{array}\right] \xrightarrow{R_3: \frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{5} & \frac{11}{5} \\ 0 & 1 & -\frac{1}{5} & -\frac{4}{5} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -2 \end{array}\right] \xrightarrow[\substack{R_2: R_2 + \frac{1}{5}R_3 \\ R_4: R_4 - 2R_3}]{R_1: R_1 + \frac{1}{5}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

no contradiction in the final matrix

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{5} \\ 1 \\ -\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 4/5 \\ -1/5 \\ 2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 4/5 \\ -1/5 \\ 2/3 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 14/15 \\ -1/15 \\ 2/3 \\ -7/3 \end{bmatrix}$$

I directly applied the same row operations in this vector

This vector is not in the span of V_1, V_2, V_3 because

$$0x_1 + 0x_2 + 0x_3 \neq -\frac{7}{3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 14/15 \\ 0 & 1 & 0 & -1/15 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & -7/3 \end{array}\right]$$

Question 4 1 pts

The two vectors $u = (-2,3,1,4)$ and $v = (1,2,0,-1)$ are orthogonal in the vector space R^4

☒ True

○ False

- Two vectors are orthogonal if their dot product is zero and they are both non-zero vectors

$$\vec{u} \cdot \vec{v} = 0$$

- Orthogonal vectors are linearly independent

$$\underset{4 \times 1}{\begin{bmatrix} -2 \\ 3 \\ 1 \\ 4 \end{bmatrix}} \cdot \underset{4 \times 1}{\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}} = -2 + 6 + 0 - 4 = 0$$

Question 5

This question has multiple correct answers!

Let V be a five-dimensional vector space, and let S be a subset of V which is a basis for V. Then S

☒ Must have exactly five elements.

☒ Must span V.

☐ Can have any number of elements (except zero).

☒ Must be linearly independent.

☐ Must be linearly dependent.

A basis of a vector space:

1. Spans the space
2. Is linearly independent
3. Has exactly n elements where n is the dimension of the space.