Equations

$1 \cdot x_1 + 2 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4 = 1$
$1 \cdot x_1 + 2 \cdot x_2 + 1 \cdot x_3 + 2 \cdot x_4 = 2$
$0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 = 1$

Using Gauss Jordan Elimination to transform the matrix to reduced row echalon form

In Gauss Jordan Elimination you use the elementary row operations (ERO)
1. Swap 2 rows
2. Multiply rows by a non-zero constant
3. Add or subtract the multiple of one row to another

The 3-row augmented matrix is:

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 1 & 2 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_2 : R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_3 : R_3 - R_2}$$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \boxed{\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]} \end{array} \xrightarrow{R_1 : R_1 - R_2} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The matrix here is in reduced row echalon form since it has 2 Pivots. It is the RREF of the system of linear equations.

A pivot is defined as
1. A row with a leading 1
2. That 1 is the first nonzero entry in the row (reading left to right)
3. It is used to eliminate entries above and below it (has zeros above and below it)

The RREF of the matrix tells us that this is a under determined system. An under determined system means there are not enough constraints to uniquely determine all variables.

Next step: The next step is to isolate the variables that correspond to the pivot columns (the leading variables or **dependent variables**) and treat the rest as **free variables** (the variables that belong to non-pivot columns)

Dependent variables: $x_1 \quad x_3$

Free variables: $x_2 \quad x_4$

The reduced system after Gauss-Jordan Elimination (or the system corresponding to RREF):

$$\begin{cases} x_1 + 2x_2 + x_4 = 1 \\ x_3 + x_4 = 1 \end{cases}$$

Isolate the variables that correspond to the pivot columns:

$$x_1 = 1 - 2x_2 - x_4$$
$$x_3 = 1 - x_4$$

General Solution:

$x_1 = 1 - 2x_2 - x_4$
$x_2 =$ free (any real number)
$x_3 = 1 - x_4$
$x_4 =$ free (any real number)

Parametric vector form of the solution set:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Question 1 1 pts

Determine whether the matrix A is in Reduced Row Echelon Form:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- [x] True
- [ ] False

Yes, Because the columns that contain pivots satisfy all the conditions required for a matrix to be in reduced row echelon form (RREF)

Question 2 1 pts

A homogeneous linear system in n unknowns, whose corresponding augmented matrix is in Reduced Row Echelon Form with r leading 1's, has n-r free variables

- [x] True
- [ ] False

- A homogeneous system of linear equations is a system where all the constant terms (from the right side of the equation) are zero.
- A non-homogeneous system of linear equations has at least one non-zero constant on the right hand side

n = number of unknown x variables that we need to solve for (the total number of x variables in the system of linear equations)
r = the number of leading 1's in the RREF of the matrix (basically, the number of dependent/ leading variables)

n-r = the number of free variables

A homogeneous system was specified here since a homogeneous system is always consistent (it will always have solutions). Whereas, there might be situations where a non-homogeneous system might not have a solution. A non-homogeneous system is not always consistent.

Example of a inconsistent non-homogeneous system: $\begin{aligned} x + y &= 2 \\ x + y &= 3 \end{aligned}$

After Gauss Jordan Elimination (or the system in RREF): $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 1 \end{array}\right]$

Question 3

If A is a 3x5 matrix, what is the largest possible value for its Rank?

- [ ] 5
- [ ] 4
- [x] 3

Rank is the number of pivots that a matrix has when it is in RREF. More formally, the rank is the number of linearly independent rows (or columns) in a matrix.

Question 4

1 pts

For what value of a does the system have nontrivial solutions:

$$x1 + 2x2 + x3 = 0$$
$$-x1 - x2 + x3 = 0$$
$$3x1 + 4x2 + a*x3 = 0$$

- ○ 1
- ✗ -1
- ○ 0

If all equations in a system of linear equations are linearly independent, then only the **trivial solution** exists.

If one or more free variable exists, then that means the system of linear equations has a **nontrivial solution**.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -1 & -1 & 1 & 0 \\ 3 & 4 & a & 0 \end{array}\right] \xrightarrow{R_3:R_3-3R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & -2 & a-3 & 0 \end{array}\right] \xrightarrow{R_2:R_2+R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & a-3 & 0 \end{array}\right] \xrightarrow{R_1:R_1-2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & a-3 & 0 \end{array}\right] \xrightarrow{R_3:R_3+2R_2} \left[\begin{array}{cccc} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & a+1 & 0 \end{array}\right]$$

value of the variable a which makes the system have nontrivial solutions: $a+1=0$, $a=-1$

value of the variable a which makes the system have trivial solutions: $a \in R, a \neq -1$

This means that a can be any real number except -1

Question 5

1 pts

We use Moore-Penrose pseudo-inverse to determine the solution of $Ax = b$ which also corresponds to the minimum norm least-squares solution

- ✗ True
- ○ False

A Moore-Penrose Pseudo-inverse is used to find the inverse of non-square matrices.

The Moor-Penrose Pseudo-inverse (when used to find x) gives a solution that minimizes the squared error and selects the error which has the smallest length (norm)

$$x = A^+ \cdot b$$

The notation of the moore penrose pseudo-inverse