Bayesian Statistics in Accounting Research

EAA PhD Forum 2018 – Milan

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Why Bayes?

"If I'm doing an experiment to save the world, I better use my prior." – Andrew Gelman

Disclaimer: I am no statistical theorist and I am still learning lots of things myself. This talk is supposed to be an applied perspective on which Bayesian tools are useful additions to our empirical tool belt.

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Things we grapple with:

1. Noisy data, lots of correlated variables
   $\rightarrow$ Regularization
2. Sparse data, need to model latent constructs or heterogeneity
   $\rightarrow$ Hierarchical Models
3. Quantify uncertainty in estimates properly for decision making
   $\rightarrow$ Posterior Distribution

Agenda for Today

1. Quick overview of classical hypothesis testing
2. Brief introduction to Bayesian inference
3. Regularization (general and Bayesian adaptive versions)
4. Hierarchical Models / Model building
5. Summary and food for thought on applications in Accounting Research

Unfortunately no time for the ins and outs of fitting Bayesian models (But code for all examples/figures on Github at github.com/hschuett/EAA2018Bayes)

Classical Hypothesis Testing

Key Problem of Inference

You want to learn something about an outcome from observations, but you cannot observe everything about the outcome

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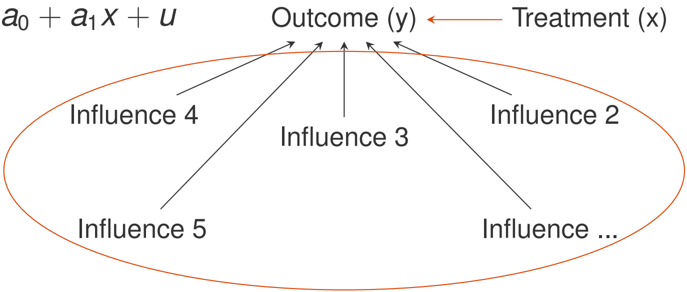
$$y = a_0 + a_1 x + u$$

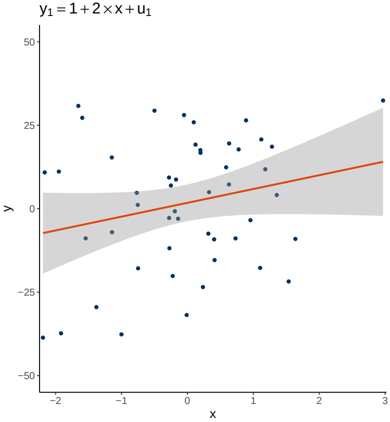


Unmeasured influences / “Noise” (u)

Nothing in the world is truly random – randomness is a vehicle to reason about unmeasured influences (e.g. a “random” coin toss)

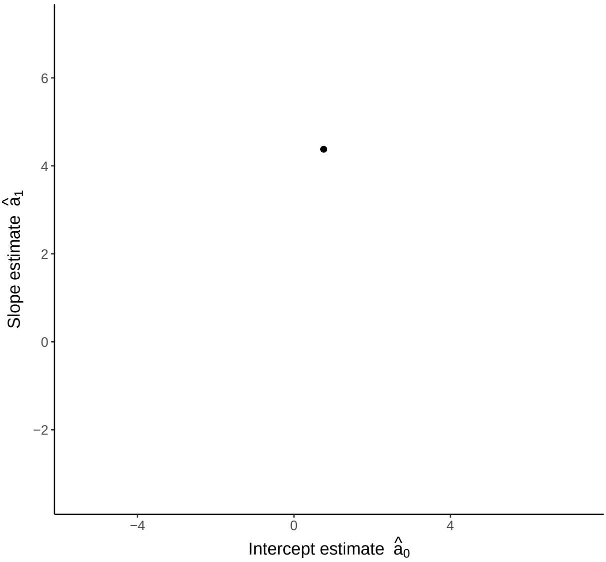
Unmeasured influences cause sampling variation

Two samples with the same $x \rightarrow y$ relation and the same x, produce different estimates because of unmeasured influences



So, how do you work with only one sample?

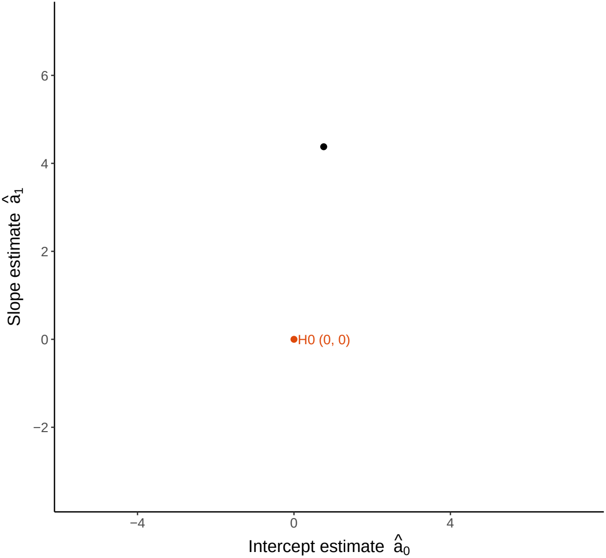
Inference: What are the true values a_0 and a_1?



Fitting $y = a_0 + a_1 x + u$

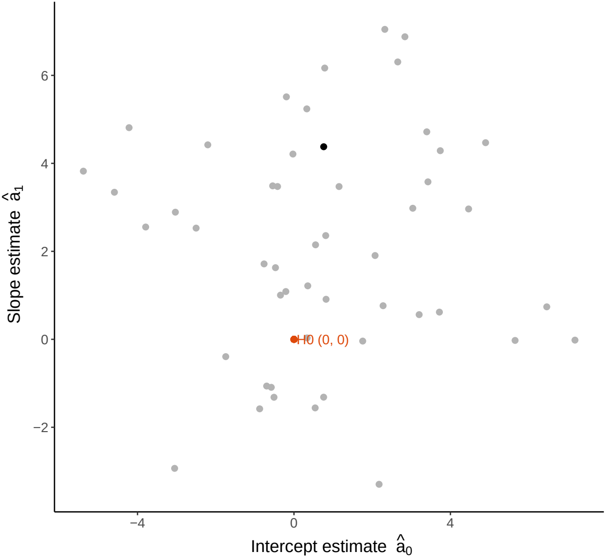
- Only one sample
- Estimates: $\hat{a}_0 = 0.8$, $\hat{a}_1 = 4.4$

Hypothesis Testing



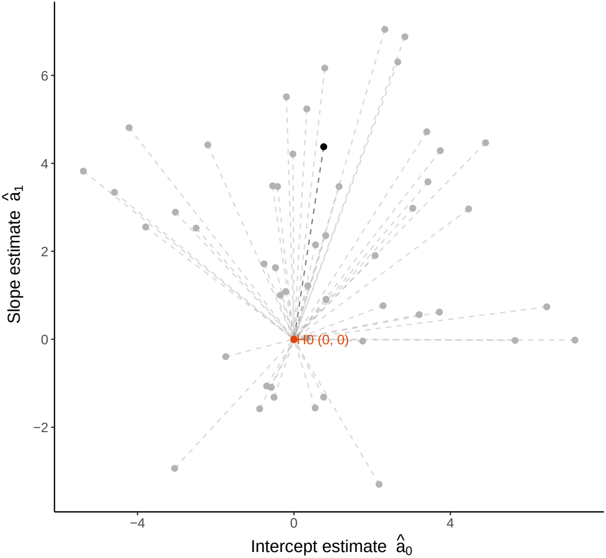
- Don't try to infer true value; Test a hypothesis *H*0
- How do we test *H*0?

Frequentist Uncertainty: Imaginary Resampling



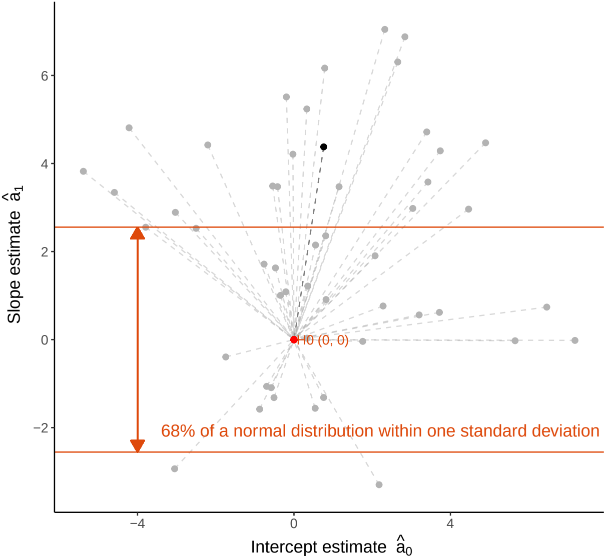
- Thought experiment. Suppose:
  1. H0 is true
  2. Regression setup correct
  3. Can draw more samples
- Constant x, variation comes from different u
- How often do we draw values like $(\hat{a}_0, \hat{a}_1)$?

Key Idea of an Hypothesis Test I



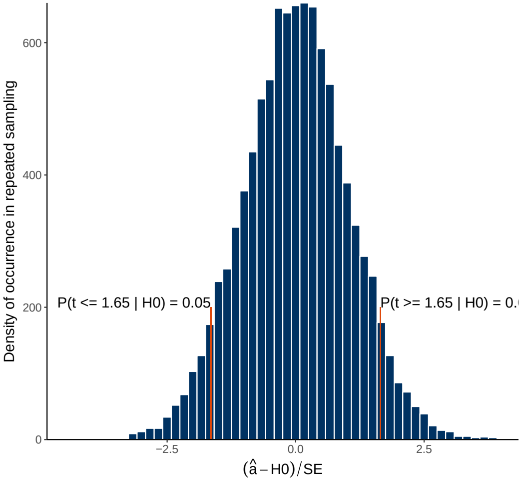
- The larger distance $\left|\hat{a}^1 - H0\right|$, the less likely $H0$
- But how unlikely?
- Can frame expected variation as a probability distribution (e.g., $\hat{a} \sim N(a, var[\hat{a}])$)
- All identification assumptions, error-term assumptions etc. are there to determine what distribution best reflects the expected variation (asymptotically)

Key Idea of an Hypothesis Test II



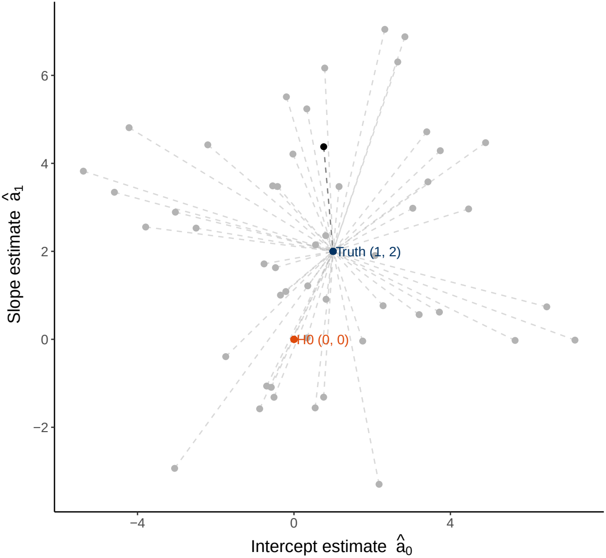
- Depending on your assumptions: $\hat{a} \sim N(a, var[\hat{a}])$
- Then
$$\frac{|\hat{a} - a|}{var[\hat{a}]} = N(0, 1)$$
- $var[\hat{a}]$ unobservable: you only have one sample and not many
- **Within sample estimate of** $var[\hat{a}]$**:** $SE(\hat{a})$

Key Idea of an Hypothesis Test III



- $\frac{|\hat{a}-a|}{var[\hat{a}]} = 1.65$
- Given expected Distribution, how often would 1.65 would occur if $H0$ true?
- Assumptions about u, which determine $SE(\hat{a})$ and form of test distribution

What are the True Values a_0 and a_1?



Solution: the true values

- $y = 1 + 2 * x + u$
- $\hat{a}_0^1 = 0.8$, $\hat{a}_1^1 = 4.4$
- Barely significantly different from zero at 10%
- True slope is 2, not 4.4
- Testing hypotheses; not inferring true values

So much noise that chance of estimates far away from true value is high

Summary

Great decision tool!

- (Only) two bits of data:
  - Estimate $\hat{a}$
  - $SE(\hat{a})$
- Fast and simple
- Effect of noise often unappreciated (i.e., "What does not kill it makes it stronger fallacy")
- Hard part is getting SEs correct and finding the test statistic distributions (i.e. asymptotic theory, etc.)

Bayesian Inference

Probability as a Measure of Plausibility

- Same setup: $y = X\beta + u$
- What if you want to know: what are the most plausible parameter values given the data?

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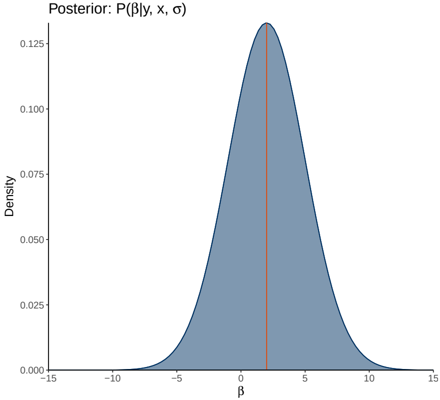
- What if you want to know: what are the most plausible parameter values given the data?
- What you are looking for is the mode of the posterior distibution of the paramters given the data

$$P(\beta, \sigma | y, X)$$

But shape of $P(\beta, \sigma | y, X)$ also summarizes uncertainty!



OLS βs the Bayesian Way

Plausible parameter values given the data?

1. Formulate OLS model as a distribution:

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3. Add priors to ”turn the likelihood around”:

$$\underbrace{p\left(\beta, \sigma | y, X\right)}_{\text{Posterior}} = \frac{\overbrace{p\left(\beta, \sigma\right)}_{\text{Prior}} \overbrace{p\left(y | X, \beta, \sigma\right)}_{\text{Likelihood}}}{\underbrace{p\left(y | X\right)}_{\text{Data}}}$$

Difference Between Posterior and Likelihood

$$\underbrace{p(\beta,\sigma|y,X)}_{\text{Posterior}} = \frac{\underbrace{p(\beta,\sigma)}_{\text{Prior}}\underbrace{p(y|X,\beta,\sigma)}_{\text{Likelihood}}}{\underbrace{p(y|X)}_{\text{Data}}}$$

- Likelihood: $p(y|X,\beta,\sigma)$
  Probability of seeing the data y given parameters
- Posterior: $p(\beta,\sigma|y,X)$
  Probability of parameter being β given the data

If you want to know the most likely parameter given data, you are looking for the mode of the posterior, not the maximum likelihood.

Core Mechanism: Bayesian Updating

Example: we want to know the ability of a manager, measured as the probability θ of making good investments. Manager has a track record of $y = 8$ out of $n = 15$ successful investments

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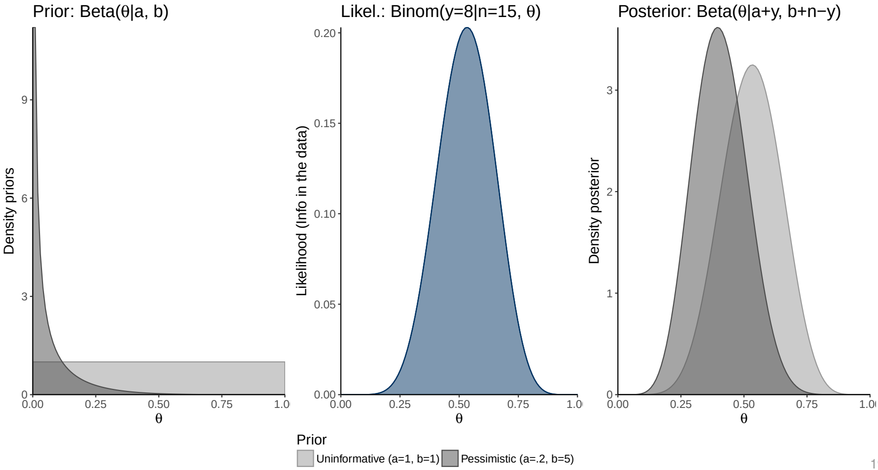

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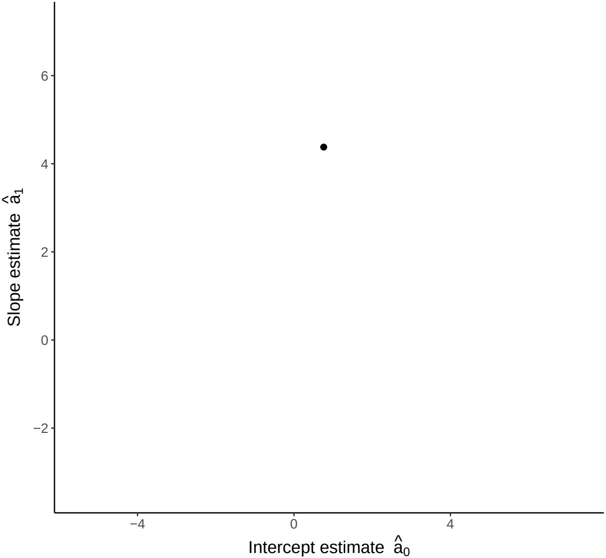
Prior

- Input: Quantifies prior state of knowledge as a distribution
- Endless source of debate: how subjective, etc.
- Nowadays weakly informative priors quite common

Bayesian Regression

Back to our original OLS estimate



- $\hat{a}_0^1 = 0.8$ and $\hat{a}_1^1 = 4.4$
- How would a Bayesian version look like?

Bayesian Regression

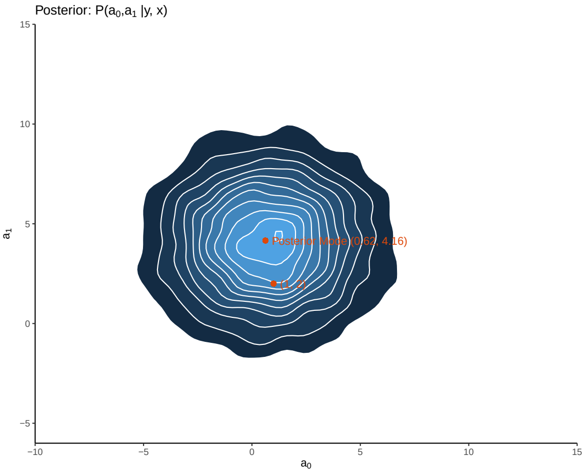
1. Specify a full model with distributions

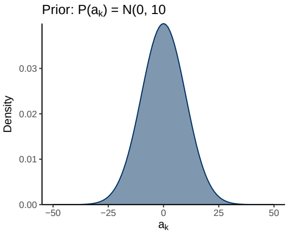
$$\begin{aligned} y &\sim N(\mu, \sigma) \\ \mu &= a_0 + a_1 * x \\ a_0 &\sim N(0, 10) \\ a_1 &\sim N(0, 10) \\ \sigma &\sim Cauchy(0, 20) \end{aligned}$$

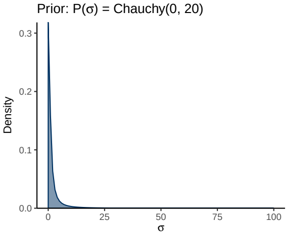
- Additional assumptions: The prior distributions
- Advantage: No more pain with inferences only holding asymptotically
- Advantage: You incorporate more information (which I argue is often a good thing)

Fitting $y = a_0 + a_1 * x + u$ with Very Weak Priors

Assume: We don't know much about what range the expected effects should be

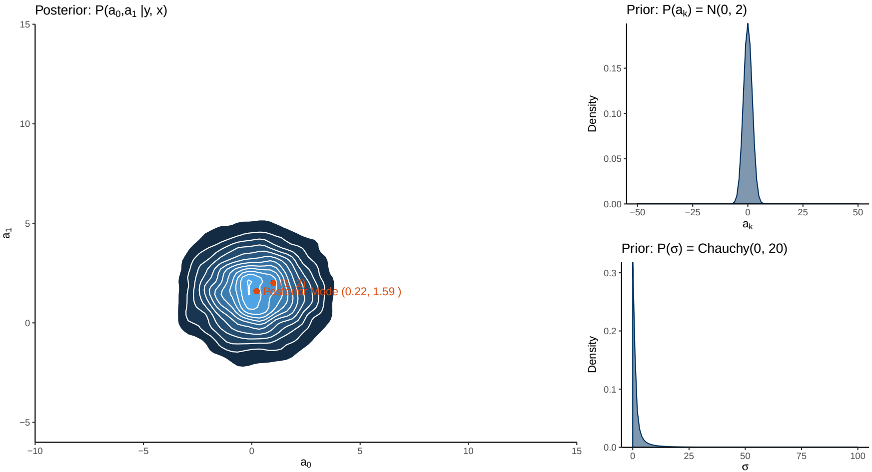






Fitting $y = a_0 + a_1 * x + u$ with Informative Priors

Assume: We are pretty confident that the effect, if present, shouldn't be bigger than 4



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- Not if you believe in the prior. Does this introduce more subjectivity?
- From a practical perspective, I'd argue you often have not very debatable, weak priors that already help a lot combating noise/overfitting, etc.

Regularization

Regularization is Everywhere in Modern Statistics

E.g., A Lasso Regression is nothing but a fast implementation of a Laplace prior

$$y \sim^{iid} N(X\beta, \sigma^2)$$
$$\beta \sim Laplace(0, \tau)$$

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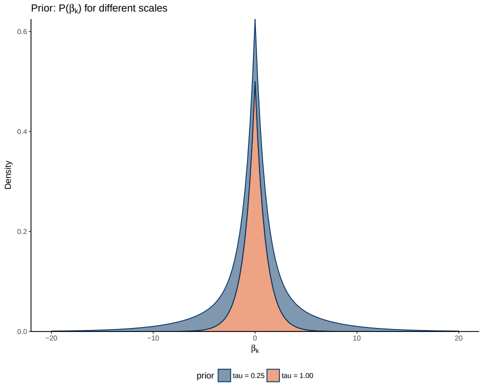

then with $\lambda = \sigma^2/\tau$, the mode of $P(\beta|y, X, \sigma)$ minimizes

$$\underbrace{(y - X\beta)'(y - X\beta)}_{MeanSquaredError} + \underbrace{\lambda \sum_i |\beta_i|}_{LassoPenalty}$$

Regularizing Example: Has Speech Become More Polarized?

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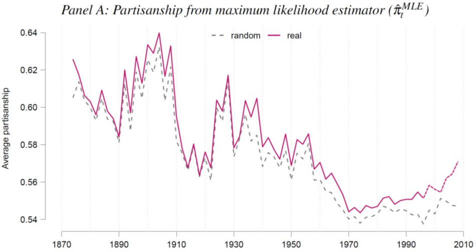
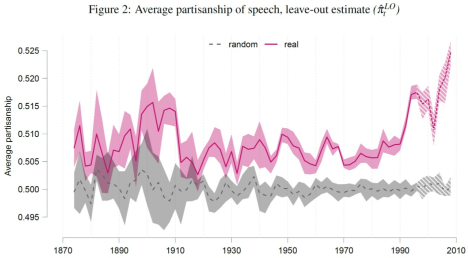
- Fig. 1 and 2:





Regularization in Modern Applications

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- Similar use for variable selection. E.g., “We show that 15 out of 49 variables used in prior work robustly explain IPO returns.” (Butler et al., 2014, e.g.,)

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- Similar use for variable selection. E.g., “We show that 15 out of 49 variables used in prior work robustly explain IPO returns.” (Butler et al., 2014, e.g.,)
- Bayesian approach the most flexible (Sometimes does not scale easily)

Automatic Regularization via Hierarchical Priors

Imagine you are interested in estimating some fixed effects

$$
\begin{aligned}
y_{i,t} &= a_i + u_{i,t} \\
a_i &\sim N(2, \sigma_a = 3) \\
u_{i,t} &\sim N(0, \sigma_u = 15)
\end{aligned}
$$

- Think of: CEO effects, firm effects, analyst effects
- Often only few observations per i (i.e., 5 years per firm)
- Use prior to discipline the a_i estimates

Regularization via Priors

Data Generating Process:

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Bayesian model:

$$\begin{aligned} y_{i,t} &= a_i + u_{i,t} \\ a_i &\sim N(\mu_a, \sigma_a) \\ u_{i,t} &\sim N(0, \sigma) \\ \mu_a &\sim N(0, 100) \\ \sigma_a &\sim \mathit{HalfN}(0, 100) \\ \sigma_u &\sim \mathit{HalfN}(0, 100) \end{aligned}$$

Tiny bit of information helps already

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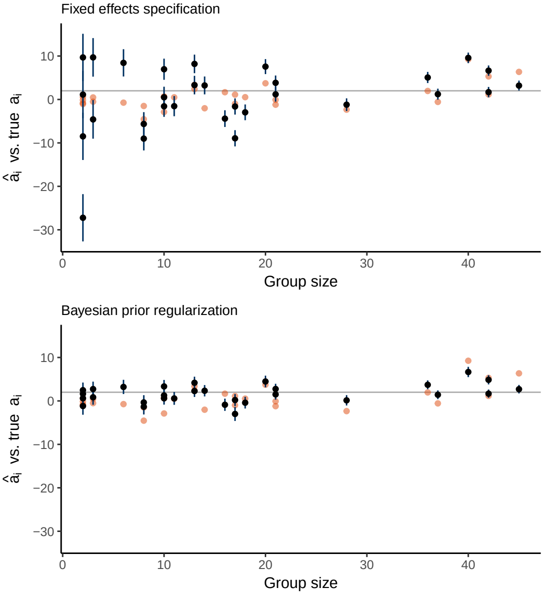

Hierarchical Models

Punchline so Far

- Bayesian inference rests on priors, which summarize prior information

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- Bayesian inference rests on priors, which summarize prior information
- Regularization uses prior information to discipline estimates
- But many other important use cases for prior information:
    - Deal with sparse data, missing values
    - Combine multiple sources of data (chain them together via updating) to identify interesting constructs

Let's look at an example

The Best Part: Flexible Hierarchical Models

“Bayesian Estimation of Population-Level Trends in Measures of Health Status”
Finucane et al. (2014), largest-ever analysis of metabolic risk factors and the first global analysis of trends

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Bayesian model to combine data from all 199 studies, drawing strength from countries with data

A Hierarchical Model for Blood Pressure

$$y_{h,i} \sim N\left(\underbrace{a_{j[i]} + b_{j[i]} t_i + u_{j[i],t_i}}_{\text{country average + time trend}} + \overbrace{X_i' \beta + \gamma_i(z_h) + e_i}^{\text{age trends, study-level covariates}}, \frac{s_{h,i}^2}{n_{h,i} + \tau_i^2} \right)$$

$$a_j = a_{country} + a_{subregion} + a_{region} + a_{global}$$

$$b_j = b_{country} + b_{subregion} + b_{region} + b_{global}$$

$$a_c \sim N(0, \kappa_c), a_s \sim N(0, \kappa_s), a_r \sim N(0, \kappa_r), a_g \sim N(0, \kappa_g)$$

$$b_c \sim N(0, \eta_c), b_s \sim N(0, \eta_s), b_r \sim N(0, \eta_r), b_g \sim N(0, \eta_g)$$

y: blood pressure, i: study, j: country, h: gender

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- a_j, b_j structure assumes that countries close to each other (same subregion, etc.) are similar
- Even if a country has no study, info about subregion AND info about distribution of country effects (e.g., $a_c \sim N(0, \kappa_c)$)!

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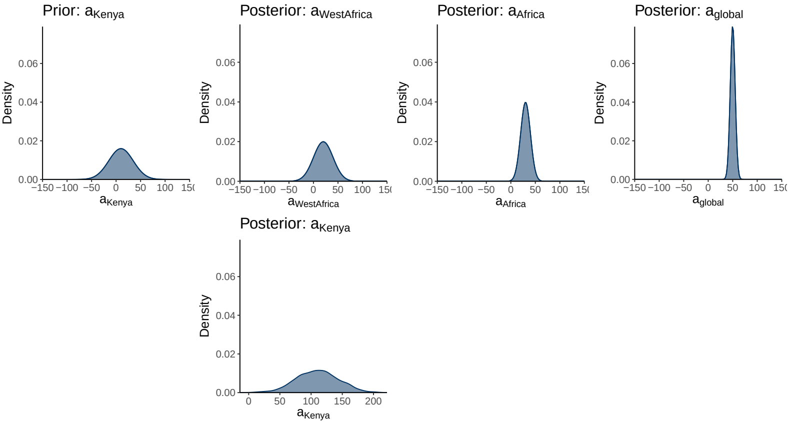
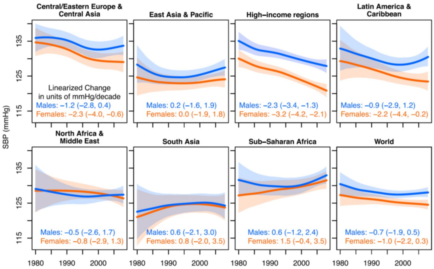
- Yes, the structure helps: $a_j = a_{country} + a_{subregion} + a_{region} + a_{global}$

Benefits of Hierarchical Structure

- Assume there is no study conducted about Kenya in a 30 year period? Can we say something about Kenya's blood pressure trends?
- Yes, the structure helps: $a_j = a_{country} + a_{subregion} + a_{region} + a_{global}$
- Chain evidence together:



Models with Impact



The results informed:

- WHO Global Status Report on noncommunicable diseases (NCDs; WHO, 2011)
- Targets for cardiovascular disease risk factors for the UN high-level meeting on NCDs
- US National Academy of Sciences Panel on International Health Differences in High Income Countries (Woolf and Aron, 2013)

Food for Thought
Use cases in Accounting

Firm-Level Heterogeneity

Let's take earnings "Persistence" as an important Accounting firm-level concept.

$$RoA_{i,t+1} = \beta_i RoA_{i,t} + u_{i,t}$$

How do we usually estimate this?

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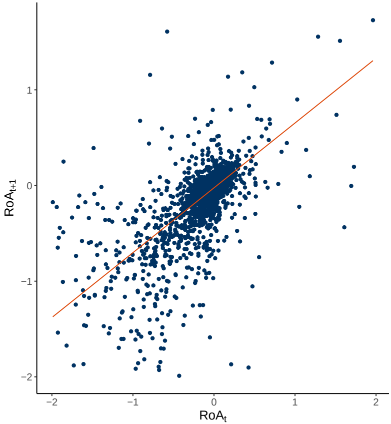

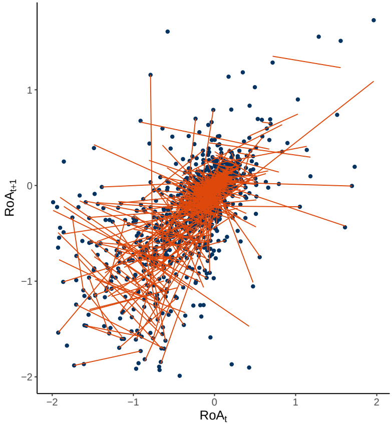

Firm-Level Heterogeneity

Can we do better? Let's frame this in a Bayesian model and use weak adaptive priors to regularize

$$
\begin{aligned}
RoA_{i,t+1} &\sim N(\beta_i RoA_{i,t}, \sigma_u) \\
\beta_i &\sim N(\mu_{firm}, \sigma_{firm}) \\
\mu_{firm} &\sim N(0, 10) \\
\sigma_{firm} &\sim HalfN(0, 10) \\
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How much shrinkage Do we get?

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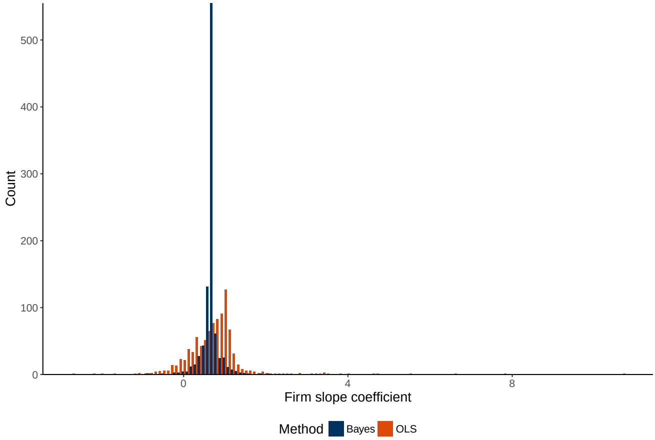
How much shrinkage Do we get?

Method	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
OLS	-48.0034	0.3493	0.7480	0.6252	0.9985	10.6649
Bayes	-0.9404	0.6479	0.6722	0.6703	0.7059	2.1198



Posterior Means:

- $\mu_{firm} = 0.67$
- $\sigma_{firm} = 0.04$
- $\sigma_u = 0.19$

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- Combing multiple data sources
  e.g., survey and archival data (Work in progress)

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- Shape of posterior distribution ideal measure of uncertainty

Are There Disadvantages?

Yes. Computational complexity

- Most posterior distributions cannot be derived analytically. Estimated by brute force Markov Chain Monte Carlo Simulation.
- Models with a lot of data and/or large number of parameters can take a long time to fit (i.e., days)
- If something is wrong with the model, it is harder to figure out what went wrong
- But: Very active area of research in speeding things up (CUDA MCMC, Variational Inference, TensorFlow Probability, etc.)

Learning Bayesian Statistics

In case you got interested

Take a course!

Textbooks:

- Gelman et al. (2013)

Software:

- Stan. Biggest momentum, easiest to use (Pystan, Rstan), very active forum and lots of case studies (examples used in this presentation are coded with Rstan)
- JAGS. Older but well optimized and established sampler
- Nimble. For the real experts, who want to write their own samplers
- Stata modules available. I have no experience with them though
- TensorFlow Probability. For large scale machine learning

Thank you for your attention

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