The Gambler's Ruin Problem:
Two gamblers start with a certain amount of money split between them and agree to play until one of the gamblers has all the money and the other has none. The game takes place over many turns, whereas on each turn, the players have same probability of winning as the turn before. The probability of winning for Gambler A is the complement of that of Gambler B. If Gambler B wins a turn, Gambler A gives one dollar to Gambler B, and vice versa.The turns continue to be played until a winner and loser are found. In this applet, the starting money amount will be $100 split between the two gamblers.

EQUAL LIKELIHOOD OF WINNING A TURN
First, we will look at the probability of Gambler A winning a game (all the money) given that the probability of winning a turn is 0.50. Set the “Probability of A winning a turn” numeric input to 0.50, if it is not already set.

1. What is the probability of Gambler A winning all the money if the two gamblers begin with equal money? Set the “Starting Money for Gambler A” to 50 and report the true probability of Gambler A winning a game.

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2. Also, what is the “Starting Money for Gambler B” and what is Gambler B’s true probability of winning a game?

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3. Now complete the table below for different starting money amounts for Gambler A.

Starting Money for Gambler A	True Probability of Winning All The Money (for Gambler A)
5	
15	
25	
35	
45	
55	
65	

Do you notice anything about the relationship between the starting money of Gambler A and his or her true probability of winning all the money?

4. Plot the points you found above in the space below and report the type of relationship you see.

Big Idea: When the probability of winning a turn is equal for both gamblers, the probability of winning a game for Gambler A is linearly related with the amount of money he has.

UNEQUAL LIKELIHOOD OF WINNING A TURN

Now we will look at what happens when the probability of winning on any given turn is not equal for the two gamblers. To do so, begin by changing the “Probability of A winning a turn” numeric input to 0.55.

5. Immediately, what did you notice about the true probability of winning for Gambler A when both gamblers begin with $50? What is that probability?

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6. Now explore different values of the “Starting Money for Gambler A” using the up and down arrows or by typing in different numbers. Notice how the true probability of gambler A winning changes at each of the values. Can you find the money amount for Gambler A to start with that would correspond with a true winning probability for the game roughly equal to that of a single turn? (Pr(A) = 0.55) Report this value.

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7. Did this value surprise you in any way? Was it higher than expected? Lower? The same? Explain.

8. Just as you have done before, complete the table of values.

Starting Money for Gambler A	True Probability of Winning All The Money (for Gambler A)
5	
15	
25	
35	
45	
55	
65	

Do you notice any differences about the relationship between starting money for Gambler A and the true probability of winning all the money in this table compared to when the probabilities of the two gamblers winning each turn were equal?

9. Plot the points found in the table above in the space below. Comment on the relationship you see.

10. Explore with more input values, changing the probability of winning a turn and the starting money amounts. What has a stronger effect on the true probability of winning a game? Do small changes in the probability of winning a turn or small changes in the starting money amounts more heavily affect the true probability of winning all the money?

    Big Idea: The probability of winning a turn for Gambler A has a much larger effect on the probability of winning the game compared to the starting Money amount. When the probability of winning a turn is not equal for the two gamblers, the relationship between the starting money and the probability of winning the game has a non-linear relationship.

UPDATED PROBABILITY OF WINNING

Next, we will look at how the probability of winning a game changes after each turn.

11. Set the "Starting Money for Gambler A" to 90 and the "Probability of A winning a turn" to 0.48. What is the true probability of winning a game for Gambler A?

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12. Now we will demonstrate how the conditional probability of Gambler A's probability of winning changes after each turn. Suppose Gambler B won three straight turns. What is the new updated probability that Gambler A will win the game? (*Hint: Click the Money for Gambler A down 3 times and report the true probability of winning.*)

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13. Suppose the next five turns take the following pattern: A wins; A wins; B wins; A wins; A wins. Click the Money for Gambler A up and down accordingly to follow this sequence of five turns. Report the true probability of winning all the money after each of the five turns.

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14. What do you notice about the money amount for Gambler A and the true probability of winning all the money after the end of these five turns?

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15. As you've done on previous exercises, continue pushing the starting money for Gambler A up and down various amounts of times in each direction, resembling the outcomes of

multiple turns in a single game. Notice how, after each turn, we are basically beginning a new game from the new money totals for each gambler.

Big Idea: Each turn is independent of the last. After each turn, the true probability is updated based on the new information we have about the game (money totals of each gambler). Thus the true probability of winning all the money <u>conditionally</u> changes based on the new information, as if you were beginning a new game at each turn.