The Gambler's Ruin Problem:
Two gamblers start with a certain amount of money split between them and agree to play until one of the gamblers has all the money and the other has none. The game takes place over many turns, whereas on each turn, the players have same probability of winning as the turn before. The probability of winning for Gambler A is the complement of that of Gambler B. If Gambler B wins a turn, Gambler A gives one dollar to Gambler B, and vice versa.The turns continue to be played until a winner and loser are found. In this applet, the starting money amount will be $100 split between the two gamblers.

IN-CLASS EXAMPLE
In pairs, you will play the gambler's ruin game. Assign one person to be Gambler A, and the other to be Gambler B. Take a dice and a 10 chocolate chips . Give Gambler A 2 chocolate chips and Gambler B 8 chocolate chips; this will represent your money. Gambler A wins a turn if the dice roll is a 1, 2, 3, or 4. Gambler B wins if the roll is a 5 or 6. Play the game until one gambler has all the money and the other has none.

1. Who won your game? Gambler A or B?

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2. Use the Gambler's Ruin Applet to find the true probability of Gambler A winning this game. Report this value.

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3. What proportion of pairs in the class would you expect to have Gambler A win the game?

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4. Now record the class data. What proportion of pairs had Gambler A win? Is this value different than what you predicted in exercise 3? How does it relate to the value found in exercise 2?

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   Big Idea: When playing one game, results may vary from one pair to another. When we put together all the results from all the pairs though, we find that the proportion of pairs in which Gambler A won is very close to the probability of Gambler A winning a single game. This is the big idea behind <u>simulation</u>, wherein we can play many games and the proportion of wins in those games will closely resemble the true probability of winning in the long run.

SIMULATION OF MANY GAMES

Our next step is to look at the probability of winning over many games using simulation. We will do this by looking at the observed probability of Gambler A winning at various numbers of simulations and how it relates to the true probability of winning for Gambler A, much like in the example explored above. To do so, start by opening the Gambler's Ruin Applet if it is not already open.

5. First, choose your own Starting Money and Probability of winning for Gambler A that makes the true probability of winning a game for Gambler A between 0.20 and 0.80. What is your true probability of winning for Gambler A? Who would you expect to win a single game?

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6. Now set "N", the number of simulations to 1. Then click the "Simulate" button to simulate <u>one game</u>. This produces a plot with a single point and a horizontal line. In this one simulation, did Gambler A win or lose? Was your prediction of a single game correct?

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7. What does the horizontal line represent?

8. Keep clicking the "Simulate" until the single point on the plot changes from 0 to 1 or vice versa. As you can see the single simulations have high variability in results and are bad estimates of overall probability of winning (being only 0 or 1).

9. Change the number of simulations, "N", from 1 to 5, and then click the "Simulate" button. What happens to the points in relation to the horizontal line as the simulation number increases?

10. Yet again, increase "N". This time to 100, and then click "Simulate". After the plot shows up, what do you notice about the points now with many more repetitions of the simulation?

11. Input more numbers for "N" and notice how the points asymptotically approach the horizontal line for different values. Are they more accurate with larger or smaller values?

Big Idea: Simulation is a great tool for estimating the long run probability of an event. As you increase the number of simulations performed, the estimate will get more accurate and approach the theoretical probability. In relation to the simulations, the variability in who wins one game, or even as many as 5 games, can be very high but that variability will decrease over many games. All in all, the probability of winning a game is the proportion of games that a gambler would win if he played many, many, many times.