Genetic algorithms

- Based on “survival of the fittest.”
- Start with “population of points.”
- Retain better points
- Based on “natural selection.”
- (as in genetic processes)

Genetic algorithms

- Maximise f(x), $x_i^L \le x_i \le x_i^u$
- Code every variable using binary string

  Eg.(00000) – (11111)

  gives 2^5 values
- The range of each variable is mapped to this range
- $x_i = x_i^L + (x_i^U - x_i^L)/(2^{l_i} - 1) * \text{decoded value}(s_i)$
- A value in this range represents actual value of variable. (eg. If $0 \le x < 8$ needs values if desired accuracy desired is 0.5)
- Every variable is represented by set of strings like this.

Objective/Fitness function

- Convert minimization problem to maximization problems
- $F(x) = 1/(1+f(x))$
- Not $F(x) = -f(x)$
- Evaluates "Fitness" of a string.

Generate a population of strings

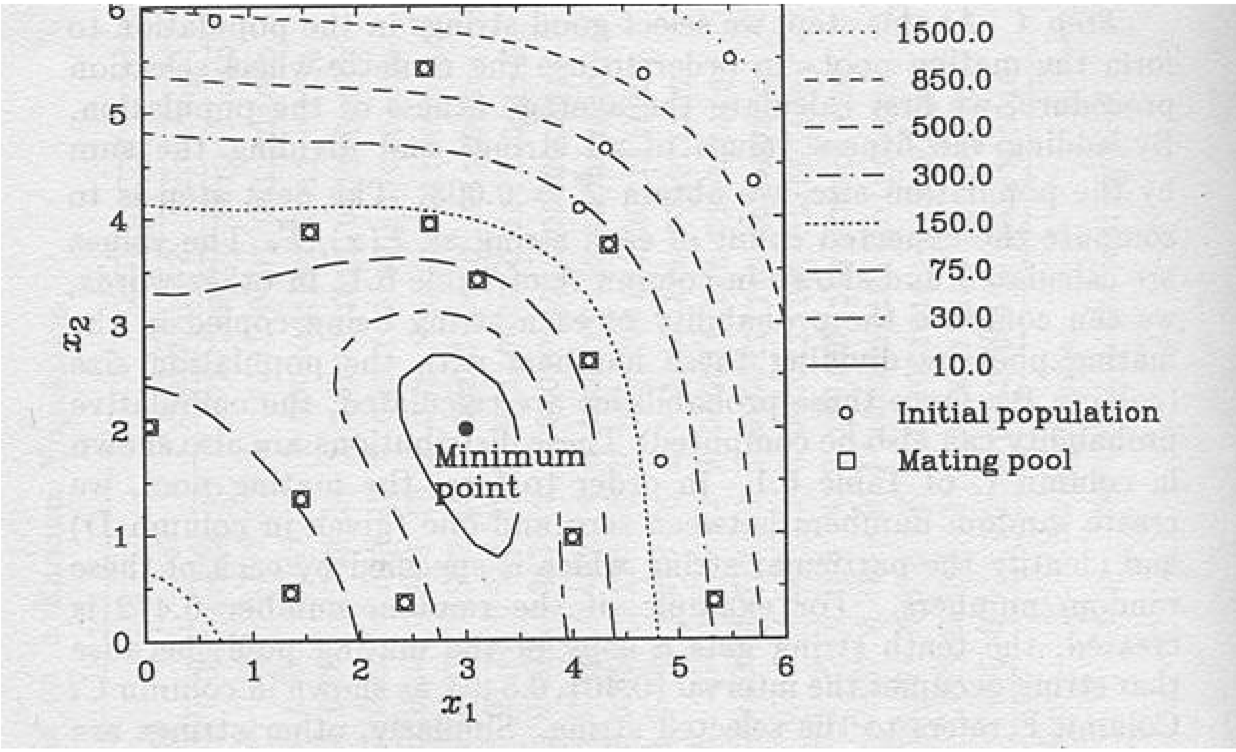
- operate using operators (reproduction, crossover, and mutation)
- get a new population.

GA operators

- Reproduction/Selection
  - select a set of above average strings
  - probabilistically add more instances of the strings to the mating pool

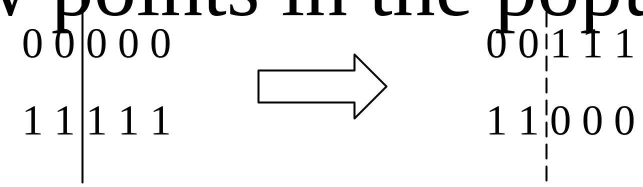
  e.g. probability of selecting i^{th} string = $\dfrac{F_i}{\sum F_i}$
- Population size remains constant at each stage

Initial Population and the mating pool



Crossover operator

- Choose two strings randomly from the mating pool.
- Choose a point in the strings randomly
- Exchange all bits to one side of the point in both strings.
- Two new points in the population

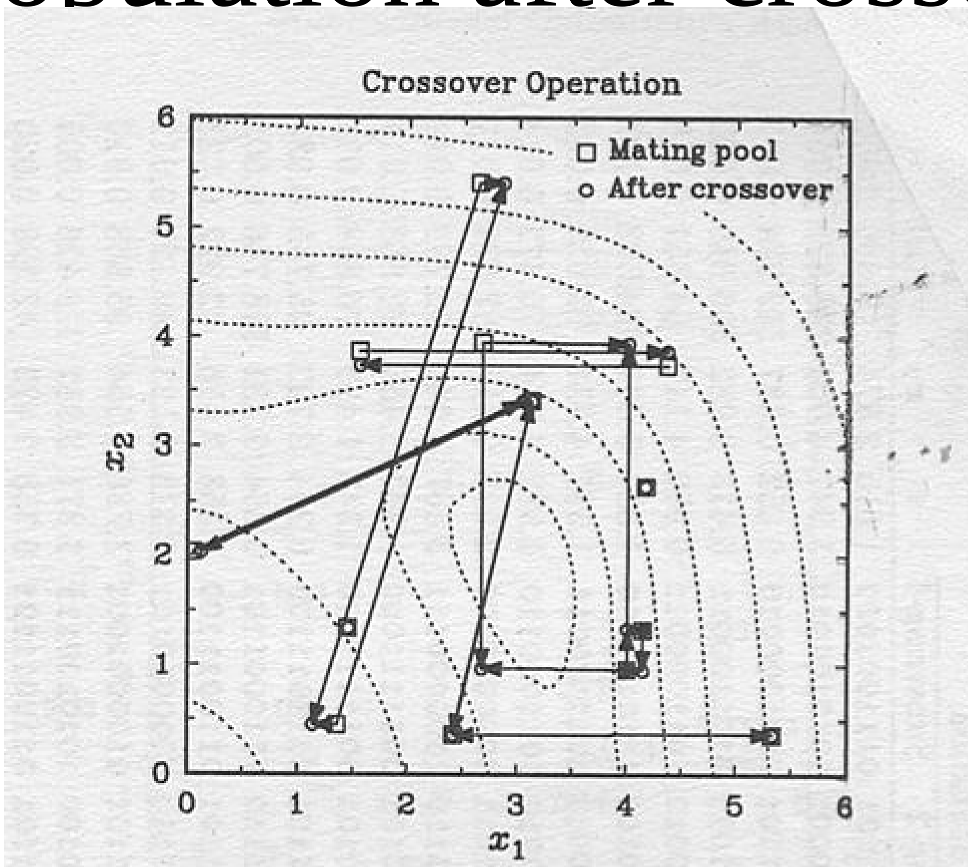


Parent string   Child string

Cross over

- Not all strings are chosen for cross over.
- Randomly chosen with a cross over probability P_c
- $N * P_c$ strings are used in crossover
- $N * (1 – P_c)$ are left unchanged

Population after crossover



Mutation operator

- Consider a population

  0 0 1 1 0 0 1

  0 1 1 0 1 1 0

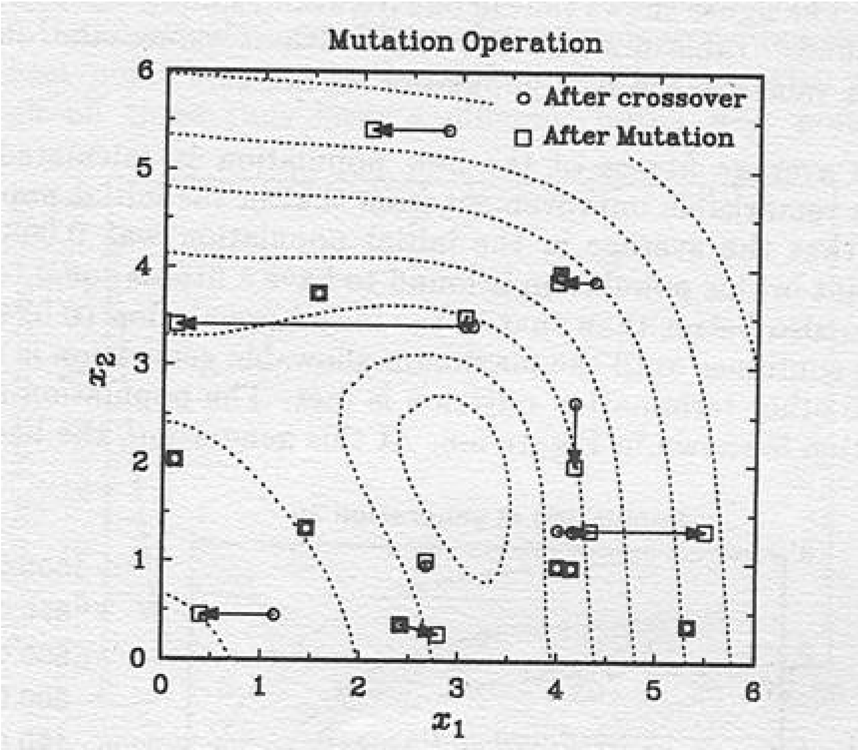
  0 1 0 1 0 1 0

  0 1 1 0 1 1 0

  0 0 1 0 1 0 1

- No amount of reproduction /crossover can change 1^{st} bit to 1: optimum may be missed.
- Change bits from 0 to 1 (or 1 to 0) with a probability of P_m.
- P_m should be very small.

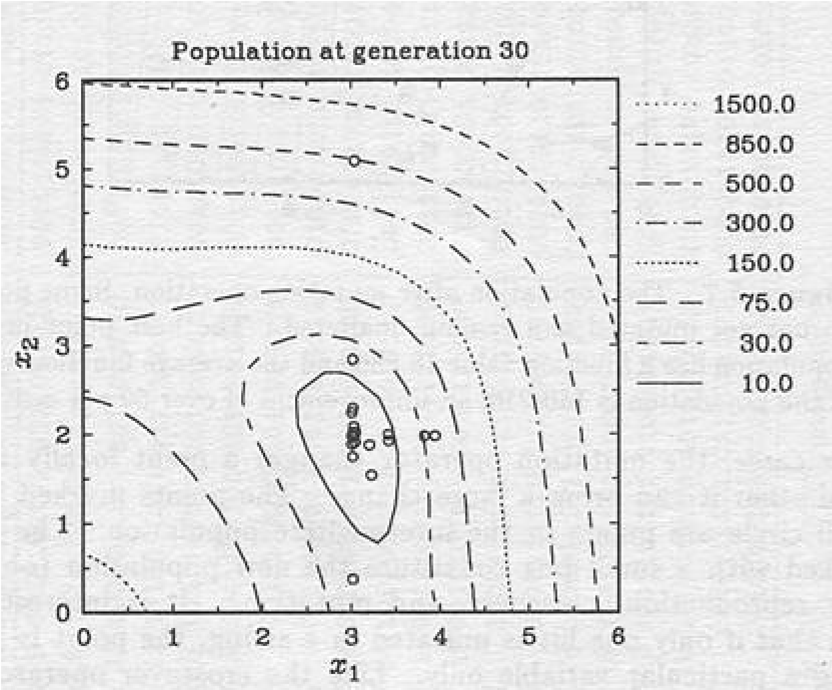
Population after mutation

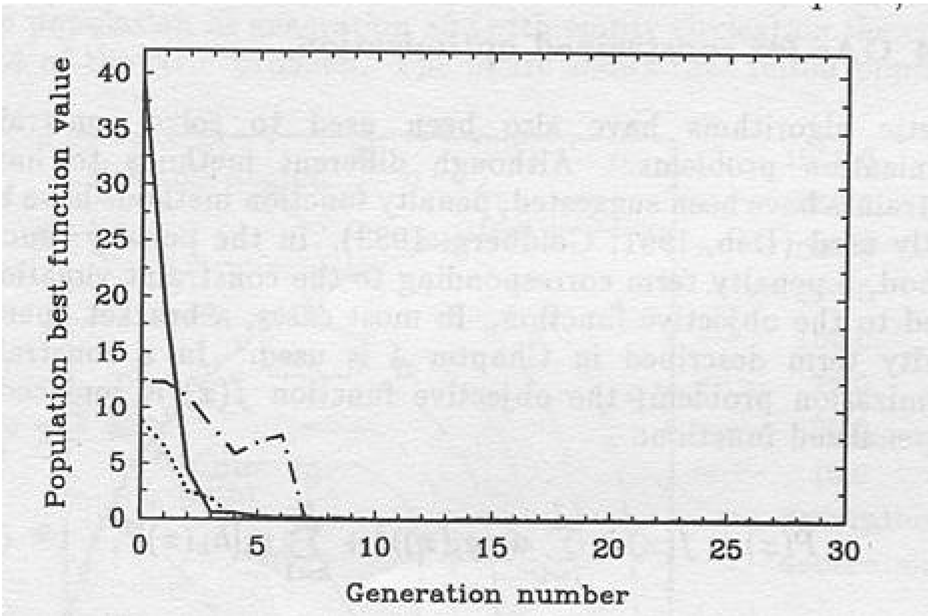


Characteristics of the operators

- Reproduction selects good strings.
- Crossover combines two good strings to come with better strings (hopefully).
- Mutation alters a string.
- Bad strings are removed in next generation (by the reproduction operator).

Result from iteration to iteration





GAs vs Traditional

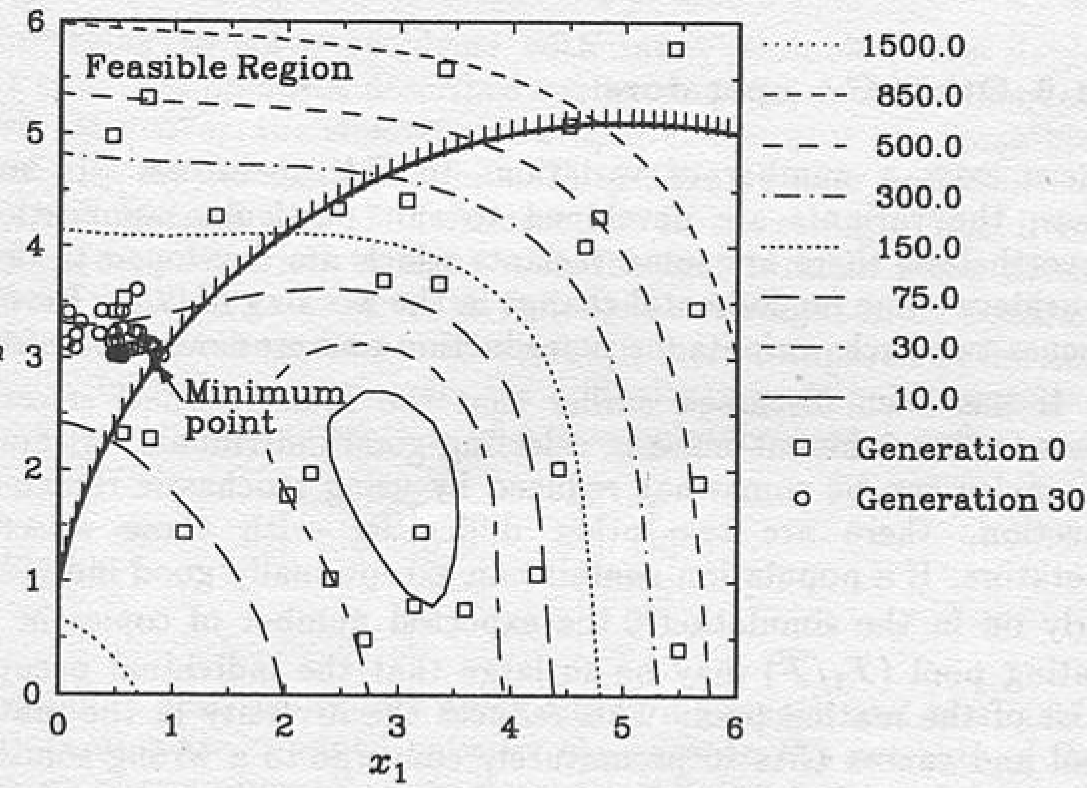
- Uses string coding of variables.
- search space becomes discrete.
- Discrete functions can also be handled easily.
- Uses a population of points.
- Larger likelihood of getting a global solution
- Multiple optimals can be found easily
- No information is needed for problem domain (like slope etc.)

Constrained Optimization problems using Gas

- Add penalty functions to minimization problem

$$P(x) = f(x) + \sum u_j \langle g_j(x) \rangle^2 + \sum v_k [h_k(x)]^2$$

- Convert to maximization problem
- F(x)=1/(1+P(x))
- Penalty parameter need not be updated from iteration to iteration
- A large value of R can be taken
- Multi modal functions can be handled easily



Feasible Region
Minimum point
x_1
x_2
0
1
2
3
4
5
6
1500.0
850.0
500.0
300.0
150.0
75.0
30.0
10.0
Generation 0
Generation 30

Modifications in GAs

- If one point in the population is very good
  - Many copies of the same after reproduction
  - Other optimal points may be difficult to get
  - Use scaling after each generation
  - $S(x)=aF(x)+b$

Modifications in Gas

- In crossover the right side bits have a greater probability of selection
- Hence use a two point crossover
- Choose two points.
- Swap bits between the two locations.

$$\begin{array}{c|ccc|c} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array} \Rightarrow \begin{array}{c|ccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}$$

Simulated Annealing

- Resembles the cooling process of molten metals through annealing
- At high temperature the atoms can move freely
- At low temperatures, movement gets restricted
- To obtain, the absolute minimum energy state, the system is cooled slowly

SA (contd)

- Introduce a Temperature like paramter
- Boltzmann Probability Distribution

$$P(E) = e^{(-\frac{E}{kT})}$$

- K – Boltzman's constant
- P – Proability of being in energy state E
- At high T, finite probability of being at any state

SA (contd)

- $P(E(t+1)) = \min [1, \exp(-\Delta E/kT)]$
  - Where $\Delta E = E(t+1) - E(t)$
- If ΔE is negative, $P = 1$, so accept new point
- Otherwise, accept only with a small probability
- Reduce T slowly over the iterations

SA (contd)

1. Choose x^0, ε, T, n (number of iterations at each T); $t = 0$
2. Calculate x^{t+1} = random point in neighborhood of x^t.
3. If $\Delta E=E(x^{t+1}-E(x^t))<0$, set $t=t+1$
    1. Else generate a random number r in (0, 1)
    2. If $r < \exp(-\Delta E/T)$ $t = t+1$ else GOTO 2
4. If $x^{t+1}-x^t < \varepsilon$ and T is small STOP
    1. Else if (t mod n) = 0 lower T
    2. GOTO 2