ACADEMY OF FINANCE

DEPARTMENT OF CORPORATE FINANCE

Chapter 3

RISK AND RETURN

CHAPTER:
RISK AND RETURN

LEARNING OBJECTIVES:

- Understand the expected returns and variances.
- Measure the expected returns and variances.
- Understand the portfolio
- Understand the systematic risk and unsystematic risk
- Understand the security market line, capital asset pricing model

Today lecture's topics

1. Returns – Definition and Measurement
2. Risk – Definition and Measurement
3. Portfolio and the impact of diversification
4. Relationship between risk and return
    - CAPM
    - SML

3.1. Expected return and variance
3.1.1. Expected return

3.1.1.1. Dollar returns

- The total dollar return on an investment is the sum of dividend and the capital gain (loss)
- Total dollar return = Dividend income + Capital gain (loss)

Dollar Returns

- You purchase an asset, your gain or loss from that investment is called the return on your investment. This return includes two components:
  - 1. some cash you received directly, eg: rent or dividend.
  - 2. capital gain or loss on your investment (because the value of assets you purchase will often change).

Dollar Returns - Example

- Suppose you purchased a share in company XYZ for $100. At the end of one year, the XYZ paid a dividend of $10 per share. Also, at the end of the year, the value of the share rises to $105. What is your Dollar Return?

Solution

- Total dollar return = Dividend Income + Capital gain or loss

  = \$10 + (\$105 - \$100) = \$15

- If you sell the share at the end of the year:

Total cash received = Initial Investment + Total return

= \$100 + \$15 = \$115

- Check:

  Cash received from selling share = Selling price + Dividend

  = \$105 + \$10 = \$115

Percentage returns

- It is **more convenient** than using dollar return, because using percentage return, **the return does not depend on how much you actually invest.**
- Percentage returns: give us the answer for the question- **How much do we get for each dollar we invest?**
- Percentage return

= (DIV + Change in market value)/beginning market value

Eg: (10 + 105 – 100)/100 = 15%

Per dollar invested, we get 15 cents in total (include 10 cents in dividend and 5 cents in capital gain)

3.1. Expected return and variance
3.1.1. Expected return

3.1.1.2. Percentage return or holding period return

The percentage return express how much we get from each dollar we invest.

$$\text{Percentage gain} = \frac{\text{Capital gain}(P1-P0)+\text{Dividend}(D1)}{\text{Initial share price }(P0)}$$

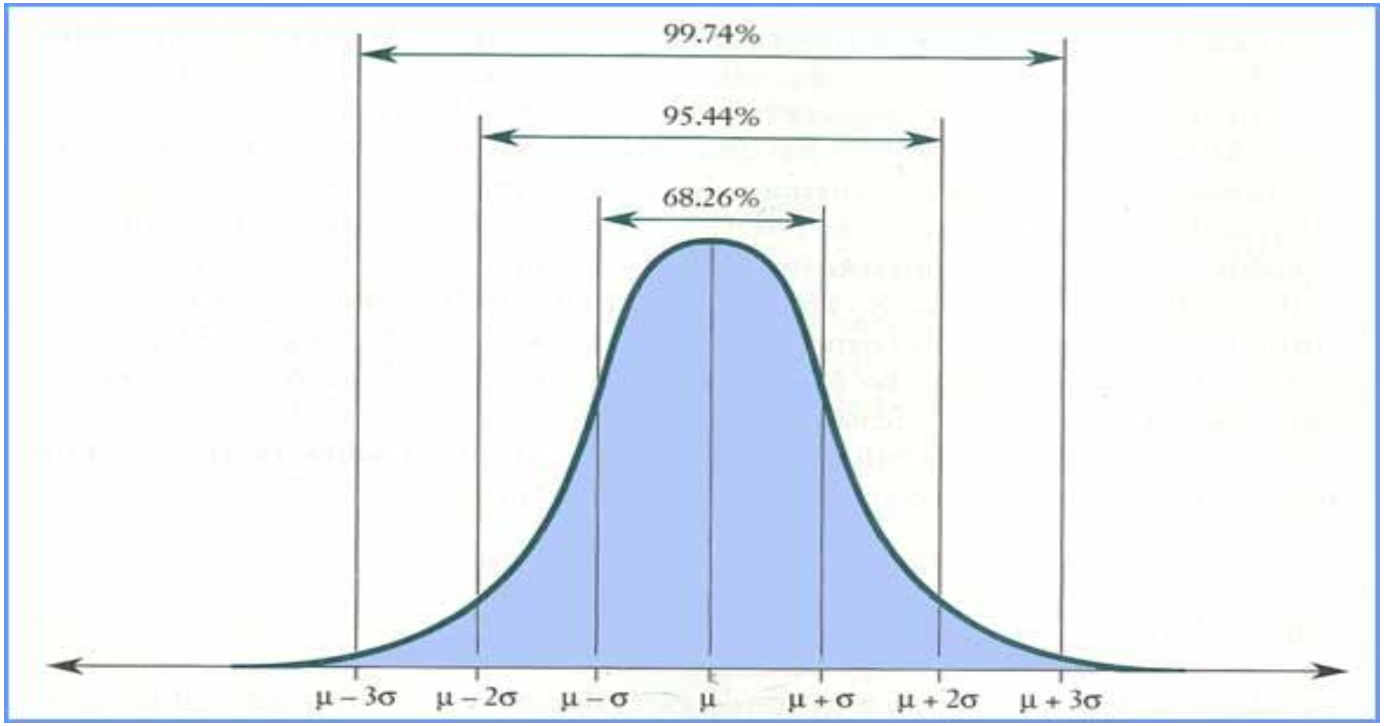
3.1. Expected return and variance
3.1.1. Expected return

3.1.1.3. Expected return

Expected return can be defined as the weighted average of the probability distribution of possible results.

Probability distribution

- **Probability:** the chance that an event may occur.
- **Probability distribution:** A list of all possible outcomes with a probability assigned to each of them.



- You flip a coin; if heads, you win $2, tails you lose $1. The expected pay-off from this gamble is equal to:
  - a) $3.00
  - b) $1.00
  - c) $0.50
  - d) none of the above
- The answer is

3.1. Expected return and variance

3.1.1. Expected return

$$\bar{r} = p_1 r_1 + p_2 r_2 + \cdots . p_n r_n = \sum_{i=1}^{n} p_i r_i$$

Here:

$\bar{r}$: expected rate of return.

ri: is the ith possible outcome

pi: is the probability of the ith outcome

n: is the number of possible outcome

ri : is a weighted average of the possible outcome with each outcome's weight being probability of occurrence.

Risk – Definition

- There is no universally agreed-upon definition of risk.
- **Risk** is the **uncertainty of outcomes.**
- Risk refers to a chance that some unfavorable events will occur.
- There is not just a possible outcome but an array of possible returns.
- An investment is more risky when the range of possible returns are large. i.e. 10% to 20% is less risky than -10% to 40%.

Risk measurement

- Most widely used measures of risk are **variance** and **standard deviation**
- Variance is a statistical measure that allows some way of comparing risk
  - $VAR = \sigma^2 = (R_1 - R)^2 + \ldots + (R_t - R)^2]1/(t-1)$
- Standard Deviation is the square root of the variance
  - It is a percentage measure
- If a security has perfect certainty
  - there is no variance - It would be "risk free"

3.1. Expected return and variance
3.1.2. Calculating the variances

- Variance

$$\delta^2 = \sum_{i=1}^{n} (r_i - \bar{r})^2 p_i$$

- Standard deviation

$$\delta = \sqrt{\sum_{i=1}^{n} (r_i - \bar{r})^2 p_i}$$

- The coefficient of variation

$$\mathrm{CV} = \frac{\delta}{\bar{r}}$$

Example - Probabilities

- An investor believes that an investment in MM Ltd will be dependent on the state of the economy
-

	Probability	Rate of Return
Strong Economy	0.18	15%
Weak Economy	0.12	-15%
No major change	0.70	10%

- Calculate the expected return of this investment

- Expected Return = $\sum(P_J * R_J)$

- If an investment has a
  - 50% chance of returning 12%,
  - 30% chance of returning 15% and
  - 20% chance of returning -5%
- What is its expected return?
- What is its standard deviation?

- E(R) =
- STD =

Example:

Economic status	Probability of occurrence	Rate of return on stock	
		A	B
Boom	0.2	17%	23%
Normal	0.6	15%	15%
Recession	0.2	13%	7%

R_A =

R_B =

Stock	r_i	$r_i - \bar{r}$	$(r_i - \bar{r})^2$	p_i	$(r_i - \bar{r})^2 p_i$
A	17%	17% - 15%	4	0.2	0.8
	15%	0	0	0.6	0
	13%	13% - 15%	4	0.2	0.8
B	23%	23% - 15%	64	0.2	12.8
	15%	0	0	0.6	0
	7%	7% - 15%	64	0.2	12.8

Standard deviation of A:

Standard deviation of B:

<u>* The coefficient of variation (CV)</u>

- 2 investments with different expected rate of returns: using σ is not reasonable
- CV: measures the risk per unit of return

$$CV = \frac{\sigma}{\bar{r}}$$

Example:

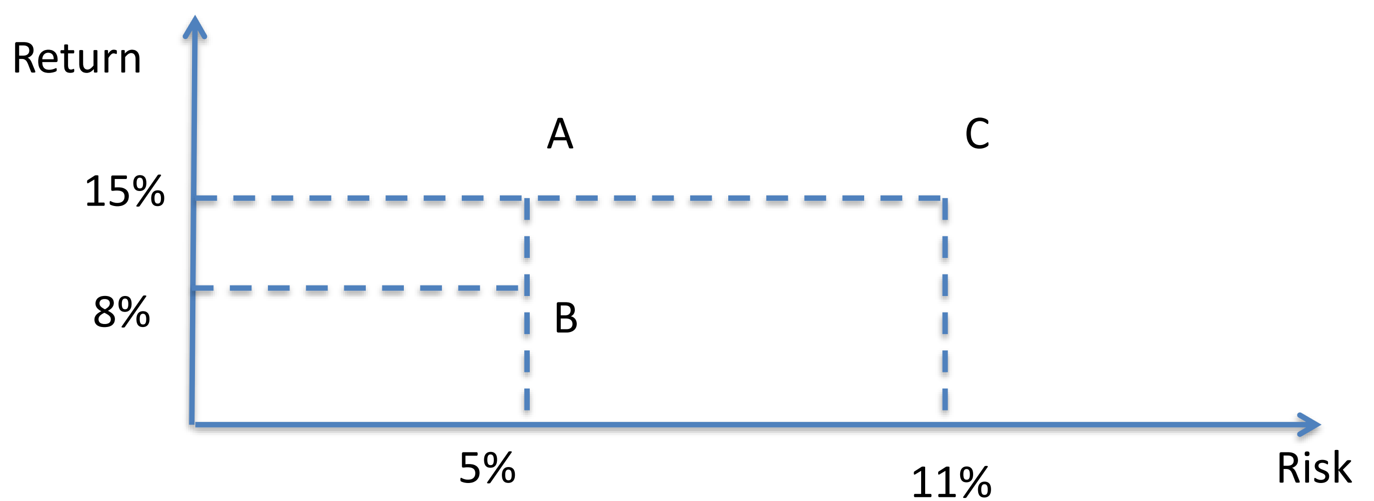
	X	Y
$\bar{r}$	15%	20%
σ	10%	12%
CV	0.67	0.6

$$CV_X = 0.67; CV_Y = 0.6$$

$\Rightarrow$ X is riskier per unit of return than Y although X has lower standard deviation.

Risk and Investors

- Assume all investors are risk averse
- Investors prefer less risk for the same return
- Given a choice between three investments Which asset would you select, A,B or C?



Risk and Asset Selection

- Just because one investment is more risky than another it does not mean investors will not invest in the more risky investment
- It depends on the investors attitude to risk – some investors are more risk averse than others
- However most investors will have more than one investment
- Investors would normally have a portfolio of assets

3.2. Portfolios

3.2.1. Portfolio and expected return of portfolio

- *A portfolio is a collection of investment securities.*
- The expected return on the portfolio is the weight average of the expected returns on the individual assets in the portfolio.

3.2. Portfolios

3.2.1. Portfolio and expected return of portfolio

- The portfolio expected return:

$$r_E = f_1 x \bar{r}_1 + f_2 x \bar{r}_2 + \ldots\ldots + f_n x \bar{r}_n = \sum_{i=1}^{n} f_i x \bar{r}_i$$

rE is the expected rate of return.

ri: is the ith possible outcome

pi: is the probability of the ith outcome

n: is the number of possible outcome

$\bar{r}$: is a weighted average of the possible outcome with each outcome's weight being probability of occurrence.

3.2. Portfolios
3.2.2. Measuring portfolio risk

- Covariance between securities A and B is calculated as follows:

$$\mathrm{COV}\,(\mathrm{A,B}) = \sum_{i=1}^{n} p_i (r_{iA} - \hat{r}_A)(r_{iB} - \hat{r}_B)$$

- Correlation coefficient

$$\rho_{AB} = \frac{COV(A,B)}{\sigma_A \sigma_B}$$

3.2. Portfolios
3.2.2. Measuring portfolio risk

- If p_{AB} = -1: the variables always move in exactly opposite directions. Returns on two investments A and B are perfectly negatively correlated. Risk of the portfolio consisting of two such investments can be eliminated completely.
- If p_{AB} = 0: two variables are not related to each other, changes in one variable are independent of changes in the other.
- If $-1 < p_{AB} < +1$: combining stocks into a portfolio reduces risk but does not eliminate it completely.

3.2. Portfolios
3.2.2. Measuring portfolio risk

Suppose a portfolio combine two stocks A and B, with fraction of portfolio for stock A: f_A and for stock B: f_B.

- Portfolio variance =

$$f_A^2 x \delta_A^2 + f_B^2 \delta_B^2 + 2 f_A x f_B x \operatorname{cov}(A, B)$$

- Portfolio standard deviation =

$$\sqrt{f_A^2 \delta_A^2 + f_B^2 \delta_A^2 + 2 f_A f_B cov}(A, B)$$

3.2. Portfolios
3.2.2. Measuring portfolio risk

- If a portfolio consists of n investments, portfolio standard deviation is calculated as follow:

$$\delta_p = \sqrt{\delta_p^2} = \sqrt{\sum_{i=1}^{n} f_i^{\,2}\delta_i^2 + 2\sum_{i=1}^{n}\sum_{j=1, i\neq j} f_i\, f_J \operatorname{cov}(i, j)}$$

σ_A: standard deviation of stock A.

σ_B: standard deviation of stock B.

Cov(A,B): covariance between stocks A and B.

Cov(A,B) = ρ_{AB} x σ_A x σ_B

ρ_{AB}: correlation coefficient between rate of return of stock A and B

3.2. Portfolios

3.2.2. Measuring portfolio risk

- If value of each investment is equal to each other, so fraction of each investment is . Investments' variance is equal ($\sigma^2_1 = \sigma^2_2 = \dots = \sigma^2_n =$)
- $\text{cov}(i,j) = p_{ij} \times \sigma_i \times \sigma_j = \overline{\text{cov}}$

$$Var_p = \frac{1}{n^2}\sum \delta_i^2 + n(n-1)\frac{1}{n^2}\overline{\text{cov}} = \frac{1}{n^2} n x \overline{\text{var}} + (n^2 - n)\frac{1}{n^2}\overline{\text{cov}} = \frac{1}{n}\overline{\text{var}} + (1 - \frac{1}{n})\overline{\text{cov}}$$

- When $n \to \infty$, $\text{var}(p) \to \overline{\text{cov}}$

A investor has a portfolio of 3 investments

What is the expected return on the portfolio?

Asset	Investment	E(R)
A	$10,000	15.5%
B	$25,000	10.0%
C	$15,000	12.5%
Tot. Invested	$50,000	

- The weight for each investment is the proportion it makes up of the total portfolio
- Weight = investment / total investment
  - for A =
  - for B =
  - for C =
- What is the expected return on the portfolio
  - E(r)=

3.3 Risk and diversification

3.3.1 Systematic versus unsystematic risk

Unsystematic risk:

- Is an individual risk of each company or each business branch.
- Can be eliminated by proper diversification.
- Is caused by such management capacity of leaders, material providing sources, strikes, winning or losing of major contracts

  and other events that are unique to a particular firm.

3.3 Risk and diversification
3.3.1 Systematic versus unsystematic risk

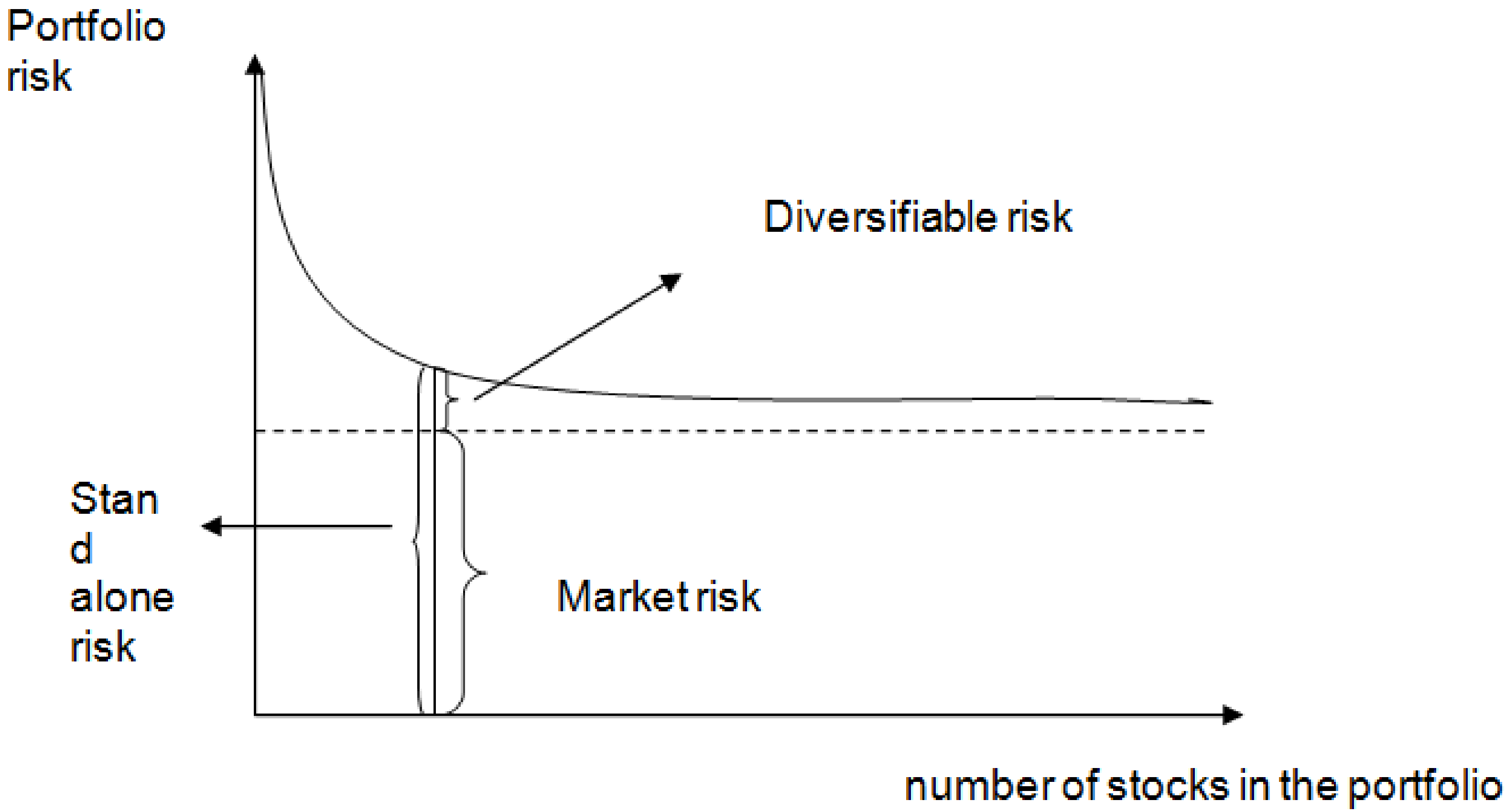
Systematic (nondiversifiable) risk

- Can be called market risk.
- Can affect most companies, and cannot be eliminated by diversification.
- Can be war, inflation, change of interest rates, exchange rate or law system. These

  conditions affect nearly all companies

  to some degree

3.3. Risk and diversification
3.3.2. The principle of diversification



3.4. The capital asset pricing model
3.4.1. Beta and risk premium

- When number of investments (securities) increase, portfolio risk reduces, but it only reduces to level of market risk.
- Market risk is a part of a security's risk that cannot eliminated.
- To know how one individual security contributes to riskiness of portfolio, it need to measure sensitivity of that security to market movements. This sensitivity is called beta.
- Beta measures the sensitivity of a security to market movements.

3.4. The capital asset pricing model
3.4.1. Beta and risk premium

$$\beta_i = \frac{\operatorname{cov}(i,m)}{\delta_m^2}$$

βi: beta coefficient of security, reflects the sensitivity of this security to market movements.

cov(i,m): covariance between return of security i and portfolio return.

δ_m^2 : portfolio variance (variance of the market return)

3.4. The capital asset pricing model
3.4.1. Beta and risk premium

- $\beta > 1$: security is more sensitive, riskier than market
- $\beta = 1$: security's change is similar to market's, which indicates that, if market moves up or falls by 10 percent, the security moves up or falls by 10 percent.
- $\beta < 1$: security is less sensitive, riskier than market

3.4. The capital asset pricing model
3.4.1. Beta and risk premium

- The beta of porfolio is calculated as follows:

$$\beta_P = \sum_{i=1}^{n} W_i x \beta_i$$

Here:

W_i : weight of investment i in a portfolio

β_P : beta of investment i

3.4. The capital asset pricing model
3.4.1. Beta and risk premium

Risk premium

- The expected return on the market can be represented as:

$$\overline{R_M} = R_F + Risk\ premium$$

- In words, the expected return on the market is the sum of the risk-free rate plus some compensation for the risk inherent in the market portfolio.

3.4. The capital asset pricing model
3.4.2. The capital asset pricing model

- The capital asset pricing model:

- $$\frac{E(R_i) - R_f}{\beta_i} = E(R_M) \text{ - Rf}$$

$$E(R_i) = R_f + [E(R_M) - R_f] x \beta_i$$

- The security market line plots the results of the CAPM for all different risks (betas).

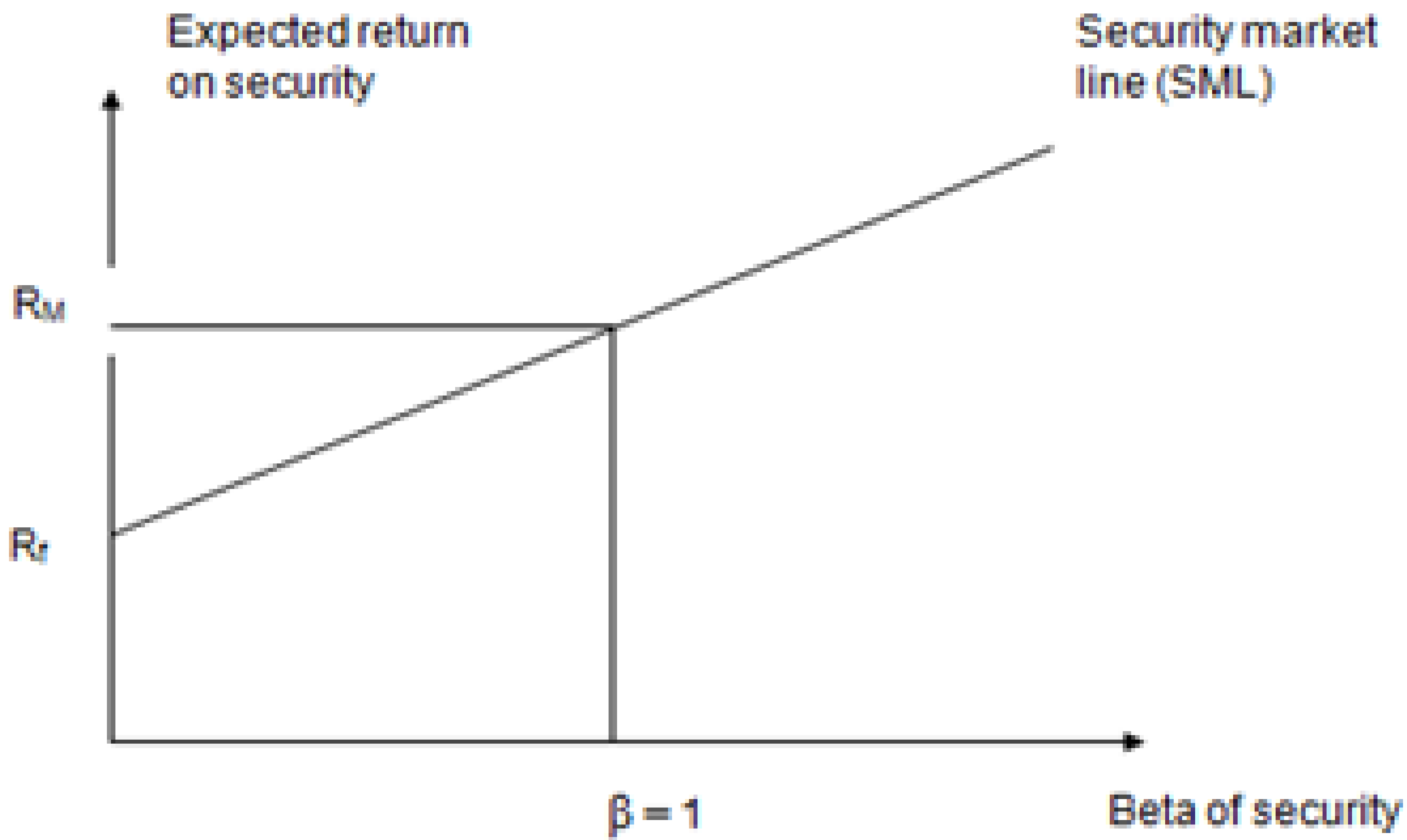
3.4. The capital asset pricing model

3.4.2. The capital asset pricing model

Model CAPM:

- Beta = 0: $\overline{R_i} = R_F$ the expected return on a security is equal to the risk free rate
- Beta = 1: $R_i = R_M$ the expected return on a security is equal to the expected return on the market.
- The higher beta, the higher expected return.

3.4. The capital asset pricing model
3.4.2. The capital asset pricing model



3.4. The capital asset pricing model
3.4.2. The capital asset pricing model

The CAPM shows that the expected return for a particular asset depends on three things:

- The pure time value of money. As measured by the risk free rate, R_f, this is reward for merely waiting for your money, without taking any risk.
- The reward for bearing systematic risk. As measured by the market risk premium, this component is the reward the market offers for bearing an average amount of systematic risk in addition to waiting.
- The amount of systematic risk measured by β_i. This is amount of systematic risk present in a particular asset, relative to that in an average asset

3.4. The capital asset pricing model
3.4.3. The security market line

- The line which is used to describe the relationship between systematic risk and expected return in financial market is usually called the security market line (SML).
- Suppose we consider a portfolio made up of all the assets in the market, that called a market portfolio. The expected return on this market portfolio is expressed as $E(R_M)$.

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3.4. The capital asset pricing model
3.4.3. The security market line

- The slope of the SML could be expressed as:

$$SML\ slope = \frac{E(R_M) - R_f}{\beta_M} = \frac{E(R_M) - R_f}{1} = E(R_M)R_f$$

- The term E(Rm).Rf is often called the market risk premium because it is the risk premium on a market portfolio. Market risk premium is the difference between the expected return on a market portfolio and the risk free rate.

Example

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Security	Amount Invested	Beta
A	$5,000	0.75
B	$10,000	1.10
C	$8,000	1.36
S	$7,000	1.88

- Suppose you invested in the four stocks
- What is the required return on the above portfolio given the risk free rate is 4% and the expected return on the market portfolio is 15%

Example - Solution

Security	Investment	Beta	Weight	B x W
A	$5,000	0.75		
B	$10,000	1.10		
C	$8,000	1.36		
S	$7,000	1.88		

Workshop Q1

- A portfolio consists of the following assets

Asset	% of Portfolio	Beta
– BHP	30%	0.80
– ASX	30%	1.10
– RIO	20%	1.50
– TIN	20%	1.60

- What is the beta and required return of the portfolio plus return on the market portfolio?
- The risk free rate is 3% and the market risk premium is 6%

Workshop Q1- Solution

Workshop Q2

- The Beta for World Corporation is 0.80 and the risk free rate is 6%. If the market risk premium is 8.5% what is the expected return for the World Corporation? What is the return on the market portfolio?

Workshop Q2 - Solution

Workshop Q3

- An investor has the following investments in shares and bonds that comprise their investment portfolio. The investor has listed the average returns over the last five years from these investments and has also recently been given the "beta's" of each investment calculated over the same period. What are the portfolio weights, return and beta?

Workshop Q3 continued

Investment	Current M. Value	Return	Beta
Rio Tinto	$6,000	16.44%	1.15
Wesfarmers	$3,000	11.76%	0.64
Telstra	$2,000	12.30%	0.70
CBA	$6,000	14.10%	0.90
News	$3,000	17.80%	1.92
Govt. Bonds	$5,000	6.00%	

Workshop Q3

Co	M. Value	Weight	Return	Beta	W x R	W x B
RIO	$6,000					
WEF	$3,000					
TEL	$2,000					
CBA	$6,000					
NEW	$3,000					
Govt	$5,000					
Total						

Summary

- - Risk can be defined as the chance that some event other than expected will occur.
- - The expected return on an investment is the mean value of its probability distribution of possible returns.
- - The higher the probability that the actual return will be significantly different from the expected return, the greater the risk associated with owning an asset.
- - A stock's risk consits of systematic risk and unsystematic risk. The unsystematic risk can be eliminated by diversificating, but the market risk can not be eliminated by diversificating.
- - Based on capital market history, there is a reward for bearing risk. This reward is the risk premium on an asset.
- - A stock's beta coefficient is a measure of the stock's market risk. A high beta stock is more volatile than an average stock while a low beta stock is less volatile than an average stock.
- - The security market line (SML) equation shows the relationship between a security's risk and its required rate of return.
- - The expected return on asset i is equal to the risk free rate plus the risk premium:
- $E(R_i) = R_f + \left[E(R_M) - R_f\right] x \beta_i$
- This is the capital asset pricing model (CAPM).