№ 4 $f(x;\theta) = \begin{cases} \dfrac{4x^3}{\theta^4} \\ \\ \end{cases}$ 1. $\hat{\theta} = \bar{X}$

1. $E[\hat{\theta}] = E[\bar{X}] = E[X_i] = \cancel{\frac{1}{n} \cdot n \cdot 1} = \frac{4}{5}\theta$

$E[X_i] = \int_0^\theta \frac{4x^3}{\theta^4} x \cdot dx = \frac{1}{\theta^4} \cdot \frac{4 \cdot x^5}{5} \Big|_0^\theta = \frac{4}{5} \cdot \frac{\theta^5}{\theta^4} = \frac{4}{5}\theta \neq \theta \Rightarrow$

оценка смещена $\Rightarrow$

$\hat{\theta} = c \cdot \theta$

$E[\hat{\theta}] = c \cdot \theta$

$E[c \cdot \hat{\theta}] = \theta$

$c \cdot \frac{4}{5}\theta = \theta \Rightarrow c = \frac{5}{4} \Rightarrow$ тогда оценка будет смещена

N 1 $f(x;\theta) = \int \frac{6x(\theta - x)}{\theta^3} dx$

$\alpha_2 = \hat{\alpha}_2$

$\alpha_2 = E[X_i^2] = \int_0^\theta x^2 \cdot \frac{6x(\theta-x)}{\theta^3} dx = \frac{1}{\theta^3} \cdot \int_0^\theta x^2 \cdot 6x \cdot \theta - x^3 \, dx =$

$= \frac{1}{\theta^3} \int_0^\theta x^3 \cdot 6\theta - x^3 \, dx = \frac{1}{\theta^3} \cdot \int_0^\theta x^3 (6\theta - 1) \, dx = \frac{6\theta - 1}{\theta^3} \cdot \frac{x^4}{4} \Big|_0^\theta =$

$= \frac{(6\theta-1) \cdot \cancel{\theta^4}}{4\cancel{\theta^3}} = \cancel{6\theta^5 - \theta^4} \quad \frac{(6\theta - 1)\cdot \theta}{4}$

$\alpha_2 = \frac{1}{n} \cdot \left(\sum_{i=1}^n X_i^2\right) = \frac{(6\theta - 1)\cdot\theta}{4}$

$\hat{\theta}_{ML} = \sqrt{4 \cdot \sum_{i=1}^n X_i^2}$

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N6 $f(x;\theta) = \frac{4x^3}{\theta^4}$

$\hat{\theta}_n = \frac{5n+3}{4n-2} \cdot \bar{X}$

$\hat{\theta}_n \xrightarrow[n \to \infty]{P} \theta$

1. ~~$E[\hat{\theta}] = \theta, E[$~~

1. $E[\hat{\theta}] = E\left[\frac{5n+3}{4n-2} \cdot \bar{X}\right] = \frac{5n+3}{4n-2} \cdot E[\bar{X}] =$

$= \frac{5n+3}{4n-2} \cdot E[X_i] = \frac{5n+3}{4n-2} \cdot \frac{4}{5}\theta \neq \theta \Rightarrow$ смещена оценка

$E[X_i] = \int_0^\theta x \cdot \frac{4x^3}{\theta^4} = \frac{1}{\theta^4} \cdot 4 \cdot \frac{x^5}{5} \Big|_0^\theta = \cancel{\frac{\theta^4}{\theta^4}} \quad \frac{4}{5} \cdot \theta$

$\lim D($

$$D(\hat{\theta}) = D\left(\frac{5n+3}{4n-2}\cdot\bar{X}\right) = \left(\frac{5n+3}{4n-2}\right)^2\cdot D(\bar{X}) = \left(\frac{5n+3}{4n-2}\right)^2\cdot D(X_i)$$

$$D(X_i) = E[\bar{X}_i^2] - (E[X])^2 = \cancel{\frac{4}{6}\theta^3 - \frac{4}{5}\theta = \frac{20\theta^3 - 24\theta}{30}} = \frac{4\theta^3}{6} -$$

$$E[X_i^2] = \int_0^\theta x^2\frac{4x^3}{\theta^3}\,dx = \frac{4}{\theta^3}\cdot\frac{x^6}{6}\Big|_0^\theta = \frac{4}{6}\cdot\theta^3 = \frac{4\theta^3}{6} - \frac{16}{25}\cdot\theta^2$$

$$\cancel{\frac{4}{6}\theta^3 - \frac{16}{25}\theta^2 = \theta^2\left(\frac{4}{6}\theta - \frac{16}{25}\right)} =$$

~~№4 f(x, θ)~~

$$\cancel{\lim_{n\to\infty}\left(\frac{5n+3}{4n-2}\right)^2}$$

$$\lim_{n\to\infty}\frac{5n+3}{4n-2}\cdot\theta - \theta = 0 \quad \checkmark$$

$$\Rightarrow \hat{\theta}_n \xrightarrow[n\to\infty]{P} \theta \text{ - состоятельна}$$

$\cancel{\lim_{n\to\infty}}$