Устная часть. Мануйленко Дарья БЭК 182

① 20 единиц

30 двоек

50 троек

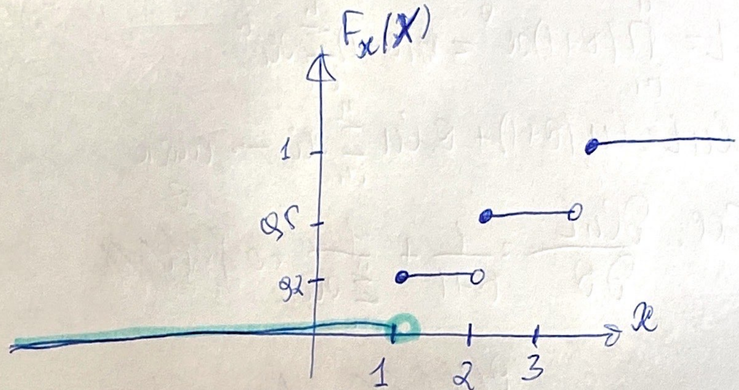
Выборочная функция распределения:

$20+30+50=100$

$\frac{20}{100}=\frac{2}{10}=0{,}2$

$\frac{30+20}{100}=\frac{50}{100}=0{,}5$

$\frac{30+20+50}{100}=\frac{100}{100}=1$



② $X=(X_1,\dots,X_n)$ - случайная выборка

$$f_X(x,\theta)=\begin{cases}\frac{6x(\theta-x)}{\theta^3}, & x\in[0,\theta]\\ 0, & \text{иначе}\end{cases}\qquad \theta>0$$

Метод моментов: Выборочные моменты = Теоретические моменты

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=Var(X)$$

$$Var(X)=E(X^2)-[E(X)]^2$$

$$E(X^2)=\int_0^\theta\frac{6x(\theta-x)}{\theta^3}\times x^2dx=\int_0^\theta\frac{6x^3\theta}{\theta^3}dx-\int_0^\theta\frac{6x^4}{\theta^3}dx=$$

$$=\frac{6}{\theta^2}\int_0^\theta x^3dx-\frac{6}{\theta^3}\int_0^\theta x^4dx=\frac{6}{\theta^2}\times\frac{\theta^4}{4}-\frac{6}{\theta^3}\times\frac{\theta^5}{5}=\frac{6\theta^2}{4}-\frac{6\theta^2}{5}=1{,}5\theta^2-1{,}2\theta^2=$$

$$=0{,}3\theta^2$$

$$E(X)=\int_0^\theta\frac{6x(\theta-x)}{\theta^3}x\,dx=\int_0^\theta\frac{6x^2}{\theta^2}dx-\int_0^\theta\frac{6x^3}{\theta^3}dx=\frac{6}{\theta^2}\times\frac{x^3}{3}\Big|_0^\theta-\frac{6}{\theta^3}\times\frac{x^4}{4}\Big|_0^\theta=$$

$= \frac{6}{\theta^2} \times \frac{\theta^3}{3} - \frac{6}{\theta^3} \times \frac{\theta^4}{4} = 2\theta^2 - 1{,}5\theta = \theta(2\theta - 1{,}5)$

$Var(\theta) = 0{,}3\theta^2 - (2\theta^2 - 1{,}5\theta)^2 = 0{,}3\theta^2 - (4\theta^4 - 6\theta -$

3

$$f(x,\theta) = \begin{cases} (\theta+1)x^\theta, & x \in (0;1) \\ 0, & \text{иначе} \end{cases}$$

$L = \prod_{i=1}^{n} (\theta+1)x^\theta = (\theta+1)\sum_{i=1}^{n} x_i^\theta$

$\ln L = \ln(\theta+1) + \theta \ln \sum_{i=1}^{n} x_i \to \max_\theta$

FOC: $\frac{\partial \ln L}{\partial \theta} = \frac{1}{\theta+1} + \frac{1}{\sum_{i=1}^{n} x_i} = 0$

$\cancel{\frac{\theta+1}{\sum_{i=1}^{n} x_i} = 0}$ $\quad \frac{1}{\theta+1} = \frac{1}{\sum_{i=1}^{n} x_i}$

$\theta + 1 = \sum_{i=1}^{n} x_i$

$\hat{\theta}_{ML} = \sum_{i=1}^{n} x_i - 1$ - оценка максимального правдоподобия

SOC: $\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{1}{(\theta+1)^2} < 0 \to \max$

4 $X_1, \ldots, X_n$ - случайная выборка

$$f(x,\theta) = \begin{cases} \frac{4x^3}{\theta^4}, & x \in [0;\theta] \\ 0, & \text{иначе} \end{cases}$$

$\hat{\theta} = \bar{X}$

а) несмещённость:

$E(\hat{\theta}) = \theta$

$E(\hat{\theta}) = E(\bar{X}) = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i\right) = \frac{1}{n} \times n \times E(x_i) = E(x_i)$

$$E(X_i) = \int_0^\theta \frac{4x^3}{\theta^4} x\,dx = \frac{4}{\theta^4}\int_0^\theta x^4 dx = \frac{4}{\theta^4} \times \frac{x^5}{5}\Big|_0^\theta = \frac{4}{\theta^4} \times \frac{\theta^5}{5} = 0{,}8\theta$$

$\Rightarrow E(\hat{\theta}) = 0{,}8\theta \Rightarrow$ оценка $\hat{\theta} = \bar{X}$ – смещённая оценка параметра θ

б) $\hat{\theta} = c\bar{X}$ была несмещённая:

$E(\hat{\theta}) = \theta \to$ несмещённая, тогда: ~~$c\bar{X} = 0{,}8\theta$~~ ~~$0{,}8\theta = \bar{X}$~~ ~~$c\hat{\theta} = 0{,}8\theta$~~ ~~$c = 0{,}8$~~

$cE(\bar{X}) = \cancel{E(\hat{\theta})}\ \theta$

$c \times 0{,}8\theta = \theta \Rightarrow 0{,}8c = 1$

$c = \frac{1}{0{,}8} = 1{,}25$

~~⑥ $X = (X_1, \dots, X_n)$ – случайная выборка из нормального распр.~~

⑦ $X = (X_1, \dots, X_n) \sim N(\mu_x; \sigma_x^2)$ $\quad \sigma_x^2 = \sigma_y^2 = \sigma_0^2 \quad \alpha = 0{,}05$

$Y = (Y_1, \dots, Y_m) \sim N(\mu_y; \sigma_y^2)$

$x_1 = 0{,}53 \quad y_1 = -0{,}8$

$x_2 = 2{,}83 \quad y_2 = 0{,}06$

$x_3 = -1{,}25 \quad y_3 = 0{,}84$

$x_4 = 1{,}86 \quad y_4 = 4{,}07$

$x_5 = 1{,}31 \quad y_5 = 3{,}26$

$$\begin{cases} H_0: \mu_x = \mu_y \\ H_1: \mu_x < \mu_y \end{cases}$$

$$\frac{\bar{x} - \bar{y} - (\mu_x - \mu_y)}{\hat{\sigma}_0 \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \sim t_{n_x + n_y - 2, \alpha/2}$$

$\bar{x} = 1{,}256 \approx 1{,}3$

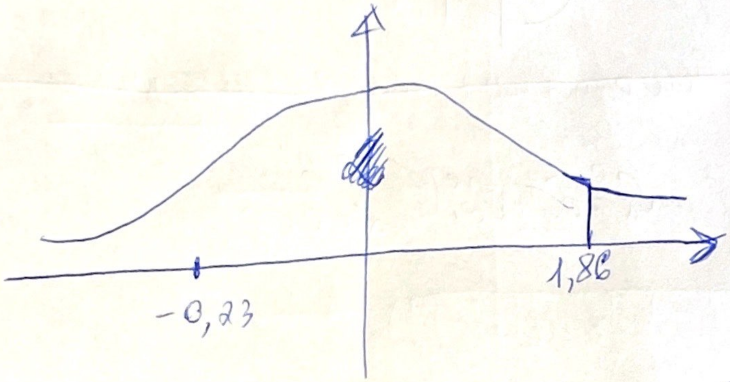
$\bar{y} = 1{,}486 \approx 1{,}5$

$$\hat{\sigma}_0^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2 + \sum_{i=1}^{m}(y_i - \bar{y})^2}{n_x + n_y - 2} = \frac{0{,}0529 + 2{,}3409 + 0{,}0025 + 0{,}3136 + 0{,}0001 + 0{,}49 + 0{,}0736 + 0{,}4356 + 6{,}6049 + 3{,}0976}{8} =$$

$= \frac{15,417}{8} = 1,926 \approx 1,93$

$$\frac{1,3-1,5}{\sqrt{1,93}\times\sqrt{\frac{1}{5}+\frac{1}{5}}} \sim t_{8;\ \cancel{0,025}\ 0,05}$$

$$\frac{-0,2}{1,39\times 0,63} = -\frac{0,2}{0,8757} \approx -0,23$$



$\Rightarrow$ Не отвергается

9) Тест на независимость событий:

$$\gamma = \sum_{i=1}^{s}\sum_{j=1}^{r}\frac{\left(n_{ij}-\frac{n_{0i}\times n_{0j}}{n}\right)^2}{\frac{n_{0i}\times n_{0j}}{n}} \overset{a.s.}{\sim} \chi^2_{(s-1)(r-1)}$$

s - ряд, r - столбец, $\alpha = 5\%$

	синяя	чёрная	фиолетовая	
«5»	100	40	50	190
«4»	65	60	50	175
	165	100	100	365 = n

$$\gamma = \frac{\left(100-\frac{190\times165}{365}\right)^2}{\frac{190\times165}{365}} + \frac{\left(140-\frac{190\times100}{365}\right)^2}{\frac{190\times100}{365}} + \frac{\left(150-\frac{190\times100}{365}\right)^2}{\frac{190\times100}{365}} +$$

$$+ \frac{\left(165-\frac{175\times165}{365}\right)^2}{\frac{175\times165}{365}} + \frac{\left(160-\frac{175\times100}{365}\right)^2}{\frac{175\times100}{365}} + \frac{\left(150-\frac{175\times100}{365}\right)^2}{\frac{175\times100}{365}} = \ldots$$