Куликовских Денис

БЭК183

Задача 1

$X_1 = 20$ (единицы)
$X_2 = 30$ (двойки)
$X_3 = 50$ (тройки

n	1	2	3
P(X=n)	20	30	50

$$F(x) = \begin{cases} 0; & -\infty < x \leq 1 \\ 0{,}2; & 1 < x \leq 2 \\ 0{,}5; & 2 < x \leq 3 \\ 1; & 3 < x < +\infty \end{cases}$$

Задача 2

$$f(x;\theta) = \begin{cases} \dfrac{6x(\theta - x)}{\theta^3} & x \in [0;\theta] \\ 0 & x \in [0;\theta] \end{cases}$$

$$E[X_i^2] = \int_0^{+\theta} x^2 \frac{6x(\theta-x)}{\theta^3} dx = \int_0^{+\theta} \frac{6x^3(\theta-x)}{\theta^3} dx$$

$$= \frac{1}{\theta^3}\left(\theta \int_0^{+\theta} 6x^3 dx - \int_0^{\theta} 6x^4 dx\right) = \frac{1}{\theta^3}\left(\theta \frac{6x^4}{4}\Big|_0^{\theta} - \frac{6x^5}{5}\Big|_0^{\theta}\right)$$

$$= \frac{1}{\theta^3}\left(\frac{6\theta^5}{4} - \frac{6\theta^5}{5}\right) = \frac{1}{\theta^3}\left(\frac{6\theta^5}{20}\right. = 0{,}3\theta^2 = (\bar{X})^2 \to$$

$$\Rightarrow \hat{\theta}_{MM} = \sqrt{\frac{(\bar{X})^2}{0{,}3}} = \frac{\bar{X}}{\sqrt{0{,}3}}$$

Задача 3

$$f(x;\theta) = \begin{cases} (\theta+1)x^{\theta} & x \in (0;1) \\ 0 & x \notin (0;1) \end{cases}$$

$$L = (\theta+1)X^{n\theta}$$

$$\ln L = l = \ln(\theta+1) + n\theta \ln X$$

$$\frac{dl}{d\theta} = \cancel{\frac{n\theta}{X} = 0 \Rightarrow} \quad \frac{1}{\theta+1} + \frac{n}{X} \Rightarrow (\theta+1)n = X$$

$$\theta n = X - n$$

$$\frac{dl^2}{d^2x} > 0 \text{ - max}$$

$$\hat{\theta}_{ML} = \frac{X}{n} - 1 = \bar{X} - 1$$

Задача 4

$X = (X_1, \dots, X_n)$

$$f(x;\theta) = \begin{cases} \frac{4x^3}{\theta^4} & x \in [0;\theta] \\ 0, & \text{иначе} \end{cases}$$

(a) $\hat{\theta} = \overline{X}$ - несмещенная - ?

$$E(X) = \int_0^\theta x \frac{4x^3}{\theta^4} dx = \frac{1}{\theta^4} \frac{4x^5}{5} \Big|_0^\theta = \frac{1}{\theta^4} \cdot \frac{4\theta^5}{5}$$

$= \frac{4}{5}\theta \Rightarrow$ оценка смещенная

(b) $\tilde{\theta} = c\overline{X} \Rightarrow c = \frac{5}{4}$

Задача 5.

$X = (X_1, \dots, X_n) \sim N(\mu, \sigma^2 > 0)$ σ^2 - известен

(a) $I_n(\mu) = -E\left(\left(\frac{\partial l}{\partial \mu}\right)^2\right)$

Задача 7.

$X = (X_1, \dots, X_n) \sim (\mu_X, \sigma_X^2)$

$Y = (Y_1, \dots, Y_m) \sim (\mu_Y, \sigma_Y^2)$

$\sigma_X^2 = \sigma_Y^2$

$\alpha = 0{,}05$

$x_1 = 1{,}53$ $x_2 = 2{,}83$ $x_3 = -1{,}25$ $x_4 = 1{,}86$ $x_5 = 1{,}31$

$y_1 = -0{,}8$ $y_2 = 0{,}06$ $y_3 = 0{,}84$ $y_4 = 4{,}07$ $y_5 = 3{,}26$

$$\begin{cases} H_0: \mu_X = \mu_Y \\ H_1: \mu_X < \mu_Y \end{cases}$$

Решение: на след странице ↓

$$T = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\left(\frac{1}{n} + \frac{1}{m}\right)\left(\frac{n-1}{n+m-2}\hat{\sigma}_X^2 + \frac{m-1}{n+m-2}\hat{\sigma}_Y^2\right)}} \sim t(n+m-2)$$

$$T = \frac{1,256 - 1,486 - 0}{\sqrt{\left(\frac{1}{5}+\frac{1}{5}\right)\left(\frac{5-1}{5+5-2}\cdot 2,3 + \frac{5-1}{5+5-2}\cdot 4,38\right)}} \approx -0,2$$

$\underbrace{\hat{\sigma}^2_{X,Y}}$ $= \frac{1}{n-1}\sum_{i=1}(x_i - \bar{x})^2$ и для Y аналогично, т.к

$$\hat{\sigma}_X^2 = \frac{1}{4}\left((1,53-1,256)^2 + (2,83-1,256)^2 + (-1,25-1,256)^2 + (1,86-1,256)^2 + (1,31-1,256)^2\right) \approx 2,3$$

$$\underbrace{\hat{\sigma}_Y^2} = \frac{1}{4}(\ldots\ldots) = 4,38$$

Область, где H_0 не отвергается:

$$[\, t_{n+m-2,\alpha}; +\infty\,] = [-1,86; +\infty)$$

$T = -0,2$ и $-0,2 \in [-1,86; +\infty)$ =>

=> основная гипотеза не отвергается.

Задача 8

$x = (X_1, \ldots, X_n)$

$\lambda > 0$ $\bar{X}_{80} = 1,7$

$\alpha = 0,1$

$$\begin{cases} H_0: \lambda = 2 \\ H_1: \lambda \neq 2 \end{cases}$$

$$L = \prod_{i=1}^{n} \frac{\lambda^{X_i}}{X_i!} e^{-\lambda}$$

$$\ln L = l = \left(\sum_{i=1}^{n} X_i\right)\ln\lambda - \sum_{i=1}^{n}\ln(X_i!) - \lambda n$$

$$\frac{\partial l}{\partial \lambda} = \frac{\sum_{i=1}^{n} X_i}{\lambda} - n = 0$$

$$\hat{\lambda}_{UR} = \frac{\sum_{i=1}^{n} X_i}{n} \Rightarrow$$

$$\hat{\lambda}_{UR} = 1,7$$

$$\hat{\lambda}_R = 2$$

на след ↓

$$l_R = (80 \cdot 1{,}7)\ln 2 - \sum_{i=1}^{n} \ln(x_i!) - 2 \cdot 80$$

$$l_{UR} = (80 \cdot 1{,}7)\ln 1{,}7 - \sum_{i=1}^{n} \ln(x_i!) - 1{,}7 \cdot 80$$

$$LR = -2(l_R - l_{UR}) = -2\left((136 \cdot \overset{0{,}693}{\cancel{9}} - 160) - (136 \cdot 0{,}53 - 136)\right) = -2(-65{,}73 + 63{,}9) \approx$$

$\approx 3{,}8$ - критическая точка $\Big| \Rightarrow \chi^2_{1;\alpha} = 2{,}706$

$\Big| \Rightarrow$ H_0 отвергается

Задача 9.

Знаю, но времени не осталось