Задача 2.

$$E(X) = \int_0^\theta x \frac{6x(\theta - x)}{\theta^3}\,dx = \int_0^\theta \frac{6x^2\theta - 6x^3}{\theta^3}\,dx =$$

$$= \frac{6x^3\theta}{3\theta^3} - \frac{6x^4}{4\theta^3}\Big|_0^\theta = \frac{6\theta^4}{3\theta^3} - \frac{6\theta^4}{4\theta^3} = \frac{1}{2}\theta - \frac{3}{2}\theta = -\theta$$

$$E(X^2) = \int_0^\theta \frac{6x^3\theta - 6x^4}{\theta^3}\,dx = \frac{6x^4\theta}{4\theta^3} - \frac{6x^5}{5\theta^3}\Big|_0^\theta = \frac{3}{2}\theta^2 - \frac{6}{5}\theta^2 =$$

$$= \frac{3}{10}\theta^2$$

$$D(X) = E(X^2) - E(X)^2 = \frac{3}{10}\theta^2 - \theta^2 = -\frac{7}{10}\theta^2 = \frac{\sum_{i=1}^n (X_i - \overline{X})^2}{n}$$

$$\boxed{\hat{\theta}_{MM} = \sqrt{\frac{\sum_{i=1}^n (X_i - \overline{X})^2}{n} \cdot \left(-\frac{10}{7}\right)}}$$

Задача 3. $L = \prod_{i=1}^n (\theta+1) x_i^\theta = (\theta+1)^n \prod_{i=1}^n x_i^\theta$

$$\ln L = n \ln(\theta+1) + \sum_{i=1}^n \ln x_i^\theta = n\ln(\theta+1) + \theta \ln \sum_{i=1}^n x_i$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta+1} + \ln \sum_{i=1}^n x_i \Big|_{\theta = \hat{\theta}_{ML}} = 0$$

$$n + (\theta+1) \ln \sum_{i=1}^n x_i = 0.$$

$$\theta \ln \sum_{i=1}^n x_i = -n - \ln \sum_{i=1}^n x_i$$

$$\boxed{\hat{\theta}_{ML} = \frac{-n - \ln \sum_{i=1}^n x_i}{\ln \sum_{i=1}^n x_i}}$$

Задача 4.

a) $\hat{\theta}=\bar{X}$?

$$E(X)=\int_0^{\theta}\frac{4x^4}{\theta^4}dx=\frac{4x^5}{5\theta^4}\Big|_0^{\theta}=\frac{4}{5}\theta=\bar{X}\Rightarrow\hat{\theta}_{мм}=\frac{5}{4}\bar{X}$$

$$E(\bar{X})=E(X_i)=\frac{4}{5}\theta\Rightarrow\boxed{\text{оценка смещенная}}$$

(иначе было бы $E(\hat{\theta})=\theta$)

b) $E(\tilde{\theta})=cE(\bar{X})=c\cdot\frac{4}{5}\theta=\theta\Rightarrow\boxed{c=\frac{5}{4}}$

Задача 5.

a) $I_n(\mu)$

$$I_n(\mu)=-E\left(\frac{\partial^2\ln L}{\partial\mu^2}\right)=$$

$$L=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

$$\ln L=-\frac{n}{2}\ln 2\pi-n\ln\sigma-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial\ln L}{\partial\mu}=\frac{\sum_{i=1}^{n}(x_i-\mu)}{\sigma^2}$$

$$\frac{\partial^2\ln L}{\partial\mu^2}=-\frac{n}{\sigma^2}$$

$$=-E\left(-\frac{n}{\sigma^2}\right)=\boxed{\frac{n}{\sigma^2}}$$

b) $E(\hat{\mu})=E(\bar{X})=E(X_i)=\mu\Rightarrow$ несмещенная

c) $D(\hat{\mu})=D(\bar{X})=\frac{n}{n^2}D(X_i)=\frac{\sigma^2}{n}$

$D(\hat{\mu})\geq I^{-1}(\hat{\mu})$ нерав-во Рао-Крамера

$\frac{\sigma^2}{n}=\frac{\sigma^2}{n}\Rightarrow$ оценка эффективная

Задача 6

$E(X_i) = \frac{4}{5}\theta = \bar{X} \Rightarrow \hat{\theta} = \frac{5}{4}\bar{X}$

1) $E(\hat{\theta}_n) = \frac{5n+3}{4n-2} E(\bar{X}_n) = \frac{5n+3}{4n-2} \cdot \frac{4n}{5}\theta$

$\lim\limits_{n\to\infty} E(\hat{\theta}_n) = \lim\limits_{n\to\infty} \frac{5n+3}{4n-2} \cdot \frac{4n}{5}\theta = \theta$

2) $D(\hat{\theta}) = \left(\frac{5n+3}{4n-2}\right)^2 D(\bar{X}_n) = \left(\frac{5n+3}{4n-2}\right)^2 \cdot \frac{n}{n^2} D(X_i) =$

$\lim\limits_{n\to\infty} \frac{25n^2+30n+9}{16n^2-16n+4} \cdot \frac{1}{n} D(X_i) = \lim\limits_{n\to\infty} \frac{0}{16} = 0.$

Да, состоятельная

Задача 7.

$\sigma_X^2 = \sigma_Y^2$, выборки X и Y независ. $\alpha = 0{,}05$

$H_0: \mu_X = \mu_Y$ $\quad \bar{X} = 1{,}256$

$H_1: \mu_X < \mu_Y$ $\quad \bar{Y} = 1{,}462$ $\quad \hat{\sigma}_\Delta = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - \bar{\Delta})^2}$

$t_p = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\hat{\sigma}_\Delta / \sqrt{n}} =$

$\hat{\sigma}_\Delta = \sqrt{\frac{1}{4}(2{,}33+0{,}206)^2 + (2{,}77+0{,}206)^2 + (-0{,}41+0{,}206)^2 + (-2{,}21+0{,}206)^2 + (-1{,}95+0{,}206)^2} = \sqrt{\frac{1}{4}(6{,}43+8{,}86+0{,}042+4{,}016)} = \sqrt{4{,}837}.$

$t_{5-1;\frac{\alpha}{2}} = -2{,}132$

$= \frac{0{,}206-0}{\sqrt{4{,}837/5}} = \frac{0{,}461}{2{,}199} \approx 0{,}21$ $\quad t_p \geq t_{4;\frac{\alpha}{2}} \Rightarrow$ H_0 не отвергается.

Задача 89.

	синяя	черная	фиол.	
пятерка	100	40	50	190
четверка	65	60	50	175
	165	100	100	365 = n.

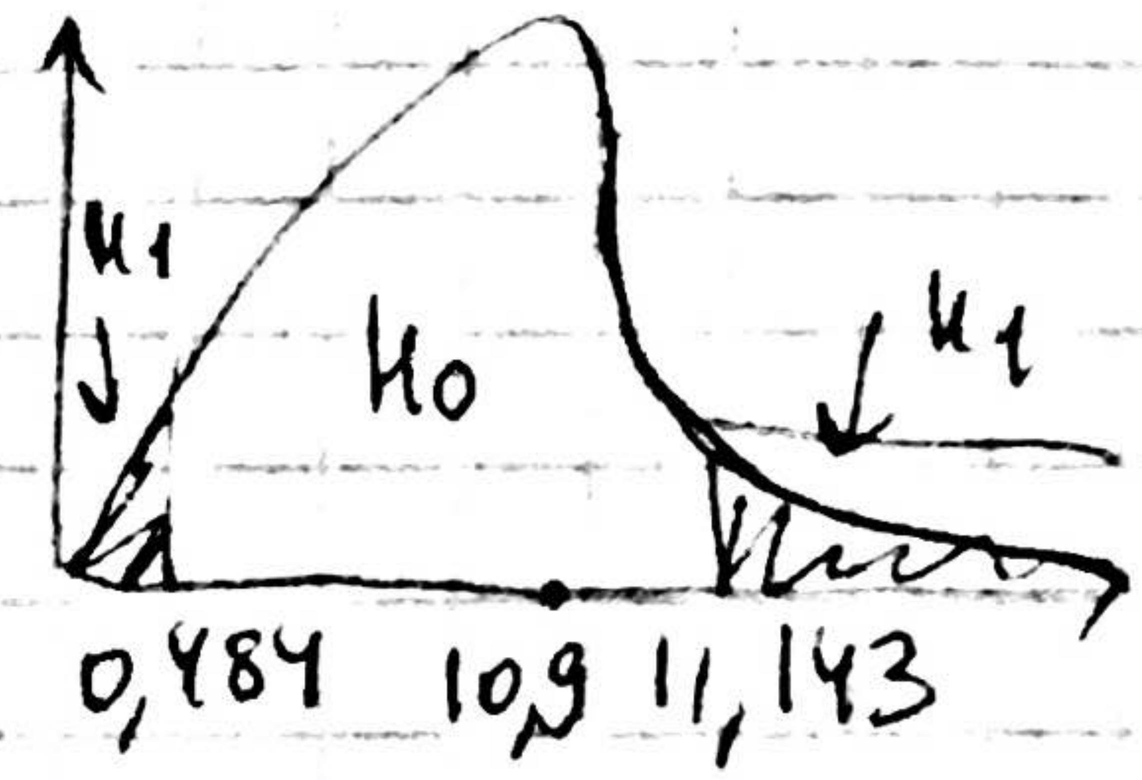
$$\gamma = \frac{\left(100 - \frac{190 \cdot 165}{365}\right)^2}{85,89} + \frac{\left(40 - \frac{190 \cdot 100}{365}\right)^2}{52,05} + \frac{\left(50 - \frac{190 \cdot 100}{365}\right)^2}{52,05} +$$

$$+ \frac{\left(65 - \frac{175 \cdot 165}{365}\right)^2}{79,11} + \frac{\left(60 - \frac{175 \cdot 100}{365}\right)^2}{47,9} + \frac{(50 - 47,9)^2}{47,9} =$$

$$= \frac{199,09}{85,89} + 2,79 + 0,081 + 2,52 + 3,05 + 0,09 =$$

$$\approx 10,9$$

$$\chi^2_{(k-1)(m-1)} = \chi^2_4$$



H_0 : признаки независимы

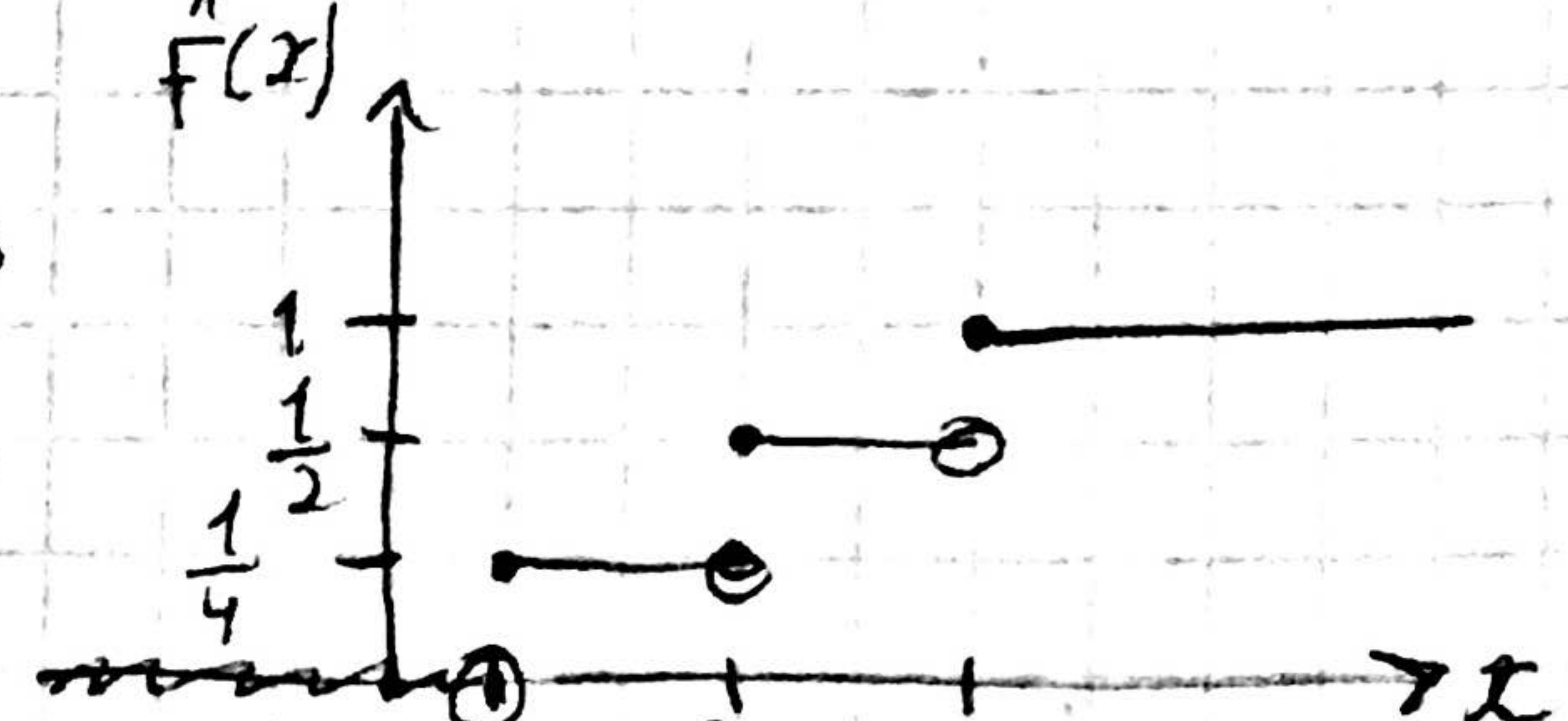
не отвергается.

Задача 1.

X_i	1	2	3
	20	30	50

$n = 100$

$$\hat{F}(x) = \begin{cases} \frac{1}{4}, & 1 \le x < 2 \\ \frac{1}{2}, & 2 \le x < 3 \\ 1, & x \ge 3 \\ 0, & x < 1 \end{cases}$$



Задача 8. $\bar{x} = \cancel{80}\ 1,7 \quad n=80 \quad \alpha=0,01$

$H_0 : \lambda = 2$

$H_1 : \lambda \neq 2$

$$f_x(x) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$L = \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!} e^{-\lambda}$$

$$\ln L = \sum_{i=1}^{n} x_i \ln\lambda - \ln \sum_{i=1}^{n} x_i! - n\lambda$$

$$\sum_{i=1}^{n} x_i = 80 \cdot 1,7 = 136.$$

$$\frac{\partial \ln L}{\partial \lambda} = \frac{\sum_{i=1}^{n} x_i}{\lambda} - n = 0 \Rightarrow \hat{\lambda}_{ML} = \frac{\sum_{i=1}^{n} x_i}{n} = 1,7.$$

$$LR = 2(\ln L(\hat{\lambda}_{ML}; x) - \ln L(\lambda_0; x)) =$$

$$\ln L(\hat{\lambda}_{ML}; x) = 136 \cdot \ln 1,7 - \ln \sum_{i=1}^{n} 1,7! - 80 \cdot 1,7$$

$$\ln L(\lambda_0; x) = 136 \ln 2 - \ln \sum_{i=1}^{n} 1,7! - 80 \cdot 2$$

$$= 2(136 \cdot \ln 1,7 - \ln \sum 1,7! - 136 - 136 \ln 2 + \ln \sum_{i=1}^{n} 1,7! + 160) =$$

= обычным калькулятором не считается