Черашева Н. А. БЭК183
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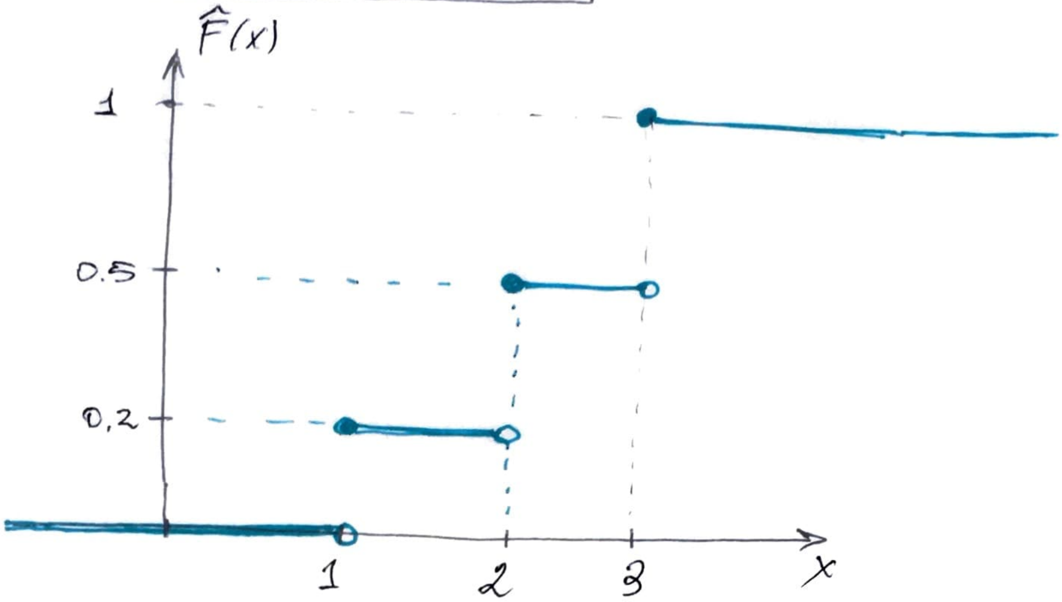
Задача 1.

X	1	2	3
P	0.20	0.30	0.5



Задача 2.

$$f(x;\theta)=\begin{cases}\dfrac{6x(\theta-x)}{\theta^3}, & x\in[0;\theta]\\ 0, & x\notin[0;\theta]\end{cases}\qquad \theta>0$$

центр. момент 2-го пор. $\longrightarrow Var(x)$

$$E(x)=\int_0^\theta \frac{6x^2(\theta-x)}{\theta^3}dx=\frac{6}{\theta^3}\cdot\int_0^\theta(\theta x^2-x^3)dx=\frac{6}{\theta^3}\cdot\frac{\theta x^3}{3}\Big|_0^\theta-\frac{6}{\theta^3}\cdot\frac{x^4}{4}\Big|_0^\theta=$$

$$=\frac{2\theta^4}{\theta^3}-\frac{3}{2}\cdot\theta=2\theta-\frac{3}{2}\theta=\frac{1}{2}\theta$$

$$E(x^2)=\int_0^\theta\frac{6x^3(\theta-x)}{\theta^3}\cdot dx=\frac{6}{\theta^3}\cdot\int_0^\theta(\theta x^3-x^4)dx=\frac{6}{\theta^3}\cdot\frac{\theta\cdot x^4}{4}\Big|_0^\theta-\frac{6}{\theta^3}\cdot\frac{\theta^5}{5}\Big|_0^\theta=$$

$$=\frac{3}{2}\cdot\frac{\theta^5}{\theta^3}-\frac{6}{5}\cdot\theta^2=\frac{15\theta^2-12\theta^2}{10}=\frac{3}{10}\theta^2$$

$$\cancel{\frac{1}{4}\theta^2}\quad\frac{\sum(x_i-\bar{X})^2}{n}=\frac{3}{10}\theta^2-\frac{1}{4}\theta^2$$

$$\frac{\Sigma(x_i-\bar{X})}{n} = \frac{6\theta^2-5\theta^2}{20} = \frac{\theta^2}{20}$$

$$\hat{\theta}_{MH} = \sqrt{\frac{\Sigma(X_i-\bar{X})}{20n}}$$

Задача 7.

i	1	2	3	4	5
X_i	1,53	2,83	-1,25	1,86	1,31
Y_i	-0.8	0.06	0.84	4.07	3.26

$X_1,\dots,X_n \sim N(\mu_x; \sigma_x^2)$

$Y_1,\dots,Y_m \sim N(\mu_y; \sigma_y^2)$

$\sigma_x^2 = \sigma_y^2$

$\alpha = 0{,}05$

~~$t_{8;\,0{,}025} = \pm 2{,}306$~~

$$\begin{cases} H_0: \mu_x = \mu_y \\ H_1: \mu_x < \mu_y \end{cases}$$

ДИ:

$$\bar{X}-\bar{Y} - t_{n+m-2;\alpha}\cdot\sigma_0\cdot\sqrt{\frac{1}{n}+\frac{1}{m}} \le \mu_x-\mu_y \le \bar{X}-\bar{Y} + t_{m+n-2;\,1-\alpha}\dots$$

$$\sigma_0 = \sqrt{\frac{\hat{\sigma}_x^2(n-1)+\hat{\sigma}_y^2(m-1)}{n+m-2}}$$

$$\bar{X} = \frac{1{,}53+2{,}83-1{,}25+1{,}86+1{,}31}{5} = 1{,}256$$

$$\bar{Y} = \frac{0.06+0.84+4{,}07+3.26-0{,}8}{5} = 1{,}486$$

$$\hat{\sigma}_x^2 = \frac{(1{,}53-1{,}256)^2+(2{,}83-1{,}256)^2+\dots+(1{,}31-1{,}256)^2}{4} = 2{,}3$$

$$\hat{\sigma}_y^2 = \frac{(-0.8-1{,}486)^2+\dots+(3{,}26-1{,}486)^2}{4} = 4{,}375$$

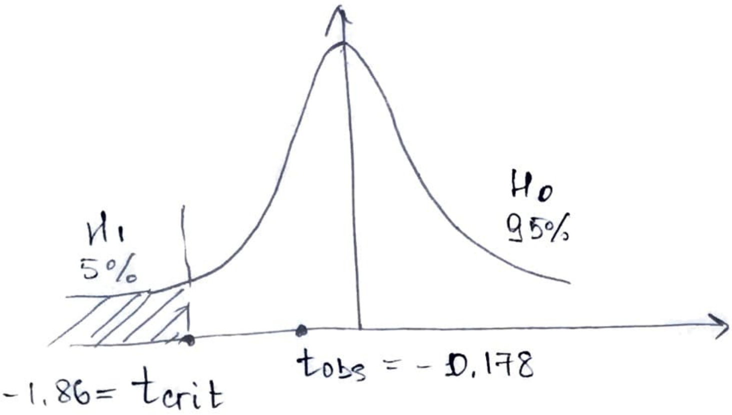
Реализация:

~~$1{,}25 - 1{,}486 - 2{,}306\dots$~~

$$\sigma_0 = \sqrt{\frac{2.3\cdot 4+4{,}375\cdot 4}{8}} = \sqrt{\frac{4}{8}(2{,}3+4{,}375)} = \sqrt{\frac{6{,}675}{2}} \approx 1{,}827$$

ой !!

$$t_{obs} = \frac{\bar{X} - \bar{y} - (\mu_x - \mu_y)}{\sigma_0 \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{1,256 - 1,486 - 0}{1,827 \cdot \sqrt{\frac{5}{2}}} = -0,178$$



Задача 3

$$L(x;\theta) = \prod (\theta+1)\, x^{\theta} = (\theta+1)^n \cdot \prod (x^{\theta})$$

$$\ln L = \cancel{\sum \ln(\theta+1)}\; n(\theta+1) + \theta \ln \sum_i x_i$$

$$\frac{\partial L}{\partial \theta} = n + \ln \sum_i X_i = 0$$