~~Задача 4~~ $\cancel{f(x;\theta)=\begin{cases}\frac{4x^3}{\theta^4}, & x\in[0;\theta]\\ 0, & x\notin[0;\theta]\end{cases}}$

$\cancel{\theta>0,\ \hat{\theta}=\bar{X}-}$

Задача 6 $f(x;\theta)=\begin{cases}\frac{4x^3}{\theta^4}, & x\in[0;\theta]\\ 0, & x\notin[0;\theta]\end{cases}$

$\theta>0$, $\hat{\theta}_n=\frac{5n+3}{4n-2}\bar{X}_n$ состоятельна?

$$E(X_i)=\int_0^\theta \frac{4x^3}{\theta^4}x\,dx=\int_0^\theta \frac{4x^4}{\theta^4}dx=\frac{4x^5}{5\theta^4}\Big|_0^\theta=\frac{4\theta^5}{5\theta^4}=\frac{4}{5}\theta$$

$$\operatorname{plim}_{n\to\infty}\hat{\theta}_n=\operatorname{plim}_{n\to\infty}\frac{5n+3}{4n-2}\bar{X}_n=\operatorname{plim}_{n\to\infty}\frac{5n+3}{4n-2}\cdot\bar{X}_n=\frac{5}{4}E(X_i)=\frac{5}{4}\cdot\frac{4}{5}\theta=\theta$$

$\Rightarrow$ состоятельна

Задача 7 $X=(X_1\dots X_n)$; $Y=(Y_1,\dots,Y_m)\sim N(\mu,\sigma^2)$

$x_1=1{,}53$, $x_2=2{,}83$, $x_3=-1{,}25$, $x_4=1{,}86$, $x_5=1{,}31$

$y_1=-0{,}80$; $y_2=0{,}06$, $y_3=0{,}84$, $y_4=4{,}07$, $y_5=3{,}26$

$\begin{cases}H_0: \mu_X=\mu_Y\\ H_1: \mu_X<\mu_Y\end{cases}$ $\sigma_X^2=\sigma_Y^2$ неизвестны, но равны

~~крит статистика:~~ $\cancel{\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}}\sim N(0,1)}$

~~t_{crit}~~ = Крит статистика: $\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{S\sqrt{\frac{1}{n}+\frac{1}{m}}}\sim t_{n+m-2}$

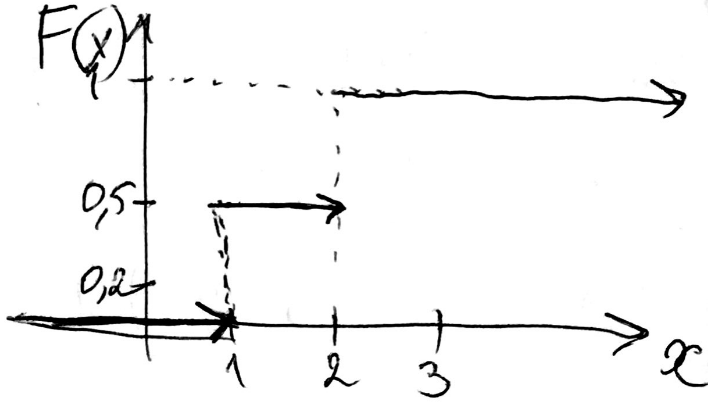
$S=\sqrt{\frac{(n-1)\sigma_X^2+(m-1)\sigma_Y^2}{n+m-2}}=(5-1)$

$\bar{X}=1{,}256$; $\bar{Y}=1{,}486$; $\hat{\sigma}_X^2=S^2=\frac{\sum(x_i-\bar{X})^2}{n-1}=\frac{\overset{0{,}0784}{(1{,}53-1{,}25)^2}+\overset{2{,}4775}{(2{,}83-1{,}256)^2}+}{4}$

$+\frac{\overset{6{,}280}{(-1{,}25-1{,}256)^2}+\overset{0{,}3648}{(1{,}86-1{,}256)}+\overset{0{,}0029}{(1{,}31-1{,}256)^2}}{4}=2{,}30162$

№1

X	1	2	3
D	20	30	50



№4 $f(x;\theta)=\begin{cases}\frac{4x^3}{\theta^4}, & x\in[0;\theta]\\ 0, & else\end{cases}$

$\theta>0$

а) $\hat{\theta}=\bar{X}$ – несмещенная?

$E(\hat{\theta})=\int_0^{\theta}\frac{4x^3}{\theta^4}x\,dx \overset{\text{из №6}}{=} \frac{4}{5}\theta\neq\theta \Rightarrow$ смещенная

б) Подобрать с, чтобы $\hat{\theta}=c\bar{X}$ была несмещ.

$c\cdot\frac{4}{5}\theta=\theta \Rightarrow c=\frac{5}{4}$

№2 $X=(X_1,\dots,X_n)$

$f(x;\theta)=\begin{cases}\frac{6x(\theta-x)}{\theta^3}, & x\in[0;\theta]\\ 0, & else\end{cases}$

$\theta>0$, исп-я ММ найти оценку для θ

• центр момент 2го порядка:

$D(X)=\frac{\sum(X_i-\bar{X})^2}{n}$; теор. мом: $E((X_i-\bar{X})^2)$

$E(X^2)=\int_0^{\theta}\frac{6x(\theta-x)}{\theta^3}x^2dx=\frac{6\theta x^4}{4\theta^3}-\frac{6x^5}{5\theta^3}\Big|_0^{\theta}=\frac{3}{2}\theta^2-\frac{6}{5}\theta^2=\frac{3}{10}\theta^2$

$D(X)=\frac{3}{10}\theta^2-\frac{1}{4}\theta^2=\frac{1}{20}\theta^2$

$\frac{1}{20}\theta^2=\left(\frac{\sum(X_i-\bar{X})^2}{n}\right)$ ← выборочный момент

$\hat{\theta}_{MM}=\sqrt{\frac{20\sum(X_i-\bar{X})^2}{n}}$

№5 $x_1, \dots, x_n$ — выборка из $N(\mu; \delta)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathcal{L} = \prod_{i=1}^{n} f(x) = \prod_{i=1} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \cdot e^{-\frac{\sum(x-\mu)^2}{2\sigma^2}}$$

$$\ln\mathcal{L} = \ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n - \frac{1}{2\delta^2}\sum(x_i-\mu)^2 = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2 =$$

$$= -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2 = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum x_i + \mu\sum x_i - \frac{n}{2}\mu^2$$

а) инфо Фишера

$$-E\left(\frac{\partial^2 \ln L}{\partial\mu}\right) \Rightarrow -E(-n) = n \Rightarrow I(\mu) = n$$

$$\frac{\partial \ln L}{\partial\mu} = \sum x_i - 2n\mu\frac{1}{2} = 0$$

$$\frac{\partial^2 \ln L}{\partial^2\mu} = -n < 0 \Rightarrow \max$$

б)

$$p\lim_{n\to\infty} |\bar{x} - \mu| > \varepsilon$$

$$\lim_{n\to\infty} \frac{\sum x_i}{n} \xrightarrow[\text{ЗБЧ}]{} E(x_i)$$

в) эффективность:

$$D(\bar{x}) = D\left(\frac{1}{n}\sum x_i\right) = \frac{1}{n^2}D\left(\sum x_i\right) = \frac{1}{n^2}\cdot n\cdot D(x_i) = \frac{1}{n}\cdot 1 = \frac{1}{n}$$

$$\boxed{D(\hat{\theta}) = \frac{1}{I(\theta)}} \quad \frac{1}{n} = \frac{1}{n} \Rightarrow \text{эффективна}$$