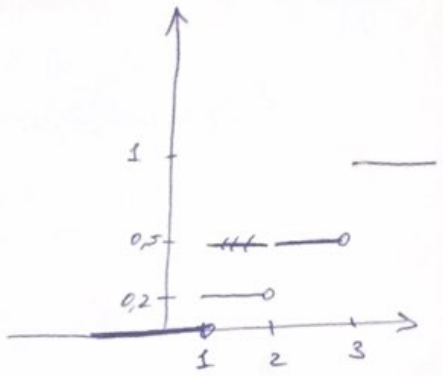
Задача 1.

	1	2	3
n	20	30	50
p	0,2	0,3	0,5

$$\begin{cases} 0; & x < 1 \\ 0{,}2; & x \in [1; 2) \\ 0{,}5; & x \in [2; 3) \\ 1; & x \geq 3 \end{cases}$$



Задача 2.

$$f(x;\theta) = \begin{cases} \frac{6x(\theta - x)}{\theta^3}; & x \in [0; \theta) \\ 0; & x \notin [0; \theta] \end{cases}$$

$$\frac{1}{n}\sum (x_i - \bar{x})^2 = E(X - E(X))^2 - \text{теор. мом.}$$

выб. мом.

$$E(x^2) = \int_0^\theta \frac{6x^3\theta - 6x^4}{\theta^3} dx = \int_0^\theta \left(\frac{6x^3}{\theta^2} - \frac{6x^4}{\theta^3}\right) dx =$$

$$= \frac{6x^4}{4\theta^2}\Big|_0^\theta - \frac{6x^5}{5\theta^3}\Big|_0^\theta = \frac{3}{2}\theta^2 - \frac{6}{5}\theta^2 = \frac{3}{10}\theta^2 = \frac{3}{10}\theta^2$$

$$E(x) = \int_0^\theta \left(\frac{6x^2\theta}{\theta^3} - \frac{6x^3}{\theta^3}\right) dx = \int_0^\theta \left(\frac{6x^2}{\theta^2} - \frac{6x^3}{\theta^3}\right) dx =$$

$$= \frac{6x^3}{3\theta^2}\Big|_0^\theta - \frac{6x^4}{4\theta^3}\Big|_0^\theta = 2\theta - \frac{3}{2}\theta = \frac{\theta}{2}$$

$$E(x)^2 = \frac{\theta^2}{4} \qquad E(x^2) - E(x)^2 = \frac{3\theta^2}{10} - \frac{\theta^2}{4} = (0{,}3 - 0{,}25)\theta^2 =$$

$$= 0{,}05\theta^2$$

$$0{,}05\theta^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$\hat{\theta} = \sqrt{\frac{\frac{1}{n}\sum (x_i - \bar{x})^2}{0{,}05}}$$

Задача 3.

~~$f(x;\theta) = \begin{cases} (\theta+1)x^\theta, & x \in (0;1) \\ 0, & \text{иначе} \end{cases}$~~

~~$L_1 = \prod (\theta+1)x^\theta$~~

Задача 4 $f(x;\theta) = \begin{cases} \frac{4x^3}{\theta^4}, & x \in [0;\theta] \\ 0 & x \notin [0;\theta] \end{cases}$

а) Несмещенность. $E(\hat{\theta}) = \theta$

$$E(\theta) = \int_0^\theta \frac{4x}{\theta^4} dx = \frac{4x^5}{5\theta^4}\Big|_0^\theta = \frac{4}{5}\theta = \bar{X}$$

$\theta = \frac{5}{4}\bar{X} \neq \bar{X}$ смещ.

б) $c = \frac{5}{4}$

Задача 7 $\sigma_X^2 = \sigma_Y^2$ $\alpha = 0{,}05$

$x_1 = 1{,}53$ $x_2 = 2{,}83$ $x_3 = -1{,}25$ $x_4 = 1{,}86$ $x_5 = 1{,}31$

$y_1 = -0{,}8$ $y_2 = 0{,}06$ $y_3 = 0{,}84$ $y_4 = 4{,}07$ $y_5 = 3{,}26$

$\bar{X} = \frac{6{,}28}{5} = 1{,}256$

$\bar{Y} = \frac{7{,}43}{5} = 1{,}486$

$H_0: \mu_X = \mu_Y = 0$

$H_1: \mu_X < \mu_Y < 0$

$$t = \frac{\bar{X} - \bar{Y} - (\mu_X + \mu_Y)}{\hat{\sigma}_0 \sqrt{\frac{1}{n_X} + \frac{1}{n_Y}}} \qquad t_{n_X+n_Y-2}$$

$$\hat{\sigma}_0^2 = \frac{\hat{\sigma}_X^2(n_X-1) + \hat{\sigma}_Y^2(n_Y-1)}{n_X + n_Y - 2}$$

$(n_X-1)\hat{\sigma}_X^2 = 0{,}075 + 2{,}4767 + 6{,}28 + 0{,}365 + 0{,}003 = 9{,}2$

$(n_Y-1)\hat{\sigma}_Y^2 = 5{,}226 + 2{,}208 + 0{,}4171 + 6{,}677 + 3{,}147 = 17{,}675$

$\hat{\sigma}_0^2 = 3{,}359$

$\hat{\sigma}_0 = 1{,}83$

$$t = \frac{-0{,}23}{1{,}83\sqrt{\frac{1}{2}}} = -0{,}177$$

$0{,}71$

$1{,}2993$

Но отвергается, если $t \in (-\infty; -1{,}86)$ $\Rightarrow$ Но не отвергается

Задача 8.

$$L = \prod (\theta+1)x^\theta = (\theta+1)^n \cdot x^{\theta n}$$

$$\ln L = \ln(\theta+1)^n + \ln x^{\theta n} = n\ln(\theta+1) + \theta n \ln x \to \max$$

$$\frac{n}{\theta+1} + n\ln x = 0 \qquad \left(\ln L''_1 = -\frac{n}{(\theta+1)^2} < 0 \Rightarrow \max\right)$$

$$\frac{1}{\theta+1} = -\ln x \qquad \theta+1 = -\frac{1}{\ln x} \qquad \hat{\theta}_{ML} = -\left(\frac{1}{\ln x} + 1\right)$$

Задача 5. $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma}}$

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{\sum(x_i-\mu)^2}{2\sigma^2}$$

$$\ln L' = +\frac{2\sum(x_i-\mu)}{2\sigma^2}$$

а) $I_n = \frac{n}{\sigma^2}$

б) $E(\hat{\mu}) = \bar{X}$