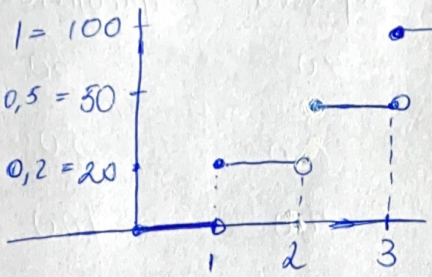
Задача 1

1	2	3
20/100	20/100	50/100
0,2	0,3	0,5



$$F(x) = \begin{cases} 0, & x < 1 \\ 0{,}2, & 1 \le x < 2 \\ 0{,}5, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$$

2

$$f(x;\theta) = \begin{cases} \frac{6x(\theta - x)}{\theta^3}, & \text{при } x \in [0;\theta] \\ x \notin [0;\theta) \end{cases}$$

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \approx E(X - E(X))^2 = Var\,X$$

$$E(X) = \int_0^{\theta} \frac{6x(\theta - x)}{\theta^3} = \frac{1}{\theta^3}\left(2x^3\theta - \frac{3}{2}x^4\right)\Big|_0^{\theta} = \frac{1}{\theta^3}\left(2\theta^4 - \frac{3}{2}\theta^4\right) = \frac{1}{\theta^3}\cdot\frac{1}{2}\theta^4 = \frac{\theta}{2}$$

$$E(X^2) = \int_0^{\theta} \frac{6x^2(\theta - x)}{\theta^3} = \frac{1}{\theta^3}\cdot\left(\frac{3}{2}\theta x^4 - \frac{6}{5}x^5\right)\Big|_0^{\theta} = \frac{1}{\theta^3}\left(\frac{3}{2}\theta^5 - \frac{6}{5}\theta^5\right) = 0{,}3\theta^2$$

$$Var\,X = 0{,}3\theta^2 - (0{,}5\theta)^2 = 0{,}05\theta^2$$

$$\Rightarrow \hat{\theta} = \sqrt{\frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}{0{,}05}}$$

3

$$f(x,\theta) = \begin{cases} (\theta+1)x^{\theta}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

$$L(X,\theta) = (\theta+1)^n (x_1 \dots x_n)^{\theta}$$

$$\ln L = n\ln(\theta+1) + \theta \underbrace{\ln(x_1 \dots x_n)}_{n\ln x} \to \max_{\theta}$$

$$\frac{1}{\theta+1} + \overline{\ln x} = 0$$

$$\theta + 1 = -\frac{1}{\ln x} \qquad \theta = -\frac{1}{\ln x} - 1$$

$$L = (\theta+1)\cdot x_1^{\theta}\cdot(\theta+)\cdot x_1^{\theta}\cdot\dots = (\theta+1)x_n^{\theta} = (\theta+1)^n\cdot(x_1\cdot x_2\cdot x_3\dots x_n)^{\theta}$$

$$\ell = n\ln(\theta+1) + \theta(\ln x_1 + \ln x_2 + \dots + \ln x_n) \to \max$$

$$\ell'_{\theta} = \frac{n}{1+\theta} + \sum \ln x_i = 0 \qquad \ell''_{\theta\theta} < 0 \Rightarrow \text{SOC} +$$

$$\frac{n}{1+\theta} = -\sum \ln x$$

$$1 + \theta = -\frac{n}{\sum \ln x}$$

$$\hat{\theta}_{ML} = \frac{-n}{\sum \ln x_i} - 1$$

④ $f(x,\theta) = \begin{cases} \frac{4x^3}{\theta^4} & x \in [0;\theta] \\ 0, \text{ иначе} \end{cases}$

$$E(X) = \int_0^{\theta} \frac{4x^4}{\theta^4} = \frac{4x^5}{5\theta^4}\Big|_0^{\theta} = \frac{4}{5}\theta \propto \bar{X} \Rightarrow \hat{\theta} = \frac{5}{4}\bar{X}$$

а) $\theta = \bar{X}$ - смещ. оценка

б) $C = \frac{4}{5}$

⑤ а) $I_n(\mu) = -E($ ...

б) $\bar{\mu} = \bar{X}$ несмещ, т.к $E(\bar{\mu}) = E(\bar{X}) = E\left(\frac{X_1 + \dots X_n}{n}\right) = \frac{n\,E(X)}{n} = E(X)$

с) $\hat{\mu} = \bar{X}$ эффект, т.к

$$Var(\bar{X}) = \frac{Var(X)}{n} \xrightarrow[n\to\infty]{} 0$$

⑥ $f(x,\theta) = \begin{cases} \frac{4x^3}{\theta^4} & x \in [0;\theta] \end{cases}$

$$E(X) = \frac{4}{5}\theta$$

$$Var(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \int_0^{\theta} \frac{4x^5}{\theta^4} = \frac{2x^6}{3\theta^4}\Big|_0^{\theta} = \frac{2}{3}\theta^2$$

$$E(X^2) = \int_0^{\theta} \frac{4x^5}{\theta^4} = \frac{2}{3}\frac{x^6}{\theta^4}\Big|_0^{\theta} = \frac{2}{3}\theta^2$$

$$Var(X) = \frac{2}{3}\theta^2 - \frac{16}{25}\theta^2 = \frac{2}{75}\theta^2$$

$$\hat{\theta}_n = \left(\frac{5n+3}{4n-2}\right)\overline{X_n} \quad n \to \infty$$

$$E(\hat{\theta}_n) \xrightarrow[n\to\infty]{} \left(\frac{5}{4}\right)E(X) = \frac{5}{4}\cdot\frac{4}{5}\theta = \theta$$

$$Var(\hat{\theta}_n) = \left(\frac{5n+3}{4n-2}\right)^2 \cdot \frac{Var(X)}{n} = 0$$

$n \to \infty$: $\left(\frac{5n+3}{4n-2}\right) \to 1$; $\frac{Var(X)}{n} \to 0$

$\Rightarrow$ оценка состоятельная

(8) $n = 80$ $\alpha = 0,01$ $H_0: \lambda = 2$ $H_1: \lambda \neq 2$

$\bar{X} = 1,7$

$\hat{\lambda}_{ML} = \bar{X}$

$\sum x_i = 136$

LR

$\frac{e^{-\lambda} \lambda^k}{k!}$ $L = \prod \frac{e^{-1,7} 1,7^k}{k!}$

$\ell = -80 \cdot 1,7 + \ln 1,7 \sum x_i - \sum \ln x_i$

$\ell_0 = -80 \cdot 1,7 + 136 \cdot \ln 1,7 - \ln 136$

$\ell \Theta = -80 \cdot 2 + 136 \ln 2 - \ln 136$

203,2 ← 249,4

χ^2_1

$\boxed{LR = 92,4}$

$LR = 2(\ell(\Theta) - \ell_0)$

(9) χ^2 Пирсона

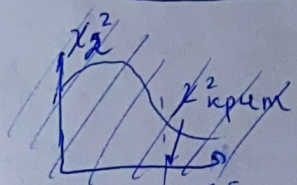
$\chi^2 = \sum ($

	c	ч	ф
5	165/2	50	50
4	165/2	50	50

$\chi^2 = \frac{(100-165/2)^2}{165/2} + \frac{(65-165/2)^2}{165/2} + \frac{(40-50)^2}{50} +$

$+ \frac{(60-50)^2}{50} + 0 + 0 \neq \chi^2$

Независимость оценки от ручки



(7) 2,477 6,28 0,365 9,029

x 1,53 2,83 −1,25 1,86 1,31 $\alpha = 0,05$

y −0,80 5,23 2,05 0,06 0,417 0,84 6,68 4,07 3,15 3,26

$\bar{x} = 1,256$

$\bar{y} = 1,486$

$\bar{x} - \bar{y} = -0,23$

$\sigma^2_x = 2,3$

$\sigma^2_y = 4,38$

$t_p = \frac{-0,23 - 0}{\sqrt{\frac{4 \cdot 2,3 + 4 \cdot 4}{8} \cdot \frac{10}{25}}} = \frac{-0,23}{1,158} \approx -0,2$ ~t8