Задачи   Бедус Юлия   БЭК 1814

Задача 2 $f(x;\theta)=\begin{cases}\frac{6x(\theta-x)}{\theta^3}\\ \end{cases}$

$$\int_0^\theta x^2\cdot\frac{6x(\theta-x)}{\theta^3}=\frac{6}{\theta^3}\int_0^\theta x^3(\theta-x)\,dx=\frac{6}{\theta^3}\int_0^\theta x^3\theta-x^4\,dx$$

$$x^3\theta-x^4$$

$$=\frac{6}{\theta^3}\left(\frac{x^4}{4}\cdot\theta-\frac{x^5}{5}\right)\Big|_0^\theta=\frac{6}{\theta^3}\left(\frac{\theta^5}{4}-\frac{\theta^5}{5}\right)=$$

$$=\frac{6}{\theta^3}\cdot\frac{\theta^5}{4}-\frac{6}{\theta^3}\cdot\frac{\theta^5}{5}=\frac{3}{2}\theta^2-\frac{6}{5}\theta^2=$$

$$=\frac{15}{10}\theta^2-\frac{12}{10}\theta^2=\frac{3}{10}\theta^2=0{,}3\theta^2$$

Задача 3 $\begin{cases}(\theta+1)x^\theta\\ 0\end{cases}$

$$L=\prod(\theta+1)x^\theta\to(\theta+1)^n\cdot\prod x^\theta\to$$

$$\to\quad L=(\theta+1)^n\cdot\sum x_i^\theta$$

$$\ln=n\ln(\theta+1)+\theta\ln\sum x_i$$

$$\frac{\ln}{\partial\theta}=\frac{n}{\theta+1}+\ln\sum x_i=0$$

$$\frac{n}{\theta+1}=-\ln\sum x_i\ |\cdot\theta+1$$

$$n=-\ln\sum x_i(\theta+1)$$

$$-\ln\sum x_i(\theta+1)=n$$

$$\theta+1=\frac{n}{-\ln\sum x_i}$$

$$\hat\theta=\frac{n}{-\ln\sum x_i}-1$$

Задача 4

$$\left\{ \frac{4x^3}{\theta^4} \right.$$

найдем матожидание:

а) $\int \frac{4x^3}{\theta^4} \cdot x dx = \int_0^\theta \frac{4x^4}{\theta^4} dx = \frac{4}{\theta^4} \cdot \frac{x^5}{5} \Big|_0^\theta =$

$= \frac{4}{\theta^4} \cdot \frac{\theta^5}{5} \overset{0}{=} \frac{4}{5}\theta \rightarrow$ смещенная

в) $\frac{4}{5}\theta = \quad c = \frac{5}{4} \rightarrow$ чтобы оценка не была смещенной

Задача 6

Состоятельность:

① $E(x) = \theta \qquad \theta = \frac{5n+3}{4n-2} \bar{X}_n$

② $Var(x) \xrightarrow[\text{при } n \to \infty]{} 0$

проверим на ассимптотич. несмещ:

$\lim \frac{5n+3}{4n-2} \bar{X}_n \rightarrow \theta$ (ассимптотич несмещенна)

$\bar{X}_n \hookrightarrow \frac{E(X_1 + \dots X_n)}{n}$

найдем $Var(x)$

$Var\left(\frac{5n+3}{4n-2}\bar{X}_n\right) = \frac{(5n+3)^2}{(4n-2)^2} \frac{Var(x_1)}{n} =$

$Var(x_1) \to E(x^2) - E^2(x)$

найдем $E(x^2)$

$\int \frac{4x^3}{\theta^4} x^2 dx = \int_0^\theta \frac{4x^5}{\theta^4} dx = \frac{4x^6}{6\theta^4} \Big|_0^\theta = \frac{4\theta^6}{6 \cdot \theta^4} = \frac{2}{3}\theta^2$

тогда var: $\frac{2}{3}\theta^2 - \frac{16}{25}\theta^2$

$= \lim\limits_{n \to \infty} \frac{(5n+3)^2 \cdot \left(\frac{2}{3}\theta^2 - \frac{16}{25}\theta^2\right)}{(4n-2)^2 \cdot n} = 0 \Rightarrow$

оценка состоятельна

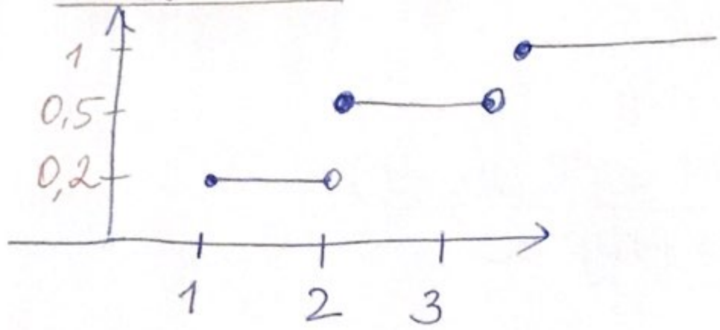
Задача 1

$n = 100$

1 – „20" $\to \frac{20}{100}$

2 – „30" $\to \frac{30}{100}$

3 – „50" $\to \frac{50}{100}$



Задача 5.

$$L = \Pi = \frac{1}{\sigma(\sqrt{2\pi})^n} e^{\left(-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\xi_i - a}{\sigma^2}\right)^2\right)}$$

$$\ln L = -n\ln\sigma - n\ln\sqrt{2\pi} - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(\xi_i = a)^2$$

Условие экстремума

$$\frac{\partial \ln L}{\partial a} = \frac{1}{2\sigma^2}\cdot 2\sum(\xi_i - a) = \frac{\sum(\xi_i - a)}{\sigma^2} = 0$$

$$\to a = \frac{1}{n}\sum \xi_i = \overline{\xi_i}$$

$$\frac{\partial \ln L}{\partial a^2} = -\frac{1}{\sigma^2} < 0$$

$$-E\left(\frac{\partial \ln L}{\partial a^2}\right) = -E\left(-\frac{1}{\sigma^2}\right) = \frac{1}{\sigma^2} = I_n$$

в) оценка $\bar{x}$ является несмещ.
поскольку $M_{ш} = M_{ши} \to M_{ши} = \bar{X}$

а. Если $E(\bar{\mu}) = \mu \to$ оценка несмещен

г) она также (оценка $\bar{x}$) является эффективной, поскольку выполнено:

1. $E(\mu) = \mu$ (см. пункт в)
2. $Var(\bar{x}) = \lim\limits_{n\to\infty}\frac{Var(x_1)}{n} \to 0 \to$ состоятельна

Задача 7. $\sigma_x^2 = \sigma_y^2 \quad \alpha = 0.05$

$H_0: \mu_x = \mu_y \qquad \sigma_x^2 = \sigma_y^2 = \sigma_0^2$

$H_1 = \mu_x < \mu_y$

$$\hat{\sigma}_0^2 = \frac{\sigma_x^2(n_x-1) + \sigma_y^2(n_y-1)}{n_x+n_y-2}$$

для x:

$$\sigma_x^2 = \frac{\overset{0.05}{(1.53-1.3)^2} + \overset{2.34}{(2.83-1.3)^2} + \overset{6.5}{(-1.25-1.3)^2} + \overset{0.31}{(1.86-1.3)^2} + \overset{0.0001}{(1.31-1.3)^2}}{5-1} =$$

$$\bar{x} = \frac{1.53+2.83+1.86-1.25+1.31}{5} = 1.256 \approx 1.3$$

$= 2.3$

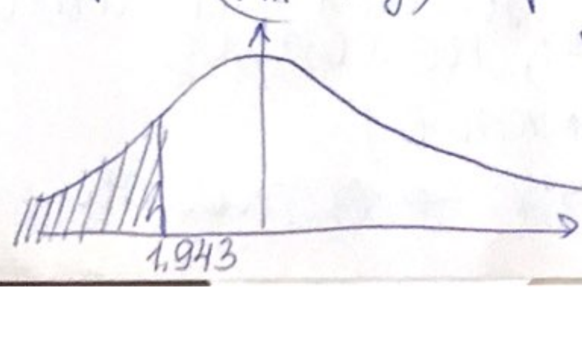
$$\sigma_y^2 = \frac{\overset{5.29}{(-0.8-1.5)^2} + \overset{2.07}{(0.06-1.5)^2} + \overset{0.43}{(0.84-1.5)^2} + \overset{6.604}{(4.07-1.5)^2} + \overset{3.09}{(3.26-1.5)^2}}{4}$$

$= 4.371$

$$\bar{y} = \frac{-0.80+0.06+0.84+4.07+3.26}{5} = 1.486$$

$$\hat{\sigma}_0^2 = \frac{2.3(4) + 1.486 \cdot 4}{4+4-2} = \frac{9.2+5.9}{6} = 2.5$$

$$\frac{\bar{x}-\bar{y}-(\mu_x-\mu_y)}{\sqrt{\hat{\sigma}_0^2\left(\frac{1}{n_x}+\frac{1}{n_y}\right)}} = \frac{1.3-1.5}{\underbrace{\sqrt{2.5\left(\frac{1}{4}+\frac{1}{4}\right)}}_{1.11}} = -0.180 \sim t_6$$



→ хипотеза Ho отвергается