Задача 1

In [150…

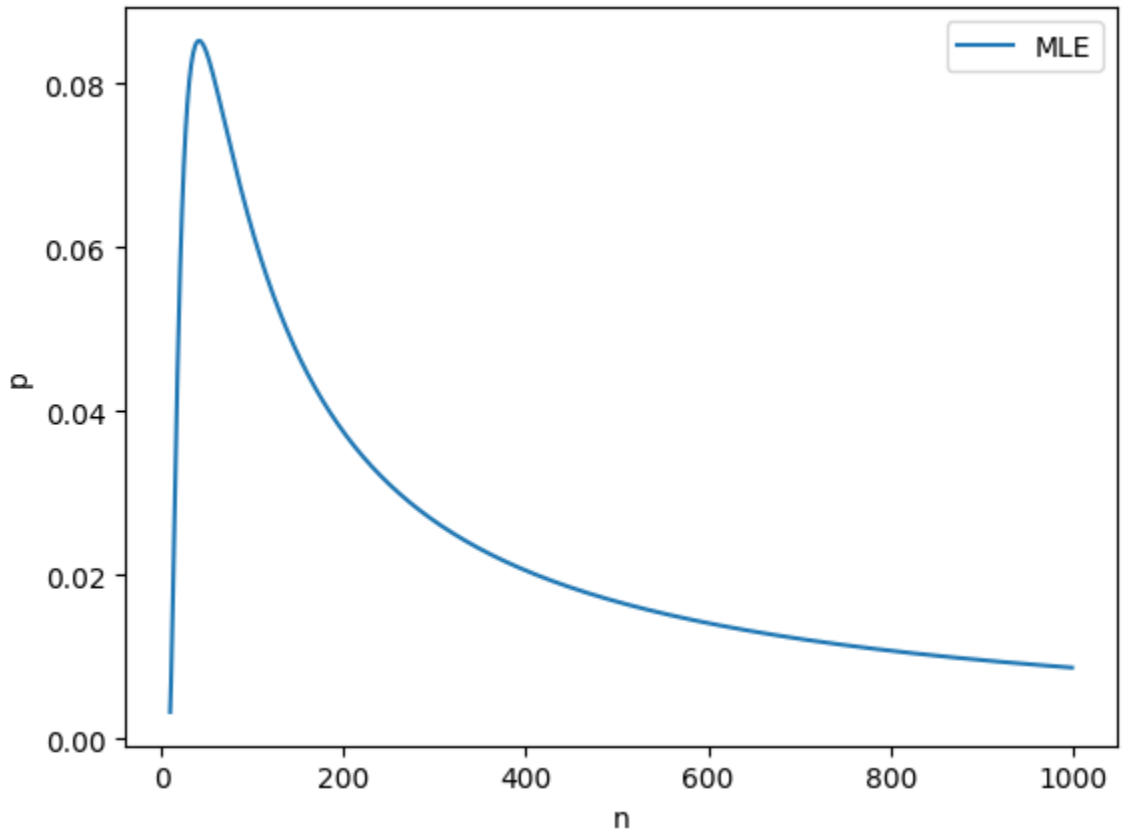
```
import pandas as pd
import numpy as np
import scipy.stats as sts
import math
from scipy.stats import norm, t, ttest_ind
import itertools
from statsmodels. stats.proportion import proportions_ztest

import matplotlib.pyplot as plt
import seaborn as sns

from collections import defaultdict
```

a)

Запишем функцию L максимального правдоподобия. Вероятность приезда уникального таксиста на втором заказе - $\frac{n-1}{n}$, на втором заказе - $\frac{n-2}{n}$, итд ...

In [151…

```
def L(n, d=10):
    res = 1
    for i in range(d - 1):
        res *= (n - i) / n
    res *= (d - 1) / n
    return res

l1 = []

for i in range(10, 1000):
    l1.append(L(i, 10))

plt.plot(range(10, 1000), l1, label='MLE')
plt.legend()
plt.xlabel('n')
plt.ylabel('p')
plt.show()

max_val = [0, 0]
for i in range(100):
    if l1[i] > max_val[0]:
        max_val[0] = l1[i]
        max_val[1] = i + 10

print('MLE:', max_val[1])
```

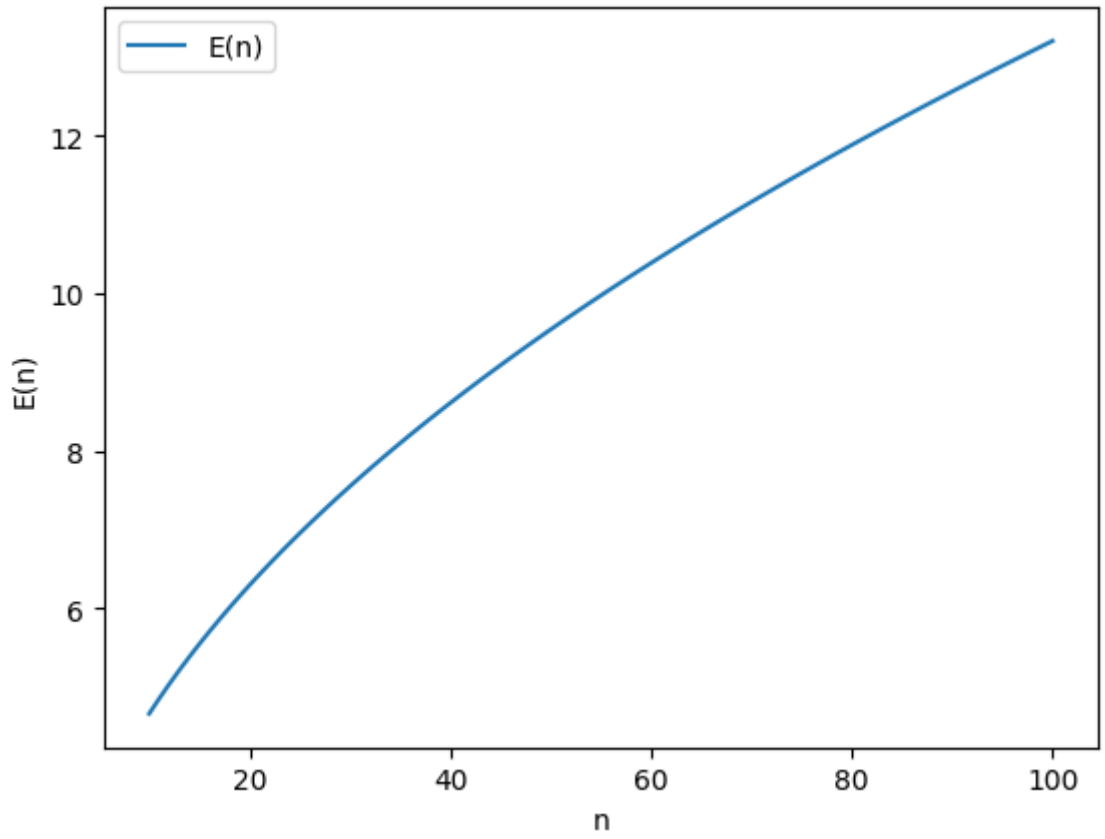



```
MLE: 42
```

б)

In [152…

```
def E(n):
    res = 0
    for i in range(1, n + 2):
        res += L(n, i) * i
    return res


l2 = []

for i in range(10, 101):
    l2.append(E(i))

plt.plot(range(10, 101), l2, label='E(n)')
plt.legend()
plt.xlabel('n')
plt.ylabel('E(n)')
plt.show()

mm = np.array([])

for i in range(10, 101):
    mm = np.append(mm, abs(E(i) - 10))

print('MM:',  range(10, 101)[np.argmin(mm)])
```




```
MM: 55
```

в)

MM:

In [153…
```
vals = []

np.random.seed(41)
for i in range(10000):
    bool_arr = [False]*100
    for j in range(101):
        cur = np.random.choice(range(100))
        if bool_arr[cur]:
            vals.append(j + 1)
            break
        else:
            bool_arr[cur] += 1
```

In [154…
```
mm_l3 = np.vectorize(E)
mm_l3 = mm_l3(range(1, 1000))
vals = np.array(vals)
mm_l3 = np.absolute(vals[:, np.newaxis] - mm_l3)
for i in range(len(mm_l3)):
    mm_l3[i] = np.argmin(mm_l3[i]) + 1
print('mm:', mm_l3.mean())
```

```
mm: 123.284
```

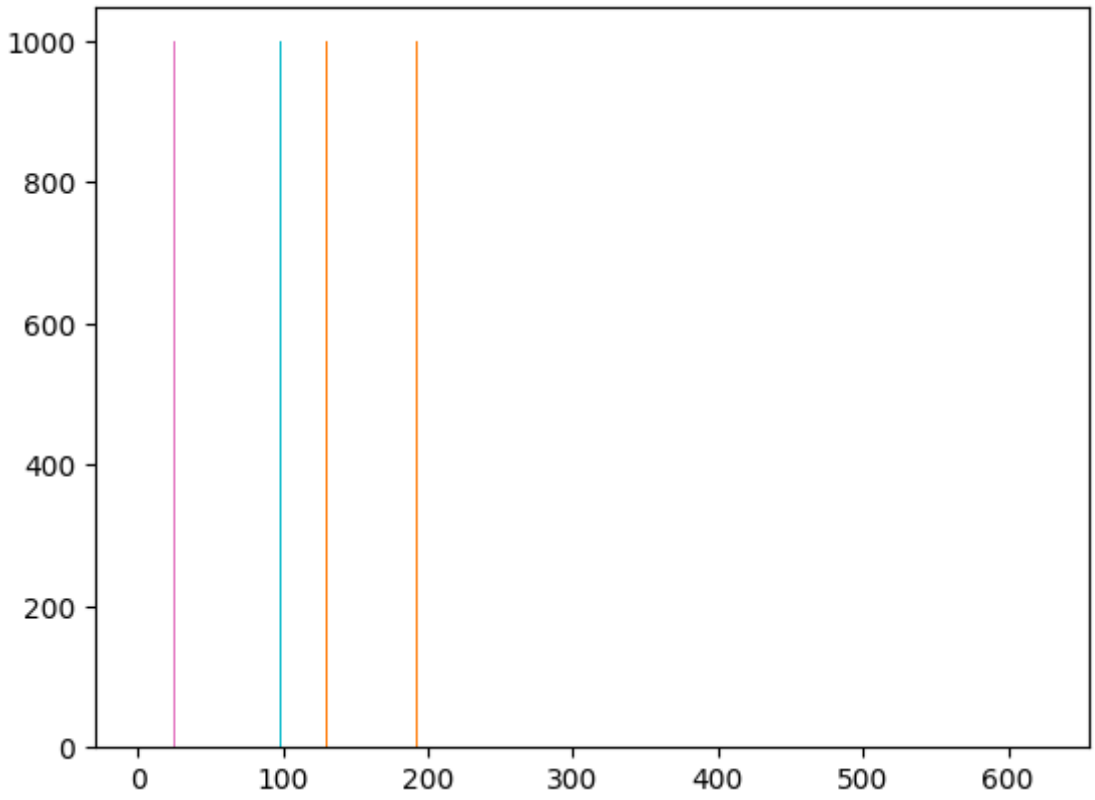
In [155…
```
print('STD', mm_l3.std(), '\nVAR',mm_l3.std()**2, '\nMEAN', mm_l3.mean(),  '\nBI
```

```
STD 114.89350261872954
VAR 13200.516944000012
MEAN 123.284
BIAS 23.284000000000006
```

In [156…
```
print(1)
plt.hist(list(mm_l3)[:200], bins = 100)
```

```
1
```

Out[156]:
```
(array([[0., 0., 0., ..., 0., 0., 0.],
        [0., 0., 0., ..., 0., 0., 0.],
        [0., 0., 0., ..., 0., 0., 0.],
        ...,
        [0., 0., 0., ..., 0., 0., 0.],
        [0., 0., 0., ..., 0., 0., 0.],
        [0., 0., 0., ..., 0., 0., 0.]]),
 array([  1.  ,   7.24,  13.48,  19.72,  25.96,  32.2 ,  38.44,  44.68,
         50.92,  57.16,  63.4 ,  69.64,  75.88,  82.12,  88.36,  94.6 ,
        100.84, 107.08, 113.32, 119.56, 125.8 , 132.04, 138.28, 144.52,
        150.76, 157.  , 163.24, 169.48, 175.72, 181.96, 188.2 , 194.44,
        200.68, 206.92, 213.16, 219.4 , 225.64, 231.88, 238.12, 244.36,
        250.6 , 256.84, 263.08, 269.32, 275.56, 281.8 , 288.04, 294.28,
        300.52, 306.76, 313.  , 319.24, 325.48, 331.72, 337.96, 344.2 ,
        350.44, 356.68, 362.92, 369.16, 375.4 , 381.64, 387.88, 394.12,
        400.36, 406.6 , 412.84, 419.08, 425.32, 431.56, 437.8 , 444.04,
        450.28, 456.52, 462.76, 469.  , 475.24, 481.48, 487.72, 493.96,
        500.2 , 506.44, 512.68, 518.92, 525.16, 531.4 , 537.64, 543.88,
        550.12, 556.36, 562.6 , 568.84, 575.08, 581.32, 587.56, 593.8 ,
        600.04, 606.28, 612.52, 618.76, 625.  ]),
 <a list of 200 BarContainer objects>)
```



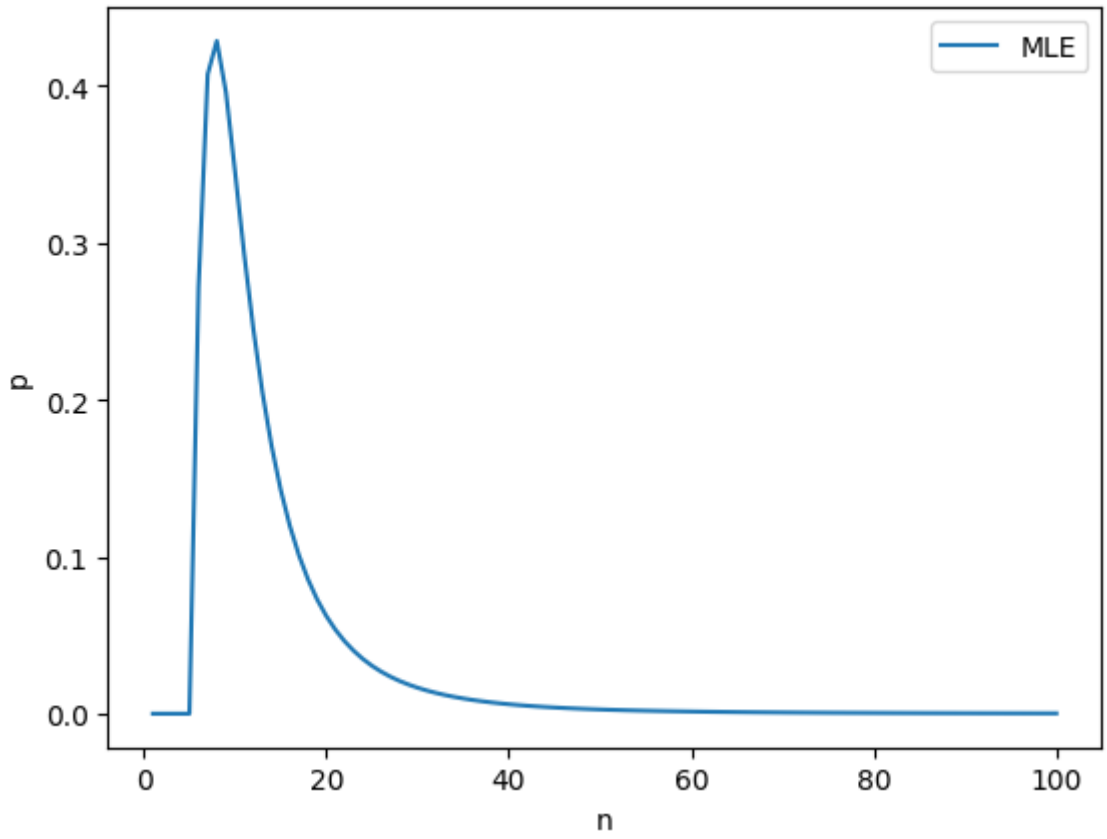
Задача 2

a)

Вероятность имеет вид: произведения только уникальных имен $\frac{n-i}{n}$ по количеству уникальных имен - 1. Вероятность встретить повторяющееся имя $= \frac{i}{n}$

In [157…

```
def L_2(n, n_unique, d):
    res = 1
    cnt = 0

    for i in range(1, n_unique):
        res *= (n-i)/n

    combs = itertools.combinations_with_replacement(np.arange(1, n_unique+1), d

    for el in combs:
        temp = 1

        for i in range(d - n_unique):
            temp *= el[i]

        cnt += temp
    res *= (cnt/(n**(d - n_unique)))
    return res

l3 = []

for i in range(1, 101):
    l3.append(L_2(i, 6, 10))

plt.plot(range(1, 101), l3, label='MLE')
plt.legend()
plt.xlabel('n')
plt.ylabel('p')
plt.show()

max_val = [0, 0]
for i in range(100):
    if l3[i] > max_val[0]:
        max_val[0] = l3[i]
        max_val[1] = i + 1

print('MLE:', max_val[1])
```

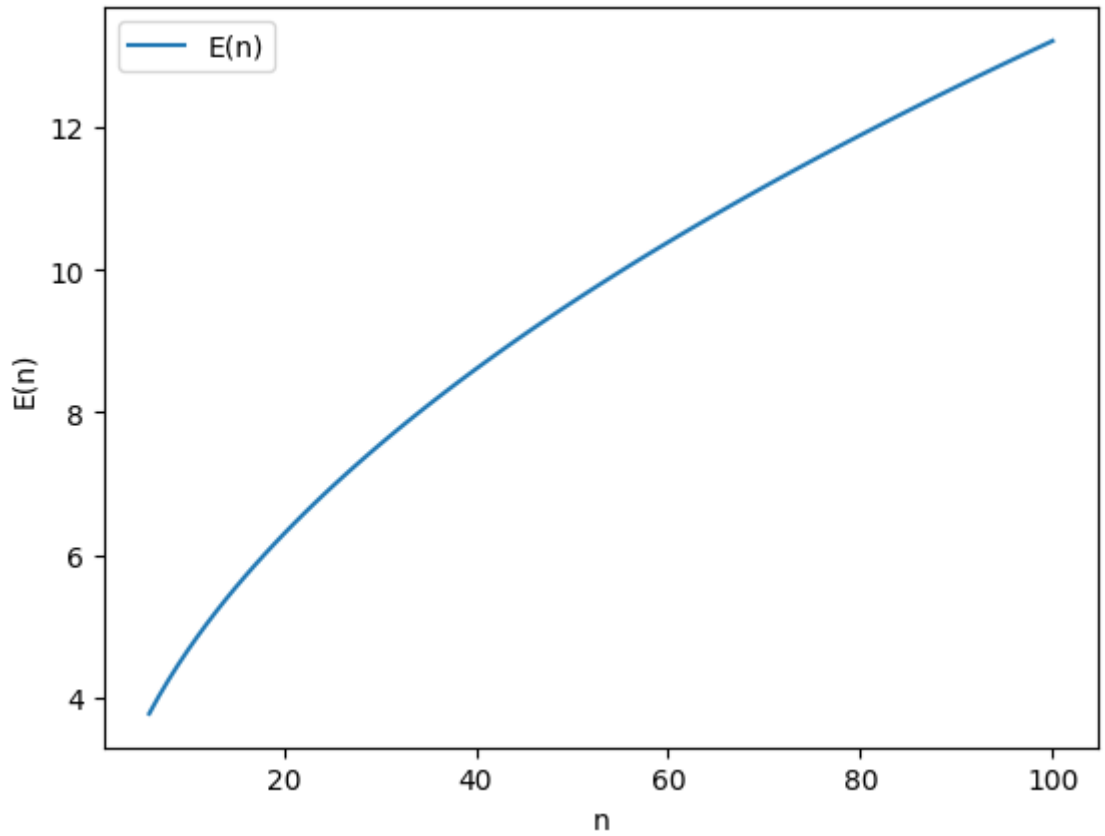



```
MLE: 8
```

б)

In [158…

```
def E_2(n, n_unique, d=10):
    res = 0
    for i in range(d):
        res += L_2(n, n_unique, d)
    return res


l4 = []

for i in range(6, 101):
    l4.append(E(i))

plt.plot(range(6, 101), l4, label='E(n)')
plt.legend()
plt.xlabel('n')
plt.ylabel('E(n)')
plt.show()

mm = np.array([])

for i in range(6, 101):
    mm = np.append(mm, abs(E_2(i, 6, 10) - 6))

print('MM:',  range(6, 101)[np.argmin(mm)])
```




```
MM: 8
```

в)

In [159…
```
vals2 = []

np.random.seed(41)
for i in range(10000):
    bool_arr = [False]*20
    for j in range(10):
        cur = np.random.choice(range(20))
        if j == 9:
            vals2.append(9)
            break
        if bool_arr[cur]:
            vals2.append(j + 1)
            break
        else:
            bool_arr[cur] += 1
```

In [161…
```
mm_l2 = np.vectorize(E_2)

vals2 = np.array(vals2)
mm_l2 = mm_l2(range(1,1000), 10)
mm_l2 = np.absolute(vals2[:, np.newaxis] - mm_l2)
for i in range(len(mm_l2)):
    mm_l2[i] = np.argmin(mm_l2[i]) + 1
print('mm:', mm_l2.mean())
```

```
mm: 157.6758
```

In [162…
```
print('STD', mm_l2.std(), '\nVAR',mm_l2.std()**2, '\nMEAN', mm_l2.mean(),  '\nBI
```

```
STD 141.23597804511422
VAR 19947.601494359988
MEAN 157.6758
BIAS 137.6758
```

Задача 3

In [163…
```
np.random.seed(33)
samples = np.random.exponential(scale=1, size=(10000, 20))
```

Асимптотический нормальный интервал

In [165…
```
l = 0

for sample in samples:
    sample_mean = np.mean(sample)
    sample_std = np.std(sample, ddof=1)

    z_alpha = norm.ppf(0.975)
    ci_classic = (sample_mean - z_alpha * sample_std / np.sqrt(len(sample)),
                  sample_mean + z_alpha * sample_std / np.sqrt(len(sample)))

    l += np.mean((ci_classic[0] <= 1) & (1 <= ci_classic[1]))

l/10000
```

Out[165]: 0.9056

Наивный бутстрап

In [166…
```
l = 0

for sample in samples:
    bootstrapped_means = np.random.choice(sample, size=(10**4, len(sample)), rep
    ci_bootstrap_naive = np.percentile(bootstrapped_means, [2.5, 97.5])
    in_ci_bootstrap_naive = (ci_bootstrap_naive[0] <= 1 <= ci_bootstrap_naive[1]
    prob_in_ci_bootstrap_naive = in_ci_bootstrap_naive.mean()
    l += prob_in_ci_bootstrap_naive

l/10000
```

Out[166]: 0.9053

Бутстрап t-статистики

In [167…
```
l = 0

for sample in samples:
    t = sts.t.ppf(0.975, df=len(sample)-1)
    bootstrapped_t = (np.random.choice(sample, size=(n_bootstraps, len(sample)),
    bootstrapped_t_means = bootstrapped_t.mean(axis=1)
    ci_bootstrap_t = (np.mean(sample) - t * np.std(bootstrapped_t_means, ddof=1)
    in_ci_bootstrap_t = (ci_bootstrap_t[0] <= 1 <= ci_bootstrap_t[1])
    l += in_ci_bootstrap_t.mean()

l/10000
```

Out[167]: 0.9603

б)

Если распределение Стьюдента:

In [168…
```
np.random.seed(33)
samples = np.random.standard_t(df=3, size=(10000, 20))
```

In [169…
```
l = 0

for sample in samples:
    sample_mean = np.mean(sample)
    sample_std = np.std(sample, ddof=1)

    z_alpha = norm.ppf(0.975)
    ci_classic = (sample_mean - z_alpha * sample_std / np.sqrt(len(sample)),
                  sample_mean + z_alpha * sample_std / np.sqrt(len(sample)))

    l += np.mean((ci_classic[0] <= 0) & (0 <= ci_classic[1]))

l/10000
```

Out[169]: 0.9398

In [170…
```
l = 0

for sample in samples:
    bootstrapped_means = np.random.choice(sample, size=(10**4, len(sample)), rep
    ci_bootstrap_naive = np.percentile(bootstrapped_means, [2.5, 97.5])
    in_ci_bootstrap_naive = (ci_bootstrap_naive[0] <= 0 <= ci_bootstrap_naive[1]
    prob_in_ci_bootstrap_naive = in_ci_bootstrap_naive.mean()
    l += prob_in_ci_bootstrap_naive

l/10000
```

Out[170]: 0.9164

In [171…
```
l = 0

for sample in samples:
    t = sts.t.ppf(0.975, df=len(sample)-1)
    bootstrapped_t = (np.random.choice(sample, size=(n_bootstraps, len(sample)),
    bootstrapped_t_means = bootstrapped_t.mean(axis=1)
    ci_bootstrap_t = (np.mean(sample) - t * np.std(bootstrapped_t_means, ddof=1)
    in_ci_bootstrap_t = (ci_bootstrap_t[0] <= 0 <= ci_bootstrap_t[1])
    l += in_ci_bootstrap_t.mean()

l/10000
```

Out[171]: 0.7864

в)

В случае экспоненциального распределения бутстрап т-статистики показал лучший результат В случае распределения Стьюдента лучшим оказался классический

интервал

Задача 4

а)

In [172…
```
df = pd.read_excel('22-23_hse_probability.xlsx', sheet_name='Exam')
```

In [173…
```
df.drop (index=range(5), axis= 0 , inplace= True )
```

In [174…
```
new_df = df[["Last name", "Unnamed: 72"]]
new_df.head(10)
```

Out[174]:

	Last name	Unnamed: 72
5	Репенкова	16.0
6	Ролдугина	0.0
7	Сафина	19.0
8	Сидоров	26.0
9	Солоухин	21.0
10	Старощук	22.0
11	Стогова	20.0
12	Торова	17.0
13	Трофимова	20.0
14	Федгинкель	21.0

In [175…
```
v_list = ['У', 'Е','Ы','А','О','Э','Я','И','Ю','Ё']
v, c = [], []

for el in new_df.index:
    if new_df.loc[el]['Last name'][0] in v_list:
        v.append([new_df.loc[el]['Last name'], new_df.loc[el]['Unnamed: 72']])
    else:
        c.append([new_df.loc[el]['Last name'], new_df.loc[el]['Unnamed: 72']])

v1 = [i[-1] for i in v]
c1 = [i[-1] for i in c]
```

$$H0 : \mu0 = \mu1$$
$$H1 : \mu0! = \mu1$$

In [176…
```
v1 = np.array(v1)
c1 = np.array(c1)
ttest_ind(v1, c1)
```

Out[176]:
```
Ttest_indResult(statistic=-0.8791005932448916, pvalue=0.3799864037939753)
```

p-value > 0.05, следовательно H0 не отвергается

б)

In [ ]:
```
```

In [177…
```
sample_v = np.random.choice(v1, size=(10000, len(v1)))
sample_c = np.random.choice(c1, size=(10000, len(c1)))
mean_v = sample_v.mean(axis=1)
mean_c = sample_c.mean(axis=1)
dif = mean_v - mean_c

np.percentile(dif, q=2.5), np.percentile(dif, q=97.5), '0 in interval -> H0 не с
```

Out[177]:
```
(-3.5750432681906705, 1.325183889810339, '0 in interval -> H0 не отвергаем')
```

в)

In [178…
```
t1 = (v1.mean() - c1.mean())/(np.sqrt(v1.var(ddof=1)/len(v1) + c1.var(ddof=1)/le
t1
```

Out[178]:
```
-0.8519661870595602
```

In [179…
```
v1_boot = np.random.choice(v1, (10000, len(v1)))
c1_boot = np.random.choice(c1, (10000, len(c1)))
t2 = ((v1_boot.mean(axis=1)  - c1_boot.mean(axis=1))-(v1.mean()-c1.mean()))/(np.
np.percentile(t2, q=2.5), np.percentile(t2, q=97.5), 't1 in interval -> H0 не от
```

Out[179]:
```
(-1.9804221705593716, 2.0699855024487013, 't1 in interval -> H0 не отвергаем')
```

г)

In [180…
```
C = np.concatenate([v1, c1])
difs = []
dif_th = v1.mean() - c1.mean()
print('dif_th:', dif_th)
for i in range(10000):
    c = np.random.permutation(C)
    difs.append(c[:50].mean() - c[50:].mean())

np.percentile(difs, q=2.5), np.percentile(difs, q=97.5), 'dif_th in interval ->
```

```
dif_th: -1.0782433114588574
```

Out[180]:
```
(-2.383829787234042,
 2.3253900709219852,
 'dif_th in interval -> H0 не отвергаем')
```

Задача 6

In [181…
```
new_df["Last name"] = new_df["Last name"].apply(lambda x: len(str(x)))
new_df.head()
```

```
C:\Users\mpara\AppData\Local\Temp\ipykernel_14928\1235541589.py:1: SettingWithC
opyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/sta
ble/user_guide/indexing.html#returning-a-view-versus-a-copy
  new_df["Last name"] = new_df["Last name"].apply(lambda x: len(str(x)))
```

Out[181]:

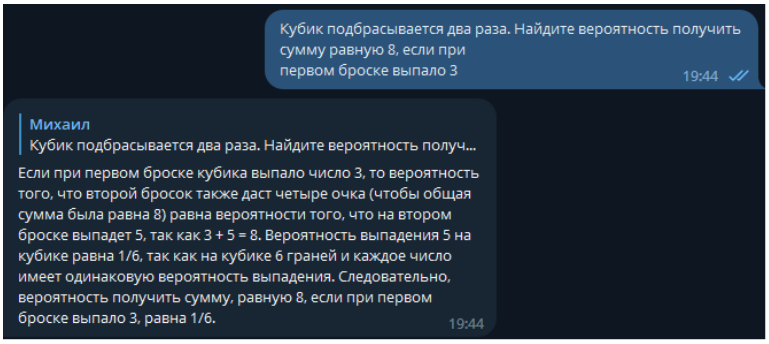
	Last name	Unnamed: 72
5	9	16.0
6	9	0.0
7	6	19.0
8	7	26.0
9	8	21.0

In [182…

```
b = new_df['Unnamed: 72'].mean() / new_df['Last name'].mean()
print('beta:', b)
print('corr coef:', np.corrcoef(new_df['Unnamed: 72'], new_df['Last name'])[1, 0
```

```
beta: 2.0613026819923372
corr coef: 0.025328052669147682
```

Задача 7



Задач 8

Канал "Лекторий ФПМИ" Именно по теории вероятностей там разбирается база, а потом идут немного не те вещи, которые мы сейчас изучаем. Из теории вероятностей я, соответственно, смотрел не так много материала. Но, например, по мат анализу лекционный материал там очень классный.

In [ ]: