Combinatorics, 2018 Fall, USTC
Homework 5

- **The due is on Thursday, Oct. 25.**
- **Please sign your name and student number.**
- **Please solve as many problems as you can.**

1. Let (G, X) be a transitive group action, that is, for any $x, y \in X$, there exists an element $g \in G$ s.t. $g * x = y$. Fix $x \in X$, let $r(x)$ be the number of different orbits of X under $Stab(x)$. Show that
$$r(x)|G| = \sum_{g \in G} |Fix(g)|^2.$$

2. Compute the cycle index of S_n.

3. Let S be a collection of regular cubes with a diagonal line in each face. Compute $|S|$.

4. Let X be a finite set and $|X| > 1$. Suppose that (G, X) is a transitive group action. Show that
$$\cup_{x \in X} Stab(x) \neq G.$$

5. Let $|A| = n$, $|C| = m$, and G be a permutation group of A. Let $\mathscr{F} = \{f \in C^A : f \text{ is an injection}\}$. Show that
   (a) Let Ω be an orbit of (G, C^A). Then either $\Omega \subset \mathscr{F}$ or $\Omega \cap \mathscr{F} = \phi$.

(b) The number of orbits of $\mathscr{F}$ under G is

$$\frac{1}{|G|}m(m-1)\cdots(m-n+1).$$