Combinatorics, 2018 Fall, USTC
Homework12

- **The due is on Thursday, Dec. 20 .**
- **Please sign your name and student number.**
- **Please solve as many problems as you can.**

1. Let $A_1, \cdots, A_n$ be a finite nonempty subsets of a field F with $|A_i| = k_i, i \in [n]$. Let $f(x_1, \cdots, x_n) \in F[x_1, \cdots, x_n] \setminus \{0\}$ and $\deg f \leq \sum_{i=1}^{n}(k_i - 1) = d$. Let $g(x_1, \cdots, x_n) = f(x_1, \cdots, x_n) \cdot (x_1 + x_2 + \cdots + x_n)^{d-degf}$. If $[x_1^{k_1-1} x_2^{k_2-1} \cdots x_n^{k_n-1}]g \neq 0$, then show that $|\{a_1 + \cdots + a_n : a_i \in A_i, f(a_1, \cdots, a_n) \neq 0\}| \geq d - degf + 1$.

   (**Hint:** Consider a polynomial $h(x_1, \cdots, x_n) = f \cdot (x_1 + x_2 + \cdots + x_n)^{d-|c|-degf} \prod_{c \in C}(x_1 + x_2 + \cdots + x_n - c)$, where $C = \{a_1 + \cdots + a_n : a_i \in A_i, f(a_1, \cdots, a_n) \neq 0\}$)

2. Let p be a prime, suppose $k < p$, let $(a_1, \cdots, a_k)$ be a sequence of not necessarily distinct members of the finite field $\mathbb{F}_p$. And let B be a subset of cardinality k of $\mathbb{F}_p$. Show that there is a numbering $\{b_1, \cdots, b_k\}$ of elements of B such that the sums $a_i + b_i$ are pairwise distinct in $\mathbb{F}_p$.

   (**Hint:** Consider a polynomial $f(x_1, \cdots, x_k) = \prod_{1 \leq i < j \leq k}(x_i - x_j)(a_i + x_i - a_j - x_j) \in \mathbb{F}_p[x_1, \cdots, x_k]$)