Combinatorics, 2018 Fall, USTC
Homework8

- **The due is on Thursday, Nov. 22 .**
- **Please sign your name and student number.**
- **Please solve as many problems as you can.**

1. Suppose five points are chosen inside an equilateral triangle with side length 1. Show that there is at least one pair of points whose distance apart is at most $\frac{1}{2}$.
2. Show that the generalized Ramsey number $R_k(s_1, \cdots, s_k) < \infty$, where $k \geq 2,\ s_i \geq 2,\ i \in [k]$.
3. Show that the condition in the Erdös-Szekers Theorem is best possible, that is, there exists a sequence of st different real numbers which has neither increasing subsequence of length $s+1$ nor decreasing subsequence of length $t+1$.
4. Show that $R(s+1, t+1) \geq st+1$.
5. Prove that $2^k < R_k(3, 3, \cdots, 3) \leq (k+1)!$.
6. For any point $p \in R^d$ in $d-$dimension,write $p = (p_1, p_2, \cdots, p_d)$. A set P of points in R^d is called good, if for each $i \in [d]$, the $i-th$ coordinates of these points are distinct. A sequence of points in R^d is called monotone if it is monotone in each of its coordinate. Show that in any good set P of $l^{2^d}+1$ points in R^d, there is a monotone subsequence of length $l+1$.

7. Use Ramsey's Theorem to prove that for every integer $k \geq 2$, there is an integer n such that every sequence of n distinct real numbers contains a monotone subsequence of k real numbers.