Combinatorics, 2018 Fall, USTC
Homework10

- **The due is on Thursday, Dec. 06 .**
- **Please sign your name and student number.**
- **Please solve as many problems as you can.**

1. Construct a $PG(3)$ and list all the points and lines.
2. Show that the complement of a (v, k, λ)-difference set is also a difference set, and determine its parameters.
3. Show that any $(q^2, q, 1)$ design (i.e. $AG(q)$) is resolvable in the following steps:
   (a) Let a parallel class be a set of mutually disjoint lines. Show that each parallel class contains q lines.
   (b) There are $q + 1$ such classes.
   (c) Any two lines from different classes meet in a point.
   (d) Lines of each parallel class cover the whole point set.
4. A set system is a pair $(X, \mathcal{D})$, where X is a finite set of points and $\mathcal{D} \subseteq 2^X$. If $(X, \mathcal{D})$ is a symmetric (v, k, λ) design with incidence matrix M, then show that the set system with incidence matrix M^{T} is also a symmetric (v, k, λ) design. (This is why we call a (v, k, λ) design with $b = v$ a symmetric design)

   [**Hint:** show $M^T M = (k - \lambda)I + \lambda J$ and $M^T J = kJ$, where I is identity matrix, $J = (a_{ij})_{v \times v}$ with $a_{ij} \equiv 1$]
5. Let S be a set of points in a $PG(q)$. Suppose no three points of S lie on a line. Prove that $|S| \leq q + 1$ if q is odd, and $|S| \leq q + 2$ if q is even. (such a set S of points is called an arc in $PG(q)$)

   [**Hint:** use the fact that each point lies on exactly $q + 1$ lines]