Combinatorics, 2018 Fall, USTC
Homework2

- **The due is on Thursday, Sep. 27 .**
- **Please sign your name and student number.**
- **Please solve as many problems as you can.**

1. Prove $e(\frac{n}{e})^n \leq n! \leq en(\frac{n}{e})^n$ by induction.

2. There are n married couples attending a dance. How many ways are there to form n pairs for dancing if no wife should dance with her husband?

3. Prove the derangement number satisfies

$$D_n = (n-1)(D_{n-2} + D_{n-1})$$

   Then prove $D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$ by induction.

4. Consider a staircase with n stairs. Let A_n be the number of ways to ascend the staircase if we can climb by 1 stair or 2 stairs in each step. Find the generating function of $\{A_n\}_{n\geq 0}$ and compute A_n

5. A and B pick logs from n logs alternatively. Each time they can pick one or two logs. If we let A pick first, how many ways of picking such that A is the last one to pick?

6. Compute the generalized binomial coefficient $\binom{\frac{1}{2}}{n}$