Combinatorics, 2018 Fall, USTC
Homework6

- **The due is on Thursday, Nov. 1 .**
- **Please sign your name and student number.**
- **Please solve as many problems as you can.**

1. Let G be a group containing all rotations of a regular octahedron. （正八面体）Compute the cycle indices of G restricted on the vertices, and on the edges, and on the faces.

2. Show that
$$P_{S_n}(\sum_{i=1}^{m} y_i, \sum_{i=1}^{m} y_i^2, \cdots, \sum_{i=1}^{m} y_i^n) = \sum_{k_1+\cdots+k_m=n} y_1^{k_1} \cdots y_m^{k_m}.$$

3. Suppose we have a n-dimensional cube. Take all the vertices as the vertex set, and all the edges as the edge set, then we get a undirected graph K. Show that $|Aut(K)| = 2^n \cdot n!$

4. Suppose we have a group action $(G, X), |X| = n$, with color set $C = [m]$. Denote b_t the number of orbits in C^X satisfying that $\sum\limits_{x \in X} f(x) = t$. Show that
$$\sum_{k=0}^{\infty} b_t x^t = P_G(\sum_{i=1}^{m} x^i, \sum_{i=1}^{m} x^{2i}, \cdots, \sum_{i=1}^{m} x^{ni}).$$

5. Show that in a group of m girls and n boys, there exist some t girls for whom husbands can be found iff any subset of the girls(say k of them) between them know at least $k+t-m$ of the boys.

6. Suppose $x_1 \in S_1, \cdots, x_m \in S_m$, where $x_1, \cdots, x_m$ are not necessarily distinct. Then we say $x_1, \cdots, x_m$ is a system of representatives (SR) of $S_1, \cdots, S_m$. Show that $S_1, \cdots, S_m$ has an SR such that each element occurs at most r times iff for any $I \subset [m]$, $r| \cup_{i \in I} S_i| \geq |I|$.