

Schwarzschild in Cylindrical Coordinates:

$$\boxed{x^\mu}$$

$$\begin{aligned} x^0 &= t, \\ x^1 &= \rho, \\ x^2 &= \phi, \\ x^3 &= z. \end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned} g_{00} &= -\frac{2m - \sqrt{z^2 + \rho^2}}{\sqrt{z^2 + \rho^2}}, \\ g_{01} &= 0, \\ g_{02} &= 0, \\ g_{03} &= 0, \\ g_{10} &= 0, \\ g_{11} &= \frac{\rho^2 \sqrt{z^2 + \rho^2} - 2mz^2 + \sqrt{z^2 + \rho^2} z^2}{(2m - \sqrt{z^2 + \rho^2})(z^2 + \rho^2)}, \\ g_{12} &= 0, \\ g_{13} &= 2\frac{\rho m z}{(2m - \sqrt{z^2 + \rho^2})(z^2 + \rho^2)}, \\ g_{20} &= 0, \\ g_{21} &= 0, \\ g_{22} &= -\rho^2, \\ g_{23} &= 0, \\ g_{30} &= 0, \\ g_{31} &= 2\frac{\rho m z}{(2m - \sqrt{z^2 + \rho^2})(z^2 + \rho^2)}, \\ g_{32} &= 0, \\ g_{33} &= \frac{\rho^2 \sqrt{z^2 + \rho^2} + \sqrt{z^2 + \rho^2} z^2 - 2\rho^2 m}{(2m - \sqrt{z^2 + \rho^2})(z^2 + \rho^2)}. \end{aligned}$$

$$\sqrt{\hspace{0.5cm}} = \sqrt{-\det(g_{\mu\nu})}$$

$$\sqrt{\hspace{0.5cm}} = \sqrt{-\frac{4\rho^2 m z^4 + 4\rho^6 m - 4\rho^2 m^2 (z^2 + \rho^2)^{\frac{3}{2}} + 8\rho^4 m z^2 - \rho^4 (z^2 + \rho^2)^{\frac{3}{2}} - \rho^2 (z^2 + \rho^2)^{\frac{3}{2}} z^2}{(2m - \sqrt{z^2 + \rho^2})^2 (z^2 + \rho^2)^{\frac{3}{2}}}}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned} g^{00} &= -\frac{(z^2 + \rho^2)^{\frac{3}{2}}}{(\rho^4 + 2\rho^2 z^2 + z^4)(2m - \sqrt{z^2 + \rho^2})}, \\ g^{01} &= 0, \\ g^{02} &= 0, \\ g^{03} &= 0, \\ g^{10} &= 0, \\ g^{11} &= -\frac{(\rho^2 \sqrt{z^2 + \rho^2} + \sqrt{z^2 + \rho^2} z^2 - 2\rho^2 m)(z^2 + \rho^2)}{\rho^4 \sqrt{z^2 + \rho^2} + \sqrt{z^2 + \rho^2} z^4 + 2\rho^2 \sqrt{z^2 + \rho^2} z^2}, \\ g^{12} &= 0, \\ g^{13} &= 2\frac{\rho m (z^2 + \rho^2) z}{\rho^4 \sqrt{z^2 + \rho^2} + \sqrt{z^2 + \rho^2} z^4 + 2\rho^2 \sqrt{z^2 + \rho^2} z^2}, \\ g^{20} &= 0, \\ g^{21} &= 0, \\ g^{22} &= -\frac{1}{\rho^2}, \\ g^{23} &= 0, \\ g^{30} &= 0, \\ g^{31} &= 2\frac{\rho m (z^2 + \rho^2) z}{\rho^4 \sqrt{z^2 + \rho^2} + \sqrt{z^2 + \rho^2} z^4 + 2\rho^2 \sqrt{z^2 + \rho^2} z^2}, \\ g^{32} &= 0, \\ g^{33} &= -\frac{(\rho^2 \sqrt{z^2 + \rho^2} - 2mz^2 + \sqrt{z^2 + \rho^2} z^2)(z^2 + \rho^2)}{\rho^4 \sqrt{z^2 + \rho^2} + \sqrt{z^2 + \rho^2} z^4 + 2\rho^2 \sqrt{z^2 + \rho^2} z^2}. \end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned} \Gamma^0_{00} &= 0, \\ \Gamma^0_{01} &= \frac{1}{2}\frac{\rho \sqrt{z^2 + \rho^2} z^2 - 2pmz^2 + \rho^3 \sqrt{z^2 + \rho^2} - \rho (z^2 + \rho^2)^{\frac{3}{2}} - 2\rho^3 m}{(\rho^4 + 2\rho^2 z^2 + z^4)(2m - \sqrt{z^2 + \rho^2})}, \\ \Gamma^0_{02} &= 0, \\ \Gamma^0_{03} &= -\frac{1}{2}\frac{2mz^3 - \sqrt{z^2 + \rho^2} z^3 + (z^2 + \rho^2)^{\frac{3}{2}} z + 2\rho^2 m z - \rho^2 \sqrt{z^2 + \rho^2} z}{(\rho^4 + 2\rho^2 z^2 + z^4)(2m - \sqrt{z^2 + \rho^2})}, \\ \Gamma^0_{10} &= \frac{1}{2}\frac{\rho \sqrt{z^2 + \rho^2} z^2 - 2pmz^2 + \rho^3 \sqrt{z^2 + \rho^2} - \rho (z^2 + \rho^2)^{\frac{3}{2}} - 2\rho^3 m}{(\rho^4 + 2\rho^2 z^2 + z^4)(2m - \sqrt{z^2 + \rho^2})}, \\ \Gamma^0_{11} &= 0, \\ \Gamma^0_{12} &= 0, \\ \Gamma^0_{13} &= 0, \\ \Gamma^0_{20} &= 0, \\ \Gamma^0_{21} &= 0, \\ \Gamma^0_{22} &= 0, \\ \Gamma^0_{23} &= 0, \\ \Gamma^0_{30} &= -\frac{1}{2}\frac{2mz^3 - \sqrt{z^2 + \rho^2} z^3 + (z^2 + \rho^2)^{\frac{3}{2}} z + 2\rho^2 m z - \rho^2 \sqrt{z^2 + \rho^2} z}{(\rho^4 + 2\rho^2 z^2 + z^4)(2m - \sqrt{z^2 + \rho^2})}, \\ \Gamma^0_{31} &= 0, \\ \Gamma^0_{32} &= 0, \\ \Gamma^0_{33} &= 0. \end{aligned}$$

