Schwarzschild Metric in spherical coordinates with a variable spherically symmetric matter density:

$\boxed{x^{\mu}}$

$x^0 = t.$
$x^1 = r.$
$x^2 = \theta.$
$x^3 = \phi.$

$\boxed{g_{\mu\nu}}$

$g_{00} = 1 - \frac{8}{3} r^2 \pi \rho(r).$
$g_{01} = 0.$
$g_{02} = 0.$
$g_{03} = 0.$
$g_{10} = 0.$
$g_{11} = 3 \frac{1}{-3 + 8 r^2 \pi \rho(r)}.$
$g_{12} = 0.$
$g_{13} = 0.$
$g_{20} = 0.$
$g_{21} = 0.$
$g_{22} = -r^2.$
$g_{23} = 0.$
$g_{30} = 0.$
$g_{31} = 0.$
$g_{32} = 0.$
$g_{33} = -r^2 \sin(\theta)^2.$

$\boxed{\surd = \sqrt{-\det(g_{\mu\nu})}}$

$\surd = \sqrt{r^4 \sin(\theta)^2}.$

$\boxed{g^{\mu\nu}}$

$g^{00} = -3 \frac{1}{-3 + 8 r^2 \pi \rho(r)}.$
$g^{01} = 0.$
$g^{02} = 0.$
$g^{03} = 0.$
$g^{10} = 0.$
$g^{11} = -1 + \frac{8}{3} r^2 \pi \rho(r).$
$g^{12} = 0.$
$g^{13} = 0.$
$g^{20} = 0.$
$g^{21} = 0.$
$g^{22} = -\frac{1}{r^2}.$
$g^{23} = 0.$
$g^{30} = 0.$
$g^{31} = 0.$
$g^{32} = 0.$
$g^{33} = -\frac{1}{r^2 \sin(\theta)^2}.$

$\boxed{\Gamma^{\sigma}_{\mu\nu}}$

$\Gamma^0_{00} = 0.$
$\Gamma^0_{01} = 4 \frac{2 r \pi \rho(r) + r^2 \rho'(r) \pi}{-3 + 8 r^2 \pi \rho(r)}.$
$\Gamma^0_{02} = 0.$
$\Gamma^0_{03} = 0.$
$\Gamma^0_{10} = 4 \frac{2 r \pi \rho(r) + r^2 \rho'(r) \pi}{-3 + 8 r^2 \pi \rho(r)}.$
$\Gamma^0_{11} = 0.$
$\Gamma^0_{12} = 0.$
$\Gamma^0_{13} = 0.$
$\Gamma^0_{20} = 0.$
$\Gamma^0_{21} = 0.$
$\Gamma^0_{22} = 0.$
$\Gamma^0_{23} = 0.$
$\Gamma^0_{30} = 0.$
$\Gamma^0_{31} = 0.$
$\Gamma^0_{32} = 0.$
$\Gamma^0_{33} = 0.$

$\Gamma^1_{00} = \frac{4}{9} (-3 + 8 r^2 \pi \rho(r)) (2 r \pi \rho(r) + r^2 \rho'(r) \pi).$
$\Gamma^1_{01} = 0.$
$\Gamma^1_{02} = 0.$
$\Gamma^1_{03} = 0.$
$\Gamma^1_{10} = 0.$
$\Gamma^1_{11} = -4 \frac{2 r \pi \rho(r) + r^2 \rho'(r) \pi}{-3 + 8 r^2 \pi \rho(r)}.$
$\Gamma^1_{12} = 0.$
$\Gamma^1_{13} = 0.$
$\Gamma^1_{20} = 0.$
$\Gamma^1_{21} = 0.$
$\Gamma^1_{22} = \frac{1}{3} r (-3 + 8 r^2 \pi \rho(r)).$
$\Gamma^1_{23} = 0.$
$\Gamma^1_{30} = 0.$
$\Gamma^1_{31} = 0.$
$\Gamma^1_{32} = 0.$
$\Gamma^1_{33} = \frac{1}{3} r \sin(\theta)^2 (-3 + 8 r^2 \pi \rho(r)).$

$\Gamma^2_{00} = 0.$
$\Gamma^2_{01} = 0.$
$\Gamma^2_{02} = 0.$
$\Gamma^2_{03} = 0.$
$\Gamma^2_{10} = 0.$
$\Gamma^2_{11} = 0.$
$\Gamma^2_{12} = \frac{1}{r}.$
$\Gamma^2_{13} = 0.$
$\Gamma^2_{20} = 0.$
$\Gamma^2_{21} = \frac{1}{r}.$
$\Gamma^2_{22} = 0.$
$\Gamma^2_{23} = 0.$
$\Gamma^2_{30} = 0.$
$\Gamma^2_{31} = 0.$
$\Gamma^2_{32} = 0.$
$\Gamma^2_{33} = -\sin(\theta)\cos(\theta).$

$\Gamma^3_{00} = 0.$
$\Gamma^3_{01} = 0.$
$\Gamma^3_{02} = 0.$
$\Gamma^3_{03} = 0.$
$\Gamma^3_{10} = 0.$
$\Gamma^3_{11} = 0.$
$\Gamma^3_{12} = 0.$
$\Gamma^3_{13} = \frac{1}{r}.$
$\Gamma^3_{20} = 0.$
$\Gamma^3_{21} = 0.$
$\Gamma^3_{22} = 0.$
$\Gamma^3_{23} = \frac{\cos(\theta)}{\sin(\theta)}.$
$\Gamma^3_{30} = 0.$
$\Gamma^3_{31} = \frac{1}{r}.$
$\Gamma^3_{32} = \frac{\cos(\theta)}{\sin(\theta)}.$
$\Gamma^3_{33} = 0.$

$\boxed{\ddot{x}^\mu = \left(\Gamma^0_{\sigma\rho}\dot{x}^\mu - \Gamma^\mu_{\sigma\rho}\right)\dot{x}^\sigma\dot{x}^\rho}$

$\ddot{x}^0 = 0.$

$$\ddot{x}^1 = -\frac{1}{9}\frac{72r\pi\rho(r) + 36r^2\rho'(r)\pi + 512r^5\pi^3\rho(r)^3 - 108r^2\rho'(r)\pi\dot{x}^2 + 192r^5\dot{z}^2\sin(\theta)^2\pi^2\rho(r)^2 - 192r^4\rho'(r)\pi^2\rho(r) + 27r\dot{z}^2\sin(\theta)^2 - 216r\pi\dot{x}^2\rho(r) - 384r^3\pi^2\rho(r)^2 + 256r^6\rho'(r)\pi^3\rho(r)^2 - 144r^3\dot{z}^2\sin(\theta)^2\pi\rho(r)}{-3 + 8r^2\pi\rho(r)}.$$

$\ddot{x}^2 = \dot{z}^2\sin(\theta)\cos(\theta).$

$\ddot{x}^3 = 2\frac{3\dot{z}\dot{x} + 4r^3\dot{z}\rho'(r)\pi\dot{x}}{r(-3 + 8r^2\pi\rho(r))}.$

$\boxed{R_{\mu\nu}}$

$R_{00} = -\frac{64}{3}r^3\rho'(r)\pi^2\rho(r) + \frac{4}{3}r^2\pi\rho''(r) - \frac{64}{3}r^2\pi^2\rho(r)^2 - \frac{32}{9}r^4\pi^2\rho''(r)\rho(r) + 8\pi\rho(r) + 8r\rho'(r)\pi.$
$R_{01} = 0.$
$R_{02} = 0.$
$R_{03} = 0.$
$R_{10} = 0.$
$R_{11} = 4\frac{r^2\pi\rho''(r) + 6\pi\rho(r) + 6r\rho'(r)\pi}{-3 + 8r^2\pi\rho(r)}.$
$R_{12} = 0.$
$R_{13} = 0.$
$R_{20} = 0.$
$R_{21} = 0.$
$R_{22} = -\frac{8}{3}r^3\rho'(r)\pi - 8r^2\pi\rho(r).$
$R_{23} = 0.$
$R_{30} = 0.$
$R_{31} = 0.$
$R_{32} = 0.$
$R_{33} = -8r^2\sin(\theta)^2\pi\rho(r) - \frac{8}{3}r^3\sin(\theta)^2\rho'(r)\pi.$

$\boxed{R^\mu_{\ \nu}}$

$R^0_{\ 0} = \frac{4}{3}r^2\pi\rho''(r) + 8\pi\rho(r) + 8r\rho'(r)\pi.$
$R^0_{\ 1} = 0.$
$R^0_{\ 2} = 0.$
$R^0_{\ 3} = 0.$
$R^1_{\ 0} = 0.$
$R^1_{\ 1} = \frac{4}{3}r^2\pi\rho''(r) + 8\pi\rho(r) + 8r\rho'(r)\pi.$
$R^1_{\ 2} = 0.$
$R^1_{\ 3} = 0.$
$R^2_{\ 0} = 0.$
$R^2_{\ 1} = 0.$
$R^2_{\ 2} = 8\pi\rho(r) + \frac{8}{3}r\rho'(r)\pi.$
$R^2_{\ 3} = 0.$
$R^3_{\ 0} = 0.$
$R^3_{\ 1} = 0.$
$R^3_{\ 2} = 0.$
$R^3_{\ 3} = 8\pi\rho(r) + \frac{8}{3}r\rho'(r)\pi.$

$\boxed{R}$

$R = \frac{8}{3}r^2\pi\rho''(r) + 32\pi\rho(r) + \frac{64}{3}r\rho'(r)\pi.$

$\boxed{G^{\mu}_{\ \nu}}$

$G^{0}_{\ 0} = -8\pi\rho(r) - \frac{8}{3}r\rho'(r)\pi.$
$G^{0}_{\ 1} = 0.$
$G^{0}_{\ 2} = 0.$
$G^{0}_{\ 3} = 0.$
$G^{1}_{\ 0} = 0.$
$G^{1}_{\ 1} = -8\pi\rho(r) - \frac{8}{3}r\rho'(r)\pi.$
$G^{1}_{\ 2} = 0.$
$G^{1}_{\ 3} = 0.$
$G^{2}_{\ 0} = 0.$
$G^{2}_{\ 1} = 0.$
$G^{2}_{\ 2} = -\frac{4}{3}r^2\pi\rho''(r) - 8\pi\rho(r) - 8r\rho'(r)\pi.$
$G^{2}_{\ 3} = 0.$
$G^{3}_{\ 0} = 0.$
$G^{3}_{\ 1} = 0.$
$G^{3}_{\ 2} = 0.$
$G^{3}_{\ 3} = -\frac{4}{3}r^2\pi\rho''(r) - 8\pi\rho(r) - 8r\rho'(r)\pi.$

$\boxed{G}$

$G = -\frac{8}{3}r^2\pi\rho''(r) - 32\pi\rho(r) - \frac{64}{3}r\rho'(r)\pi.$

$\boxed{G^{\mu}_{\ \nu:\mu} = 0}$

$G^{\mu}_{\ 0:\mu} = 0.$
$G^{\mu}_{\ 1:\mu} = 0.$
$G^{\mu}_{\ 2:\mu} = 0.$
$G^{\mu}_{\ 3:\mu} = 0.$

$\boxed{g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu} = 0?}$

$g^{\mu\nu}\,\Gamma^{0}_{\mu\nu} = 0.$
$g^{\mu\nu}\,\Gamma^{1}_{\mu\nu} = 9\frac{r^3}{\left(-3+8r^2\pi\rho(r)\right)^2} - 24\frac{r^2\rho'(r)\pi}{\left(-3+8r^2\pi\rho(r)\right)^2} + \frac{8192}{9}\frac{r^8\rho'(r)\pi^4\rho(r)^3}{\left(-3+8r^2\pi\rho(r)\right)^2} - 72\frac{r^5\sin(\theta)^4\pi\rho(r)}{\left(-3+8r^2\pi\rho(r)\right)^2} + \frac{16384}{9}\frac{r^7\pi^4\rho(r)^4}{\left(-3+8r^2\pi\rho(r)\right)^2} - 1024\frac{r^5\pi^3\rho(r)^3}{\left(-3+8r^2\pi\rho(r)\right)^2} + 192\frac{r^7\sin(\theta)^4\pi^2\rho(r)^2}{\left(-3+8r^2\pi\rho(r)\right)^2} + 128\frac{r^4\rho'(r)\pi^2\rho(r)}{\left(-3+8r^2\pi\rho(r)\right)^2} - 72\frac{r^5\pi\rho(r)}{\left(-3+8r^2\pi\rho(r)\right)^2} - 512\frac{r^6\rho'(r)\pi^3\rho(r)^2}{\left(-3+8r^2\pi\rho(r)\right)^2} - \frac{512}{3}\frac{r^9\pi^3\rho(r)^3}{\left(-3+8r^2\pi\rho(r)\right)^2} + 256\frac{r^3\pi^2\rho(r)^2}{\left(-3+8r^2\pi\rho(r)\right)^2} + 192\frac{r^7\pi^2\rho(r)^2}{\left(-3+8r^2\pi\rho(r)\right)^2} - \frac{512}{3}\frac{r^9\sin(\theta)^4\pi^3\rho(r)^3}{\left(-3+8r^2\pi\rho(r)\right)^2} + 9\frac{r^3\sin(\theta)^4}{\left(-3+8r^2\pi\rho(r)\right)^2} - \frac{32768}{27}\frac{r^9\pi^5\rho(r)^5}{\left(-3+8r^2\pi\rho(r)\right)^2} - \frac{16384}{27}\frac{r^{10}\rho'(r)\pi^5\rho(r)^4}{\left(-3+8r^2\pi\rho(r)\right)^2} - 48\frac{r\pi\rho(r)}{\left(-3+8r^2\pi\rho(r)\right)^2}.$
$g^{\mu\nu}\,\Gamma^{2}_{\mu\nu} = r^2\sin(\theta)^3\cos(\theta).$
$g^{\mu\nu}\,\Gamma^{3}_{\mu\nu} = 0.$