

Andere Kugelkoordinaten:

$$\boxed{x^\mu}$$

$$x^0 = r.$$

$$x^1 = a.$$

$$x^2 = b.$$

$$\boxed{g_{\mu\nu}}$$

$$g_{00} = \frac{b^2 r^2 \sin(\frac{a}{r})^2 + b^2 a^2 \cos(\frac{a}{r})^2 + a^2 r^2 \sin(\frac{a}{r})^2 + r^4 \sin(\frac{a}{r})^2 - 2 b^2 a r \cos(\frac{a}{r}) \sin(\frac{a}{r})}{r^4 \sin(\frac{a}{r})^2}.$$

$$g_{01} = -\frac{b^2 a \cos(\frac{a}{r})^2 + a r^2 \sin(\frac{a}{r})^2 - b^2 r \cos(\frac{a}{r}) \sin(\frac{a}{r})}{r^3 \sin(\frac{a}{r})^2}.$$

$$g_{02} = \frac{b(a \cos(\frac{a}{r}) - r \sin(\frac{a}{r}))}{r^2 \sin(\frac{a}{r})}.$$

$$g_{10} = -\frac{b^2 a \cos(\frac{a}{r})^2 + a r^2 \sin(\frac{a}{r})^2 - b^2 r \cos(\frac{a}{r}) \sin(\frac{a}{r})}{r^3 \sin(\frac{a}{r})^2}.$$

$$g_{11} = \frac{r^2 \sin(\frac{a}{r})^2 + b^2 \cos(\frac{a}{r})^2}{r^2 \sin(\frac{a}{r})^2}.$$

$$g_{12} = -\frac{b \cos(\frac{a}{r})}{r \sin(\frac{a}{r})}.$$

$$g_{20} = \frac{b(a \cos(\frac{a}{r}) - r \sin(\frac{a}{r}))}{r^2 \sin(\frac{a}{r})}.$$

$$g_{21} = -\frac{b \cos(\frac{a}{r})}{r \sin(\frac{a}{r})}$$

$$g_{22} = 1.$$

$$\boxed{\sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{-\det(g_{\mu\nu})} = i.$$

$$\boxed{g^{\mu\nu}}$$

$$g^{00} = 1.$$

$$g^{01} = \frac{a}{r}.$$

$$g^{02} = \frac{b}{r}.$$

$$g^{10} = \frac{a}{r}.$$

$$g^{11} = \frac{a^2 + r^2}{r^2}.$$

$$g^{12} = \frac{b r \cos(\frac{a}{r}) + b a \sin(\frac{a}{r})}{r^2 \sin(\frac{a}{r})}.$$

$$g^{20} = \frac{b}{r}.$$

$$g^{21} = \frac{b r \cos(\frac{a}{r}) + b a \sin(\frac{a}{r})}{r^2 \sin(\frac{a}{r})}.$$

$$g^{22} = \frac{r^2 \sin(\frac{a}{r})^2 + b^2 \sin(\frac{a}{r})^2 + b^2 \cos(\frac{a}{r})^2}{r^2 \sin(\frac{a}{r})^2}.$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\Gamma^0_{00} = -\frac{b^2 r^2 \sin(\frac{a}{r})^2 + b^2 a^2 \cos(\frac{a}{r})^2 + a^2 r^2 \sin(\frac{a}{r})^2 - 2 b^2 a r \cos(\frac{a}{r}) \sin(\frac{a}{r})}{r^5 \sin(\frac{a}{r})^2}.$$

$$\Gamma^0_{01} = \frac{b^2 a \cos(\frac{a}{r})^2 + a r^2 \sin(\frac{a}{r})^2 - b^2 r \cos(\frac{a}{r}) \sin(\frac{a}{r})}{r^4 \sin(\frac{a}{r})^2}.$$

$$\Gamma^0_{02} = \frac{b r \sin(\frac{a}{r}) - b a \cos(\frac{a}{r})}{r^3 \sin(\frac{a}{r})}.$$

$$\Gamma^0_{10} = \frac{b^2 a \cos(\frac{a}{r})^2 + a r^2 \sin(\frac{a}{r})^2 - b^2 r \cos(\frac{a}{r}) \sin(\frac{a}{r})}{r^4 \sin(\frac{a}{r})^2}.$$

$$\Gamma^0_{11} = -\frac{r^2 \sin(\frac{a}{r})^2 + b^2 \cos(\frac{a}{r})^2}{r^3 \sin(\frac{a}{r})^2}.$$

$$\Gamma^0_{12} = \frac{b \cos(\frac{a}{r})}{r^2 \sin(\frac{a}{r})}.$$

$$\Gamma^0_{20} = \frac{b r \sin(\frac{a}{r}) - b a \cos(\frac{a}{r})}{r^3 \sin(\frac{a}{r})}.$$

$$\Gamma^0_{21} = \frac{b \cos(\frac{a}{r})}{r^2 \sin(\frac{a}{r})}.$$

$$\Gamma^0_{22} = -\frac{1}{r}.$$

$$\Gamma^1_{00} = -\frac{b^2 a^3 \cos(\frac{a}{r})^2 \sin(\frac{a}{r}) + a^3 r^2 \sin(\frac{a}{r})^3 - 2 b^2 a^2 r \cos(\frac{a}{r}) \sin(\frac{a}{r})^2 + b^2 a^2 r \cos(\frac{a}{r})^3 + b^2 a r^2 \sin(\frac{a}{r})^3 + b^2 r^3 \cos(\frac{a}{r}) \sin(\frac{a}{r})^2 - 2 b^2 a r^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r})}{r^6 \sin(\frac{a}{r})^3}.$$

$$\Gamma^1_{01} = \frac{b^2 a r \cos(\frac{a}{r})^3 + a^2 r^2 \sin(\frac{a}{r})^3 + b^2 a^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r}) - b^2 a r \cos(\frac{a}{r}) \sin(\frac{a}{r})^2 - b^2 r^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r})}{r^5 \sin(\frac{a}{r})^3}.$$

$$\Gamma^1_{02} = \frac{b r^2 \cos(\frac{a}{r}) \sin(\frac{a}{r}) - b a r \cos(\frac{a}{r})^2 - b a^2 \cos(\frac{a}{r}) \sin(\frac{a}{r}) + b a r \sin(\frac{a}{r})^2}{r^4 \sin(\frac{a}{r})^3}.$$

$$\Gamma^1_{10} = \frac{b^2 a r \cos(\frac{a}{r})^3 + a^2 r^2 \sin(\frac{a}{r})^3 + b^2 a^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r}) - b^2 a r \cos(\frac{a}{r}) \sin(\frac{a}{r})^2 - b^2 r^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r})}{r^5 \sin(\frac{a}{r})^3}.$$

$$\Gamma^1_{11} = -\frac{a r^2 \sin(\frac{a}{r})^3 + b^2 a \cos(\frac{a}{r})^2 \sin(\frac{a}{r}) + b^2 r \cos(\frac{a}{r})^3}{r^4 \sin(\frac{a}{r})^3}.$$

$$\Gamma^1_{12} = \frac{b a \cos(\frac{a}{r}) \sin(\frac{a}{r}) + b r \cos(\frac{a}{r})^2}{r^3 \sin(\frac{a}{r})^2}.$$

$$\Gamma^1_{20} = \frac{b r^2 \cos(\frac{a}{r}) \sin(\frac{a}{r}) - b a r \cos(\frac{a}{r})^2 - b a^2 \cos(\frac{a}{r}) \sin(\frac{a}{r}) + b a r \sin(\frac{a}{r})^2}{r^4 \sin(\frac{a}{r})^2}.$$

$$\Gamma^1_{21} = \frac{b a \cos(\frac{a}{r}) \sin(\frac{a}{r}) + b r \cos(\frac{a}{r})^2}{r^3 \sin(\frac{a}{r})^2}.$$

$$\Gamma^1_{22} = -\frac{a \sin(\frac{a}{r})}{r^2 \sin(\frac{a}{r})} + r \cos(\frac{a}{r}).$$

$$\Gamma^2_{00} = \frac{2 b^3 a r \cos(\frac{a}{r}) \sin(\frac{a}{r})^3 - b^3 r^2 \sin(\frac{a}{r})^4 + 2 b^3 a r \cos(\frac{a}{r})^3 \sin(\frac{a}{r}) - b^3 a^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r})^2 - b^3 r^2 \cos(\frac{a}{r})^2 \sin(\frac{a}{r})^2 - b^3 a^2 \cos(\frac{a}{r})^4}{r^6 \sin(\frac{a}{r})^4}.$$

$$\Gamma^2_{01} = \frac{b^3 a \cos(\frac{a}{r})^4 - b^3 r \cos(\frac{a}{r})^3 \sin(\frac{a}{r}) + b^3 a \cos(\frac{a}{r})^2 \sin(\frac{a}{r})^2 - b^3 r \cos(\frac{a}{r}) \sin(\frac{a}{r})^3}{r^5 \sin(\frac{a}{r})^4}.$$

$$\Gamma^2_{02} = \frac{b^2 r \cos(\frac{a}{r})^2 \sin(\frac{a}{r}) - b^2 a \cos(\frac{a}{r})^3 + b^2 r \sin(\frac{a}{r})^3 - b^2 a \cos(\frac{a}{r}) \sin(\frac{a}{r})^2}{r^4 \sin(\frac{a}{r})^3}.$$

$$\Gamma^2_{10} = \frac{b^3 a \cos(\frac{a}{r})^4 - b^3 r \cos(\frac{a}{r})^3 \sin(\frac{a}{r}) + b^3 a \cos(\frac{a}{r})^2 \sin(\frac{a}{r})^2 - b^3 r \cos(\frac{a}{r}) \sin(\frac{a}{r})^3}{r^5 \sin(\frac{a}{r})^4}.$$

$$\Gamma^2_{11} = -\frac{b^3 \cos(\frac{a}{r})^4 + b^3 \cos(\frac{a}{r})^2 \sin(\frac{a}{r})^2}{r^4 \sin(\frac{a}{r})^4}.$$

$$\Gamma^2_{12} = \frac{b^2 \cos(\frac{a}{r}) \sin(\frac{a}{r})^2 + b^2 \cos(\frac{a}{r})^3}{r^3 \sin(\frac{a}{r})^3}.$$

$$\Gamma^2_{20} = \frac{b^2 r \cos(\frac{a}{r})^2 \sin(\frac{a}{r}) - b^2 a \cos(\frac{a}{r})^3 + b^2 r \sin(\frac{a}{r})^3 - b^2 a \cos(\frac{a}{r}) \sin(\frac{a}{r})^2}{r^4 \sin(\frac{a}{r})^3}.$$

$$\Gamma^2_{21} = \frac{b^2 \cos(\frac{a}{r}) \sin(\frac{a}{r})^2 + b^2 \cos(\frac{a}{r})^3}{r^3 \sin(\frac{a}{r})^3}.$$

$$\Gamma^2_{22} = -\frac{b \cos(\frac{a}{r})^2 + b \sin(\frac{a}{r})^2}{r^2 \sin(\frac{a}{r})^2}.$$

$$R_{\mu\nu}$$

$$\begin{aligned}R_{00} &= 0, \\ R_{01} &= 0, \\ R_{02} &= 0, \\ R_{10} &= 0, \\ R_{11} &= 0, \\ R_{12} &= 0, \\ R_{20} &= 0, \\ R_{21} &= 0, \\ R_{22} &= 0.\end{aligned}$$

$$R^{\mu}{}_{\nu}$$

$$\begin{aligned}R^0_0 &= 0, \\ R^0_1 &= 0, \\ R^0_2 &= 0, \\ R^1_0 &= 0, \\ R^1_1 &= 0, \\ R^1_2 &= 0, \\ R^2_0 &= 0, \\ R^2_1 &= 0, \\ R^2_2 &= 0.\end{aligned}$$

$$\overline{R}$$

$$R=0.$$

$$G^{\mu}{}_{\nu}$$

$$\begin{aligned}G^0_0 &= 0, \\ G^0_1 &= 0, \\ G^0_2 &= 0, \\ G^1_0 &= 0, \\ G^1_1 &= 0, \\ G^1_2 &= 0, \\ G^2_0 &= 0, \\ G^2_1 &= 0, \\ G^2_2 &= 0.\end{aligned}$$

$$\overline{G}$$

$$G=0.$$

$$G^{\mu}{}_{\nu;\mu}=0$$

$$\begin{aligned}G^{\mu}{}_{0;\mu} &= 0, \\ G^{\mu}{}_{1;\mu} &= 0, \\ G^{\mu}{}_{2;\mu} &= 0.\end{aligned}$$

$$g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu}=0?$$

$$\begin{aligned}g^{\mu\nu}\,\Gamma^0_{\mu\nu} &= -2\frac{b^4a^2\cos(\frac{a}{r})^4}{r^7\sin(\frac{a}{r})^4}-3\frac{b^2}{r^3}+4\frac{b^4a\cos(\frac{a}{r})}{r^6\sin(\frac{a}{r})}-2\frac{b^2a^2}{r^5}-4\frac{b^2\cos(\frac{a}{r})^2}{r^3\sin(\frac{a}{r})^2}-\frac{b^4a^4\cos(\frac{a}{r})^4}{r^9\sin(\frac{a}{r})^4}-3\frac{a^2}{r^3}-7\frac{b^2a^2\cos(\frac{a}{r})^2}{r^5\sin(\frac{a}{r})^2}-2\frac{b^2a^4\cos(\frac{a}{r})^2}{r^7\sin(\frac{a}{r})^2}+4\frac{b^4a\cos(\frac{a}{r})^3}{r^6\sin(\frac{a}{r})^3}-2\frac{b^4\cos(\frac{a}{r})^2}{r^5\sin(\frac{a}{r})^2}-2\frac{1}{r}-\frac{b^4\cos(\frac{a}{r})^4}{r^5\sin(\frac{a}{r})^4}+10\frac{b^2a\cos(\frac{a}{r})}{r^4\sin(\frac{a}{r})}-\frac{a^4}{r^5}-\frac{b^4}{r^5}+4\frac{b^4a^3\cos(\frac{a}{r})^3}{r^8\sin(\frac{a}{r})^3}+4\frac{b^2a^3\cos(\frac{a}{r})}{r^6\sin(\frac{a}{r})}-6\frac{b^4a^2\cos(\frac{a}{r})^2}{r^7\sin(\frac{a}{r})^2}.\\ g^{\mu\nu}\,\Gamma^{1}_{\mu\nu} &= -2\frac{b^4\cos(\frac{a}{r})^3}{r^5\sin(\frac{a}{r})^3}-\frac{b^2a^4\cos(\frac{a}{r})^3}{r^7\sin(\frac{a}{r})^3}+2\frac{b^4a\cos(\frac{a}{r})^2}{r^6\sin(\frac{a}{r})^2}-2\frac{b^2a^3}{r^6}-5\frac{b^2a^2\cos(\frac{a}{r})^3}{r^5\sin(\frac{a}{r})^3}-3\frac{b^2\cos(\frac{a}{r})^3}{r^3\sin(\frac{a}{r})^3}-2\frac{b^4a^2\cos(\frac{a}{r})^5}{r^7\sin(\frac{a}{r})^5}+9\frac{b^2a^2\cos(\frac{a}{r})}{r^5\sin(\frac{a}{r})}+2\frac{b^4a^3\cos(\frac{a}{r})^4}{r^8\sin(\frac{a}{r})^4}-3\frac{b^2\cos(\frac{a}{r})}{r^5\sin(\frac{a}{r})}-\frac{b^4\cos(\frac{a}{r})}{r^9\sin(\frac{a}{r})^5}-3\frac{a^3}{r^4}+4\frac{b^2a^4\cos(\frac{a}{r})}{r^7\sin(\frac{a}{r})}+4\frac{b^2a\cos(\frac{a}{r})^2}{r^4\sin(\frac{a}{r})^2}-\frac{a^5}{r^6}-2\frac{b^4a^2\cos(\frac{a}{r})^3}{r^7\sin(\frac{a}{r})^3}-2\frac{a}{r^2}-\frac{b^4\cos(\frac{a}{r})^5}{r^5\sin(\frac{a}{r})^5}-6\frac{b^4a^3\cos(\frac{a}{r})^2}{r^8\sin(\frac{a}{r})^2}-3\frac{b^2a}{r^4}-2\frac{b^2a^5\cos(\frac{a}{r})^2}{r^8\sin(\frac{a}{r})^2}-\frac{\cos(\frac{a}{r})}{r\sin(\frac{a}{r})}+4\frac{b^4a^4\cos(\frac{a}{r})^3}{r^9\sin(\frac{a}{r})^3}-\frac{b^4a}{r^6}+3\frac{b^4a\cos(\frac{a}{r})^4}{r^6\sin(\frac{a}{r})^4}+4\frac{b^4a^2\cos(\frac{a}{r})}{r^7\sin(\frac{a}{r})}-5\frac{b^2a^3\cos(\frac{a}{r})^2}{r^6\sin(\frac{a}{r})^2}-\frac{b^4a^5\cos(\frac{a}{r})^4}{r^{10}\sin(\frac{a}{r})^4}.\\ g^{\mu\nu}\,\Gamma^2_{\mu\nu} &= 8\frac{b^3a\cos(\frac{a}{r})}{r^5\sin(\frac{a}{r})^3}+8\frac{b^5a\cos(\frac{a}{r})^3}{r^7\sin(\frac{a}{r})^3}-\frac{b^5a^4\cos(\frac{a}{r})^6}{r^{10}\sin(\frac{a}{r})^6}-3\frac{b^3\cos(\frac{a}{r})^4}{r^4\sin(\frac{a}{r})^4}-3\frac{b^2}{r^4}+4\frac{b^5a^3\cos(\frac{a}{r})^3}{r^9\sin(\frac{a}{r})^3}+2\frac{b^3a^3\cos(\frac{a}{r})^3}{r^7\sin(\frac{a}{r})^3}-3\frac{b^5\cos(\frac{a}{r})^2}{r^6\sin(\frac{a}{r})^2}-8\frac{b^5a^2\cos(\frac{a}{r})^4}{r^8\sin(\frac{a}{r})^4}+2\frac{b^3a^3\cos(\frac{a}{r})}{r^7\sin(\frac{a}{r})}-6\frac{b^3a^2\cos(\frac{a}{r})^2}{r^6\sin(\frac{a}{r})^2}-2\frac{b^5a^2\cos(\frac{a}{r})^6}{r^8\sin(\frac{a}{r})^6}-\frac{b\cos(\frac{a}{r})^2}{r^2\sin(\frac{a}{r})^2}-\frac{b^2a^4\cos(\frac{a}{r})^4}{r^8\sin(\frac{a}{r})^4}+8\frac{b^3a\cos(\frac{a}{r})^3}{r^5\sin(\frac{a}{r})^3}-\frac{b^5a^4\cos(\frac{a}{r})^4}{r^{10}\sin(\frac{a}{r})^4}+4\frac{b^5a\cos(\frac{a}{r})}{r^7\sin(\frac{a}{r})}-\frac{b^3a^4\cos(\frac{a}{r})^2}{r^8\sin(\frac{a}{r})^2}+4\frac{b^5a^3\cos(\frac{a}{r})^5}{r^9\sin(\frac{a}{r})^5}-\frac{b^5\cos(\frac{a}{r})^6}{r^6\sin(\frac{a}{r})^6}+4\frac{b^5a\cos(\frac{a}{r})^5}{r^7\sin(\frac{a}{r})^5}-5\frac{b^3a^2\cos(\frac{a}{r})^4}{r^6\sin(\frac{a}{r})^4}-3\frac{b^5\cos(\frac{a}{r})^4}{r^6\sin(\frac{a}{r})^4}-\frac{b}{r^2}-6\frac{b^5a^2\cos(\frac{a}{r})^2}{r^8\sin(\frac{a}{r})^2}-\frac{b^3a^2}{r^6}-6\frac{b^3\cos(\frac{a}{r})^2}{r^4\sin(\frac{a}{r})^2}-\frac{b^5}{r^6}.\end{aligned}$$