

Schwarzschild Metric in spherical coordinates with a variable spherically symmetric matter density:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= t. \\ x^1 &= r. \\ x^2 &= \theta. \\ x^3 &= \phi.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= \frac{r-2m}{r}. \\ g_{01} &= 0. \\ g_{02} &= 0. \\ g_{03} &= 0. \\ g_{10} &= 0. \\ g_{11} &= -\frac{r}{r-2m}. \\ g_{12} &= 0. \\ g_{13} &= 0. \\ g_{20} &= 0. \\ g_{21} &= 0. \\ g_{22} &= -r^2. \\ g_{23} &= 0. \\ g_{30} &= 0. \\ g_{31} &= 0. \\ g_{32} &= 0. \\ g_{33} &= -r^2\sin(\theta)^2.\end{aligned}$$

$$\boxed{\sqrt{=}\sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{=}\sqrt{r^4\sin(\theta)^2}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= \frac{r}{r-2m}. \\ g^{01} &= 0. \\ g^{02} &= 0. \\ g^{03} &= 0. \\ g^{10} &= 0. \\ g^{11} &= -\frac{r-2m}{r}. \\ g^{12} &= 0. \\ g^{13} &= 0. \\ g^{20} &= 0. \\ g^{21} &= 0. \\ g^{22} &= -\frac{1}{r^2}. \\ g^{23} &= 0. \\ g^{30} &= 0. \\ g^{31} &= 0. \\ g^{32} &= 0. \\ g^{33} &= -\frac{1}{r^2\sin(\theta)^2}.\end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 0. \\ \Gamma^0_{01} &= \frac{m}{(r-2m)r}. \\ \Gamma^0_{02} &= 0. \\ \Gamma^0_{03} &= 0. \\ \Gamma^0_{10} &= \frac{m}{(r-2m)r}. \\ \Gamma^0_{11} &= 0. \\ \Gamma^0_{12} &= 0. \\ \Gamma^0_{13} &= 0. \\ \Gamma^0_{20} &= 0. \\ \Gamma^0_{21} &= 0. \\ \Gamma^0_{22} &= 0. \\ \Gamma^0_{23} &= 0. \\ \Gamma^0_{30} &= 0. \\ \Gamma^0_{31} &= 0. \\ \Gamma^0_{32} &= 0. \\ \Gamma^0_{33} &= 0.\end{aligned}$$

$$\begin{aligned}\Gamma^1_{00} &= \frac{(r-2m)m}{r^3}. \\ \Gamma^1_{01} &= 0. \\ \Gamma^1_{02} &= 0. \\ \Gamma^1_{03} &= 0. \\ \Gamma^1_{10} &= 0. \\ \Gamma^1_{11} &= -\frac{m}{(r-2m)r}. \\ \Gamma^1_{12} &= 0. \\ \Gamma^1_{13} &= 0. \\ \Gamma^1_{20} &= 0. \\ \Gamma^1_{21} &= 0. \\ \Gamma^1_{22} &= -r+2m. \\ \Gamma^1_{23} &= 0. \\ \Gamma^1_{30} &= 0. \\ \Gamma^1_{31} &= 0. \\ \Gamma^1_{32} &= 0. \\ \Gamma^1_{33} &= -(r-2m)\sin(\theta)^2.\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^2 &= 0, \\ \Gamma_{01}^2 &= 0, \\ \Gamma_{02}^2 &= 0, \\ \Gamma_{03}^2 &= 0, \\ \Gamma_{10}^2 &= 0, \\ \Gamma_{11}^2 &= 0, \\ \Gamma_{12}^2 &= \frac{1}{r}, \\ \Gamma_{13}^2 &= 0, \\ \Gamma_{20}^2 &= 0, \\ \Gamma_{21}^2 &= \frac{1}{r}, \\ \Gamma_{22}^2 &= 0, \\ \Gamma_{23}^2 &= 0, \\ \Gamma_{30}^2 &= 0, \\ \Gamma_{31}^2 &= 0, \\ \Gamma_{32}^2 &= 0, \\ \Gamma_{33}^2 &= -\cos(\theta)\sin(\theta).\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^3 &= 0, \\ \Gamma_{01}^3 &= 0, \\ \Gamma_{02}^3 &= 0, \\ \Gamma_{03}^3 &= 0, \\ \Gamma_{10}^3 &= 0, \\ \Gamma_{11}^3 &= 0, \\ \Gamma_{12}^3 &= 0, \\ \Gamma_{13}^3 &= \frac{1}{r}, \\ \Gamma_{20}^3 &= 0, \\ \Gamma_{21}^3 &= 0, \\ \Gamma_{22}^3 &= 0, \\ \Gamma_{23}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\ \Gamma_{30}^3 &= 0, \\ \Gamma_{31}^3 &= \frac{1}{r}, \\ \Gamma_{32}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\ \Gamma_{33}^3 &= 0.\end{aligned}$$

$$\ddot{x}^\mu = \left(\Gamma_{\sigma\rho}^0 \dot{x}^\mu - \Gamma_{\sigma\rho}^\mu\right) \dot{x}^\sigma \dot{x}^\rho$$

$$\begin{aligned}\ddot{x}^0 &= 0, \\ \ddot{x}^1 &= \frac{4r^3m^2\dot{z}^2\sin(\theta)^2+4rm^2+3\dot{x}^2r^2m-4r^4m\dot{z}^2\sin(\theta)^2-4m^3-r^2m+r^5\dot{z}^2\sin(\theta)^2}{(r-2m)r^3},\end{aligned}$$

$$\ddot{x}^2 = \cos(\theta)\dot{z}^2\sin(\theta).$$

$$\ddot{x}^3 = -2\frac{\dot{x}r\dot{z}-3\dot{x}m\dot{z}}{(r-2m)r}.$$

$$g^{\mu\nu}\Gamma_{\mu\nu}^\lambda=0?$$

$$\begin{aligned}g^{\mu\nu}\Gamma_{\mu\nu}^0 &= 0, \\ g^{\mu\nu}\Gamma_{\mu\nu}^1 &= -6\frac{r^4m\sin(\theta)^4}{(r-2m)^2}-8\frac{m^2}{(r-2m)^2}r+\frac{r^5\sin(\theta)^4}{(r-2m)^2}+12\frac{r^3m^2\sin(\theta)^4}{(r-2m)^2}+12\frac{r^3m^2}{(r-2m)^2}-8\frac{r^2m^3}{(r-2m)^2}+2\frac{m}{(r-2m)^2}-32\frac{m^4}{(r-2m)^2r^3}+16\frac{m^5}{(r-2m)^2r^4}+\frac{r^5}{(r-2m)^2}-6\frac{r^4m}{(r-2m)^2}+24\frac{m^3}{(r-2m)^2r^2}-8\frac{r^2m^3\sin(\theta)^4}{(r-2m)^2}, \\ g^{\mu\nu}\Gamma_{\mu\nu}^2 &= \cos(\theta)r^2\sin(\theta)^3, \\ g^{\mu\nu}\Gamma_{\mu\nu}^3 &= 0.\end{aligned}$$