Schwarzschild Metric in spherical Abraham-Rössler-R coordinates with a variable spherically symmetric matter density:

$\boxed{x^\mu}$

$x^0 = t.$

$x^1 = R.$

$x^2 = \theta.$

$x^3 = \phi.$

$\boxed{g_{\mu\nu}}$

$g_{00} = 1 - \frac{8}{3} r(R)^2 \rho(t, R)\pi.$

$g_{01} = 0.$

$g_{02} = 0.$

$g_{03} = 0.$

$g_{10} = 0.$

$g_{11} = -1 + \frac{8}{3} r(R)^2 \rho(t, R)\pi.$

$g_{12} = 0.$

$g_{13} = 0.$

$g_{20} = 0.$

$g_{21} = 0.$

$g_{22} = -r(R)^2.$

$g_{23} = 0.$

$g_{30} = 0.$

$g_{31} = 0.$

$g_{32} = 0.$

$g_{33} = -r(R)^2 \sin(\theta)^2.$

$\boxed{\surd = \sqrt{-\det(g_{\mu\nu})}}$

$$\surd = \sqrt{\frac{64}{9} r(R)^8 \rho(t, R)^2 \sin(\theta)^2 \pi^2 - \frac{16}{3} r(R)^6 \rho(t, R) \sin(\theta)^2 \pi + r(R)^4 \sin(\theta)^2}.$$

$\boxed{g^{\mu\nu}}$

$$g^{00} = -3\frac{1}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$g^{01} = 0.$$
$$g^{02} = 0.$$
$$g^{03} = 0.$$
$$g^{10} = 0.$$
$$g^{11} = 3\frac{1}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$g^{12} = 0.$$
$$g^{13} = 0.$$
$$g^{20} = 0.$$
$$g^{21} = 0.$$
$$g^{22} = -\frac{1}{r(R)^2}.$$
$$g^{23} = 0.$$
$$g^{30} = 0.$$
$$g^{31} = 0.$$
$$g^{32} = 0.$$
$$g^{33} = -\frac{1}{r(R)^2\sin(\theta)^2}.$$

$\boxed{\Gamma^{\sigma}_{\mu\nu}}$

$$\Gamma^0_{00} = 4\frac{r(R)^2\pi\dot{\rho}(t,R)}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^0_{01} = 4\frac{2r(R)\rho(t,R)r'(R)\pi + r(R)^2\rho'(t,R)\pi}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^0_{02} = 0.$$
$$\Gamma^0_{03} = 0.$$
$$\Gamma^0_{10} = 4\frac{2r(R)\rho(t,R)r'(R)\pi + r(R)^2\rho'(t,R)\pi}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^0_{11} = 4\frac{r(R)^2\pi\dot{\rho}(t,R)}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^0_{12} = 0.$$
$$\Gamma^0_{13} = 0.$$
$$\Gamma^0_{20} = 0.$$
$$\Gamma^0_{21} = 0.$$
$$\Gamma^0_{22} = 0.$$
$$\Gamma^0_{23} = 0.$$
$$\Gamma^0_{30} = 0.$$
$$\Gamma^0_{31} = 0.$$
$$\Gamma^0_{32} = 0.$$
$$\Gamma^0_{33} = 0.$$

$$\Gamma^1_{00} = 4\frac{2r(R)\rho(t,R)r'(R)\pi + r(R)^2\rho'(t,R)\pi}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^1_{01} = 4\frac{r(R)^2\pi\dot{\rho}(t,R)}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^1_{02} = 0.$$
$$\Gamma^1_{03} = 0.$$
$$\Gamma^1_{10} = 4\frac{r(R)^2\pi\dot{\rho}(t,R)}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^1_{11} = 4\frac{2r(R)\rho(t,R)r'(R)\pi + r(R)^2\rho'(t,R)\pi}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^1_{12} = 0.$$
$$\Gamma^1_{13} = 0.$$
$$\Gamma^1_{20} = 0.$$
$$\Gamma^1_{21} = 0.$$
$$\Gamma^1_{22} = 3\frac{r(R)r'(R)}{-3 + 8r(R)^2\rho(t,R)\pi}.$$
$$\Gamma^1_{23} = 0.$$
$$\Gamma^1_{30} = 0.$$
$$\Gamma^1_{31} = 0.$$
$$\Gamma^1_{32} = 0.$$
$$\Gamma^1_{33} = 3\frac{r(R)r'(R)\sin(\theta)^2}{-3 + 8r(R)^2\rho(t,R)\pi}.$$

$$\Gamma^2_{00} = 0.$$
$$\Gamma^2_{01} = 0.$$
$$\Gamma^2_{02} = 0.$$
$$\Gamma^2_{03} = 0.$$
$$\Gamma^2_{10} = 0.$$
$$\Gamma^2_{11} = 0.$$
$$\Gamma^2_{12} = \frac{r'(R)}{r(R)}.$$
$$\Gamma^2_{13} = 0.$$
$$\Gamma^2_{20} = 0.$$
$$\Gamma^2_{21} = \frac{r'(R)}{r(R)}.$$
$$\Gamma^2_{22} = 0.$$
$$\Gamma^2_{23} = 0.$$
$$\Gamma^2_{30} = 0.$$
$$\Gamma^2_{31} = 0.$$
$$\Gamma^2_{32} = 0.$$
$$\Gamma^2_{33} = -\cos(\theta)\sin(\theta).$$

$\Gamma^3_{00} = 0.$
$\Gamma^3_{01} = 0.$
$\Gamma^3_{02} = 0.$
$\Gamma^3_{03} = 0.$
$\Gamma^3_{10} = 0.$
$\Gamma^3_{11} = 0.$
$\Gamma^3_{12} = 0.$
$\Gamma^3_{13} = \frac{r'(R)}{r(R)}.$
$\Gamma^3_{20} = 0.$
$\Gamma^3_{21} = 0.$
$\Gamma^3_{22} = 0.$
$\Gamma^3_{23} = \frac{\cos(\theta)}{\sin(\theta)}.$
$\Gamma^3_{30} = 0.$
$\Gamma^3_{31} = \frac{r'(R)}{r(R)}.$
$\Gamma^3_{32} = \frac{\cos(\theta)}{\sin(\theta)}.$
$\Gamma^3_{33} = 0.$

$\boxed{R_{\mu\nu}}$

$$R_{00} = 32\frac{r(R)^4\ddot{\rho}(t,R)\rho(t,R)\pi^2}{(-3+8r(R)^2\rho(t,R)\pi)^2} + 72\frac{r(R)r'(R)\rho'(t,R)\pi}{(-3+8r(R)^2\rho(t,R)\pi)^2} - 64\frac{r(R)^3\rho(t,R)^2\pi^2 r''(R)}{(-3+8r(R)^2\rho(t,R)\pi)^2} + 72\frac{\rho(t,R)r'(R)^2\pi}{(-3+8r(R)^2\rho(t,R)\pi)^2} -$$

$$R_{01} = -8\frac{r(R)r'(R)\pi\dot{\rho}(t,R)}{-3+8r(R)^2\rho(t,R)\pi}.$$

$R_{02} = 0.$
$R_{03} = 0.$

$$R_{10} = -8\frac{r(R)r'(R)\pi\dot{\rho}(t,R)}{-3+8r(R)^2\rho(t,R)\pi}.$$

$$R_{11} = -32\frac{r(R)^4\ddot{\rho}(t,R)\rho(t,R)\pi^2}{(-3+8r(R)^2\rho(t,R)\pi)^2} - 24\frac{r(R)r'(R)\rho'(t,R)\pi}{(-3+8r(R)^2\rho(t,R)\pi)^2} + 192\frac{r(R)^3\rho(t,R)^2\pi^2 r''(R)}{(-3+8r(R)^2\rho(t,R)\pi)^2} + 24\frac{\rho(t,R)r'(R)^2\pi}{(-3+8r(R)^2\rho(t,R)\pi)}$$

$R_{12} = 0.$
$R_{13} = 0.$
$R_{20} = 0.$
$R_{21} = 0.$

$$R_{22} = -8\frac{r(R)^2\rho(t,R)\pi}{-3+8r(R)^2\rho(t,R)\pi} - 3\frac{r(R)r''(R)}{-3+8r(R)^2\rho(t,R)\pi} - 3\frac{r'(R)^2}{-3+8r(R)^2\rho(t,R)\pi} + 3\frac{1}{-3+8r(R)^2\rho(t,R)\pi}.$$

$R_{23} = 0.$
$R_{30} = 0.$
$R_{31} = 0.$
$R_{32} = 0.$

$$R_{33} = -8\frac{r(R)^2\rho(t,R)\sin(\theta)^2\pi}{-3+8r(R)^2\rho(t,R)\pi} + 3\frac{\sin(\theta)^2}{-3+8r(R)^2\rho(t,R)\pi} - 3\frac{r'(R)^2\sin(\theta)^2}{-3+8r(R)^2\rho(t,R)\pi} - 3\frac{r(R)\sin(\theta)^2 r''(R)}{-3+8r(R)^2\rho(t,R)\pi}.$$

$\boxed{R^{\mu}_{\ \nu}}$

$$R^0_{\ 0} = 36\frac{r(R)^2\ddot{\rho}(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 216\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 96\frac{r(R)^4\pi^2\dot{\rho}(t,R)^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 192\frac{r(R)^2\rho(t,R)^2r'(R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3}$$

$$R^0_{\ 1} = 24\frac{r(R)r'(R)\pi\dot{\rho}(t,R)}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2}.$$

$$R^0_{\ 2} = 0.$$

$$R^0_{\ 3} = 0.$$

$$R^1_{\ 0} = -24\frac{r(R)r'(R)\pi\dot{\rho}(t,R)}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2}.$$

$$R^1_{\ 1} = 36\frac{r(R)^2\ddot{\rho}(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 72\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 96\frac{r(R)^4\pi^2\dot{\rho}(t,R)^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 576\frac{r(R)^2\rho(t,R)^2r'(R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3}$$

$$R^1_{\ 2} = 0.$$

$$R^1_{\ 3} = 0.$$

$$R^2_{\ 0} = 0.$$

$$R^2_{\ 1} = 0.$$

$$R^2_{\ 2} = -3\frac{1}{r(R)^2(-3+8r(R)^2\rho(t,R)\pi)} + 8\frac{\rho(t,R)\pi}{-3+8r(R)^2\rho(t,R)\pi} + 3\frac{r''(R)}{r(R)(-3+8r(R)^2\rho(t,R)\pi)} + 3\frac{r'(R)^2}{r(R)^2(-3+8r(R)^2\rho(t,}$$

$$R^2_{\ 3} = 0.$$

$$R^3_{\ 0} = 0.$$

$$R^3_{\ 1} = 0.$$

$$R^3_{\ 2} = 0.$$

$$R^3_{\ 3} = -3\frac{1}{r(R)^2(-3+8r(R)^2\rho(t,R)\pi)} + 8\frac{\rho(t,R)\pi}{-3+8r(R)^2\rho(t,R)\pi} + 3\frac{r''(R)}{r(R)(-3+8r(R)^2\rho(t,R)\pi)} + 3\frac{r'(R)^2}{r(R)^2(-3+8r(R)^2\rho(t,}$$

$\boxed{R}$

$$R = 72\frac{r(R)^2\ddot{\rho}(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 432\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 192\frac{r(R)^4\pi^2\dot{\rho}(t,R)^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 1152\frac{r(R)^2\rho(t,R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)}$$

$\boxed{G^{\mu}_{\ \nu}}$

$$G^{0}_{\ 0} = -64\frac{r(R)^2\rho(t,R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2} + 24\frac{r(R)r'(R)\rho'(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2} + 24\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2} + 9\frac{r'(R)^2}{r(R)^2(-3+8r(R)^2\rho(t,R}$$

$$G^{0}_{\ 1} = 24\frac{r(R)r'(R)\pi\dot{\rho}(t,R)}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2}.$$

$$G^{0}_{\ 2} = 0.$$

$$G^{0}_{\ 3} = 0.$$

$$G^{1}_{\ 0} = -24\frac{r(R)r'(R)\pi\dot{\rho}(t,R)}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2}.$$

$$G^{1}_{\ 1} = -64\frac{r(R)^2\rho(t,R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2} - 24\frac{r(R)r'(R)\rho'(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2} - 72\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^2} + 9\frac{r'(R)^2}{r(R)^2(-3+8r(R)^2\rho(t,R}$$

$$G^{1}_{\ 2} = 0.$$

$$G^{1}_{\ 3} = 0.$$

$$G^{2}_{\ 0} = 0.$$

$$G^{2}_{\ 1} = 0.$$

$$G^{2}_{\ 2} = -36\frac{r(R)^2\ddot{\rho}(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 72\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 96\frac{r(R)^4\pi^2\dot{\rho}(t,R)^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 192\frac{r(R)^2\rho(t,R)^2r'(R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^{3}}$$

$$G^{2}_{\ 3} = 0.$$

$$G^{3}_{\ 0} = 0.$$

$$G^{3}_{\ 1} = 0.$$

$$G^{3}_{\ 2} = 0.$$

$$G^{3}_{\ 3} = -36\frac{r(R)^2\ddot{\rho}(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 72\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 96\frac{r(R)^4\pi^2\dot{\rho}(t,R)^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 192\frac{r(R)^2\rho(t,R)^2r'(R)^2\pi^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^{3}}$$

$\boxed{G}$

$$G = -72\frac{r(R)^2\ddot{\rho}(t,R)\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 432\frac{\rho(t,R)r'(R)^2\pi}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} - 192\frac{r(R)^4\pi^2\dot{\rho}(t,R)^2}{\left(-3+8r(R)^2\rho(t,R)\pi\right)^3} + 1152\frac{r(R)^2\rho(t,R)^2\pi^2}{(-3+8r(R)^2\rho(t,R)\pi}$$

$\boxed{G^{\mu}_{\ \nu:\mu} = 0}$

$$G^{\mu}_{\ 0:\mu} = 0.$$

$$G^{\mu}_{\ 1:\mu} = 0.$$

$$G^{\mu}_{\ 2:\mu} = 0.$$

$$G^{\mu}_{\ 3:\mu} = 0.$$

$\boxed{g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu} = 0?}$

$$g^{\mu\nu}\,\Gamma^{0}_{\mu\nu} = 0.$$

$$g^{\mu\nu}\,\Gamma^{1}_{\mu\nu} = -3\frac{r(R)^3r'(R)\sin(\theta)^4}{-3+8r(R)^2\rho(t,R)\pi} - 3\frac{r(R)^3r'(R)}{-3+8r(R)^2\rho(t,R)\pi}.$$

$$g^{\mu\nu}\,\Gamma^{2}_{\mu\nu} = \cos(\theta)r(R)^2\sin(\theta)^3.$$

$$g^{\mu\nu}\,\Gamma^{3}_{\mu\nu} = 0.$$