

Cylindrical Coordinates for LASER:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= t. \\ x^1 &= \rho. \\ x^2 &= \phi. \\ x^3 &= z.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= \exp(2A(\rho)). \\ g_{01} &= 0. \\ g_{02} &= 0. \\ g_{03} &= 0. \\ g_{10} &= 0. \\ g_{11} &= -\exp(2B(\rho)). \\ g_{12} &= 0. \\ g_{13} &= 0. \\ g_{20} &= 0. \\ g_{21} &= 0. \\ g_{22} &= -\rho^2 \exp(2C(\rho)). \\ g_{23} &= 0. \\ g_{30} &= 0. \\ g_{31} &= 0. \\ g_{32} &= 0. \\ g_{33} &= -\exp(2D(\rho)).\end{aligned}$$

$$\boxed{\sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{-\det(g_{\mu\nu})} = \sqrt{\exp(2A(\rho))\rho^2 \exp(2D(\rho)) \exp(2C(\rho)) \exp(2B(\rho))}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= \frac{1}{\exp(2A(\rho))}. \\ g^{01} &= 0. \\ g^{02} &= 0. \\ g^{03} &= 0. \\ g^{10} &= 0. \\ g^{11} &= -\frac{1}{\exp(2B(\rho))}. \\ g^{12} &= 0. \\ g^{13} &= 0. \\ g^{20} &= 0. \\ g^{21} &= 0. \\ g^{22} &= -\frac{1}{\rho^2 \exp(2C(\rho))}. \\ g^{23} &= 0. \\ g^{30} &= 0. \\ g^{31} &= 0. \\ g^{32} &= 0. \\ g^{33} &= -\frac{1}{\exp(2D(\rho))}.\end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 0. \\ \Gamma^0_{01} &= A'(\rho). \\ \Gamma^0_{02} &= 0. \\ \Gamma^0_{03} &= 0. \\ \Gamma^0_{10} &= A'(\rho). \\ \Gamma^0_{11} &= 0. \\ \Gamma^0_{12} &= 0. \\ \Gamma^0_{13} &= 0. \\ \Gamma^0_{20} &= 0. \\ \Gamma^0_{21} &= 0. \\ \Gamma^0_{22} &= 0. \\ \Gamma^0_{23} &= 0. \\ \Gamma^0_{30} &= 0. \\ \Gamma^0_{31} &= 0. \\ \Gamma^0_{32} &= 0. \\ \Gamma^0_{33} &= 0.\end{aligned}$$

$$\begin{aligned}\Gamma^1_{00} &= \frac{A'(\rho) \exp(2A(\rho))}{\exp(2B(\rho))}. \\ \Gamma^1_{01} &= 0. \\ \Gamma^1_{02} &= 0. \\ \Gamma^1_{03} &= 0. \\ \Gamma^1_{10} &= 0. \\ \Gamma^1_{11} &= B'(\rho). \\ \Gamma^1_{12} &= 0. \\ \Gamma^1_{13} &= 0. \\ \Gamma^1_{20} &= 0. \\ \Gamma^1_{21} &= 0. \\ \Gamma^1_{22} &= -\frac{\rho^2 \exp(2C(\rho))C'(\rho) + \rho \exp(2C(\rho))}{\exp(2B(\rho))}. \\ \Gamma^1_{23} &= 0. \\ \Gamma^1_{30} &= 0. \\ \Gamma^1_{31} &= 0. \\ \Gamma^1_{32} &= 0. \\ \Gamma^1_{33} &= -\frac{\exp(2D(\rho))D'(\rho)}{\exp(2B(\rho))}.\end{aligned}$$

$$\Gamma_{00}^2 = 0.$$

$$\Gamma_{01}^2 = 0.$$

$$\Gamma_{02}^2 = 0.$$

$$\Gamma_{03}^2 = 0.$$

$$\Gamma_{10}^2 = 0.$$

$$\Gamma_{11}^2 = 0.$$

$$\Gamma_{12}^2 = \frac{1 + \rho C''(\rho)}{\rho}.$$

$$\Gamma_{13}^2 = 0.$$

$$\Gamma_{20}^2 = 0.$$

$$\Gamma_{21}^2 = \frac{1 + \rho C''(\rho)}{\rho}.$$

$$\Gamma_{22}^2 = 0.$$

$$\Gamma_{23}^2 = 0.$$

$$\Gamma_{30}^2 = 0.$$

$$\Gamma_{31}^2 = 0.$$

$$\Gamma_{32}^2 = 0.$$

$$\Gamma_{33}^2 = 0.$$

$$\Gamma_{00}^3 = 0.$$

$$\Gamma_{01}^3 = 0.$$

$$\Gamma_{02}^3 = 0.$$

$$\Gamma_{03}^3 = 0.$$

$$\Gamma_{10}^3 = 0.$$

$$\Gamma_{11}^3 = 0.$$

$$\Gamma_{12}^3 = 0.$$

$$\Gamma_{13}^3 = D'(\rho).$$

$$\Gamma_{20}^3 = 0.$$

$$\Gamma_{21}^3 = 0.$$

$$\Gamma_{22}^3 = 0.$$

$$\Gamma_{23}^3 = 0.$$

$$\Gamma_{30}^3 = 0.$$

$$\Gamma_{31}^3 = D'(\rho).$$

$$\Gamma_{32}^3 = 0.$$

$$\Gamma_{33}^3 = 0.$$

$$\boxed{R_{\mu\nu}}$$

$$R_{00} = \frac{A'(\rho) \exp(2A(\rho)) \rho B'(\rho) - \exp(2A(\rho)) \rho A''(\rho) - A'(\rho) \exp(2A(\rho)) \rho C'(\rho) - A'(\rho)^2 \exp(2A(\rho)) \rho - A'(\rho) \exp(2A(\rho)) - A'(\rho) \exp(2A(\rho)) \rho D'(\rho)}{\rho \exp(2B(\rho))}.$$

$$R_{01} = 0.$$

$$R_{02} = 0.$$

$$R_{03} = 0.$$

$$R_{10} = 0.$$

$$R_{11} = \frac{A'(\rho)^2 \rho - A'(\rho) \rho B'(\rho) + \rho D''(\rho) + \rho A''(\rho) + \rho D'(\rho)^2 + C''(\rho) \rho - \rho D'(\rho) B'(\rho) + \rho C'(\rho)^2 - B'(\rho) - \rho B'(\rho) C'(\rho) + 2C'(\rho)}{\rho}.$$

$$R_{12} = 0.$$

$$R_{13} = 0.$$

$$R_{20} = 0.$$

$$R_{21} = 0.$$

$$R_{22} = -\frac{\rho \exp(2C(\rho)) B'(\rho) - A'(\rho) \rho \exp(2C(\rho)) - A'(\rho) \rho^2 \exp(2C(\rho)) C'(\rho) - C''(\rho) \rho^2 \exp(2C(\rho)) - \rho^2 \exp(2C(\rho)) D'(\rho) C'(\rho) - 2\rho \exp(2C(\rho)) C'(\rho) + \rho^2 \exp(2C(\rho)) B'(\rho) C'(\rho) - \rho \exp(2C(\rho)) D'(\rho) - \rho^2 \exp(2C(\rho)) C'(\rho)^2}{\exp(2B(\rho))}.$$

$$R_{23} = 0.$$

$$R_{30} = 0.$$

$$R_{31} = 0.$$

$$R_{32} = 0.$$

$$R_{33} = \frac{A'(\rho) \rho \exp(2D(\rho)) D'(\rho) + \exp(2D(\rho)) D'(\rho) + \rho \exp(2D(\rho)) D'(\rho) C'(\rho) + \rho D''(\rho) \exp(2D(\rho)) + \rho \exp(2D(\rho)) D'(\rho)^2 - \rho \exp(2D(\rho)) D'(\rho) B'(\rho)}{\rho \exp(2B(\rho))}.$$

$$\boxed{R^\mu{}_\nu}$$

$$R^0_0 = -\frac{A'(\rho) C'(\rho)}{\exp(2B(\rho))} - \frac{A'(\rho) D'(\rho)}{\exp(2B(\rho))} - \frac{A'(\rho)^2}{\exp(2B(\rho))} + \frac{A'(\rho) B'(\rho)}{\exp(2B(\rho))} - \frac{A'(\rho)}{\rho \exp(2B(\rho))} - \frac{A''(\rho)}{\exp(2B(\rho))}.$$

$$R^0_1 = 0.$$

$$R^0_2 = 0.$$

$$R^0_3 = 0.$$

$$R^1_0 = 0.$$

$$R^1_1 = -\frac{C'(\rho)^2}{\exp(2B(\rho))} - \frac{C''(\rho)}{\exp(2B(\rho))} + \frac{D'(\rho) B'(\rho)}{\exp(2B(\rho))} - 2\frac{C'(\rho)}{\rho \exp(2B(\rho))} - \frac{D'(\rho)^2}{\exp(2B(\rho))} - \frac{A'(\rho)^2}{\exp(2B(\rho))} + \frac{B'(\rho)}{\rho \exp(2B(\rho))} + \frac{B'(\rho) C'(\rho)}{\exp(2B(\rho))} + \frac{A'(\rho) B'(\rho)}{\exp(2B(\rho))} - \frac{D''(\rho)}{\exp(2B(\rho))} - \frac{A''(\rho)}{\exp(2B(\rho))}.$$

$$R^1_2 = 0.$$

$$R^1_3 = 0.$$

$$R^2_0 = 0.$$

$$R^2_1 = 0.$$

$$R^2_2 = -\frac{D'(\rho) C'(\rho)}{\exp(2B(\rho))} - \frac{C'(\rho)^2}{\exp(2B(\rho))} - \frac{C''(\rho)}{\exp(2B(\rho))} - 2\frac{C'(\rho)}{\rho \exp(2B(\rho))} - \frac{A'(\rho) C'(\rho)}{\exp(2B(\rho))} - \frac{D'(\rho)}{\rho \exp(2B(\rho))} + \frac{B'(\rho)}{\rho \exp(2B(\rho))} + \frac{B'(\rho) C'(\rho)}{\exp(2B(\rho))} - \frac{A'(\rho)}{\rho \exp(2B(\rho))}.$$

$$R^2_3 = 0.$$

$$R^3_0 = 0.$$

$$R^3_1 = 0.$$

$$R^3_2 = 0.$$

$$R^3_3 = -\frac{D'(\rho) C'(\rho)}{\exp(2B(\rho))} + \frac{D'(\rho) B'(\rho)}{\exp(2B(\rho))} - \frac{D'(\rho)^2}{\exp(2B(\rho))} - \frac{D'(\rho)}{\rho \exp(2B(\rho))} - \frac{A'(\rho) D'(\rho)}{\exp(2B(\rho))} - \frac{D''(\rho)}{\exp(2B(\rho))}.$$

$$\boxed{R}$$

$$R = -2\frac{D'(\rho) C'(\rho)}{\exp(2B(\rho))} - 2\frac{C'(\rho)^2}{\exp(2B(\rho))} - 2\frac{C''(\rho)}{\exp(2B(\rho))} + 2\frac{D'(\rho) B'(\rho)}{\exp(2B(\rho))} - 4\frac{C'(\rho)}{\rho \exp(2B(\rho))} - 2\frac{D'(\rho)^2}{\exp(2B(\rho))} - 2\frac{A'(\rho) C'(\rho)}{\exp(2B(\rho))} - 2\frac{D'(\rho)}{\rho \exp(2B(\rho))} - 2\frac{A'(\rho) D'(\rho)}{\exp(2B(\rho))} - 2\frac{A'(\rho)^2}{\exp(2B(\rho))} + 2\frac{B'(\rho)}{\rho \exp(2B(\rho))} + 2\frac{B'(\rho) C'(\rho)}{\exp(2B(\rho))} + 2\frac{A'(\rho) B'(\rho)}{\exp(2B(\rho))} - 2\frac{A'(\rho)}{\rho \exp(2B(\rho))} - 2\frac{D''(\rho)}{\exp(2B(\rho))} - 2\frac{A''(\rho)}{\exp(2B(\rho))}.$$

$$G^{\mu}_{\nu}$$

$$G^0_0=\frac{D'(\rho)C'(\rho)}{\exp(2B(\rho))}+\frac{C'(\rho)^2}{\exp(2B(\rho))}+\frac{C''(\rho)}{\exp(2B(\rho))}-\frac{D'(\rho)B'(\rho)}{\exp(2B(\rho))}+2\frac{C'(\rho)}{\rho\exp(2B(\rho))}+\frac{D'(\rho)^2}{\exp(2B(\rho))}+\frac{D'(\rho)}{\rho\exp(2B(\rho))}-\frac{B'(\rho)}{\rho\exp(2B(\rho))}-\frac{B'(\rho)C'(\rho)}{\exp(2B(\rho))}+\frac{D''(\rho)}{\exp(2B(\rho))}.$$

$$G^0_1=0.$$

$$G^1_2=0.$$

$$G^0_3=0.$$

$$G^1_0=0.$$

$$G^1_1=\frac{D'(\rho)C'(\rho)}{\exp(2B(\rho))}+\frac{A'(\rho)C'(\rho)}{\exp(2B(\rho))}+\frac{D'(\rho)}{\rho\exp(2B(\rho))}+\frac{A'(\rho)D'(\rho)}{\exp(2B(\rho))}+\frac{A'(\rho)}{\rho\exp(2B(\rho))}.$$

$$G^1_2=0.$$

$$G^1_3=0.$$

$$G^0_0=0.$$

$$G^2_1=0.$$

$$G^2_2=-\frac{D'(\rho)B'(\rho)}{\exp(2B(\rho))}+\frac{D'(\rho)^2}{\exp(2B(\rho))}+\frac{A'(\rho)D'(\rho)}{\exp(2B(\rho))}+\frac{A'(\rho)^2}{\exp(2B(\rho))}-\frac{A'(\rho)B'(\rho)}{\exp(2B(\rho))}+\frac{D''(\rho)}{\exp(2B(\rho))}+\frac{A''(\rho)}{\exp(2B(\rho))}.$$

$$G^2_3=0.$$

$$G^3_0=0.$$

$$G^3_1=0.$$

$$G^3_2=0.$$

$$G^3_3=\frac{C'(\rho)^2}{\exp(2B(\rho))}+\frac{C''(\rho)}{\exp(2B(\rho))}+2\frac{C'(\rho)}{\rho\exp(2B(\rho))}+\frac{A'(\rho)C'(\rho)}{\exp(2B(\rho))}+\frac{A'(\rho)^2}{\exp(2B(\rho))}-\frac{B'(\rho)}{\rho\exp(2B(\rho))}-\frac{B'(\rho)C'(\rho)}{\exp(2B(\rho))}-\frac{A'(\rho)B'(\rho)}{\exp(2B(\rho))}+\frac{A'(\rho)}{\rho\exp(2B(\rho))}+\frac{A''(\rho)}{\exp(2B(\rho))}.$$

$$G$$

$$G=2\frac{D'(\rho)C'(\rho)}{\exp(2B(\rho))}+2\frac{C'(\rho)^2}{\exp(2B(\rho))}+2\frac{C''(\rho)}{\exp(2B(\rho))}-2\frac{D'(\rho)B'(\rho)}{\exp(2B(\rho))}+4\frac{C'(\rho)}{\rho\exp(2B(\rho))}+2\frac{D'(\rho)^2}{\exp(2B(\rho))}+2\frac{A'(\rho)C'(\rho)}{\exp(2B(\rho))}+2\frac{D'(\rho)}{\rho\exp(2B(\rho))}+2\frac{A'(\rho)D'(\rho)}{\exp(2B(\rho))}+2\frac{A'(\rho)^2}{\exp(2B(\rho))}-2\frac{B'(\rho)}{\rho\exp(2B(\rho))}-2\frac{B'(\rho)C'(\rho)}{\exp(2B(\rho))}-2\frac{A'(\rho)B'(\rho)}{\exp(2B(\rho))}+2\frac{A'(\rho)}{\rho\exp(2B(\rho))}+2\frac{D''(\rho)}{\exp(2B(\rho))}+2\frac{A''(\rho)}{\exp(2B(\rho))}.$$

$$G^{\mu}_{\nu;\mu}=0$$

$$G^{\mu}_{6;\mu}=0.$$

$$G^4_{1;\mu}=0.$$

$$G^{\mu}_{2;\mu}=0.$$

$$G^{\mu}_{3;\mu}=0.$$

$$g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu}=0?$$

$$g^{\mu\nu}\,\Gamma^0_{\mu\nu}=0.$$

$$g^{\mu\nu}\,\Gamma^1_{\mu\nu}=\frac{\rho^4\exp(2C(\rho))^2C'(\rho)}{\exp(2B(\rho))}+\frac{A'(\rho)\exp(2A(\rho))^2}{\exp(2B(\rho))}-\exp(2B(\rho))B'(\rho)+\frac{\exp(2D(\rho))^2D'(\rho)}{\exp(2B(\rho))}+\frac{\rho^3\exp(2C(\rho))^2}{\exp(2B(\rho))}.$$

$$g^{\mu\nu}\,\Gamma^2_{\mu\nu}=0.$$

$$g^{\mu\nu}\,\Gamma^3_{\mu\nu}=0.$$