Schwarzschild Metric in spherical Abraham-Rössler-R coordinates:

$\boxed{x^\mu}$

$x^0 = t.$
$x^1 = R.$
$x^2 = \theta.$
$x^3 = \phi.$

$\boxed{g_{\mu\nu}}$

$g_{00} = -\frac{2m - r(R)}{r(R)}.$
$g_{01} = 0.$
$g_{02} = 0.$
$g_{03} = 0.$
$g_{10} = 0.$
$g_{11} = \frac{2m - r(R)}{r(R)}.$
$g_{12} = 0.$
$g_{13} = 0.$
$g_{20} = 0.$
$g_{21} = 0.$
$g_{22} = -r(R)^2.$
$g_{23} = 0.$
$g_{30} = 0.$
$g_{31} = 0.$
$g_{32} = 0.$
$g_{33} = -\sin(\theta)^2 r(R)^2.$

$\boxed{\sqrt{} = \sqrt{-\det(g_{\mu\nu})}}$

$\sqrt{} = \sqrt{4m^2 \sin(\theta)^2 r(R)^2 + \sin(\theta)^2 r(R)^4 - 4m \sin(\theta)^2 r(R)^3}.$

$\boxed{g^{\mu\nu}}$

$g^{00} = -\frac{r(R)}{2m - r(R)}.$
$g^{01} = 0.$
$g^{02} = 0.$
$g^{03} = 0.$
$g^{10} = 0.$
$g^{11} = \frac{r(R)}{2m - r(R)}.$
$g^{12} = 0.$
$g^{13} = 0.$
$g^{20} = 0.$
$g^{21} = 0.$
$g^{22} = -\frac{1}{r(R)^2}.$
$g^{23} = 0.$
$g^{30} = 0.$
$g^{31} = 0.$
$g^{32} = 0.$
$g^{33} = -\frac{1}{\sin(\theta)^2 r(R)^2}.$

$\boxed{\Gamma^\sigma_{\mu\nu}}$

$\Gamma^0_{00} = 0.$
$\Gamma^0_{01} = -\frac{r'(R)m}{(2m - r(R))r(R)}.$
$\Gamma^0_{02} = 0.$
$\Gamma^0_{03} = 0.$
$\Gamma^0_{10} = -\frac{r'(R)m}{(2m - r(R))r(R)}.$
$\Gamma^0_{11} = 0.$
$\Gamma^0_{12} = 0.$
$\Gamma^0_{13} = 0.$
$\Gamma^0_{20} = 0.$
$\Gamma^0_{21} = 0.$
$\Gamma^0_{22} = 0.$
$\Gamma^0_{23} = 0.$
$\Gamma^0_{30} = 0.$
$\Gamma^0_{31} = 0.$
$\Gamma^0_{32} = 0.$
$\Gamma^0_{33} = 0.$

$\Gamma^1_{00} = -\frac{r'(R)m}{(2m - r(R))r(R)}.$
$\Gamma^1_{01} = 0.$
$\Gamma^1_{02} = 0.$
$\Gamma^1_{03} = 0.$
$\Gamma^1_{10} = 0.$
$\Gamma^1_{11} = -\frac{r'(R)m}{(2m - r(R))r(R)}.$
$\Gamma^1_{12} = 0.$
$\Gamma^1_{13} = 0.$
$\Gamma^1_{20} = 0.$
$\Gamma^1_{21} = 0.$
$\Gamma^1_{22} = \frac{r'(R)r(R)^2}{2m - r(R)}.$
$\Gamma^1_{23} = 0.$
$\Gamma^1_{30} = 0.$
$\Gamma^1_{31} = 0.$
$\Gamma^1_{32} = 0.$
$\Gamma^1_{33} = \frac{r'(R)\sin(\theta)^2 r(R)^2}{2m - r(R)}.$

$\Gamma^2_{00} = 0.$
$\Gamma^2_{01} = 0.$
$\Gamma^2_{02} = 0.$
$\Gamma^2_{03} = 0.$
$\Gamma^2_{10} = 0.$
$\Gamma^2_{11} = 0.$
$\Gamma^2_{12} = \frac{r'(R)}{r(R)}.$
$\Gamma^2_{13} = 0.$
$\Gamma^2_{20} = 0.$
$\Gamma^2_{21} = \frac{r'(R)}{r(R)}.$
$\Gamma^2_{22} = 0.$
$\Gamma^2_{23} = 0.$
$\Gamma^2_{30} = 0.$
$\Gamma^2_{31} = 0.$
$\Gamma^2_{32} = 0.$
$\Gamma^2_{33} = -\cos(\theta)\sin(\theta).$

$\Gamma^3_{00} = 0.$
$\Gamma^3_{01} = 0.$
$\Gamma^3_{02} = 0.$
$\Gamma^3_{03} = 0.$
$\Gamma^3_{10} = 0.$
$\Gamma^3_{11} = 0.$
$\Gamma^3_{12} = 0.$
$\Gamma^3_{13} = \frac{r'(R)}{r(R)}.$
$\Gamma^3_{20} = 0.$
$\Gamma^3_{21} = 0.$
$\Gamma^3_{22} = 0.$
$\Gamma^3_{23} = \frac{\cos(\theta)}{\sin(\theta)}.$
$\Gamma^3_{30} = 0.$
$\Gamma^3_{31} = \frac{r'(R)}{r(R)}.$
$\Gamma^3_{32} = \frac{\cos(\theta)}{\sin(\theta)}.$
$\Gamma^3_{33} = 0.$

$\boxed{R_{\mu\nu}}$

$$R_{00} = \frac{2r'(R)^2m^2 - mr(R)^2r''(R) + 2m^2r(R)r''(R)}{(2m - r(R))^2 r(R)^2}.$$

$R_{01} = 0.$
$R_{02} = 0.$
$R_{03} = 0.$
$R_{10} = 0.$

$$R_{11} = -\frac{4r'(R)^2mr(R) - 6r'(R)^2m^2 + 7mr(R)^2r''(R) - 2r(R)^3r''(R) - 6m^2r(R)r''(R)}{(2m - r(R))^2 r(R)^2}.$$

$R_{12} = 0.$
$R_{13} = 0.$
$R_{20} = 0.$
$R_{21} = 0.$

$$R_{22} = -\frac{2m - r(R) + r'(R)^2r(R) + r(R)^2r''(R)}{2m - r(R)}.$$

$R_{23} = 0.$
$R_{30} = 0.$
$R_{31} = 0.$
$R_{32} = 0.$

$$R_{33} = -\frac{r'(R)^2\sin(\theta)^2r(R) + \sin(\theta)^2r(R)^2r''(R) - \sin(\theta)^2r(R) + 2m\sin(\theta)^2}{2m - r(R)}.$$

$\boxed{R^{\mu}_{\ \nu}}$

$$R^0_{\ 0} = -2\frac{m^2r''(R)}{(2m - r(R))^3} + \frac{mr(R)r''(R)}{(2m - r(R))^3} - 2\frac{r'(R)^2m^2}{(2m - r(R))^3 r(R)}.$$

$R^0_{\ 1} = 0.$
$R^0_{\ 2} = 0.$
$R^0_{\ 3} = 0.$
$R^1_{\ 0} = 0.$

$$R^1_{\ 1} = 6\frac{m^2r''(R)}{(2m - r(R))^3} + 2\frac{r(R)^2r''(R)}{(2m - r(R))^3} - 7\frac{mr(R)r''(R)}{(2m - r(R))^3} - 4\frac{r'(R)^2m}{(2m - r(R))^3} + 6\frac{r'(R)^2m^2}{(2m - r(R))^3 r(R)}.$$

$R^1_{\ 2} = 0.$
$R^1_{\ 3} = 0.$
$R^2_{\ 0} = 0.$
$R^2_{\ 1} = 0.$

$$R^2_{\ 2} = -\frac{1}{(2m - r(R))r(R)} + \frac{r''(R)}{2m - r(R)} + 2\frac{m}{(2m - r(R))r(R)^2} + \frac{r'(R)^2}{(2m - r(R))r(R)}.$$

$R^2_{\ 3} = 0.$
$R^3_{\ 0} = 0.$
$R^3_{\ 1} = 0.$
$R^3_{\ 2} = 0.$

$$R^3_{\ 3} = -\frac{1}{(2m - r(R))r(R)} + \frac{r''(R)}{2m - r(R)} + 2\frac{m}{(2m - r(R))r(R)^2} + \frac{r'(R)^2}{(2m - r(R))r(R)}.$$

$\boxed{R}$

$$R = -24\frac{m^2}{(2m - r(R))^3 r(R)} + 12\frac{m}{(2m - r(R))^3} - 2\frac{r(R)}{(2m - r(R))^3} + 16\frac{m^3}{(2m - r(R))^3 r(R)^2} + 12\frac{m^2r''(R)}{(2m - r(R))^3} + 4\frac{r(R)^2r''(R)}{(2m - r(R))^3} - 14\frac{mr(R)r''(R)}{(2m - r(R))^3} + 2\frac{r'(R)^2r(R)}{(2m - r(R))^3} - 12\frac{r'(R)^2m}{(2m - r(R))^3} + 12\frac{r'(R)^2m^2}{(2m - r(R))^3 r(R)}.$$

$\boxed{G^{\mu}{}_{\nu}}$

$$G^{0}{}_{0} = \frac{r'(R)^2}{(2m - r(R))^2} - 4\frac{r'(R)^2 m}{(2m - r(R))^2 r(R)} - 4\frac{m^2}{(2m - r(R))^2 r(R)^2} + 4\frac{m}{(2m - r(R))^2 r(R)} - \frac{1}{(2m - r(R))^2} + 2\frac{r(R) r''(R)}{(2m - r(R))^2} - 4\frac{m r''(R)}{(2m - r(R))^2}.$$

$G^{0}{}_{1} = 0.$
$G^{0}{}_{2} = 0.$
$G^{0}{}_{3} = 0.$
$G^{1}{}_{0} = 0.$

$$G^{1}{}_{1} = \frac{r'(R)^2}{(2m - r(R))^2} - 4\frac{m^2}{(2m - r(R))^2 r(R)^2} + 4\frac{m}{(2m - r(R))^2 r(R)} - \frac{1}{(2m - r(R))^2}.$$

$G^{1}{}_{2} = 0.$
$G^{1}{}_{3} = 0.$
$G^{2}{}_{0} = 0.$
$G^{2}{}_{1} = 0.$

$$G^{2}{}_{2} = -2\frac{m^2 r''(R)}{(2m - r(R))^3} - \frac{r(R)^2 r''(R)}{(2m - r(R))^3} + 3\frac{m r(R) r''(R)}{(2m - r(R))^3} + 2\frac{r'(R)^2 m}{(2m - r(R))^3} - 2\frac{r'(R)^2 m^2}{(2m - r(R))^3 r(R)}.$$

$G^{2}{}_{3} = 0.$
$G^{3}{}_{0} = 0.$
$G^{3}{}_{1} = 0.$
$G^{3}{}_{2} = 0.$

$$G^{3}{}_{3} = -2\frac{m^2 r''(R)}{(2m - r(R))^3} - \frac{r(R)^2 r''(R)}{(2m - r(R))^3} + 3\frac{m r(R) r''(R)}{(2m - r(R))^3} + 2\frac{r'(R)^2 m}{(2m - r(R))^3} - 2\frac{r'(R)^2 m^2}{(2m - r(R))^3 r(R)}.$$

$\boxed{G}$

$$G = 24\frac{m^2}{(2m - r(R))^3 r(R)} - 12\frac{m}{(2m - r(R))^3} + 2\frac{r(R)}{(2m - r(R))^3} - 16\frac{m^3}{(2m - r(R))^3 r(R)^2} - 12\frac{m^2 r''(R)}{(2m - r(R))^3} - 4\frac{r(R)^2 r''(R)}{(2m - r(R))^3} + 14\frac{m r(R) r''(R)}{(2m - r(R))^3} - 2\frac{r'(R)^2 r(R)}{(2m - r(R))^3} + 12\frac{r'(R)^2 m}{(2m - r(R))^3} - 12\frac{r'(R)^2 m^2}{(2m - r(R))^3 r(R)}.$$

$\boxed{G^{\mu}{}_{\nu:\mu} = 0}$

$G^{\mu}{}_{0:\mu} = 0.$
$G^{\mu}{}_{1:\mu} = 0.$
$G^{\mu}{}_{2:\mu} = 0.$
$G^{\mu}{}_{3:\mu} = 0.$

$\boxed{g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu} = 0?}$

$g^{\mu\nu}\,\Gamma^{0}_{\mu\nu} = 0.$

$$g^{\mu\nu}\,\Gamma^{1}_{\mu\nu} = -\frac{r'(R) r(R)^4}{2m - r(R)} - \frac{r'(R)\sin(\theta)^4 r(R)^4}{2m - r(R)}.$$

$g^{\mu\nu}\,\Gamma^{2}_{\mu\nu} = \cos(\theta)\sin(\theta)^3 r(R)^2.$
$g^{\mu\nu}\,\Gamma^{3}_{\mu\nu} = 0.$