

Schwarzschild Metric in spherical coordinates with a variable spherically symmetric matter density:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= t. \\ x^1 &= r. \\ x^2 &= \theta. \\ x^3 &= \phi.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= \frac{r-2m}{r}. \\ g_{01} &= 0. \\ g_{02} &= 0. \\ g_{03} &= 0. \\ g_{10} &= 0. \\ g_{11} &= -\frac{r}{r-2m}. \\ g_{12} &= 0. \\ g_{13} &= 0. \\ g_{20} &= 0. \\ g_{21} &= 0. \\ g_{22} &= -r^2. \\ g_{23} &= 0. \\ g_{30} &= 0. \\ g_{31} &= 0. \\ g_{32} &= 0. \\ g_{33} &= -\sin(\theta)^2r^2.\end{aligned}$$

$$\boxed{\sqrt{= \sqrt{-\det(g_{\mu\nu})}}}$$

$$\sqrt{= \sqrt{\sin(\theta)^2r^4}}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= \frac{r}{r-2m}. \\ g^{01} &= 0. \\ g^{02} &= 0. \\ g^{03} &= 0. \\ g^{10} &= 0. \\ g^{11} &= -\frac{r-2m}{r}. \\ g^{12} &= 0. \\ g^{13} &= 0. \\ g^{20} &= 0. \\ g^{21} &= 0. \\ g^{22} &= -\frac{1}{r^2}. \\ g^{23} &= 0. \\ g^{30} &= 0. \\ g^{31} &= 0. \\ g^{32} &= 0. \\ g^{33} &= -\frac{1}{\sin(\theta)^2r^2}.\end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 0. \\ \Gamma^0_{01} &= \frac{m}{(r-2m)r}. \\ \Gamma^0_{02} &= 0. \\ \Gamma^0_{03} &= 0. \\ \Gamma^0_{10} &= -\frac{m}{(r-2m)r}. \\ \Gamma^0_{11} &= 0. \\ \Gamma^0_{12} &= 0. \\ \Gamma^0_{13} &= 0. \\ \Gamma^0_{20} &= 0. \\ \Gamma^0_{21} &= 0. \\ \Gamma^0_{22} &= 0. \\ \Gamma^0_{23} &= 0. \\ \Gamma^0_{30} &= 0. \\ \Gamma^0_{31} &= 0. \\ \Gamma^0_{32} &= 0. \\ \Gamma^0_{33} &= 0.\end{aligned}$$

$$\begin{aligned}\Gamma^1_{00} &= \frac{(r-2m)m}{r^3}. \\ \Gamma^1_{01} &= 0. \\ \Gamma^1_{02} &= 0. \\ \Gamma^1_{03} &= 0. \\ \Gamma^1_{10} &= 0. \\ \Gamma^1_{11} &= -\frac{m}{(r-2m)r}. \\ \Gamma^1_{12} &= 0. \\ \Gamma^1_{13} &= 0. \\ \Gamma^1_{20} &= 0. \\ \Gamma^1_{21} &= 0. \\ \Gamma^1_{22} &= -r+2m. \\ \Gamma^1_{23} &= 0. \\ \Gamma^1_{30} &= 0. \\ \Gamma^1_{31} &= 0. \\ \Gamma^1_{32} &= 0. \\ \Gamma^1_{33} &= -(r-2m)\sin(\theta)^2.\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^2 &= 0, \\ \Gamma_{01}^2 &= 0, \\ \Gamma_{02}^2 &= 0, \\ \Gamma_{03}^2 &= 0, \\ \Gamma_{10}^2 &= 0, \\ \Gamma_{11}^2 &= 0, \\ \Gamma_{12}^2 &= \frac{1}{r}, \\ \Gamma_{13}^2 &= 0, \\ \Gamma_{20}^2 &= 0, \\ \Gamma_{21}^2 &= \frac{1}{r}, \\ \Gamma_{22}^2 &= 0, \\ \Gamma_{23}^2 &= 0, \\ \Gamma_{30}^2 &= 0, \\ \Gamma_{31}^2 &= 0, \\ \Gamma_{32}^2 &= 0, \\ \Gamma_{33}^2 &= -\cos(\theta)\sin(\theta).\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^3 &= 0, \\ \Gamma_{01}^3 &= 0, \\ \Gamma_{02}^3 &= 0, \\ \Gamma_{03}^3 &= 0, \\ \Gamma_{10}^3 &= 0, \\ \Gamma_{11}^3 &= 0, \\ \Gamma_{12}^3 &= 0, \\ \Gamma_{13}^3 &= \frac{1}{r}, \\ \Gamma_{20}^3 &= 0, \\ \Gamma_{21}^3 &= 0, \\ \Gamma_{22}^3 &= 0, \\ \Gamma_{23}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\ \Gamma_{30}^3 &= 0, \\ \Gamma_{31}^3 &= \frac{1}{r}, \\ \Gamma_{32}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\ \Gamma_{33}^3 &= 0.\end{aligned}$$

$$\boxed{R_{\mu\nu}}$$

$$\begin{aligned}R_{00} &= 0, \\ R_{01} &= 0, \\ R_{02} &= 0, \\ R_{03} &= 0, \\ R_{10} &= 0, \\ R_{11} &= 0, \\ R_{12} &= 0, \\ R_{13} &= 0, \\ R_{20} &= 0, \\ R_{21} &= 0, \\ R_{22} &= 0, \\ R_{23} &= 0, \\ R_{30} &= 0, \\ R_{31} &= 0, \\ R_{32} &= 0, \\ R_{33} &= 0.\end{aligned}$$

$$\boxed{R^\mu{}_\nu}$$

$$\begin{aligned}R^0{}_0 &= 0, \\ R^0{}_1 &= 0, \\ R^0{}_2 &= 0, \\ R^0{}_3 &= 0, \\ R^1{}_0 &= 0, \\ R^1{}_1 &= 0, \\ R^1{}_2 &= 0, \\ R^1{}_3 &= 0, \\ R^2{}_0 &= 0, \\ R^2{}_1 &= 0, \\ R^2{}_2 &= 0, \\ R^2{}_3 &= 0, \\ R^3{}_0 &= 0, \\ R^3{}_1 &= 0, \\ R^3{}_2 &= 0, \\ R^3{}_3 &= 0.\end{aligned}$$

$$\boxed{R}$$

$$R=0.$$

$$\boxed{G^\mu{}_\nu}$$

$$\begin{aligned}G^0{}_0 &= 0, \\ G^0{}_1 &= 0, \\ G^0{}_2 &= 0, \\ G^0{}_3 &= 0, \\ G^1{}_0 &= 0, \\ G^1{}_1 &= 0, \\ G^1{}_2 &= 0, \\ G^1{}_3 &= 0, \\ G^2{}_0 &= 0, \\ G^2{}_1 &= 0, \\ G^2{}_2 &= 0, \\ G^2{}_3 &= 0, \\ G^3{}_0 &= 0, \\ G^3{}_1 &= 0, \\ G^3{}_2 &= 0, \\ G^3{}_3 &= 0.\end{aligned}$$

$$\boxed{G}$$

$$G=0.$$

$$\boxed{G^{\mu}_{\nu;\mu} = 0}$$

$$G^{\mu}_{0;\mu} = 0.$$

$$G^{\mu}_{1;\mu} = 0.$$

$$G^{\mu}_{2;\mu} = 0.$$

$$G^{\mu}_{3;\mu} = 0.$$

$$\boxed{g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu} = 0?}$$

$$g^{\mu\nu}\,\Gamma^0_{\mu\nu} = 0.$$

$$g^{\mu\nu}\,\Gamma^1_{\mu\nu} = -6\frac{r^4m}{(r-2m)^2} + 2\frac{m}{(r-2m)^2} + 16\frac{m^5}{(r-2m)^2r^4} + \frac{r^5}{(r-2m)^2} - 6\frac{\sin(\theta)^4r^4m}{(r-2m)^2} + 24\frac{m^3}{(r-2m)^2r^2} + 12\frac{r^3m^2}{(r-2m)^2} - 8\frac{\sin(\theta)^4r^2m^3}{(r-2m)^2} - 8\frac{m^2}{(r-2m)^2r} - 32\frac{m^4}{(r-2m)^2r^3} + 12\frac{\sin(\theta)^4r^3m^2}{(r-2m)^2} + \frac{\sin(\theta)^4r^5}{(r-2m)^2} - 8\frac{r^2m^3}{(r-2m)^2}.$$

$$g^{\mu\nu}\,\Gamma^2_{\mu\nu} = \cos(\theta)\sin(\theta)^3r^2.$$

$$g^{\mu\nu}\,\Gamma^3_{\mu\nu} = 0.$$