axially symmetric photon field:

$\boxed{x^\mu}$

$x^0 = r.$
$x^1 = \theta.$
$x^2 = v.$
$x^3 = u.$

$\boxed{g_{\mu\nu}}$

$g_{00} = -1.$
$g_{01} = 0.$
$g_{02} = 0.$
$g_{03} = 0.$
$g_{10} = 0.$
$g_{11} = -r^2.$
$g_{12} = 0.$
$g_{13} = 0.$
$g_{20} = 0.$
$g_{21} = 0.$
$g_{22} = 0.$
$g_{23} = 1.$
$g_{30} = 0.$
$g_{31} = 0.$
$g_{32} = 1.$
$g_{33} = 2a(r, u).$

$\boxed{\sqrt{} = \sqrt{-\det(g_{\mu\nu})}}$

$\sqrt{} = \sqrt{r^2}.$

$\boxed{g^{\mu\nu}}$

$g^{00} = -1.$
$g^{01} = 0.$
$g^{02} = 0.$
$g^{03} = 0.$
$g^{10} = 0.$
$g^{11} = -\frac{1}{r^2}.$
$g^{12} = 0.$
$g^{13} = 0.$
$g^{20} = 0.$
$g^{21} = 0.$
$g^{22} = -2a(r, u).$
$g^{23} = 1.$
$g^{30} = 0.$
$g^{31} = 0.$
$g^{32} = 1.$
$g^{33} = 0.$

$\boxed{\Gamma^\sigma_{\mu\nu}}$

$\Gamma^0_{00} = 0.$
$\Gamma^0_{01} = 0.$
$\Gamma^0_{02} = 0.$
$\Gamma^0_{03} = 0.$
$\Gamma^0_{10} = 0.$
$\Gamma^0_{11} = -r.$
$\Gamma^0_{12} = 0.$
$\Gamma^0_{13} = 0.$
$\Gamma^0_{20} = 0.$
$\Gamma^0_{21} = 0.$
$\Gamma^0_{22} = 0.$
$\Gamma^0_{23} = 0.$
$\Gamma^0_{30} = 0.$
$\Gamma^0_{31} = 0.$
$\Gamma^0_{32} = 0.$
$\Gamma^0_{33} = \dot{a}(r, u).$

$\Gamma^1_{00} = 0.$
$\Gamma^1_{01} = \frac{1}{r}.$
$\Gamma^1_{02} = 0.$
$\Gamma^1_{03} = 0.$
$\Gamma^1_{10} = \frac{1}{r}.$
$\Gamma^1_{11} = 0.$
$\Gamma^1_{12} = 0.$
$\Gamma^1_{13} = 0.$
$\Gamma^1_{20} = 0.$
$\Gamma^1_{21} = 0.$
$\Gamma^1_{22} = 0.$
$\Gamma^1_{23} = 0.$
$\Gamma^1_{30} = 0.$
$\Gamma^1_{31} = 0.$
$\Gamma^1_{32} = 0.$
$\Gamma^1_{33} = 0.$

$\Gamma^2_{00} = 0.$
$\Gamma^2_{01} = 0.$
$\Gamma^2_{02} = 0.$
$\Gamma^2_{03} = \dot{a}(r,u).$
$\Gamma^2_{10} = 0.$
$\Gamma^2_{11} = 0.$
$\Gamma^2_{12} = 0.$
$\Gamma^2_{13} = 0.$
$\Gamma^2_{20} = 0.$
$\Gamma^2_{21} = 0.$
$\Gamma^2_{22} = 0.$
$\Gamma^2_{23} = 0.$
$\Gamma^2_{30} = \dot{a}(r,u).$
$\Gamma^2_{31} = 0.$
$\Gamma^2_{32} = 0.$
$\Gamma^2_{33} = a'(r,u).$

$\Gamma^3_{00} = 0.$
$\Gamma^3_{01} = 0.$
$\Gamma^3_{02} = 0.$
$\Gamma^3_{03} = 0.$
$\Gamma^3_{10} = 0.$
$\Gamma^3_{11} = 0.$
$\Gamma^3_{12} = 0.$
$\Gamma^3_{13} = 0.$
$\Gamma^3_{20} = 0.$
$\Gamma^3_{21} = 0.$
$\Gamma^3_{22} = 0.$
$\Gamma^3_{23} = 0.$
$\Gamma^3_{30} = 0.$
$\Gamma^3_{31} = 0.$
$\Gamma^3_{32} = 0.$
$\Gamma^3_{33} = 0.$

$\boxed{R_{\mu\nu}}$

$R_{00} = 0.$
$R_{01} = 0.$
$R_{02} = 0.$
$R_{03} = 0.$
$R_{10} = 0.$
$R_{11} = 0.$
$R_{12} = 0.$
$R_{13} = 0.$
$R_{20} = 0.$
$R_{21} = 0.$
$R_{22} = 0.$
$R_{23} = 0.$
$R_{30} = 0.$
$R_{31} = 0.$
$R_{32} = 0.$
$R_{33} = -\frac{\dot{a}(r,u) + \ddot{a}(r,u)r}{r}.$

$\boxed{R^{\mu}_{\nu}}$

$R^0_0 = 0.$
$R^0_1 = 0.$
$R^0_2 = 0.$
$R^0_3 = 0.$
$R^1_0 = 0.$
$R^1_1 = 0.$
$R^1_2 = 0.$
$R^1_3 = 0.$
$R^2_0 = 0.$
$R^2_1 = 0.$
$R^2_2 = 0.$
$R^2_3 = -\frac{\dot{a}(r,u)}{r} - \ddot{a}(r,u).$
$R^3_0 = 0.$
$R^3_1 = 0.$
$R^3_2 = 0.$
$R^3_3 = 0.$

$\boxed{R}$

$R = 0.$

$\boxed{G^{\mu}_{\nu}}$

$G^0_0 = 0.$
$G^0_1 = 0.$
$G^0_2 = 0.$
$G^0_3 = 0.$
$G^1_0 = 0.$
$G^1_1 = 0.$
$G^1_2 = 0.$
$G^1_3 = 0.$
$G^2_0 = 0.$
$G^2_1 = 0.$
$G^2_2 = 0.$
$G^2_3 = -\frac{\dot{a}(r,u)}{r} - \ddot{a}(r,u).$
$G^3_0 = 0.$
$G^3_1 = 0.$
$G^3_2 = 0.$
$G^3_3 = 0.$

$\boxed{G}$

$G = 0.$

$\boxed{G^{\mu}_{\nu;\mu} = 0}$

$G^{\mu}_{0;\mu} = 0.$
$G^{\mu}_{1;\mu} = 0.$
$G^{\mu}_{2;\mu} = 0.$
$G^{\mu}_{3;\mu} = 0.$

$\boxed{g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu} = 0?}$

$g^{\mu\nu}\,\Gamma^{0}_{\mu\nu} = 2a(r,u)\dot{a}(r,u) + r^3.$
$g^{\mu\nu}\,\Gamma^{1}_{\mu\nu} = 0.$
$g^{\mu\nu}\,\Gamma^{2}_{\mu\nu} = 2a(r,u)a'(r,u).$
$g^{\mu\nu}\,\Gamma^{3}_{\mu\nu} = 0.$