axially symmetric photon field:

$\boxed{x^\mu}$

$x^0 = r.$
$x^1 = \theta.$
$x^2 = z.$
$x^3 = t.$

$\boxed{g_{\mu\nu}}$

$g_{00} = -1.$
$g_{01} = 0.$
$g_{02} = 0.$
$g_{03} = 0.$
$g_{10} = 0.$
$g_{11} = -r^2.$
$g_{12} = 0.$
$g_{13} = 0.$
$g_{20} = 0.$
$g_{21} = 0.$
$g_{22} = -1 + a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$g_{23} = -a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$g_{30} = 0.$
$g_{31} = 0.$
$g_{32} = -a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$g_{33} = 1 + a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$

$\boxed{\surd = \sqrt{-\det(g_{\mu\nu})}}$

$\surd = \sqrt{r^2}.$

$\boxed{g^{\mu\nu}}$

$g^{00} = -1.$
$g^{01} = 0.$
$g^{02} = 0.$
$g^{03} = 0.$
$g^{10} = 0.$
$g^{11} = -\frac{1}{r^2}.$
$g^{12} = 0.$
$g^{13} = 0.$
$g^{20} = 0.$
$g^{21} = 0.$
$g^{22} = -1 - a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$g^{23} = -a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$g^{30} = 0.$
$g^{31} = 0.$
$g^{32} = -a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$g^{33} = 1 - a(r, \frac{1}{2}\sqrt{2}(t - z - r)).$

$\boxed{\Gamma^\sigma_{\mu\nu}}$

$\Gamma^0_{00} = 0.$
$\Gamma^0_{01} = 0.$
$\Gamma^0_{02} = 0.$
$\Gamma^0_{03} = 0.$
$\Gamma^0_{10} = 0.$
$\Gamma^0_{11} = -r.$
$\Gamma^0_{12} = 0.$
$\Gamma^0_{13} = 0.$
$\Gamma^0_{20} = 0.$
$\Gamma^0_{21} = 0.$
$\Gamma^0_{22} = \frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t - z - r)) - \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$\Gamma^0_{23} = -\frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t - z - r)) + \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$\Gamma^0_{30} = 0.$
$\Gamma^0_{31} = 0.$
$\Gamma^0_{32} = -\frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t - z - r)) + \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t - z - r)).$
$\Gamma^0_{33} = \frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t - z - r)) - \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t - z - r)).$

$\Gamma^1_{00} = 0.$
$\Gamma^1_{01} = \frac{1}{r}.$
$\Gamma^1_{02} = 0.$
$\Gamma^1_{03} = 0.$
$\Gamma^1_{10} = \frac{1}{r}.$
$\Gamma^1_{11} = 0.$
$\Gamma^1_{12} = 0.$
$\Gamma^1_{13} = 0.$
$\Gamma^1_{20} = 0.$
$\Gamma^1_{21} = 0.$
$\Gamma^1_{22} = 0.$
$\Gamma^1_{23} = 0.$
$\Gamma^1_{30} = 0.$
$\Gamma^1_{31} = 0.$
$\Gamma^1_{32} = 0.$
$\Gamma^1_{33} = 0.$

$\Gamma^2_{00} = 0.$
$\Gamma^2_{01} = 0.$
$\Gamma^2_{02} = -\frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{03} = \frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{10} = 0.$
$\Gamma^2_{11} = 0.$
$\Gamma^2_{12} = 0.$
$\Gamma^2_{13} = 0.$
$\Gamma^2_{20} = -\frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{21} = 0.$
$\Gamma^2_{22} = \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{23} = -\frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{30} = \frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{31} = 0.$
$\Gamma^2_{32} = -\frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^2_{33} = \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$

$\Gamma^3_{00} = 0.$
$\Gamma^3_{01} = 0.$
$\Gamma^3_{02} = -\frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{03} = \frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{10} = 0.$
$\Gamma^3_{11} = 0.$
$\Gamma^3_{12} = 0.$
$\Gamma^3_{13} = 0.$
$\Gamma^3_{20} = -\frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{21} = 0.$
$\Gamma^3_{22} = \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{23} = -\frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{30} = \frac{1}{2}\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{31} = 0.$
$\Gamma^3_{32} = -\frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$
$\Gamma^3_{33} = \frac{1}{4}\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$

$\boxed{R_{\mu\nu}}$

$R_{00} = 0.$
$R_{01} = 0.$
$R_{02} = 0.$
$R_{03} = 0.$
$R_{10} = 0.$
$R_{11} = 0.$
$R_{12} = 0.$
$R_{13} = 0.$
$R_{20} = 0.$
$R_{21} = 0.$
$R_{22} = \frac{1}{4}\frac{2\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}r - 2\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))r - a''(r, \frac{1}{2}\sqrt{2}(t-z-r))r - 2\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$
$R_{23} = -\frac{1}{4}\frac{2\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}r - 2\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))r - a''(r, \frac{1}{2}\sqrt{2}(t-z-r))r - 2\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$
$R_{30} = 0.$
$R_{31} = 0.$
$R_{32} = -\frac{1}{4}\frac{2\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}r - 2\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))r - a''(r, \frac{1}{2}\sqrt{2}(t-z-r))r - 2\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$
$R_{33} = \frac{1}{4}\frac{2\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}r - 2\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))r - a''(r, \frac{1}{2}\sqrt{2}(t-z-r))r - 2\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$

$\boxed{R^{\mu}_{\ \nu}}$

$R^0_{\ 0} = 0.$
$R^0_{\ 1} = 0.$
$R^0_{\ 2} = 0.$
$R^0_{\ 3} = 0.$
$R^1_{\ 0} = 0.$
$R^1_{\ 1} = 0.$
$R^1_{\ 2} = 0.$
$R^1_{\ 3} = 0.$
$R^2_{\ 0} = 0.$
$R^2_{\ 1} = 0.$
$R^2_{\ 2} = -\frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} + \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} + \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$
$R^2_{\ 3} = \frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} - \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} - \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$
$R^3_{\ 0} = 0.$
$R^3_{\ 1} = 0.$
$R^3_{\ 2} = -\frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} + \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} + \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$
$R^3_{\ 3} = \frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} - \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} - \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$

$\boxed{R}$

$R = 0.$

$\boxed{G^{\mu}_{\ \nu}}$

$G^{0}_{\ 0} = 0.$
$G^{0}_{\ 1} = 0.$
$G^{0}_{\ 2} = 0.$
$G^{0}_{\ 3} = 0.$
$G^{1}_{\ 0} = 0.$
$G^{1}_{\ 1} = 0.$
$G^{1}_{\ 2} = 0.$
$G^{1}_{\ 3} = 0.$
$G^{2}_{\ 0} = 0.$
$G^{2}_{\ 1} = 0.$

$$G^{2}_{\ 2} = -\frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} + \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} + \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$$

$$G^{2}_{\ 3} = \frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} - \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} - \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$$

$G^{3}_{\ 0} = 0.$
$G^{3}_{\ 1} = 0.$

$$G^{3}_{\ 2} = -\frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} + \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} + \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$$

$$G^{3}_{\ 3} = \frac{1}{2}\dot{a}'(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2} - \frac{1}{2}\ddot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)) - \frac{1}{2}\frac{\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r} - \frac{1}{4}a''(r, \frac{1}{2}\sqrt{2}(t-z-r)) + \frac{1}{4}\frac{\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r))}{r}.$$

$\boxed{G}$

$G = 0.$

$\boxed{G^{\mu}_{\ \nu:\mu} = 0}$

$G^{\mu}_{\ 0:\mu} = 0.$
$G^{\mu}_{\ 1:\mu} = 0.$
$G^{\mu}_{\ 2:\mu} = 0.$
$G^{\mu}_{\ 3:\mu} = 0.$

$\boxed{g^{\mu\nu}\,\Gamma^{\lambda}_{\ \mu\nu} = 0?}$

$$g^{\mu\nu}\,\Gamma^{0}_{\ \mu\nu} = r^3 - a(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)) + 2a(r, \frac{1}{2}\sqrt{2}(t-z-r))\dot{a}(r, \frac{1}{2}\sqrt{2}(t-z-r)).$$

$g^{\mu\nu}\,\Gamma^{1}_{\ \mu\nu} = 0.$

$$g^{\mu\nu}\,\Gamma^{2}_{\ \mu\nu} = a(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$$

$$g^{\mu\nu}\,\Gamma^{3}_{\ \mu\nu} = a(r, \frac{1}{2}\sqrt{2}(t-z-r))\sqrt{2}a'(r, \frac{1}{2}\sqrt{2}(t-z-r)).$$