Robertson-Walker-Schwarzschild Metric with c(t) in cartesian coordinates:

x^{μ} $x^0 = t.$ $x^1 = x$. $x^2 = y.$ $x^3 = z.$ $g_{\mu u}$ $g_{00} = c(t)^2.$ $g_{01}=0.$ $g_{02} = 0.$ $g_{03}=0.$ $g_{10}=0.$ $g_{11} = -\frac{R(t)^2(-1+z^2\kappa+y^2\kappa)}{-1+z^2\kappa+x^2\kappa+y^2\kappa}.$ $g_{12} = \frac{yxR(t)^2\kappa}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}.$ $g_{13} = \frac{zxR(t)^2\kappa}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}.$ $g_{20}=0.$ $g_{21} = \frac{yxR(t)^2\kappa}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}.$ $g_{22} = -\frac{(-1 + z^2\kappa + x^2\kappa)R(t)^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}.$ $g_{23} = \frac{zyR(t)^2\kappa}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}.$ $g_{30}=0.$ $g_{31} = \frac{zxR(t)^2 \kappa}{-1 + z^2 \kappa + x^2 \kappa + y^2 \kappa}.$ $g_{32} = \frac{zyR(t)^2 \kappa}{-1 + z^2 \kappa + x^2 \kappa + y^2 \kappa}.$ $g_{33} = -\frac{(-1 + x^2 \kappa + y^2 \kappa) R(t)^2}{-1 + z^2 \kappa + x^2 \kappa + y^2 \kappa}.$ $\sqrt{-\det(g_{\mu\nu})}$ $g^{23} = \frac{zy\kappa}{R(t)^2}$ $g^{30} = 0.$ $g^{31} = \frac{zx\kappa}{R(t)^2}$ $g^{32} = \frac{zy\kappa}{R(t)^2}$ $g^{33} = \frac{-1 + z^2\kappa}{R(t)^2}$ $\Gamma^0_{00} = \frac{\dot{c}(t)}{c(t)}$ $\Gamma^0_{00} = 0$ $\Gamma^{0}_{01} = 0.$ $\Gamma^{0}_{02} = 0.$ $\Gamma^{0}_{03} = 0.$ $\Gamma_{10}^{0} = 0.$ $\Gamma_{11}^{0} = \frac{\dot{R}(t)R(t)(-1+z^{2}\kappa+y^{2}\kappa)}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma^{0}_{12} = -\frac{yx\dot{R}(t)R(t)\kappa}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma^{0}_{13} = -\frac{zx\dot{R}(t)R(t)\kappa}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma_{20}^0 = 0.$ $\Gamma^{0}_{21} = -\frac{yx\dot{R}(t)R(t)\kappa}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma^{0}_{22} = \frac{(-1 + z^{2}\kappa + x^{2}\kappa)\dot{R}(t)R(t)}{(-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa)c(t)^{2}}.$ $\Gamma^{0}_{23} = -\frac{zy\dot{R}(t)R(t)\kappa}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma_{30}^0 = 0.$ $\Gamma^{0}_{31} = -\frac{zx\dot{R}(t)R(t)\kappa}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma^{0}_{32} = -\frac{zy\dot{R}(t)R(t)\kappa}{(-1+z^{2}\kappa+x^{2}\kappa+y^{2}\kappa)c(t)^{2}}.$ $\Gamma^{0}_{33} = \frac{(-1 + x^{2}\kappa + y^{2}\kappa)\dot{R}(t)R(t)}{(-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa)c(t)^{2}}.$

 $\Gamma^{1}_{00} = 0.$ $\Gamma^{1}_{01} = \frac{\dot{R}(t)}{R(t)}.$ $\Gamma^{1}_{02} = 0.$ $\Gamma^{1}_{03} = 0.$
$$\begin{split} &\Gamma_{10}^{1} = 0. \\ &\Gamma_{10}^{1} = \frac{\dot{R}(t)}{R(t)}. \\ &\Gamma_{11}^{1} = \frac{z^{2}x\kappa^{2} - x\kappa + y^{2}x\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{12}^{1} = -\frac{yx^{2}\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{13}^{1} = -\frac{zx^{2}\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{20}^{1} = 0. \\ &\Gamma_{21}^{1} = -\frac{yx^{2}\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{22}^{1} = \frac{\dot{z}^{2}x\kappa^{2} - x\kappa + x^{3}\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{31}^{1} = -\frac{zyx\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{31}^{1} = 0. \\ &\Gamma_{31}^{1} = -\frac{zx^{2}\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{32}^{1} = -\frac{zy\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{33}^{1} = -\frac{x\kappa - x^{3}\kappa^{2} - y^{2}x\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \\ &\Gamma_{33}^{1} = -\frac{x\kappa - x^{3}\kappa^{2} - y^{2}x\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}. \end{split}$$
 $\Gamma_{00}^2 = 0.$ $\Gamma_{01}^2 = 0.$ $\Gamma_{02}^2 = \frac{\dot{R}(t)}{R(t)}.$ $\Gamma_{03}^2 = 0.$ $\Gamma_{10}^2 = 0.$
$$\begin{split} &\Gamma_{11}^2 = 0. \\ &\Gamma_{11}^2 = -\frac{y\kappa - z^2y\kappa^2 - y^3\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{12}^2 = -\frac{y^2x\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{13}^2 = -\frac{zyx\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{20}^2 = \frac{\dot{R}(t)}{R(t)}. \\ &\Gamma_{21}^2 = -\frac{y^2x\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{22}^2 = -\frac{y\kappa - z^2y\kappa^2 - yx^2\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{23}^2 = -\frac{zy^2\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{30}^2 = 0. \\ &\Gamma_{31}^2 = -\frac{zyx\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{32}^2 = -\frac{zy^2\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{33}^2 = -\frac{y\kappa - yx^2\kappa^2 - y^3\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \\ &\Gamma_{33}^2 = -\frac{y\kappa - yx^2\kappa^2 - y^3\kappa^2}{-1 + z^2\kappa + x^2\kappa + y^2\kappa}. \end{split}$$
 $\Gamma_{00}^{3} = 0.$ $\Gamma_{01}^{3} = 0.$ $\Gamma_{02}^{3} = 0.$ $\Gamma_{03}^{3} = \frac{\dot{R}(t)}{R(t)}.$ $\Gamma_{10}^{3} = 0.$ $\Gamma_{11}^{3} = \frac{zy^{2}\kappa^{2} + z^{3}\kappa^{2} - z\kappa}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{12}^{3} = -\frac{zyx\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{13}^{3} = -\frac{z^{2}x\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{20}^{3} = 0.$ $\Gamma_{21}^{3} = -\frac{zyx\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{22}^{3} = \frac{z^{3}\kappa^{2} + zx^{2}\kappa^{2} - z\kappa}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{23}^{3} = -\frac{z^{2}y\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{30}^{3} = \frac{\dot{R}(t)}{R(t)}.$ $\Gamma_{31}^{3} = -\frac{z^{2}x\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{32}^{3} = -\frac{z^{2}x\kappa^{2}}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{33}^{3} = \frac{\dot{R}(t)}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{33}^{3} = \frac{z^{2}x\kappa^{2} - z\kappa}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{33}^{3} = \frac{z^{2}x\kappa^{2} - z\kappa}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$ $\Gamma_{33}^{3} = \frac{z^{2}x\kappa^{2} - z\kappa}{-1 + z^{2}\kappa + x^{2}\kappa + y^{2}\kappa}.$

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R_{00} = -3\frac{\dot{c}(t)\dot{R}(t) - \ddot{R}(t)c(t)}{2}
          R_{02}=0.
          R_{03} = 0.
          R_{10} = 0.
          (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
                                               \dot{c}(t)yx\dot{R}(t)R(t)\kappa - \ddot{R}(t)yxR(t)\kappa c(t) - 2yx\kappa^2c(t)^3 - 2yx\dot{R}(t)^2\kappa c(t)
                                                                                                                           (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
                                     2zx\dot{R}(t)^{2}\kappa c(t) + 2zx\kappa^{2}c(t)^{3} + \ddot{R}(t)zxR(t)\kappa c(t) - \dot{c}(t)zx\dot{R}(t)R(t)\kappa
                                                                                                                    (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
          R_{20}=0.
                                               \dot{c}(t)yx\dot{R}(t)R(t)\kappa - \ddot{R}(t)yxR(t)\kappa c(t) - 2yx\kappa^2c(t)^3 - 2yx\dot{R}(t)^2\kappa c(t)
                                                                                                                           (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
                                               2x^2\dot{R}(t)^2\kappa c(t) + 2x^2\kappa^2 c(t)^3 + \ddot{R}(t)z^2R(t)\kappa c(t) + \dot{c}(t)\dot{R}(t)R(t) + 2z^2\kappa^2 c(t)^3 - \dot{c}(t)x^2\dot{R}(t)R(t)\kappa + 2z^2\dot{R}(t)^2\kappa c(t) - \ddot{R}(t)R(t)c(t) - 2\dot{R}(t)^2c(t) - \dot{c}(t)z^2\dot{R}(t)R(t)\kappa c(t) - 2\kappa c(t)^3 + \dot{R}(t)z^2R(t)\kappa c(t) + \dot{R
                                                                                                                                                                                                                                                                                                                                                                                    (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
                                        2zy\kappa^2c(t)^3 + 2zy\dot{R}(t)^2\kappa c(t) + \ddot{R}(t)zyR(t)\kappa c(t) - \dot{c}(t)zy\dot{R}(t)R(t)\kappa
                                                                                                                 (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
          R_{30} = 0.
                                     2zx\dot{R}(t)^{2}\kappa c(t) + 2zx\kappa^{2}c(t)^{3} + \ddot{R}(t)zxR(t)\kappa c(t) - \dot{c}(t)zx\dot{R}(t)R(t)\kappa
                                                                                                                 (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
                                     -\frac{2zy\kappa^2c(t)^3+2zy\dot{R}(t)^2\kappa c(t)+\ddot{R}(t)zyR(t)\kappa c(t)-\dot{c}(t)zy\dot{R}(t)R(t)\kappa}{2zy\kappa^2c(t)^3+2zy\dot{R}(t)^2\kappa c(t)+\ddot{R}(t)zyR(t)\kappa c(t)-\dot{c}(t)zy\dot{R}(t)R(t)\kappa}
                                                                                                                 (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
                                               2x^2\dot{R}(t)^2\kappa c(t) + 2x^2\kappa^2 c(t)^3 - \dot{c}(t)y^2\dot{R}(t)R(t)\kappa + \dot{c}(t)\dot{R}(t)R(t) + 2y^2\dot{R}(t)^2\kappa c(t) - \dot{c}(t)x^2\dot{R}(t)R(t)\kappa + 2y^2\kappa^2 c(t)^3 - \ddot{R}(t)R(t)c(t) - 2\dot{R}(t)^2c(t) + \ddot{R}(t)y^2R(t)\kappa c(t) + \ddot{R}(t)x^2R(t)\kappa c(t) - 2\kappa c(t)^3 - \ddot{R}(t)R(t)\kappa c(t) + \ddot{R}(t)x^2R(t)\kappa c(t) + \ddot{R}(t)x^2R
                                                                                                                                                                                                                                                                                                                                                                                  (-1+z^2\kappa+x^2\kappa+y^2\kappa)c(t)^3
R^{\mu}_{\ \nu}
          R_0^0 = 3\frac{\ddot{R}(t)}{R(t)c(t)^2} - 3\frac{\dot{c}(t)\dot{R}(t)}{R(t)c(t)^3}
       R_2^0 = 0.
          R_{1}^{1} = \frac{\ddot{R}(t)}{R(t)c(t)^{2}} + 2\frac{\dot{R}(t)^{2}}{R(t)^{2}c(t)^{2}} - \frac{\dot{c}(t)\dot{R}(t)}{R(t)c(t)^{3}} + 2\frac{\kappa}{R(t)^{2}}.
       R^{1}_{2} = 0.
R^{1}_{3} = 0.
R^{2}_{0} = 0.
          R_2^2 = \frac{\ddot{R}(t)}{R(t)c(t)^2} + 2\frac{\dot{R}(t)^2}{R(t)^2c(t)^2} - \frac{\dot{c}(t)\dot{R}(t)}{R(t)c(t)^3} + 2\frac{\kappa}{R(t)^2}.
          R_3^3 = \frac{\ddot{R}(t)}{R(t)c(t)^2} + 2\frac{\dot{R}(t)^2}{R(t)^2c(t)^2} - \frac{\dot{c}(t)\dot{R}(t)}{R(t)c(t)^3} + 2\frac{\kappa}{R(t)^2}.
      G^0_{\ 0} = -3\frac{\dot{R}(t)^2 + \kappa c(t)^2}{R(t)^2 c(t)^2}.
       G_1^0 = 0.
       G_2^0 = 0.
          G_3^0 = 0.
          G_0^1 = 0.
       G_{1}^{1} = \frac{2\dot{c}(t)\dot{R}(t)R(t) - 2\ddot{R}(t)R(t)c(t) - \dot{R}(t)^{2}c(t) - \kappa c(t)^{3}}{2}
       G_2^1 = 0.
       G_3^1 = 0.
       G_0^2 = 0.
          G_1^2 = 0.
      G_{2}^{2} = \frac{2\dot{c}(t)\dot{R}(t)R(t) - 2\ddot{R}(t)R(t)c(t) - \dot{R}(t)^{2}c(t) - \kappa c(t)^{3}}{2}
          G_3^2 = 0.
          G_0^3 = 0.
       G_1^3 = 0.
          G_{2}^{3}=0.
       G_3^3 = \frac{2\dot{c}(t)\dot{R}(t)R(t) - 2\ddot{R}(t)R(t)c(t) - \dot{R}(t)^2c(t) - \kappa c(t)^3}{R(t)^2c(t)^3}.
\boxed{G^{\mu}_{\ \nu:\mu}=0}
       G^{\mu}_{1:\mu} = 0.
       G^{\mu}_{2:\mu} = 0.
     G^{\mu}_{3:\mu} = 0.
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