

Robertson-Walker Metric of a flat universe in Cartesian coordinates:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= t. \\ x^1 &= x. \\ x^2 &= y. \\ x^3 &= z.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= c^2. \\ g_{01} &= 0. \\ g_{02} &= 0. \\ g_{03} &= 0. \\ g_{10} &= 0. \\ g_{11} &= -\frac{a(t)^2(-1+kx^2+kz^2+ky^2)}{-1+kx^2+kz^2+ky^2}. \\ g_{12} &= \frac{k y x a(t)^2}{-1+kx^2+kz^2+ky^2}. \\ g_{13} &= \frac{k z x a(t)^2}{-1+kx^2+kz^2+ky^2}. \\ g_{20} &= 0. \\ g_{21} &= \frac{k y x a(t)^2}{-1+kx^2+kz^2+ky^2}. \\ g_{22} &= -\frac{a(t)^2(-1+kx^2+kz^2)}{-1+kx^2+kz^2+ky^2}. \\ g_{23} &= \frac{k z y a(t)^2}{-1+kx^2+kz^2+ky^2}. \\ g_{30} &= 0. \\ g_{31} &= \frac{k z x a(t)^2}{-1+kx^2+kz^2+ky^2}. \\ g_{32} &= \frac{k z y a(t)^2}{-1+kx^2+kz^2+ky^2}. \\ g_{33} &= -\frac{(-1+kx^2+ky^2)a(t)^2}{-1+kx^2+kz^2+ky^2}.\end{aligned}$$

$$\boxed{\sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{-\det(g_{\mu\nu})} = \sqrt{-\frac{a(t)^6c^2}{-1+kx^2+kz^2+ky^2}}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= \frac{1}{c^2}. \\ g^{01} &= 0. \\ g^{02} &= 0. \\ g^{03} &= 0. \\ g^{10} &= 0. \\ g^{11} &= \frac{-1+kx^2}{a(t)^2}. \\ g^{12} &= \frac{k y x}{a(t)^2}. \\ g^{13} &= \frac{k z x}{a(t)^2}. \\ g^{20} &= 0. \\ g^{21} &= \frac{k y x}{a(t)^2}. \\ g^{22} &= \frac{-1+ky^2}{a(t)^2}. \\ g^{23} &= \frac{k z y}{a(t)^2}. \\ g^{30} &= 0. \\ g^{31} &= \frac{k z x}{a(t)^2}. \\ g^{32} &= \frac{k z y}{a(t)^2}. \\ g^{33} &= \frac{-1+kz^2}{a(t)^2}.\end{aligned}$$

$$\boxed{\Gamma^\sigma{}_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 0. \\ \Gamma^0_{01} &= 0. \\ \Gamma^0_{02} &= 0. \\ \Gamma^0_{03} &= 0. \\ \Gamma^0_{10} &= 0. \\ \Gamma^0_{11} &= \frac{\dot{a}(t)a(t)(-1+kz^2+ky^2)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{12} &= -\frac{k\dot{a}(t)yxa(t)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{13} &= -\frac{k\dot{a}(t)zxa(t)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{20} &= 0. \\ \Gamma^0_{21} &= -\frac{k\dot{a}(t)yxa(t)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{22} &= \frac{\dot{a}(t)a(t)(-1+kx^2+kz^2)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{23} &= -\frac{k\dot{a}(t)zya(t)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{30} &= 0. \\ \Gamma^0_{31} &= -\frac{k\dot{a}(t)zxa(t)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{32} &= -\frac{k\dot{a}(t)zya(t)}{(-1+kx^2+kz^2+ky^2)c^2}. \\ \Gamma^0_{33} &= \frac{\dot{a}(t)(-1+kx^2+ky^2)a(t)}{(-1+kx^2+kz^2+ky^2)c^2}.\end{aligned}$$

$$\begin{aligned}
\Gamma_{00}^1 &= 0, \\
\Gamma_{11}^1 &= \frac{\dot{a}(t)}{a(t)}, \\
\Gamma_{22}^1 &= 0, \\
\Gamma_{33}^1 &= 0, \\
\Gamma_{10}^1 &= \frac{\dot{a}(t)}{a(t)}, \\
\Gamma_{11}^1 &= -\frac{kx - k^2y^2x - k^2z^2x}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{12}^1 &= -\frac{k^2yx^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{13}^1 &= -\frac{k^2zx^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{20}^1 &= 0, \\
\Gamma_{21}^1 &= -\frac{k^2yx^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{22}^1 &= -\frac{kx - k^2x^3 - k^2z^2x}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{23}^1 &= -\frac{k^2zyx}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{30}^1 &= 0, \\
\Gamma_{31}^1 &= -\frac{k^2zx^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{32}^1 &= -\frac{k^2zyx}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{33}^1 &= -\frac{kx - k^2y^2x - k^2x^3}{-1 + kx^2 + kz^2 + ky^2},
\end{aligned}$$

$$\begin{aligned}
\Gamma_{00}^2 &= 0, \\
\Gamma_{11}^2 &= 0, \\
\Gamma_{22}^2 &= \frac{\dot{a}(t)}{a(t)}, \\
\Gamma_{33}^2 &= 0, \\
\Gamma_{10}^2 &= 0, \\
\Gamma_{11}^2 &= \frac{k^2z^2y + k^2y^3 - ky}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{12}^2 &= -\frac{k^2y^2x}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{13}^2 &= -\frac{k^2zyx}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{20}^2 &= \frac{\dot{a}(t)}{a(t)}, \\
\Gamma_{21}^2 &= -\frac{k^2y^2x}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{22}^2 &= \frac{k^2yx^2 + k^2z^2y - ky}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{23}^2 &= -\frac{k^2zy^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{30}^2 &= 0, \\
\Gamma_{31}^2 &= -\frac{k^2zyx}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{32}^2 &= -\frac{k^2zy^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{33}^2 &= \frac{k^2yx^2 + k^2y^3 - ky}{-1 + kx^2 + kz^2 + ky^2},
\end{aligned}$$

$$\begin{aligned}
\Gamma_{00}^3 &= 0, \\
\Gamma_{01}^3 &= 0, \\
\Gamma_{02}^3 &= 0, \\
\Gamma_{03}^3 &= \frac{\dot{a}(t)}{a(t)}, \\
\Gamma_{10}^3 &= 0, \\
\Gamma_{11}^3 &= \frac{k^2z^3 + k^2zy^2 - kz}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{12}^3 &= -\frac{k^2zyx}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{13}^3 &= -\frac{k^2z^2x}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{20}^3 &= 0, \\
\Gamma_{21}^3 &= -\frac{k^2zyx}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{22}^3 &= \frac{k^2z^3 + k^2zx^2 - kz}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{23}^3 &= -\frac{k^2z^2y}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{30}^3 &= \frac{\dot{a}(t)}{a(t)}, \\
\Gamma_{31}^3 &= -\frac{k^2z^2x}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{32}^3 &= -\frac{k^2z^2y}{-1 + kx^2 + kz^2 + ky^2}, \\
\Gamma_{33}^3 &= \frac{k^2zx^2 + k^2zy^2 - kz}{-1 + kx^2 + kz^2 + ky^2}.
\end{aligned}$$

$$\hat{x}^\mu = \left(\Gamma_{\sigma\rho}^0 \dot{x}^\mu - \Gamma_{\sigma\rho}^\mu \right) \dot{x}^\sigma \dot{x}^\rho$$

$$\begin{aligned}
\ddot{x}^0 &= 0, \\
\ddot{x}^1 &= \frac{k\dot{a}(t)y^2a(t)^2\dot{x}^3 + k\dot{a}(t)z^2x^2a(t)^2\dot{x} - k^2z^2x^3a(t)c^2 - k^2z^2y^2xa(t)c^2 - k^2y^2xa(t)c^2\dot{x}^2 + 2k\dot{a}(t)z^2a(t)c^2\dot{x} + kiz^2xa(t)c^2 - 2k\dot{a}(t)y^2c^2\dot{x} + kxa(t)c^2\dot{x}^2 + 2\dot{a}(t)c^2\dot{x} + k\dot{a}(t)\dot{z}^2y^2a(t)^2\dot{x} - 2k\dot{a}(t)z^2c^2\dot{x} - k^2z^2xa(t)c^2\dot{x}^2 - \dot{a}(t)a(t)^2\dot{x}^3 - 2k\dot{a}(t)\dot{z}zxa(t)^2\dot{x}^2 - 2k\dot{a}(t)x^2c^2\dot{x} - \dot{a}(t)\dot{z}^2a(t)^2\dot{x} + k\dot{a}(t)z^2a(t)^2\dot{x}^3}{(-1 + kx^2 + kz^2 + ky^2)a(t)c^2}, \\
\ddot{x}^2 &= \frac{k^2z^2y\dot{x}^2 - k\dot{z}^2y - 2k\dot{z}^2zyx\dot{x} + k^2\dot{z}^2y^3 - ky\dot{x}^2 + k^2y^3\dot{x}^2 + k^2z^2yx^2}{-1 + kx^2 + kz^2 + ky^2}, \\
\ddot{x}^3 &= -\frac{\dot{a}(t)\dot{z}^3a(t)^2 - k\dot{a}(t)\dot{z}^3x^2a(t)^2 + k^2z^3a(t)c^2\dot{x}^2 - k\dot{z}^2za(t)c^2 + k^2\dot{z}^2zy^2a(t)c^2 + k^2zy^2a(t)c^2\dot{x}^2 - k\dot{a}(t)\dot{z}y^2a(t)^2\dot{x}^2 + \dot{a}(t)\dot{z}a(t)^2\dot{x}^2 + 2k\dot{a}(t)\dot{z}x^2c^2 + 2k\dot{a}(t)\dot{z}^2zxa(t)^2\dot{x} + 2k\dot{a}(t)\dot{z}y^2c^2 + k^2\dot{z}^2zx^2a(t)c^2 - k\dot{a}(t)\dot{z}z^2a(t)^2\dot{x}^2 - 2\dot{a}(t)\dot{z}c^2 - k\dot{a}(t)\dot{z}^3y^2a(t)^2 + 2k\dot{a}(t)\dot{z}z^2c^2 - kza(t)c^2\dot{x}^2 - 2k^2\dot{z}z^2xa(t)c^2\dot{x}}{(-1 + kx^2 + kz^2 + ky^2)a(t)c^2}.
\end{aligned}$$

$$R_{\mu\nu}$$

$$\begin{aligned} R_{00} &= 3\frac{\ddot{a}(t)}{a(t)}, \\ R_{01} &= 0, \\ R_{02} &= 0, \\ R_{03} &= 0, \\ R_{10} &= 0, \\ R_{11} &= \frac{2kc^2-2k\dot{a}(t)^2z^2-2k\dot{a}(t)^2y^2+2\dot{a}(t)^2-k\ddot{a}(t)y^2a(t)-2k^2y^2c^2-kz^2\ddot{a}(t)a(t)+\ddot{a}(t)a(t)-2k^2z^2c^2}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{12} &= \frac{2k\dot{a}(t)^2yx+k\ddot{a}(t)yx a(t)+2k^2yxc^2}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{13} &= \frac{2k^2zxc^2+kz\ddot{a}(t)xa(t)+2k\dot{a}(t)^2zx}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{20} &= 0, \\ R_{21} &= \frac{2k\dot{a}(t)^2yx+k\ddot{a}(t)yx a(t)+2k^2yxc^2}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{22} &= \frac{2kc^2-2k\dot{a}(t)^2z^2-2k^2x^2c^2-k\ddot{a}(t)x^2a(t)+2\dot{a}(t)^2-2k\dot{a}(t)^2x^2-kz^2\ddot{a}(t)a(t)+\ddot{a}(t)a(t)-2k^2z^2c^2}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{23} &= \frac{kz\ddot{a}(t)ya(t)+2k\dot{a}(t)^2zy+2k^2zyc^2}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{30} &= 0, \\ R_{31} &= \frac{2k^2zxc^2+kz\ddot{a}(t)xa(t)+2k\dot{a}(t)^2zx}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{32} &= \frac{kz\ddot{a}(t)ya(t)+2k\dot{a}(t)^2zy+2k^2zyc^2}{(-1+kx^2+kz^2+ky^2)c^2}, \\ R_{33} &= \frac{2kc^2-2k^2x^2c^2-k\ddot{a}(t)x^2a(t)-2k\dot{a}(t)^2y^2+2\dot{a}(t)^2-k\ddot{a}(t)y^2a(t)-2k^2y^2c^2-2k\dot{a}(t)^2x^2+\ddot{a}(t)a(t)}{(-1+kx^2+kz^2+ky^2)c^2}, \end{aligned}$$

$$R^\mu{}_\nu$$

$$\begin{aligned} R^0_0 &= 3\frac{\ddot{a}(t)}{a(t)c^2}, \\ R^0_1 &= 0, \\ R^0_2 &= 0, \\ R^0_3 &= 0, \\ R^1_0 &= 0, \\ R^1_1 &= \frac{\ddot{a}(t)}{a(t)c^2}+2\frac{k}{a(t)^2}+2\frac{\dot{a}(t)^2}{a(t)^2c^2}, \\ R^1_2 &= 0, \\ R^1_3 &= 0, \\ R^2_0 &= 0, \\ R^2_1 &= 0, \\ R^2_2 &= \frac{\ddot{a}(t)}{a(t)c^2}+2\frac{k}{a(t)^2}+2\frac{\dot{a}(t)^2}{a(t)^2c^2}, \\ R^2_3 &= 0, \\ R^3_0 &= 0, \\ R^3_1 &= 0, \\ R^3_2 &= 0, \\ R^3_3 &= \frac{\ddot{a}(t)}{a(t)c^2}+2\frac{k}{a(t)^2}+2\frac{\dot{a}(t)^2}{a(t)^2c^2}, \end{aligned}$$

$$R$$

$$R=6\frac{\ddot{a}(t)}{a(t)c^2}+6\frac{k}{a(t)^2}+6\frac{\dot{a}(t)^2}{a(t)^2c^2},$$

$$G^\mu{}_\nu$$

$$\begin{aligned} G^0_0 &= -3\frac{kc^2+\dot{a}(t)^2}{a(t)^2c^2}, \\ G^0_1 &= 0, \\ G^0_2 &= 0, \\ G^0_3 &= 0, \\ G^1_0 &= 0, \\ G^1_1 &= -\frac{kc^2+\dot{a}(t)^2+2\ddot{a}(t)a(t)}{a(t)^2c^2}, \\ G^1_2 &= 0, \\ G^1_3 &= 0, \\ G^2_0 &= 0, \\ G^2_1 &= 0, \\ G^2_2 &= -\frac{kc^2+\dot{a}(t)^2+2\ddot{a}(t)a(t)}{a(t)^2c^2}, \\ G^2_3 &= 0, \\ G^3_0 &= 0, \\ G^3_1 &= 0, \\ G^3_2 &= 0, \\ G^3_3 &= -\frac{kc^2+\dot{a}(t)^2+2\ddot{a}(t)a(t)}{a(t)^2c^2}, \end{aligned}$$

$$G$$

$$G=-6\frac{\ddot{a}(t)}{a(t)c^2}-6\frac{k}{a(t)^2}-6\frac{\dot{a}(t)^2}{a(t)^2c^2}.$$

$$G^\mu{}_{\nu;\mu}=0$$

$$\begin{aligned} G^a{}_{0;a} &= 0, \\ G^{\mu}{}_{1;\mu} &= 0, \\ G^a{}_{2;a} &= 0, \\ G^a{}_{3;a} &= 0. \end{aligned}$$

$$g^{\mu\nu}\,\Gamma^\lambda_{\mu\nu}=0?$$

$$\begin{aligned} g^{\mu\nu}\,\Gamma^0_{\mu\nu} &= -4\frac{k^2\dot{a}(t)y^2x^2a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}-2\frac{k^2\dot{a}(t)x^4a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}-3\frac{\dot{a}(t)a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}-2\frac{k^2\dot{a}(t)y^4a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}+4\frac{k\ddot{a}(t)z^2a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}+4\frac{k\dot{a}(t)y^2a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}-2\frac{k^2\dot{a}(t)z^4a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}-4\frac{k^2\dot{a}(t)z^2y^2a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}+4\frac{k\dot{a}(t)x^2a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}-4\frac{k^2\dot{a}(t)z^2x^2a(t)^3}{(-1+kx^2+kz^2+ky^2)^2c^2}, \\ g^{\mu\nu}\,\Gamma^1_{\mu\nu} &= -2\frac{k^3x^5a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-2\frac{k^3z^4xa(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3y^2x^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-3\frac{kxa(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2z^2xa(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2y^2xa(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^2y^2xa(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^2x^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-2\frac{k^3y^4xa(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2x^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}, \\ g^{\mu\nu}\,\Gamma^2_{\mu\nu} &= -2\frac{k^3yx^4a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^2y^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2y^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-3\frac{kya(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-2\frac{k^3z^4ya(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-2\frac{k^3y^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2z^2ya(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2yx^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3y^3x^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^2yx^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^2y^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}, \\ g^{\mu\nu}\,\Gamma^3_{\mu\nu} &= -2\frac{k^3zy^4a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^3x^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2zx^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3zy^2x^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-2\frac{k^3z^5a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-3\frac{kza(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2z^3a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}+4\frac{k^2zy^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-2\frac{k^3zx^4a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}-4\frac{k^3z^3y^2a(t)^2}{(-1+kx^2+kz^2+ky^2)^2}. \end{aligned}$$