

Schwarzschild Metric in spherical coordinates in its flatter form:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= T, \\ x^1 &= R, \\ x^2 &= \theta, \\ x^3 &= \phi.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= \frac{r(R)}{r(R)-2m}, \\ g_{01} &= 0, \\ g_{02} &= 0, \\ g_{03} &= 0, \\ g_{10} &= 0, \\ g_{11} &= -\frac{r(R)-2m}{r(R)}, \\ g_{12} &= 0, \\ g_{13} &= 0, \\ g_{20} &= 0, \\ g_{21} &= 0, \\ g_{22} &= -r(R)^2, \\ g_{23} &= 0, \\ g_{30} &= 0, \\ g_{31} &= 0, \\ g_{32} &= 0, \\ g_{33} &= -r(R)^2\sin(\theta)^2.\end{aligned}$$

$$\boxed{\sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{-} = \sqrt{r(R)^4\sin(\theta)^2}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= \frac{r(R)-2m}{r(R)}, \\ g^{01} &= 0, \\ g^{02} &= 0, \\ g^{03} &= 0, \\ g^{10} &= 0, \\ g^{11} &= -\frac{r(R)}{r(R)-2m}, \\ g^{12} &= 0, \\ g^{13} &= 0, \\ g^{20} &= 0, \\ g^{21} &= 0, \\ g^{22} &= -\frac{1}{r(R)^2}, \\ g^{23} &= 0, \\ g^{30} &= 0, \\ g^{31} &= 0, \\ g^{32} &= 0, \\ g^{33} &= -\frac{1}{r(R)^2\sin(\theta)^2}.\end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 0, \\ \Gamma^0_{01} &= -\frac{r'(R)m}{r(R)(r(R)-2m)}, \\ \Gamma^0_{02} &= 0, \\ \Gamma^0_{03} &= 0, \\ \Gamma^0_{10} &= -\frac{r'(R)m}{r(R)(r(R)-2m)}, \\ \Gamma^0_{11} &= 0, \\ \Gamma^0_{12} &= 0, \\ \Gamma^0_{13} &= 0, \\ \Gamma^0_{20} &= 0, \\ \Gamma^0_{21} &= 0, \\ \Gamma^0_{22} &= 0, \\ \Gamma^0_{23} &= 0, \\ \Gamma^0_{30} &= 0, \\ \Gamma^0_{31} &= 0, \\ \Gamma^0_{32} &= 0, \\ \Gamma^0_{33} &= 0,\end{aligned}$$

$$\begin{aligned}\Gamma^1_{00} &= -\frac{r'(R)r(R)m}{(r(R)-2m)^3}, \\ \Gamma^1_{01} &= 0, \\ \Gamma^1_{02} &= 0, \\ \Gamma^1_{03} &= 0, \\ \Gamma^1_{10} &= 0, \\ \Gamma^1_{11} &= \frac{r'(R)m}{r(R)(r(R)-2m)}, \\ \Gamma^1_{12} &= 0, \\ \Gamma^1_{13} &= 0, \\ \Gamma^1_{20} &= 0, \\ \Gamma^1_{21} &= 0, \\ \Gamma^1_{22} &= -\frac{r'(R)r(R)^2}{r(R)-2m}, \\ \Gamma^1_{23} &= 0, \\ \Gamma^1_{30} &= 0, \\ \Gamma^1_{31} &= 0, \\ \Gamma^1_{32} &= 0, \\ \Gamma^1_{33} &= -\frac{r'(R)r(R)^2\sin(\theta)^2}{r(R)-2m}.\end{aligned}$$

$$\begin{aligned}
\Gamma_{00}^2 &= 0, \\
\Gamma_{01}^2 &= 0, \\
\Gamma_{02}^2 &= 0, \\
\Gamma_{03}^2 &= 0, \\
\Gamma_{10}^2 &= 0, \\
\Gamma_{11}^2 &= 0, \\
\Gamma_{12}^2 &= \frac{r'(R)}{r(R)}, \\
\Gamma_{13}^2 &= 0, \\
\Gamma_{20}^2 &= 0, \\
\Gamma_{21}^2 &= \frac{r'(R)}{r(R)}, \\
\Gamma_{22}^2 &= 0, \\
\Gamma_{23}^2 &= 0, \\
\Gamma_{30}^2 &= 0, \\
\Gamma_{31}^2 &= 0, \\
\Gamma_{32}^2 &= 0, \\
\Gamma_{33}^2 &= -\cos(\theta)\sin(\theta).
\end{aligned}$$

$$\begin{aligned}
\Gamma_{00}^3 &= 0, \\
\Gamma_{01}^3 &= 0, \\
\Gamma_{02}^3 &= 0, \\
\Gamma_{03}^3 &= 0, \\
\Gamma_{10}^3 &= 0, \\
\Gamma_{11}^3 &= 0, \\
\Gamma_{12}^3 &= 0, \\
\Gamma_{13}^3 &= \frac{r'(R)}{r(R)}, \\
\Gamma_{20}^3 &= 0, \\
\Gamma_{21}^3 &= 0, \\
\Gamma_{22}^3 &= 0, \\
\Gamma_{23}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\
\Gamma_{30}^3 &= 0, \\
\Gamma_{31}^3 &= \frac{r'(R)}{r(R)}, \\
\Gamma_{32}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\
\Gamma_{33}^3 &= 0.
\end{aligned}$$

$$\boxed{R_{\mu\nu}}$$

$$R_{00} = -\frac{r''(R)r(R)^2m - 4r'(R)^2m^2 - 2r''(R)r(R)m^2}{(r(R) - 2m)^4}.$$

$$R_{01} = 0.$$

$$R_{02} = 0.$$

$$R_{03} = 0.$$

$$R_{10} = 0.$$

$$R_{11} = -\frac{9r''(R)r(R)^2m - 4r'(R)^2m^2 - 2r''(R)r(R)^3 - 10r''(R)r(R)m^2}{r(R)^2(r(R) - 2m)^2}.$$

$$R_{12} = 0.$$

$$R_{13} = 0.$$

$$R_{20} = 0.$$

$$R_{21} = 0.$$

$$R_{22} = -\frac{2r''(R)r(R)^2m - r'(R)^2r(R)^2 + 4r'(R)^2r(R)m + r(R)^2 - r''(R)r(R)^3 - 4r(R)m + 4m^2}{(r(R) - 2m)^2}.$$

$$R_{23} = 0.$$

$$R_{30} = 0.$$

$$R_{31} = 0.$$

$$R_{32} = 0.$$

$$R_{33} = \frac{r''(R)r(R)^3\sin(\theta)^2 - 4m^2\sin(\theta)^2 + 4r(R)m\sin(\theta)^2 - 2r''(R)r(R)^2m\sin(\theta)^2 + r'(R)^2r(R)^2\sin(\theta)^2 - r(R)^2\sin(\theta)^2 - 4r'(R)^2r(R)m\sin(\theta)^2}{(r(R) - 2m)^2}.$$

$$\boxed{R^\mu{}_\nu}$$

$$R^0_0 = -4\frac{r'(R)^2m^2}{r(R)(r(R) - 2m)^3} + \frac{r''(R)r(R)m}{(r(R) - 2m)^3} - 2\frac{r''(R)m^2}{(r(R) - 2m)^3}.$$

$$R^0_1 = 0.$$

$$R^0_2 = 0.$$

$$R^0_3 = 0.$$

$$R^1_0 = 0.$$

$$R^1_1 = -4\frac{r'(R)^2m^2}{r(R)(r(R) - 2m)^3} - 2\frac{r''(R)r(R)^2}{(r(R) - 2m)^3} + 9\frac{r''(R)r(R)m}{(r(R) - 2m)^3} - 10\frac{r''(R)m^2}{(r(R) - 2m)^3}.$$

$$R^1_2 = 0.$$

$$R^1_3 = 0.$$

$$R^2_0 = 0.$$

$$R^2_1 = 0.$$

$$R^2_2 = 4\frac{m^2}{r(R)^2(r(R) - 2m)^2} + 4\frac{r'(R)^2m}{r(R)(r(R) - 2m)^2} + \frac{1}{(r(R) - 2m)^2} - \frac{r'(R)^2}{(r(R) - 2m)^2} - 4\frac{m}{r(R)(r(R) - 2m)^2} - \frac{r''(R)r(R)}{(r(R) - 2m)^2} + 2\frac{r''(R)m}{(r(R) - 2m)^2}.$$

$$R^2_3 = 0.$$

$$R^3_0 = 0.$$

$$R^3_1 = 0.$$

$$R^3_2 = 0.$$

$$R^3_3 = 4\frac{m^2}{r(R)^2(r(R) - 2m)^2} + 4\frac{r'(R)^2m}{r(R)(r(R) - 2m)^2} + \frac{1}{(r(R) - 2m)^2} - \frac{r'(R)^2}{(r(R) - 2m)^2} - 4\frac{m}{r(R)(r(R) - 2m)^2} - \frac{r''(R)r(R)}{(r(R) - 2m)^2} + 2\frac{r''(R)m}{(r(R) - 2m)^2}.$$

$$\boxed{R}$$

$$R = -24\frac{r'(R)^2m^2}{r(R)(r(R) - 2m)^3} + 2\frac{r(R)}{(r(R) - 2m)^3} - 12\frac{m}{(r(R) - 2m)^3} - 2\frac{r'(R)^2r(R)}{(r(R) - 2m)^3} + 12\frac{r'(R)^2m}{(r(R) - 2m)^3} - 16\frac{m^3}{r(R)^2(r(R) - 2m)^3} - 4\frac{r''(R)r(R)^2}{(r(R) - 2m)^3} + 24\frac{m^2}{r(R)(r(R) - 2m)^3} + 18\frac{r''(R)r(R)m}{(r(R) - 2m)^3} - 20\frac{r''(R)m^2}{(r(R) - 2m)^3}.$$

$$G^{\mu}_{\nu}$$

$$G^0_0=-4\frac{m^2}{r(R)^2(r(R)-2m)^2}-4\frac{r'(R)^2m}{r(R)(r(R)-2m)^2}-\frac{1}{(r(R)-2m)^2}+\frac{r'(R)^2}{(r(R)-2m)^2}+4\frac{m}{r(R)(r(R)-2m)^2}+2\frac{r''(R)r(R)}{(r(R)-2m)^2}-4\frac{r''(R)m}{(r(R)-2m)^2}.$$

$$G^0_1=0.$$

$$G^0_2=0.$$

$$G^0_3=0.$$

$$G^1_0=0.$$

$$G^1_1=-4\frac{m^2}{r(R)^2(r(R)-2m)^2}-4\frac{r'(R)^2m}{r(R)(r(R)-2m)^2}-\frac{1}{(r(R)-2m)^2}+\frac{r'(R)^2}{(r(R)-2m)^2}+4\frac{m}{r(R)(r(R)-2m)^2}.$$

$$G^1_2=0.$$

$$G^1_3=0.$$

$$G^2_0=0.$$

$$G^2_1=0.$$

$$G^2_2=4\frac{r'(R)^2m^2}{r(R)(r(R)-2m)^3}+\frac{r''(R)r(R)^2}{(r(R)-2m)^3}-5\frac{r''(R)r(R)m}{(r(R)-2m)^3}+6\frac{r''(R)m^2}{(r(R)-2m)^3}.$$

$$G^2_3=0.$$

$$G^3_0=0.$$

$$G^3_1=0.$$

$$G^3_2=0.$$

$$G^3_3=4\frac{r'(R)^2m^2}{r(R)(r(R)-2m)^3}+\frac{r''(R)r(R)^2}{(r(R)-2m)^3}-5\frac{r''(R)r(R)m}{(r(R)-2m)^3}+6\frac{r''(R)m^2}{(r(R)-2m)^3}.$$

$$G$$

$$G=24\frac{r'(R)^2m^2}{r(R)(r(R)-2m)^3}-2\frac{r(R)}{(r(R)-2m)^3}+12\frac{m}{(r(R)-2m)^3}+2\frac{r'(R)^2r(R)}{(r(R)-2m)^3}-12\frac{r'(R)^2m}{(r(R)-2m)^3}+16\frac{m^3}{r(R)^2(r(R)-2m)^3}+4\frac{r''(R)r(R)^2}{(r(R)-2m)^3}-24\frac{m^2}{r(R)(r(R)-2m)^3}-18\frac{r''(R)r(R)m}{(r(R)-2m)^3}+20\frac{r''(R)m^2}{(r(R)-2m)^3}.$$

$$G^{\mu}_{\nu;\mu}=0$$

$$G^{\mu}_{0;\mu}=0.$$

$$G^{\mu}_{1;\mu}=0.$$

$$G^{\mu}_{2;\mu}=0.$$

$$G^{\mu}_{3;\mu}=0.$$

$$g^{\mu\nu}\,\Gamma^{\lambda}_{\mu\nu}=0?$$

$$g^{\mu\nu}\,\Gamma^0_{\mu\nu}=0.$$

$$g^{\mu\nu}\,\Gamma^1_{\mu\nu}=-6\frac{r'(R)r(R)^6m}{(r(R)-2m)^4}+12\frac{r'(R)r(R)^5m^2\sin(\theta)^4}{(r(R)-2m)^4}-24\frac{r'(R)m^3}{(r(R)-2m)^4}-6\frac{r'(R)r(R)^6m\sin(\theta)^4}{(r(R)-2m)^4}+8\frac{r'(R)r(R)m^2}{(r(R)-2m)^4}-16\frac{r'(R)m^5}{r(R)^2(r(R)-2m)^4}+12\frac{r'(R)r(R)^5m^2}{(r(R)-2m)^4}+32\frac{r'(R)m^4}{r(R)(r(R)-2m)^4}-8\frac{r'(R)r(R)^4m^3}{(r(R)-2m)^4}+\frac{r'(R)r(R)^2}{(r(R)-2m)^4}-2\frac{r'(R)r(R)^2m}{(r(R)-2m)^4}+\frac{r'(R)r(R)^7\sin(\theta)^4}{(r(R)-2m)^4}-8\frac{r'(R)r(R)^4m^3\sin(\theta)^4}{(r(R)-2m)^4}.$$

$$g^{\mu\nu}\,\Gamma^2_{\mu\nu}=\cos(\theta)r(R)^2\sin(\theta)^3.$$

$$g^{\mu\nu}\,\Gamma^3_{\mu\nu}=0.$$