

Schwarzschild Metric in spherical coordinates:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= t. \\ x^1 &= r. \\ x^2 &= \theta. \\ x^3 &= \phi.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= \exp(A(r)). \\ g_{01} &= 0. \\ g_{02} &= 0. \\ g_{03} &= 0. \\ g_{10} &= 0. \\ g_{11} &= -\exp(B(r)). \\ g_{12} &= 0. \\ g_{13} &= 0. \\ g_{20} &= 0. \\ g_{21} &= 0. \\ g_{22} &= -r^2. \\ g_{23} &= 0. \\ g_{30} &= 0. \\ g_{31} &= 0. \\ g_{32} &= 0. \\ g_{33} &= -\sin(\theta)^2r^2.\end{aligned}$$

$$\boxed{\sqrt{\hspace{0.1cm}} = \sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{\hspace{0.1cm}} = \sqrt{\sin(\theta)^2\exp(B(r))\exp(A(r))r^4}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= \frac{1}{\exp(A(r))}. \\ g^{01} &= 0. \\ g^{02} &= 0. \\ g^{03} &= 0. \\ g^{10} &= 0. \\ g^{11} &= -\frac{1}{\exp(B(r))}. \\ g^{12} &= 0. \\ g^{13} &= 0. \\ g^{20} &= 0. \\ g^{21} &= 0. \\ g^{22} &= -\frac{1}{r^2}. \\ g^{23} &= 0. \\ g^{30} &= 0. \\ g^{31} &= 0. \\ g^{32} &= 0. \\ g^{33} &= -\frac{1}{\sin(\theta)^2r^2}.\end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 0. \\ \Gamma^0_{01} &= \frac{1}{2}A'(r). \\ \Gamma^0_{02} &= 0. \\ \Gamma^0_{03} &= 0. \\ \Gamma^0_{10} &= \frac{1}{2}A'(r). \\ \Gamma^0_{11} &= 0. \\ \Gamma^0_{12} &= 0. \\ \Gamma^0_{13} &= 0. \\ \Gamma^0_{20} &= 0. \\ \Gamma^0_{21} &= 0. \\ \Gamma^0_{22} &= 0. \\ \Gamma^0_{23} &= 0. \\ \Gamma^0_{30} &= 0. \\ \Gamma^0_{31} &= 0. \\ \Gamma^0_{32} &= 0. \\ \Gamma^0_{33} &= 0.\end{aligned}$$

$$\begin{aligned}\Gamma^1_{00} &= \frac{1}{2}\frac{A'(r)\exp(A(r))}{\exp(B(r))}. \\ \Gamma^1_{01} &= 0. \\ \Gamma^1_{02} &= 0. \\ \Gamma^1_{03} &= 0. \\ \Gamma^1_{10} &= 0. \\ \Gamma^1_{11} &= \frac{1}{2}B'(r). \\ \Gamma^1_{12} &= 0. \\ \Gamma^1_{13} &= 0. \\ \Gamma^1_{20} &= 0. \\ \Gamma^1_{21} &= 0. \\ \Gamma^1_{22} &= -\frac{r}{\exp(B(r))}. \\ \Gamma^1_{23} &= 0. \\ \Gamma^1_{30} &= 0. \\ \Gamma^1_{31} &= 0. \\ \Gamma^1_{32} &= 0. \\ \Gamma^1_{33} &= -\frac{\sin(\theta)^2r}{\exp(B(r))}.\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^2 &= 0, \\ \Gamma_{01}^2 &= 0, \\ \Gamma_{02}^2 &= 0, \\ \Gamma_{03}^2 &= 0, \\ \Gamma_{10}^2 &= 0, \\ \Gamma_{11}^2 &= 0, \\ \Gamma_{12}^2 &= -\frac{1}{r}, \\ \Gamma_{13}^2 &= 0, \\ \Gamma_{20}^2 &= 0, \\ \Gamma_{21}^2 &= \frac{1}{r}, \\ \Gamma_{22}^2 &= 0, \\ \Gamma_{23}^2 &= 0, \\ \Gamma_{30}^2 &= 0, \\ \Gamma_{31}^2 &= 0, \\ \Gamma_{32}^2 &= 0, \\ \Gamma_{33}^2 &= -\cos(\theta)\sin(\theta).\end{aligned}$$

$$\begin{aligned}\Gamma_{00}^3 &= 0, \\ \Gamma_{01}^3 &= 0, \\ \Gamma_{02}^3 &= 0, \\ \Gamma_{03}^3 &= 0, \\ \Gamma_{10}^3 &= 0, \\ \Gamma_{11}^3 &= 0, \\ \Gamma_{12}^3 &= 0, \\ \Gamma_{13}^3 &= -\frac{1}{r}, \\ \Gamma_{20}^3 &= 0, \\ \Gamma_{21}^3 &= 0, \\ \Gamma_{22}^3 &= 0, \\ \Gamma_{23}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\ \Gamma_{30}^3 &= 0, \\ \Gamma_{31}^3 &= \frac{1}{r}, \\ \Gamma_{32}^3 &= \frac{\cos(\theta)}{\sin(\theta)}, \\ \Gamma_{33}^3 &= 0.\end{aligned}$$

$$\ddot{x}^\mu = \left(\Gamma_{\sigma\rho}^0 \dot{x}^\mu - \Gamma_{\sigma\rho}^\mu\right) \dot{x}^\sigma \dot{x}^\rho$$

$$\begin{aligned}\ddot{x}^0 &= 0, \\ \ddot{x}^1 &= \frac{1}{2} \frac{2\sin(\theta)^2 \dot{z}^2 r - \dot{x}^2 B'(r) \exp(B(r)) - A'(r) \exp(A(r)) + 2\dot{x}^2 A'(r) \exp(B(r))}{\exp(B(r))}, \\ \ddot{x}^2 &= \cos(\theta)\sin(\theta)\dot{z}^2, \\ \ddot{x}^3 &= \frac{\dot{x} A'(r) \dot{z} r - 2\dot{x} \dot{z}}{r}.\end{aligned}$$

$$R_{\mu\nu}$$

$$\begin{aligned}R_{00} &= -\frac{1}{4} \frac{4A'(r)\exp(A(r)) + 2\exp(A(r))A''(r)r + A'(r)^2\exp(A(r))r - A'(r)B'(r)\exp(A(r))r}{\exp(B(r))r}, \\ R_{01} &= 0, \\ R_{02} &= 0, \\ R_{03} &= 0, \\ R_{10} &= 0, \\ R_{11} &= \frac{1}{4} \frac{A'(r)^2 r + 2A''(r)r - 4B'(r) - A'(r)B'(r)r}{r}, \\ R_{12} &= 0, \\ R_{13} &= 0, \\ R_{20} &= 0, \\ R_{21} &= 0, \\ R_{22} &= -\frac{1}{2} \frac{-2 + B'(r)r + 2\exp(B(r)) - A'(r)r}{\exp(B(r))}, \\ R_{23} &= 0, \\ R_{30} &= 0, \\ R_{31} &= 0, \\ R_{32} &= 0, \\ R_{33} &= -\frac{1}{2} \frac{B'(r)\sin(\theta)^2 r + 2\sin(\theta)^2 \exp(B(r)) - A'(r)\sin(\theta)^2 r - 2\sin(\theta)^2}{\exp(B(r))}.\end{aligned}$$

$$R^{\mu}_{\nu}$$

$$\begin{aligned}R^0_0 &= -\frac{1}{4} \frac{A'(r)^2}{\exp(B(r))} + \frac{1}{4} \frac{A'(r)B'(r)}{\exp(B(r))} - \frac{1}{2} \frac{A''(r)}{\exp(B(r))} - \frac{A'(r)}{\exp(B(r))r}, \\ R^0_1 &= 0, \\ R^0_2 &= 0, \\ R^0_3 &= 0, \\ R^1_0 &= 0, \\ R^1_1 &= -\frac{1}{4} \frac{A'(r)^2}{\exp(B(r))} + \frac{1}{4} \frac{A'(r)B'(r)}{\exp(B(r))} - \frac{1}{2} \frac{A''(r)}{\exp(B(r))} + \frac{B'(r)}{\exp(B(r))r}, \\ R^1_2 &= 0, \\ R^1_3 &= 0, \\ R^2_0 &= 0, \\ R^2_1 &= 0, \\ R^2_2 &= \frac{1}{r^2} - \frac{1}{2} \frac{A'(r)}{\exp(B(r))r} - \frac{1}{\exp(B(r))r^2} + \frac{1}{2} \frac{B'(r)}{\exp(B(r))r}, \\ R^2_3 &= 0, \\ R^3_0 &= 0, \\ R^3_1 &= 0, \\ R^3_2 &= 0, \\ R^3_3 &= \frac{1}{r^2} - \frac{1}{2} \frac{A'(r)}{\exp(B(r))r} - \frac{1}{\exp(B(r))r^2} + \frac{1}{2} \frac{B'(r)}{\exp(B(r))r}.\end{aligned}$$

$$R$$

$$R = 2\frac{1}{r^2} - \frac{1}{2} \frac{A'(r)^2}{\exp(B(r))} + \frac{1}{2} \frac{A'(r)B'(r)}{\exp(B(r))} - \frac{A''(r)}{\exp(B(r))} - 2\frac{A'(r)}{\exp(B(r))r} - 2\frac{1}{\exp(B(r))r^2} + 2\frac{B'(r)}{\exp(B(r))r}.$$

$$\boxed{G_{\nu}^{\mu}}$$

$$G_0^0=-\frac{-1+B'(r)r+\exp(B(r))}{\exp(B(r))r^2}.$$

$$G_1^0=0.$$

$$G_2^0=0.$$

$$G_3^0=0.$$

$$G_0^1=0.$$

$$G_1^1=-\frac{-1+\exp(B(r))-A'(r)r}{\exp(B(r))r^2}.$$

$$G_2^1=0.$$

$$G_3^1=0.$$

$$G_0^2=0.$$

$$G_1^2=0.$$

$$G_2^2=\frac{1}{4}\frac{2A'(r)+A'(r)^2r+2A''(r)r-2B'(r)-A'(r)B'(r)r}{\exp(B(r))r}.$$

$$G_3^2=0.$$

$$G_0^3=0.$$

$$G_1^3=0.$$

$$G_2^3=0.$$

$$G_3^3=\frac{1}{4}\frac{2A'(r)+A'(r)^2r+2A''(r)r-2B'(r)-A'(r)B'(r)r}{\exp(B(r))r}.$$

$$\boxed{G}$$

$$G=-2\frac{1}{r^2}+\frac{1}{2}\frac{A'(r)^2}{\exp(B(r))}-\frac{1}{2}\frac{A'(r)B'(r)}{\exp(B(r))}+\frac{A''(r)}{\exp(B(r))}+2\frac{A'(r)}{\exp(B(r))r}+2\frac{1}{\exp(B(r))r^2}-2\frac{B'(r)}{\exp(B(r))r}.$$

$$\boxed{G_{\nu;\mu}^{\mu}=0}$$

$$G_{6;\mu}^{\mu}=0.$$

$$G_{4;\mu}^{\mu}=0.$$

$$G_{2;\mu}^{\mu}=0.$$

$$G_{3;\mu}^{\mu}=0.$$

$$\boxed{g^{\mu\nu}\,\Gamma_{\mu\nu}^{\lambda}=0?}$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^0=0.$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^1=\frac{\sin(\theta)^4r^3}{\exp(B(r))}+\frac{r^3}{\exp(B(r))}-\frac{1}{2}B'(r)\exp(B(r))+\frac{1}{2}\frac{A'(r)\exp(A(r))^2}{\exp(B(r))}.$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^2=\cos(\theta)\sin(\theta)^3r^2.$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^3=0.$$