

Schwarzschild Metric in spherical coordinates with a variable spherically symmetric matter density:

$$\boxed{x^\mu}$$

$$\begin{aligned}x^0 &= t. \\ x^1 &= r. \\ x^2 &= \theta. \\ x^3 &= \phi.\end{aligned}$$

$$\boxed{g_{\mu\nu}}$$

$$\begin{aligned}g_{00} &= 1 - \frac{8}{3}r^2\pi\rho(r,t). \\ g_{01} &= 0. \\ g_{02} &= 0. \\ g_{03} &= 0. \\ g_{10} &= 0. \\ g_{11} &= 3\frac{1}{-3 + 8r^2\pi\rho(r,t)}. \\ g_{12} &= 0. \\ g_{13} &= 0. \\ g_{20} &= 0. \\ g_{21} &= 0. \\ g_{22} &= -r^2. \\ g_{23} &= 0. \\ g_{30} &= 0. \\ g_{31} &= 0. \\ g_{32} &= 0. \\ g_{33} &= -\sin(\theta)^2r^2.\end{aligned}$$

$$\boxed{\sqrt{\hspace{1cm}} = \sqrt{-\det(g_{\mu\nu})}}$$

$$\sqrt{\hspace{1cm}} = \sqrt{\sin(\theta)^2r^4}.$$

$$\boxed{g^{\mu\nu}}$$

$$\begin{aligned}g^{00} &= -3\frac{1}{-3 + 8r^2\pi\rho(r,t)}. \\ g^{01} &= 0. \\ g^{02} &= 0. \\ g^{03} &= 0. \\ g^{10} &= 0. \\ g^{11} &= -1 + \frac{8}{3}r^2\pi\rho(r,t). \\ g^{12} &= 0. \\ g^{13} &= 0. \\ g^{20} &= 0. \\ g^{21} &= 0. \\ g^{22} &= -\frac{1}{r^2}. \\ g^{23} &= 0. \\ g^{30} &= 0. \\ g^{31} &= 0. \\ g^{32} &= 0. \\ g^{33} &= -\frac{1}{\sin(\theta)^2r^2}.\end{aligned}$$

$$\boxed{\Gamma^\sigma_{\mu\nu}}$$

$$\begin{aligned}\Gamma^0_{00} &= 4\frac{\rho'(r,t)r^2\pi}{-3 + 8r^2\pi\rho(r,t)}. \\ \Gamma^0_{01} &= 4\frac{2r\pi\rho(r,t) + \dot{\rho}(r,t)r^2\pi}{-3 + 8r^2\pi\rho(r,t)}. \\ \Gamma^0_{02} &= 0. \\ \Gamma^0_{03} &= 0. \\ \Gamma^0_{10} &= 4\frac{2r\pi\rho(r,t) + \dot{\rho}(r,t)r^2\pi}{-3 + 8r^2\pi\rho(r,t)}. \\ \Gamma^0_{11} &= -36\frac{\rho'(r,t)r^2\pi}{(-3 + 8r^2\pi\rho(r,t))^3}. \\ \Gamma^0_{12} &= 0. \\ \Gamma^0_{13} &= 0. \\ \Gamma^0_{20} &= 0. \\ \Gamma^0_{21} &= 0. \\ \Gamma^0_{22} &= 0. \\ \Gamma^0_{23} &= 0. \\ \Gamma^0_{30} &= 0. \\ \Gamma^0_{31} &= 0. \\ \Gamma^0_{32} &= 0. \\ \Gamma^0_{33} &= 0.\end{aligned}$$

$$\begin{aligned}\Gamma^1_{00} &= \frac{4}{9}(-3 + 8r^2\pi\rho(r,t))(2r\pi\rho(r,t) + \dot{\rho}(r,t)r^2\pi). \\ \Gamma^1_{01} &= -4\frac{\rho'(r,t)r^2\pi}{-3 + 8r^2\pi\rho(r,t)}. \\ \Gamma^1_{02} &= 0. \\ \Gamma^1_{03} &= 0. \\ \Gamma^1_{10} &= -4\frac{\rho'(r,t)r^2\pi}{-3 + 8r^2\pi\rho(r,t)}. \\ \Gamma^1_{11} &= -4\frac{2r\pi\rho(r,t) + \dot{\rho}(r,t)r^2\pi}{-3 + 8r^2\pi\rho(r,t)}. \\ \Gamma^1_{12} &= 0. \\ \Gamma^1_{13} &= 0. \\ \Gamma^1_{20} &= 0. \\ \Gamma^1_{21} &= 0. \\ \Gamma^1_{22} &= \frac{1}{3}r(-3 + 8r^2\pi\rho(r,t)). \\ \Gamma^1_{23} &= 0. \\ \Gamma^1_{30} &= 0. \\ \Gamma^1_{31} &= 0. \\ \Gamma^1_{32} &= 0. \\ \Gamma^1_{33} &= \frac{1}{3}\sin(\theta)^2r(-3 + 8r^2\pi\rho(r,t)).\end{aligned}$$

$$\left[G_\nu^\mu\right]$$

$$G_0^0=-\frac{8}{3}\dot{\rho}(r,t)r\pi-8\pi\rho(r,t).$$

$$G_1^0=-24\frac{\rho'(r,t)r\pi}{\left(-3+8r^2\pi\rho(r,t)\right)^2}.$$

$$G_2^0=0.$$

$$G_3^0=0.$$

$$G_0^1=\frac{8}{3}\rho'(r,t)r\pi.$$

$$G_1^1=-\frac{8}{3}\dot{\rho}(r,t)r\pi-8\pi\rho(r,t).$$

$$G_2^1=0.$$

$$G_3^1=0.$$

$$G_0^2=0.$$

$$G_3^2=0.$$

$$G_2^2=\frac{4}{3}\frac{3456\dot{\rho}(r,t)r^5\pi^3\rho(r,t)^2-72\rho''(r,t)r^4\pi^2\rho(r,t)+144\dot{\rho}'(r,t)^2r^4\pi^2-1296r^2\pi^2\rho(r,t)^2+162\dot{\rho}(r,t)r\pi+3456r^4\pi^3\rho(r,t)^3-3072\dot{\rho}(r,t)r^7\pi^4\rho(r,t)^3+576r^6\dot{\rho}(r,t)\pi^3\rho(r,t)^2-1296\dot{\rho}(r,t)r^3\pi^2\rho(r,t)+27\rho''(r,t)r^2\pi+162\pi\rho(r,t)-216r^4\dot{\rho}(r,t)\pi^2\rho(r,t)+27r^2\ddot{\rho}(r,t)\pi-3072r^6\pi^4\rho(r,t)^4-512r^8\ddot{\rho}(r,t)\pi^4\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^3}.$$

$$G_3^2=0.$$

$$G_0^3=0.$$

$$G_3^3=0.$$

$$G_3^3=\frac{4}{3}\frac{3456\dot{\rho}(r,t)r^5\pi^3\rho(r,t)^2-72\rho''(r,t)r^4\pi^2\rho(r,t)+144\dot{\rho}'(r,t)^2r^4\pi^2-1296r^2\pi^2\rho(r,t)^2+162\dot{\rho}(r,t)r\pi+3456r^4\pi^3\rho(r,t)^3-3072\dot{\rho}(r,t)r^7\pi^4\rho(r,t)^3+576r^6\dot{\rho}(r,t)\pi^3\rho(r,t)^2-1296\dot{\rho}(r,t)r^3\pi^2\rho(r,t)+27\rho''(r,t)r^2\pi+162\pi\rho(r,t)-216r^4\dot{\rho}(r,t)\pi^2\rho(r,t)+27r^2\ddot{\rho}(r,t)\pi-3072r^6\pi^4\rho(r,t)^4-512r^8\ddot{\rho}(r,t)\pi^4\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^3}.$$

$$\left[G\right]$$

$$G=18432\frac{r^4\pi^3\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+384\frac{\rho'(r,t)^2r^4\pi^2}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-16384\frac{r^6\pi^4\rho(r,t)^4}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-\frac{32768}{3}\frac{\dot{\rho}(r,t)r^7\pi^4\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-\frac{4096}{3}\frac{r^8\dot{\rho}(r,t)\pi^4\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+72\frac{\rho''(r,t)r^2\pi}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+72\frac{r^2\dot{\rho}(r,t)\pi}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+576\frac{\dot{\rho}(r,t)r\pi}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+12288\frac{\dot{\rho}(r,t)r^5\pi^3\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-576\frac{r^4\dot{\rho}(r,t)\pi^2\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+1536\frac{r^6\dot{\rho}(r,t)\pi^3\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-192\frac{\rho''(r,t)r^4\pi^2\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^3}+864\frac{\pi\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-4608\frac{\dot{\rho}(r,t)r^3\pi^2\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^3}-6912\frac{r^2\pi^2\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^3}.$$

$$\left[G_{\nu;\mu}^\mu=0\right]$$

$$G_{0;\mu}^\mu=0.$$

$$G_{1;\mu}^\mu=0.$$

$$G_{2;\mu}^\mu=0.$$

$$G_{3;\mu}^\mu=0.$$

$$\left[g^{\mu\nu}\,\Gamma_{\mu\nu}^\lambda=0\right]$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^0=8192\frac{\rho'(r,t)r^8\pi^4\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^4}-\frac{16384}{3}\frac{\rho'(r,t)r^{10}\pi^5\rho(r,t)^4}{\left(-3+8r^2\pi\rho(r,t)\right)^4}-216\frac{\rho'(r,t)r^2\pi}{\left(-3+8r^2\pi\rho(r,t)\right)^4}-4608\frac{\rho'(r,t)r^6\pi^3\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^4}+1152\frac{\rho'(r,t)r^4\pi^2\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^4},$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^1=-\frac{512}{3}\frac{\sin(\theta)^4r^6\pi^3\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+256\frac{r^3\pi^2\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+192\frac{r^7\pi^2\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-24\frac{\dot{\rho}(r,t)r^2\pi}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-72\frac{\sin(\theta)^4r^5\pi\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-1024\frac{r^5\pi^3\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-48\frac{r\pi\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+9\frac{\sin(\theta)^4r^3}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+\frac{16384}{9}\frac{r^7\pi^4\rho(r,t)^4}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+9\frac{r^3}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-512\frac{\dot{\rho}(r,t)r^6\pi^3\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-\frac{16384}{27}\frac{\dot{\rho}(r,t)r^{10}\pi^5\rho(r,t)^4}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+128\frac{\dot{\rho}(r,t)r^4\pi^2\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-\frac{32768}{27}\frac{r^9\pi^5\rho(r,t)^5}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+192\frac{\sin(\theta)^4r^7\pi^2\rho(r,t)^2}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-72\frac{r^5\pi\rho(r,t)}{\left(-3+8r^2\pi\rho(r,t)\right)^2}-\frac{512}{3}\frac{r^9\pi^3\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^2}+\frac{8192}{9}\frac{\dot{\rho}(r,t)r^8\pi^4\rho(r,t)^3}{\left(-3+8r^2\pi\rho(r,t)\right)^2}.$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^2=\cos(\theta)\sin(\theta)^3r^2.$$

$$g^{\mu\nu}\,\Gamma_{\mu\nu}^3=0.$$