“열역학적” 기후 Proxy

Atmospheric Thermodynamics

6조

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- Theoretical background : MEP & 2 boxes
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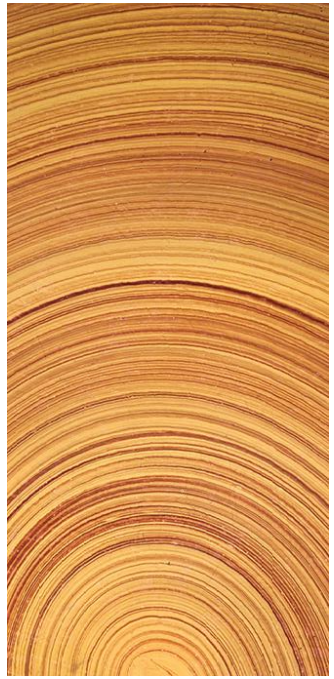
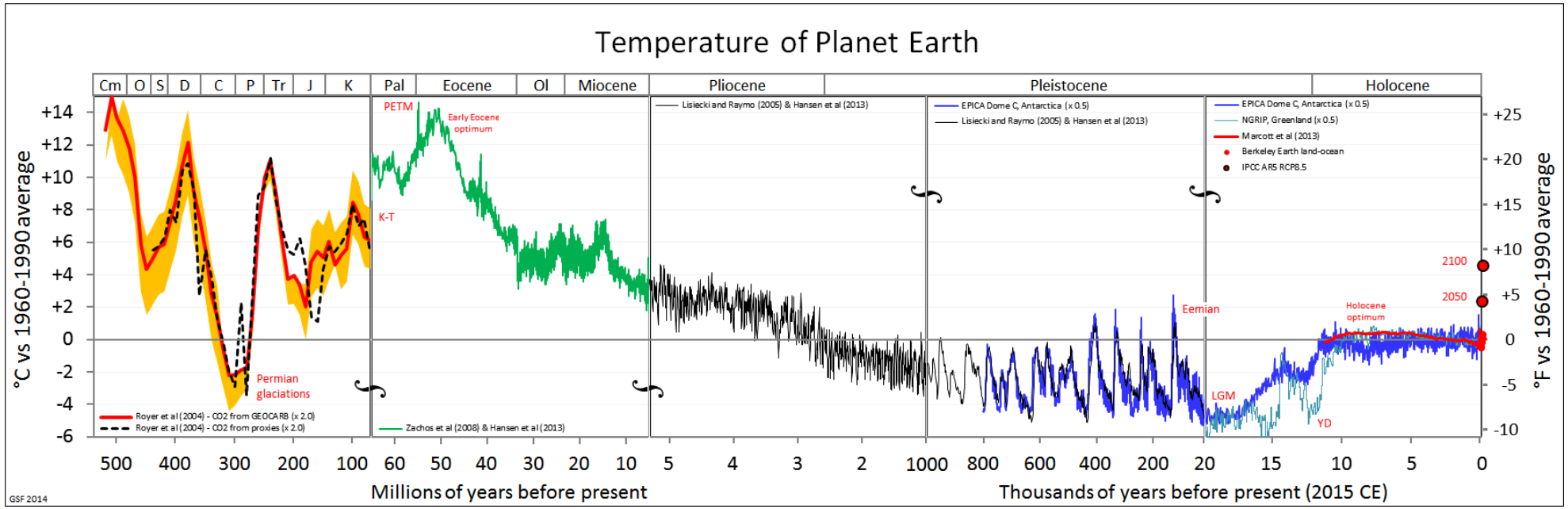
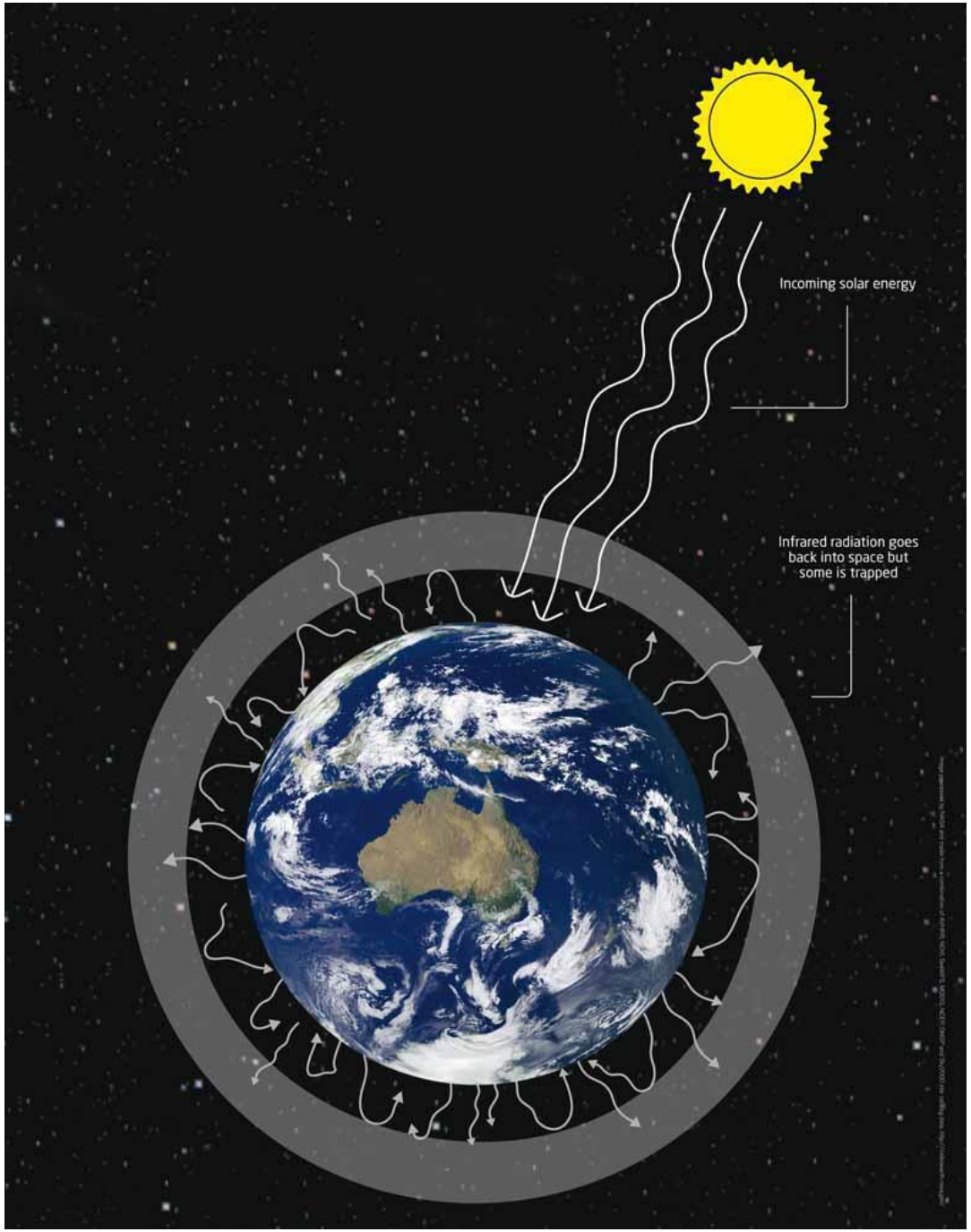
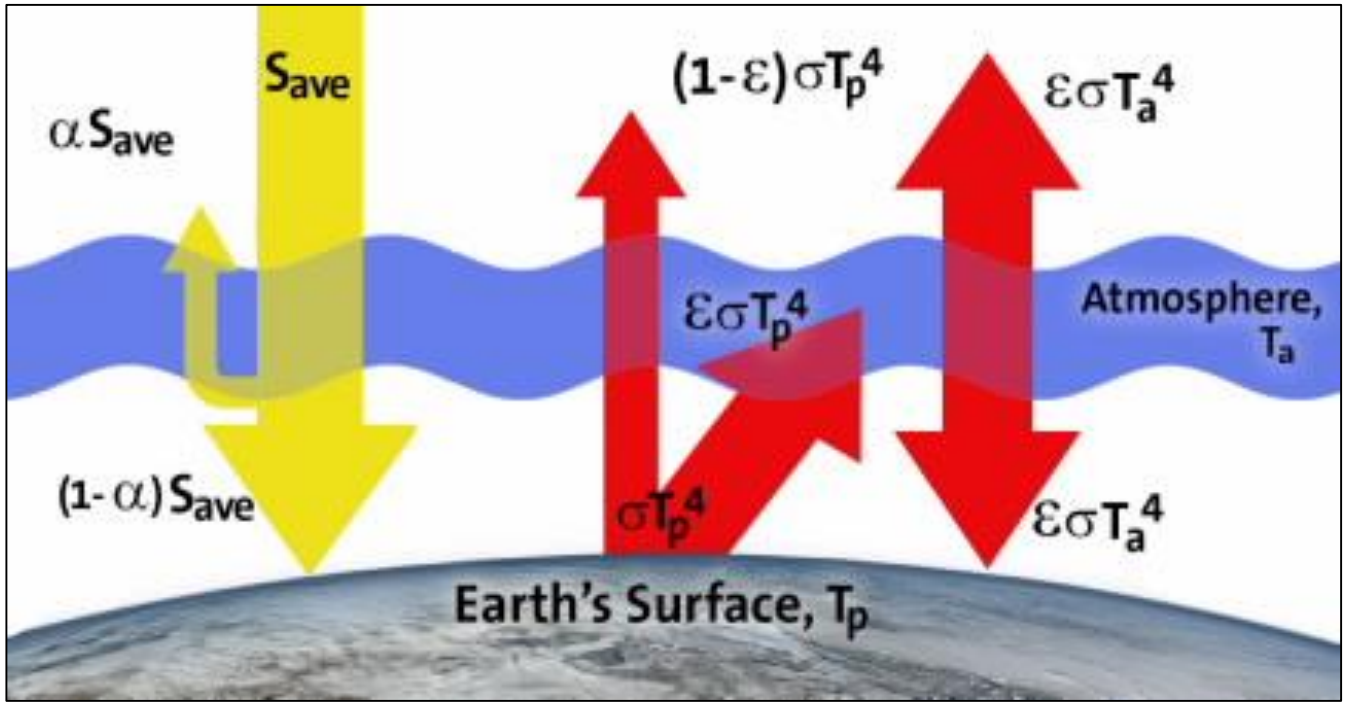
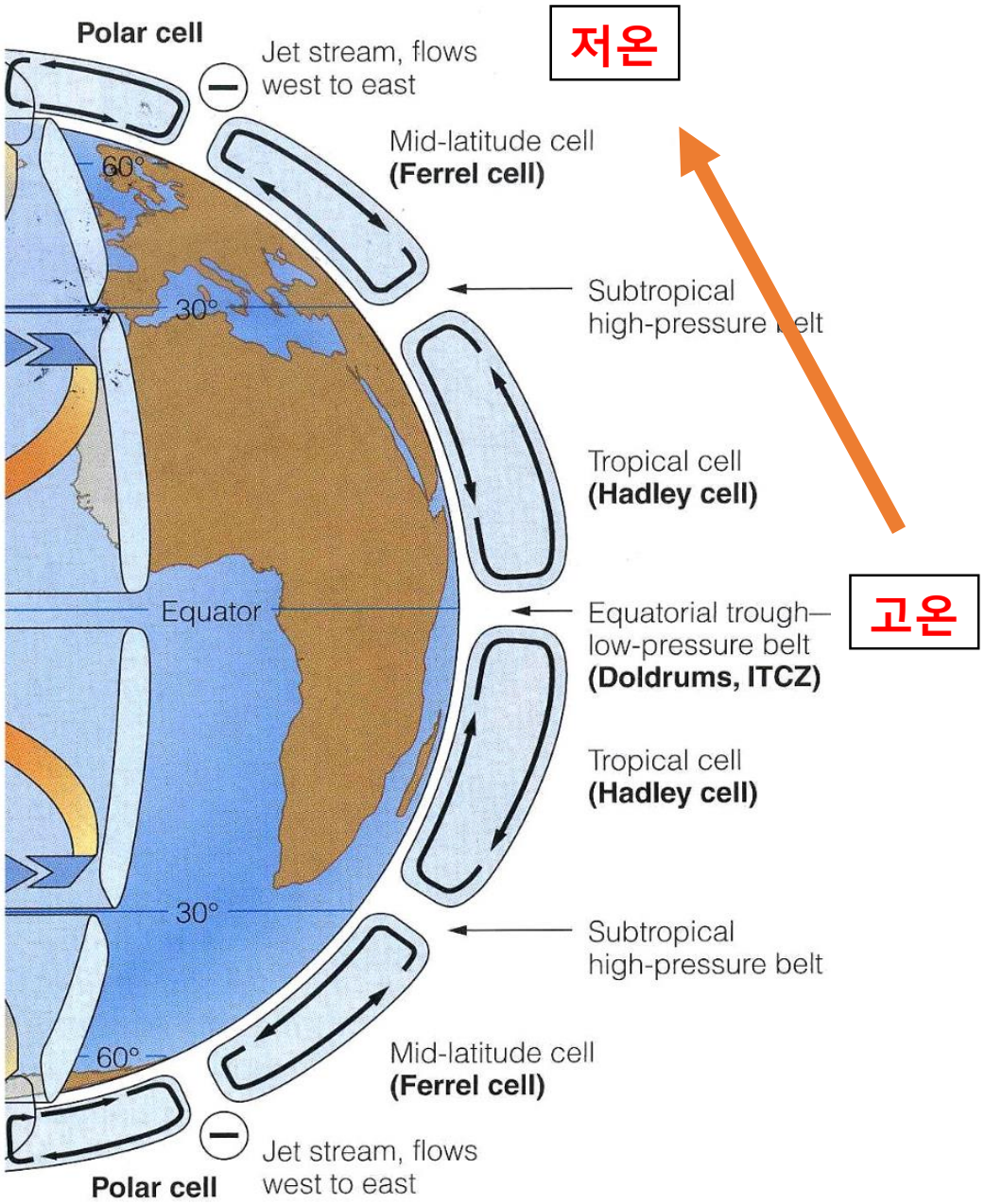



Temperature of Planet Earth
Cm
O
S
D
C
P
Tr
J
K
Pal
Eocene
Ol
Miocene
Pliocene
Pleistocene
Holocene
PETM
Early Eocene optimum
K-T
Permian glaciations
Lisiecki and Raymo (2005) & Hansen et al (2013)
EPICA Dome C, Antarctica (x 0.5)
Lisiecki and Raymo (2005) & Hansen et al (2013)
EPICA Dome C, Antarctica (x 0.5)
NGRIP, Greenland (x 0.5)
Marcott et al (2013)
Berkeley Earth land-ocean
IPCC AR5 RCP8.5
Eemian
Holocene optimum
LGM
YD
2100
2050
Royer et al (2004) - CO2 from GEOCARB (x 2.0)
Royer et al (2004) - CO2 from proxies (x 2.0)
Zachos et al (2008) & Hansen et al (2013)
°C vs 1960-1990 average
°F vs 1960-1990 average
+14
+12
+10
+8
+6
+4
+2
0
-2
-4
-6
+25
+20
+15
+10
+5
0
-5
-10
500
400
300
200
100
60
50
40
30
20
10
5
4
3
2
1000
800
600
400
200
20
15
10
5
0
Millions of years before present
Thousands of years before present (2015 CE)
GSF 2014


Incoming solar energy
Infrared radiation goes back into space but some is trapped


S_{ave}
αS_{ave}
$(1-\varepsilon)\sigma T_p^4$
$\varepsilon\sigma T_a^4$
$\varepsilon\sigma T_p^4$
Atmosphere, T_a
$(1-\alpha) S_{ave}$
σT_p^4
$\varepsilon\sigma T_a^4$
Earth's Surface, T_p


Polar cell
Jet stream, flows west to east
Mid-latitude cell (Ferrel cell)
60°
30°
Subtropical high-pressure belt
Tropical cell (Hadley cell)
Equator
Equatorial trough—low-pressure belt (Doldrums, ITCZ)
Tropical cell (Hadley cell)
30°
Subtropical high-pressure belt
60°
Mid-latitude cell (Ferrel cell)
Jet stream, flows west to east
Polar cell

저온

고온

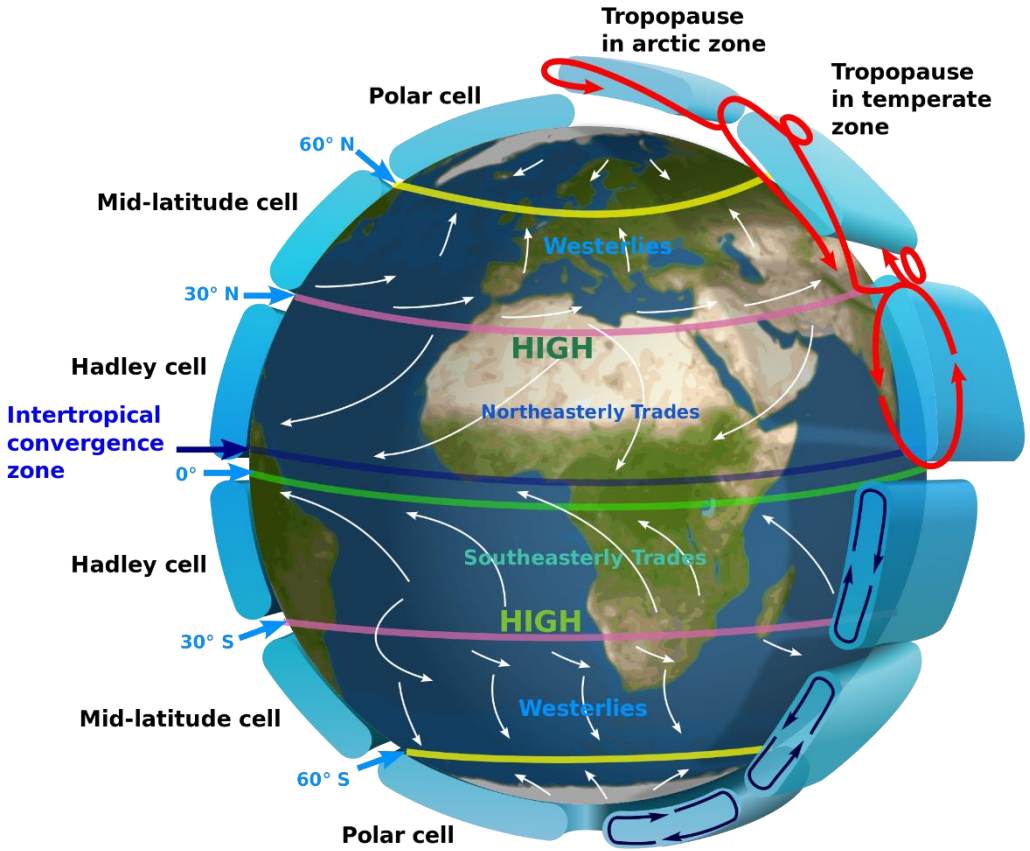
Can we find any new method or climate proxies?

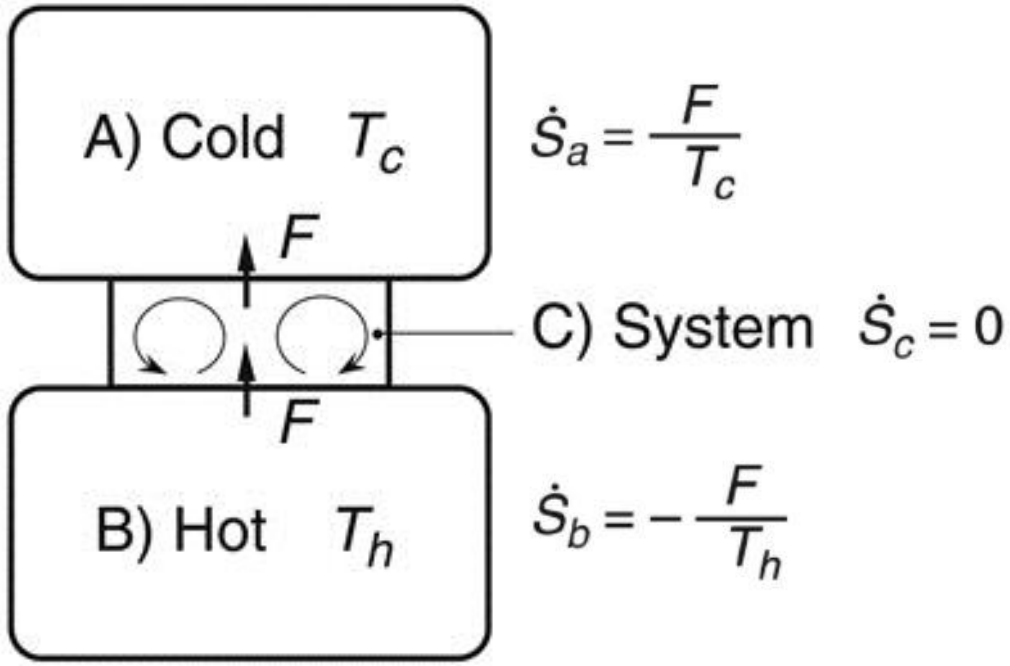
We must attribute to heat the great movements that we observe all about us on the Earth. Heat is the cause of currents in the atmosphere, of the rising motion of clouds, of the falling of rain and of other atmospheric phenomena ... .

Sadi Carnot (1824)

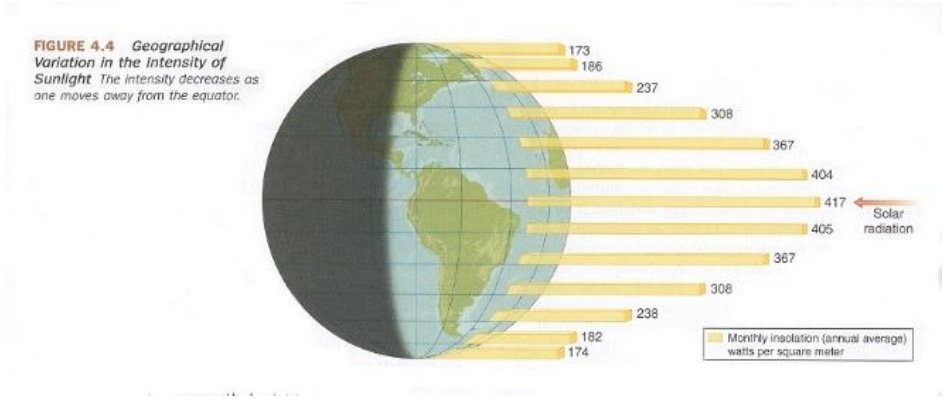
Maximum Entropy Production (MEP)

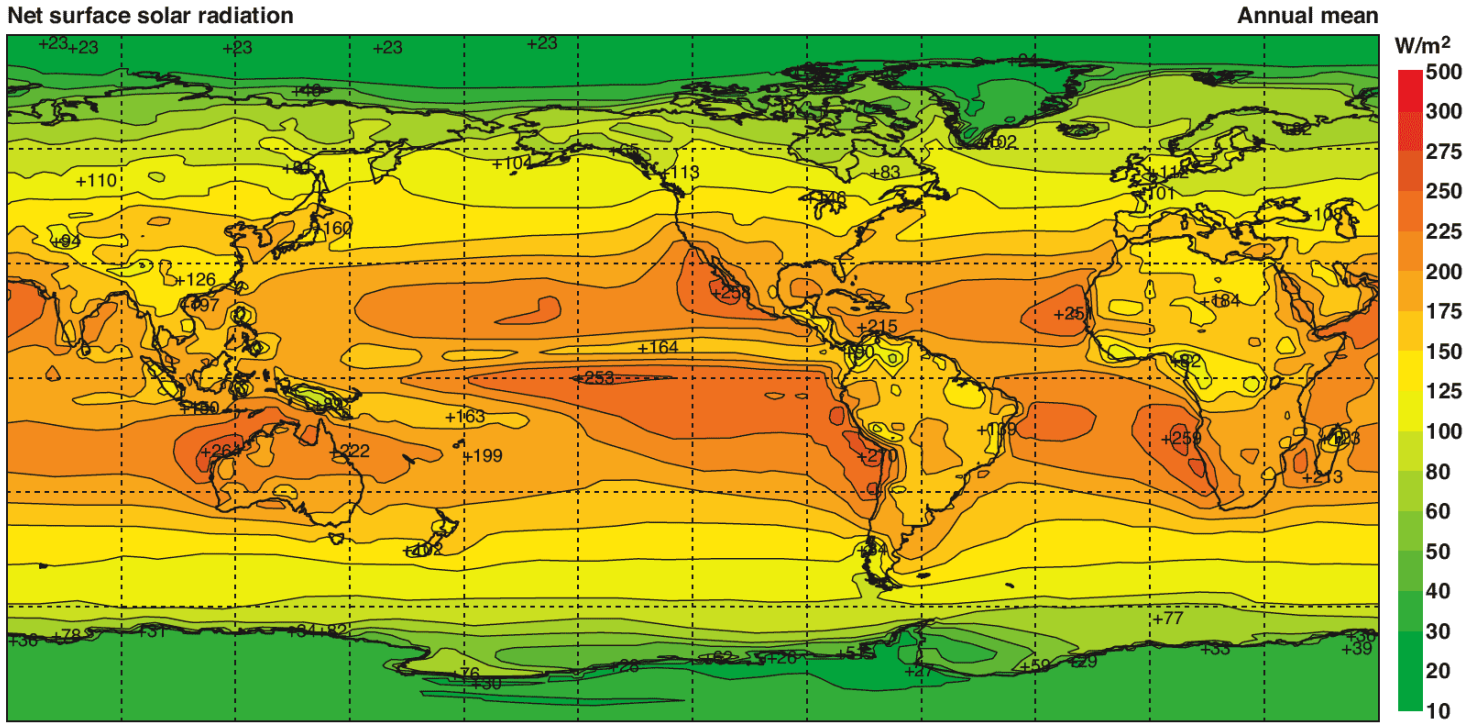
- Thermodynamical approach



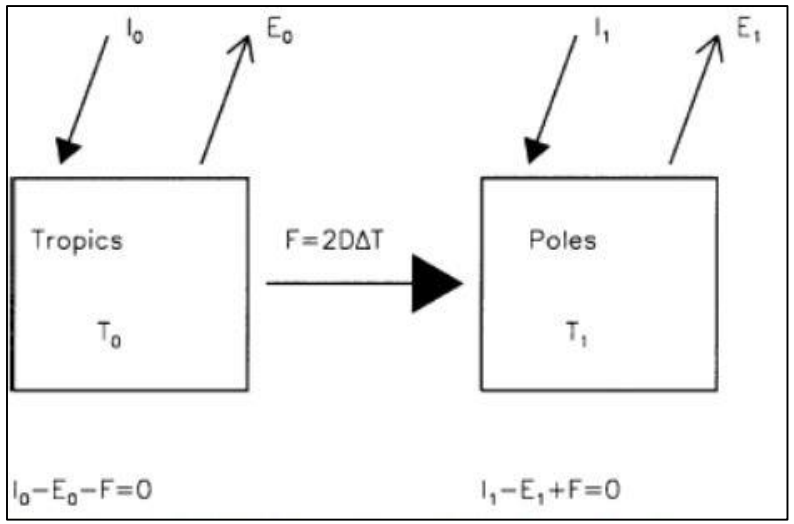


$$\frac{dS_{turb}}{dt} = -\frac{F}{T_h} + \frac{F}{T_c} = maximum$$





2 Box model



Assumptions

1. 온도에 대한 지구복사에너지 식을 선형으로 근사 (grey approximation).

2. T_{avg}(지구 표면의 평균 온도) = 288K.

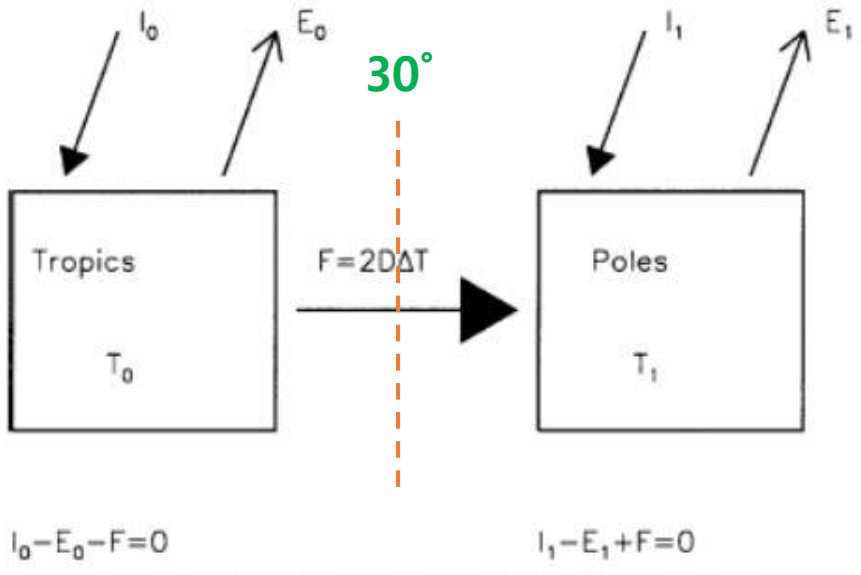
2 Box model - equations

Lorenz, 2002

$$\begin{cases} I_0 = E_0 + F \\ I_1 + F = E_1 \end{cases}$$

$$E_n = A + BT_n$$

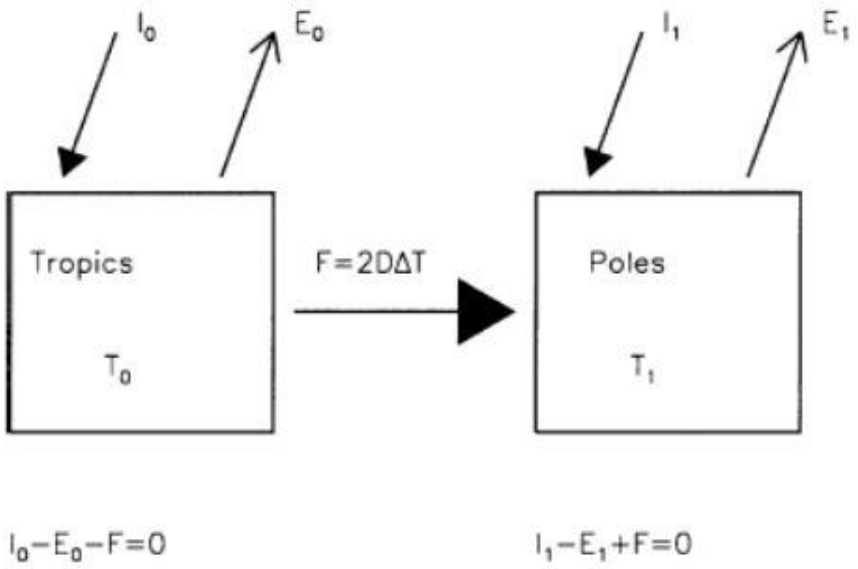
$$F = 2D\Delta T$$



$$F = \frac{d}{dx}\left[\begin{cases} I_0 = E_0 + F \\ I_1 + F = E_1 \end{cases}\right] \frac{dT}{dT_n} 2D\Delta T$$

2 Box model - equations

Lorenz, 2002



$$\begin{bmatrix} B+2D & -2D \\ -2D & B+2D \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \end{bmatrix} = \begin{bmatrix} I_0 - A \\ I_1 - A \end{bmatrix}$$

$$\begin{cases} T_0 = T_{avg} + \dfrac{I_0 - I_1}{2(B+4D)} \\ T_1 = T_{avg} - \dfrac{I_0 - I_1}{2(B+4D)} \end{cases}$$

$$\frac{dS_{turb}}{dt} = -\frac{F}{T_0} + \frac{F}{T_1} = max$$

2 Box model

- calculation

$$\frac{dS_{turb}}{dt} = -\frac{F}{T_0} + \frac{F}{T_1}$$

$$F = 2D\Delta T$$

I_0	240
I_1	140
T_{avg}	288
B	3.234489
A	-741.533

D	ΔT	F	T_0	T_1	dS_{turb}/dt
0.01	30.53911	0.610782	303.2696	272.7304	0.000226
0.02	30.17056	1.206822	303.0853	272.9147	0.00044
0.03	29.8108	1.788648	302.9054	273.0946	0.000645
0.04	29.45951	2.356761	302.7298	273.2702	0.000839
0.05	29.11641	2.911641	302.5582	273.4418	0.001025
0.06	28.78121	3.453745	302.3906	273.6094	0.001201
0.07	28.45364	3.983509	302.2268	273.7732	0.00137
0.08	28.13344	4.50135	302.0667	273.9333	0.00153
0.09	27.82036	5.007666	301.9102	274.0898	0.001684
0.1	27.51418	5.502837	301.7571	274.2429	0.00183
0.11	27.21467	5.987227	301.6073	274.3927	0.001969
0.12	26.9216	6.461185	301.4608	274.5392	0.002102
0.13	26.63478	6.925043	301.3174	274.6826	0.002229
0.14	26.35401	7.379122	301.177	274.823	0.00235
0.15	26.07909	7.823728	301.0395	274.9605	0.002465
0.16	25.80985	8.259153	300.9049	275.0951	0.002575
0.17	25.54612	8.68568	300.7731	275.2269	0.00268
0.18	25.28772	9.103578	300.6439	275.3561	0.002781
0.19	25.03449	9.513106	300.5172	275.4828	0.002877
0.2	24.78629	9.914514	300.3931	275.6069	0.002968
0.21	24.54295	10.30804	300.2715	275.7285	0.003056
0.22	24.30435	10.69392	300.1522	275.8478	0.003139
0.23	24.07035	11.07236	300.0352	275.9648	0.003219
0.24	23.84081	11.44359	299.9204	276.0796	0.003295
0.25	23.6156	11.8078	299.8078	276.1922	0.003368
0.26	23.39461	12.1652	299.6973	276.3027	0.003437
0.27	23.17771	12.51597	299.5889	276.4111	0.003503
0.28	22.96481	12.86029	299.4824	276.5176	0.003566
0.29	22.75577	13.19835	299.3779	276.6221	0.003627
0.3	22.55051	13.53031	299.2753	276.7247	0.003684
0.31	22.34892	13.85633	299.1745	276.8255	0.003739
0.32	22.1509	14.17658	299.0755	276.9245	0.003792
0.33	21.95636	14.4912	298.9782	277.0218	0.003842
0.34	21.76521	14.80034	298.8826	277.1174	0.003889

Sheet1 Sheet2 Sheet3

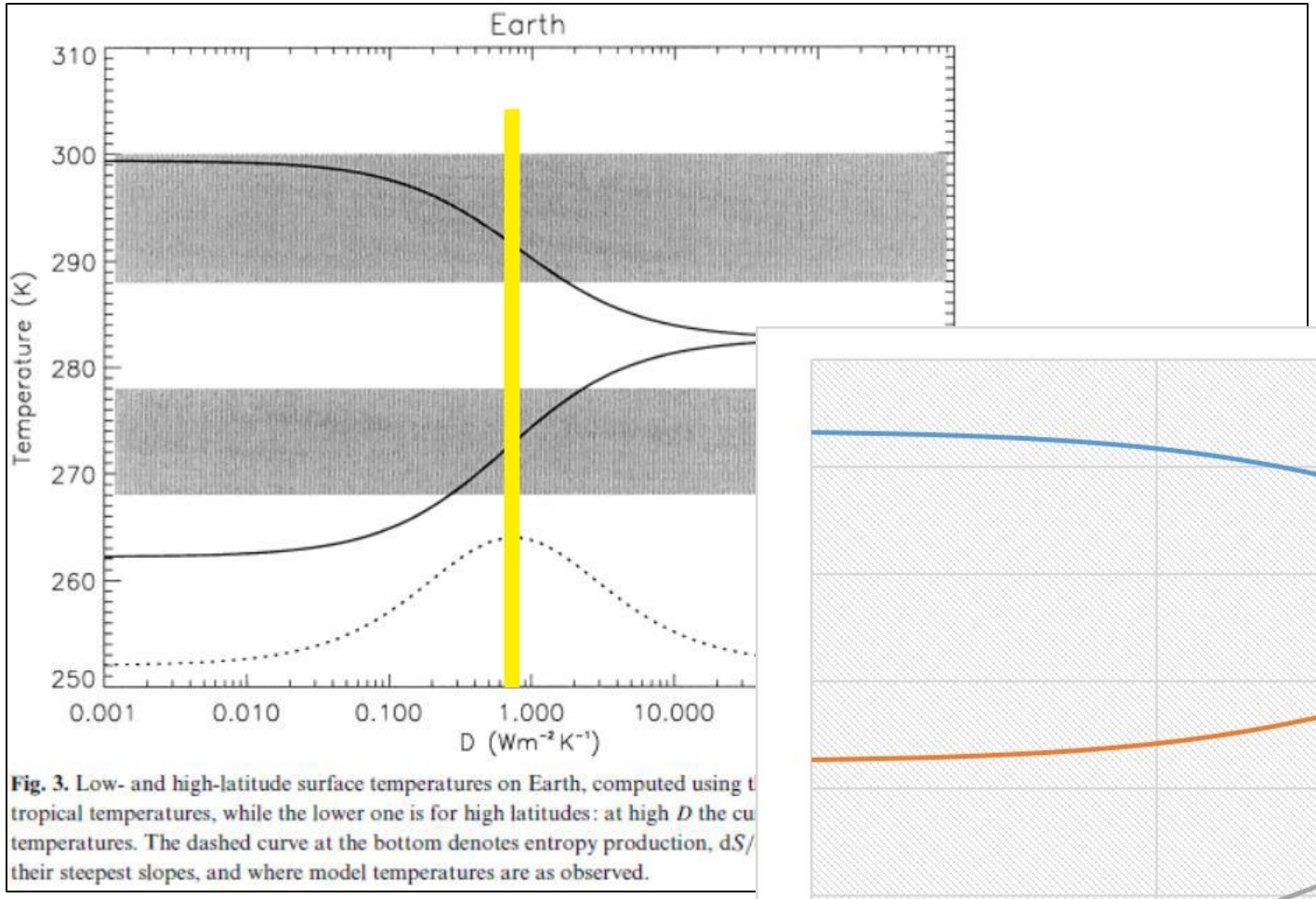
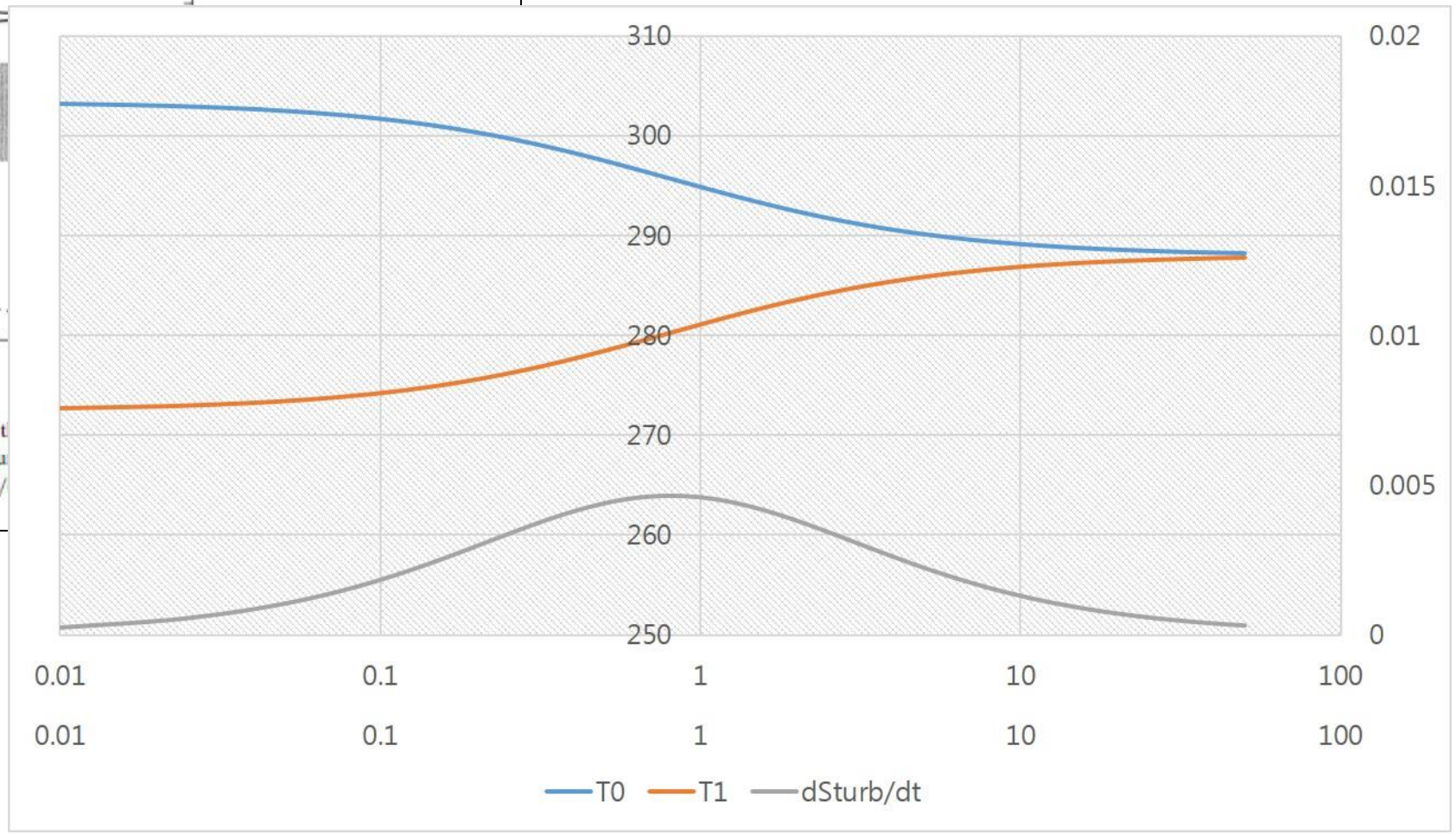
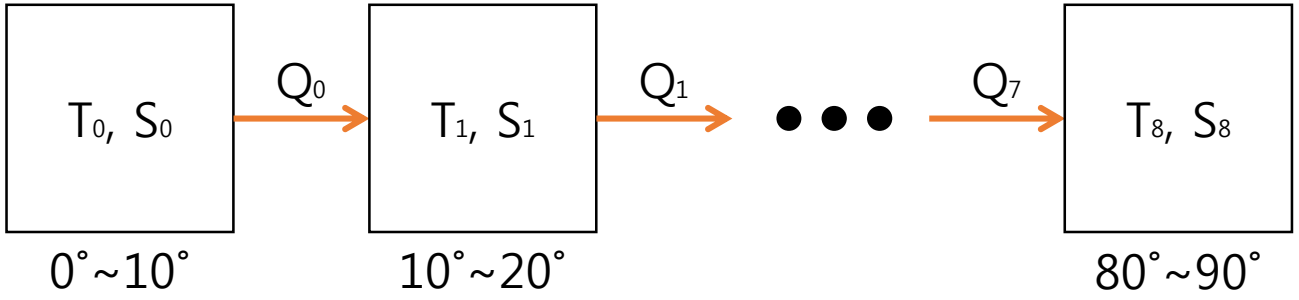


Fig. 3. Low- and high-latitude surface temperatures on Earth, computed using t
tropical temperatures, while the lower one is for high latitudes: at high *D* the cu
temperatures. The dashed curve at the bottom denotes entropy production, d*S*/
their steepest slopes, and where model temperatures are as observed.



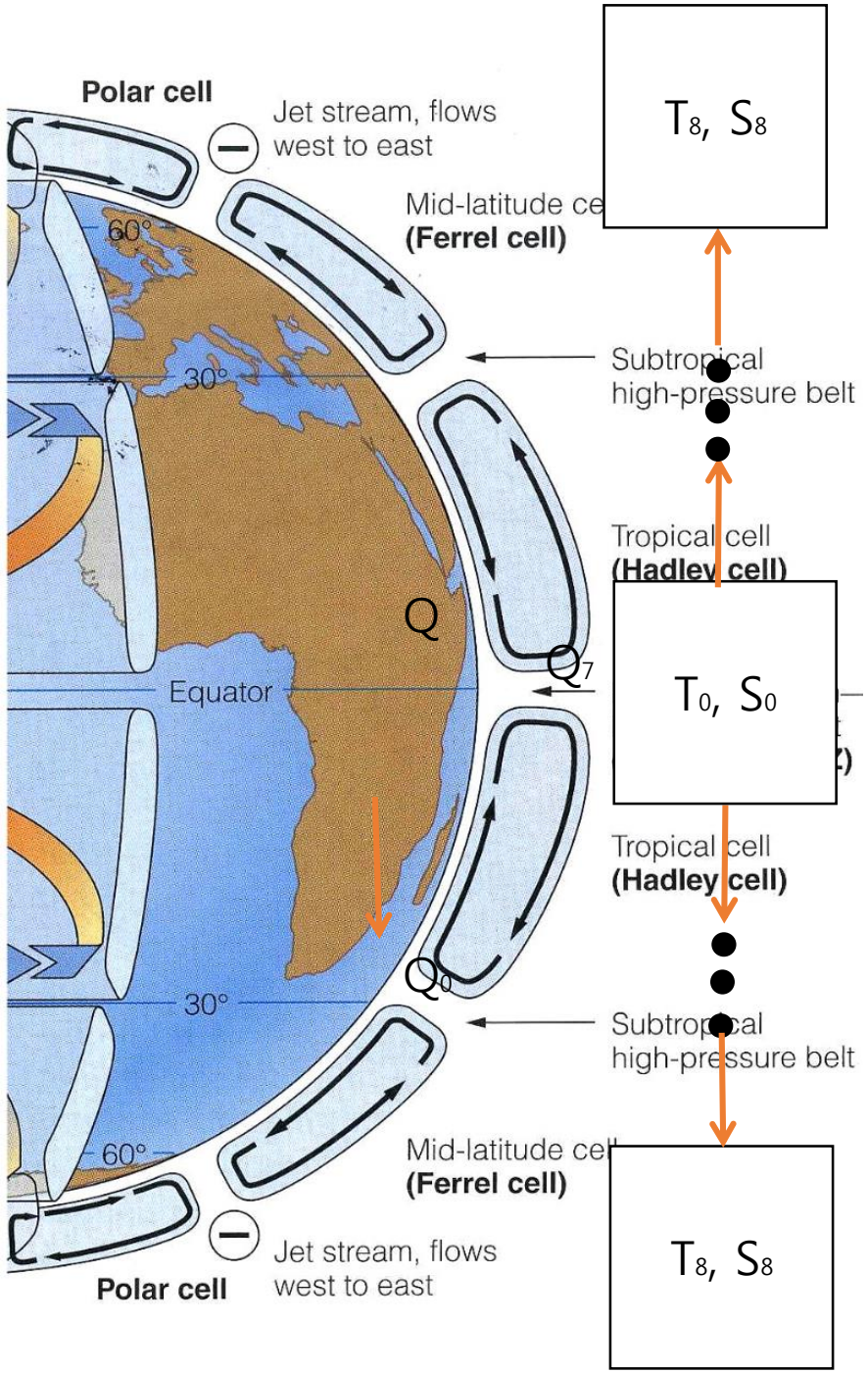
2 Box model
- Results

9 Box model

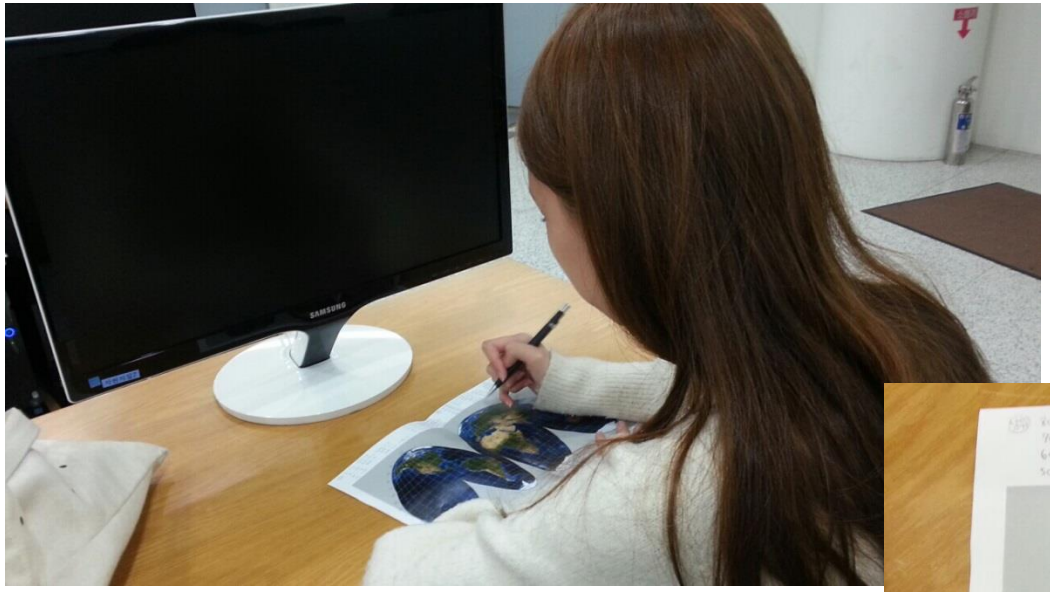


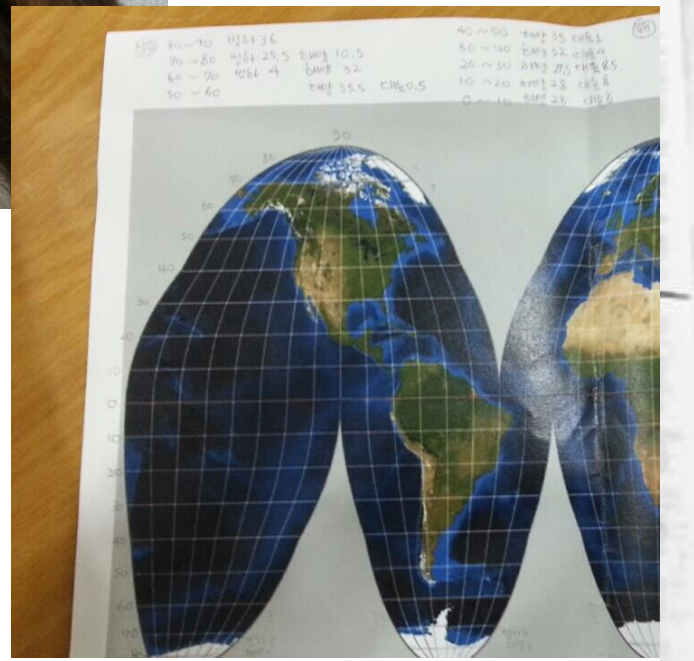
Assumptions

1. 2-box model**의 가정을 동일하게 적용.**
2. **상자 간 전달되는 열은** heat flux**와 평균 면적의 곱.** $(Q_n = F_n \cdot \overline{S_n})$
3. (Heat transfer parameter) D = Constant
4. $T \propto (1-\alpha)^{1/4}$, α = Albedo



Derivation of insolation





반사율 0.06
흡수율 0.5
복사율 0.2

위도	인해지구	
북반구	80-90	0.091
	70-80	0.147
	60-70	0.174
	50-60	0.138
	40-50	0.131
	30-40	0.118
	20-30	0.114
	10-20	0.091
	0-10	0.089
남반구	80-90	0.5
	70-80	0.372
	60-70	0.109
	50-60	0.062
	40-50	0.064
	30-40	0.096
	20-30	0.093
	10-20	0.091
	0-10	0.091

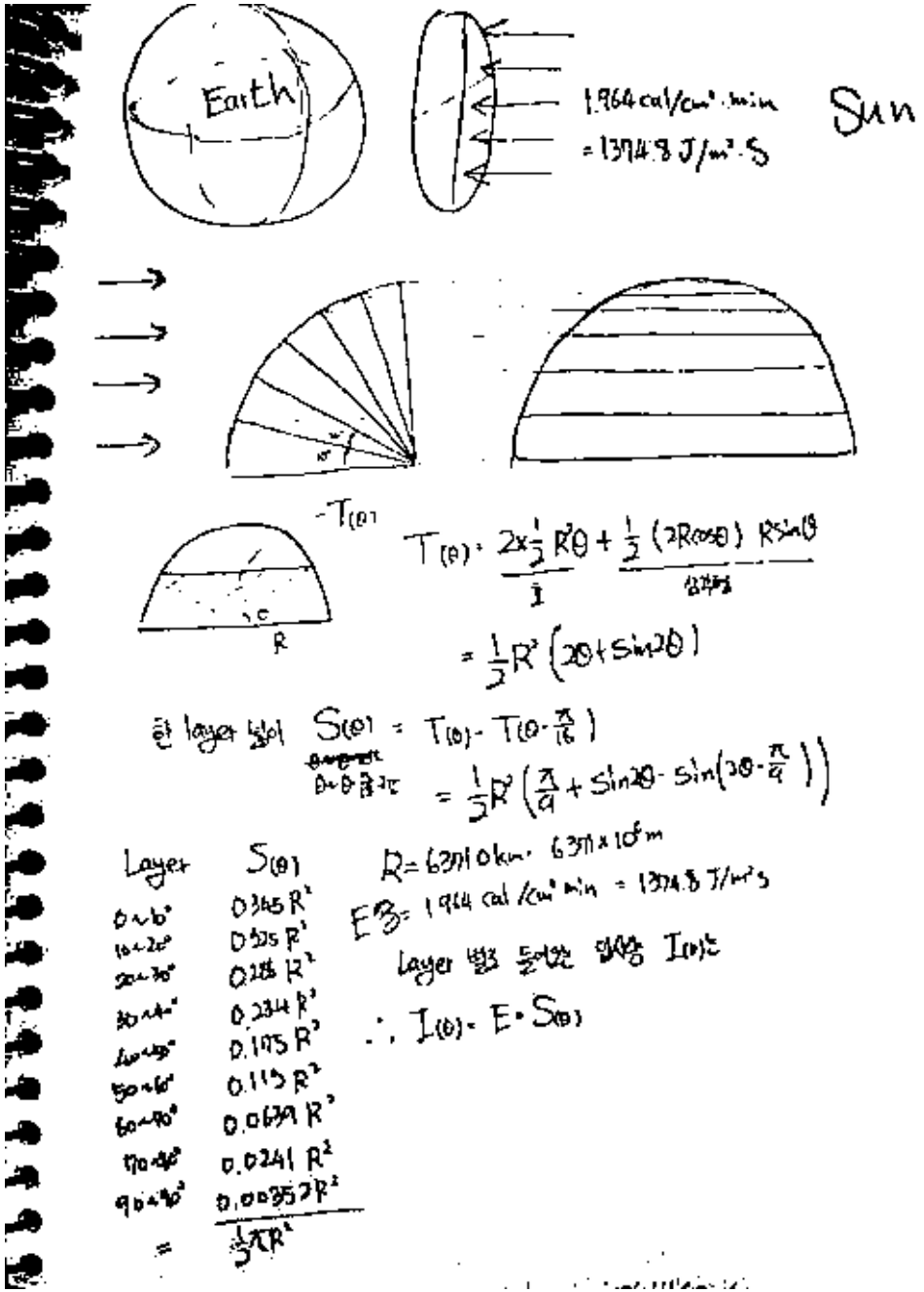
Derivation of insolation

$$C_{sol}R^2\left(\frac{\pi}{18}+\sin\varphi_2-\sin\varphi_1\right)(1-\alpha)$$

$$\alpha=\alpha_s+\alpha_a$$

(α_s = 지표의 알베도, α_a = 대기의 알베도)

$$\alpha_a\approx 0.88\times 0.3$$ (Donohoe & Battisti, 2011)



9 Box model

$$Q_n = k_n D(T_n - T_{n+1})$$

$$\begin{bmatrix} S_0B + k_0D & -k_0D & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_0D & S_1B + (k_0 + k_1)D & -k_1D & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_1D & S_2B + (k_1 + k_2)D & -k_2D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_2D & S_3B + (k_2 + k_3)D & -k_3D & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_3D & S_4B + (k_3 + k_4)D & -k_4D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_4D & S_5B + (k_4 + k_5)D & -k_5D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_5D & S_6B + (k_5 + k_6)D & -k_6D & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_6D & S_7B + (k_6 + k_7)D & -k_7D \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_7D & S_8B + k_7D \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{bmatrix} = \begin{bmatrix} I_0 - S_0A \\ I_1 - S_1A \\ I_2 - S_2A \\ I_3 - S_3A \\ I_4 - S_4A \\ I_5 - S_5A \\ I_6 - S_6A \\ I_7 - S_7A \\ I_8 - S_8A \end{bmatrix}$$

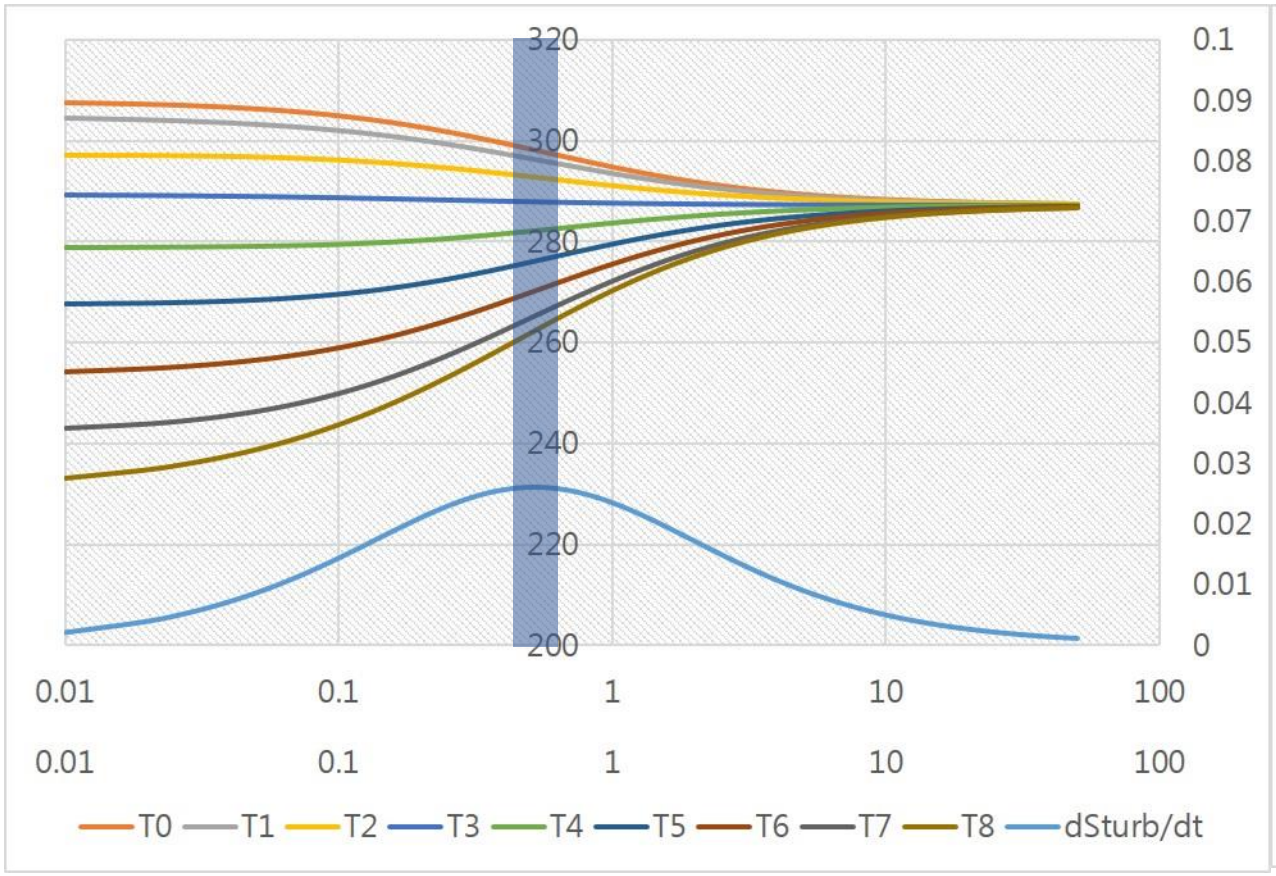
$$\frac{dS_{turb}}{dt} = \frac{-Q_0}{T_0} + \frac{Q_0 - Q_1}{T_1} + \frac{Q_1 - Q_2}{T_2} + \frac{Q_2 - Q_3}{T_3} + \frac{Q_3 - Q_4}{T_4} + \frac{Q_4 - Q_5}{T_5} + \frac{Q_5 - Q_6}{T_6} + \frac{Q_6 - Q_7}{T_7} + \frac{Q_7}{T_8}$$

9 Box model

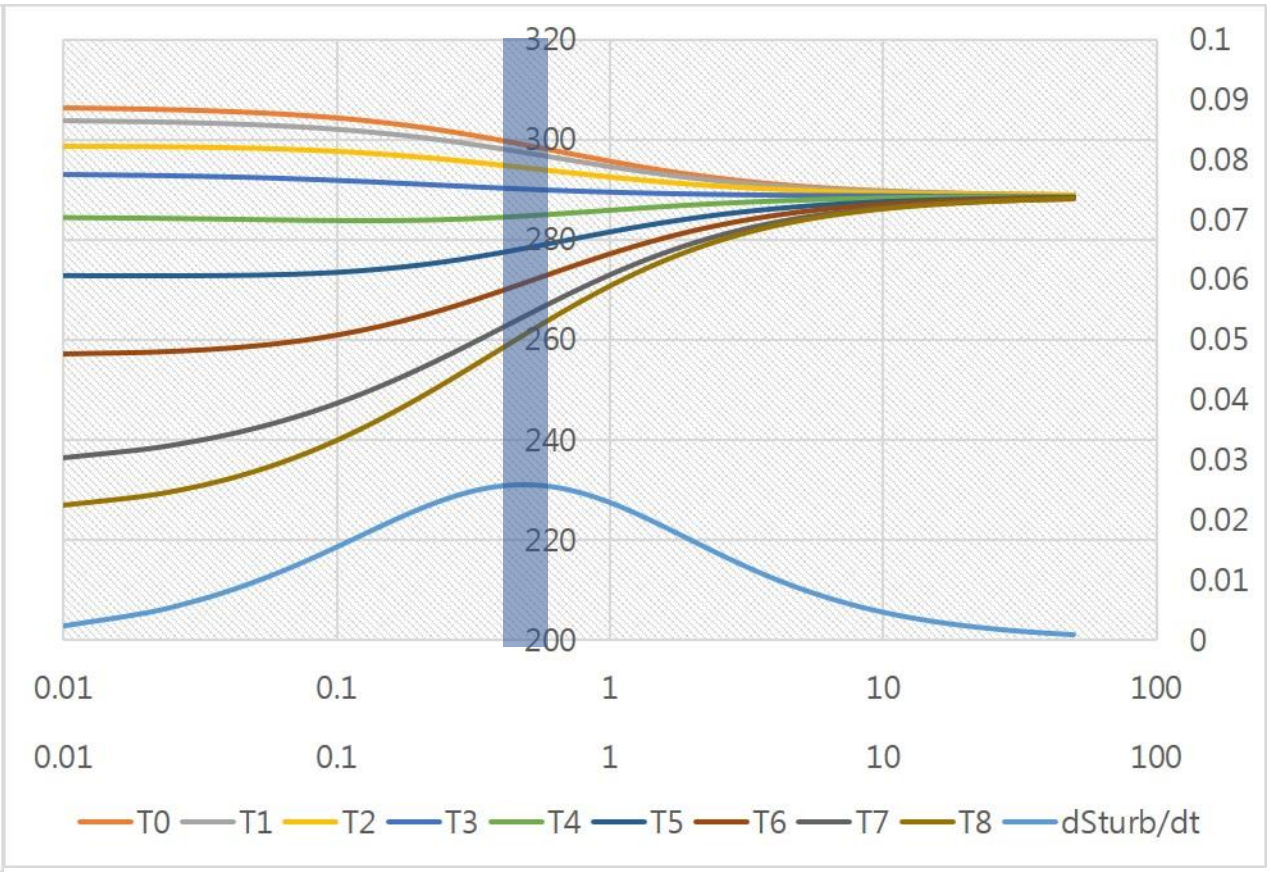
	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1																						
2	D	Matrix									Inverse									C	T	Q
3	0.01	4.068781	-0.35273	0	0	0	0	0	0	0	0.247549135	0.020477538	0.001722666	0.000147771	1.29896E-05	1.18126E-06	1.13438E-07	1.21881E-08	1.95724E-09	1159.258	311.6569	1.125
4		-0.35273	4.292445	-0.33657	0	0	0	0	0	0	0.020477538	0.236208315	0.019870941	0.001704535	0.000149835	1.36258E-05	1.3085E-06	1.40589E-07	2.25767E-08	1111.671	308.4649	1.355
5		0	-0.33657	4.02751	-0.31019	0	0	0	0	0	0.001722666	0.019870941	0.251615261	0.021583635	0.00189728	0.000172537	1.65689E-05	1.78021E-06	2.85877E-07	1030.063	304.4386	2.187
6		0	0	-0.31019	3.640202	-0.27438	0	0	0	0	0.000147771	0.001704535	0.021583635	0.278393773	0.02447182	0.002225446	0.000213711	2.29618E-05	3.68735E-06	909.4651	297.3851	2.952
7		0	0	0	-0.27438	3.142288	-0.23023	0	0	0	1.29896E-05	0.000149835	0.00189728	0.02447182	0.32252528	0.02933017	0.0028166	0.000302624	4.85972E-05	756.4379	286.6243	3.367
8		0	0	0	0	-0.23023	2.548897	-0.17909	0	0	1.18126E-06	1.36258E-05	0.000172537	0.002225446	0.02933017	0.397658946	0.038187508	0.004102977	0.000658882	581.0896	271.9979	2.496
9		0	0	0	0	0	-0.17909	1.878059	-0.1225	0	1.13438E-07	1.3085E-06	1.65689E-05	0.000213711	0.0028166	0.038187508	0.539889877	0.058007335	0.009315186	405.5012	258.0592	1.175
10		0	0	0	0	0	0	-0.1225	1.150157	-0.0622	1.21881E-08	1.40589E-07	1.78021E-06	2.29618E-05	0.000302624	0.004102977	0.058007335	0.883295259	0.141845165	239.245	248.465	0.537
11		0	0	0	0	0	0	0	-0.0622	0.387308	1.95724E-09	2.25767E-08	2.85877E-07	3.68735E-06	4.85972E-05	0.000658882	0.009315186	0.141845165	2.604699961	77.42965	239.8174	
12	0.02	4.421514	-0.70547	0	0	0	0	0	0	0	0.231503496	0.033447329	0.004914528	0.00073635	0.000113068	1.79648E-05	3.01598E-06	5.68221E-07	1.57246E-07	1159.258	311.3839	2.140
13		-0.70547	4.981753	-0.67315	0	0	0	0	0	0	0.033447329	0.209631069	0.030801797	0.004615074	0.000708656	0.000112595	1.89026E-05	3.56132E-06	9.85536E-07	1111.671	308.3499	2.784
14		0	-0.67315	4.674273	-0.62038	0	0	0	0	0	0.004914528	0.030801797	0.222803441	0.033382933	0.005126032	0.000814448	0.000136731	2.57607E-05	7.12883E-06	1030.063	304.2138	4.377
15		0	0	-0.62038	4.224769	-0.54876	0	0	0	0	0.00073635	0.004615074	0.033382933	0.246518199	0.037853482	0.006014337	0.001009701	0.000190231	5.26433E-05	909.4651	297.158	5.836
16		0	0	0	-0.54876	3.646896	-0.46046	0	0	0	0.000113068	0.000708656	0.005126032	0.037853482	0.285631705	0.045382494	0.007618917	0.001435431	0.000397231	756.4379	286.523	6.518
17		0	0	0	0	-0.46046	2.958215	-0.35818	0	0	1.79648E-05	0.000112595	0.000814448	0.006014337	0.045382494	0.352266194	0.059139251	0.011142044	0.003083373	581.0896	272.3675	4.856
18		0	0	0	0	0	-0.35818	2.17965	-0.24501	0	3.01598E-06	1.89026E-05	0.000136731	0.001009701	0.007618917	0.059139251	0.478644123	0.090178245	0.024955309	405.5012	258.8098	2.351
19		0	0	0	0	0	0	-0.24501	1.334857	-0.12439	5.68221E-07	3.56132E-06	2.57607E-05	0.000190231	0.001435431	0.011142044	0.090178245	0.78596452	0.217502431	239.245	249.2111	0.993
20		0	0	0	0	0	0	0	-0.12439	0.449505	1.57246E-07	9.85536E-07	7.12883E-06	5.26433E-05	0.000397231	0.003083373	0.024955309	0.217502431	2.284860061	77.42965	241.2204	
21	0.03	4.774248	-1.0582	0	0	0	0	0	0	0	0.218836273	0.042315779	0.008321734	0.001668898	0.00034304	7.29779E-05	1.64176E-05	4.15676E-06	1.51574E-06	1159.258	311.1282	3.090
22		-1.0582	5.671061	-1.00972	0	0	0	0	0	0	0.042315779	0.190914634	0.037544881	0.007529511	0.001547682	0.000329252	7.40706E-05	1.87539E-05	6.83852E-06	1111.671	308.2076	4.247
23		0	-1.00972	5.321037	-0.93057	0	0	0	0	0	0.008321734	0.037544881	0.202147675	0.040540099	0.008332968	0.001772744	0.000398808	0.000100974	3.68197E-05	1030.063	304.0014	6.558
24		0	0	-0.93057	4.809335	-0.82313	0	0	0	0	0.001668898	0.007529511	0.040540099	0.22364088	0.045969111	0.009779406	0.002200038	0.000557027	0.000203117	909.4651	296.954	8.639
25		0	0	0	-0.82313	4.151505	-0.69069	0	0	0	0.00034304	0.001547682	0.008332968	0.045969111	0.25916372	0.055134139	0.012403328	0.003140395	0.00114513	756.4379	286.4577	9.494
26		0	0	0	0	-0.69069	3.367533	-0.53726	0	0	7.29779E-05	0.000329252	0.001772744	0.009779406	0.055134139	0.31973736	0.07193016	0.018211977	0.006640907	581.0896	272.7114	7.096
27		0	0	0	0	0	-0.53726	2.48124	-0.36751	0	1.64176E-05	7.40706E-05	0.000398808	0.002200038	0.012403328	0.07193016	0.43490892	0.110114469	0.040152694	405.5012	259.5027	3.500
28		0	0	0	0	0	0	-0.36751	1.519556	-0.18659	4.15676E-06	1.87539E-05	0.000100974	0.000557027	0.003140395	0.018211977	0.110114469	0.716814096	0.261382698	239.245	249.9791	1.400
29		0	0	0	0	0	0	0	-0.18659	0.511701	1.51574E-06	6.83852E-06	3.68197E-05	0.000203117	0.00114513	0.006640907	0.040152694	0.261382698	2.049576887	77.42965	242.4717	
30	0.04	5.126981	-1.41093	0	0	0	0	0	0	0	0.208445417	0.048688139	0.011566335	0.002802192	0.000695905	0.000178919	4.86886E-05	1.49448E-05	6.47861E-06	1159.258	310.8839	3.998
31		-1.41093	6.360369	-1.3463	0	0	0	0	0	0	0.048688139	0.176920478	0.042029159	0.010182463	0.002528743	0.000650148	0.000176922	5.43057E-05	2.35416E-05	1111.671	308.0499	5.723
32		0	-1.3463	5.9678	-1.24075	0	0	0	0	0	0.011566335	0.042029159	0.186438397	0.045168691	0.011217325	0.002884012	0.000784814	0.000240896	0.000104429	1030.063	303.7986	8.720
33		0	0	-1.24075	5.393902	-1.09751	0	0	0	0	0.002802192	0.010182463	0.045168691	0.206204375	0.051209397	0.013166108	0.003582838	0.001099742	0.00047674	909.4651	296.7706	11.36
34		0	0	0	-1.09751	4.656113	-0.92092	0	0	0	0.000695905	0.002528743	0.011217325	0.051209397	0.238995563	0.061446562	0.016721194	0.005132522	0.002224958	756.4379	286.4175	12.32
35		0	0	0	0	-0.92092	3.776851	-0.71635	0	0	0.000178919	0.000650148	0.002884012	0.013166108	0.061446562	0.294978501	0.080271257	0.024639028	0.010681066	581.0896	273.0363	9.229
36		0	0	0	0	0	-0.71635	2.782831	-0.49001	0	4.86886E-05	0.000176922	0.000784814	0.003582838	0.016721194	0.080271257	0.401722049	0.123307413	0.053454002	405.5012	260.1517	4.610

Modeling | Present North | Present South | Present Earth | Pangaea North | Pangaea South | Pangaea Earth

9 Box model

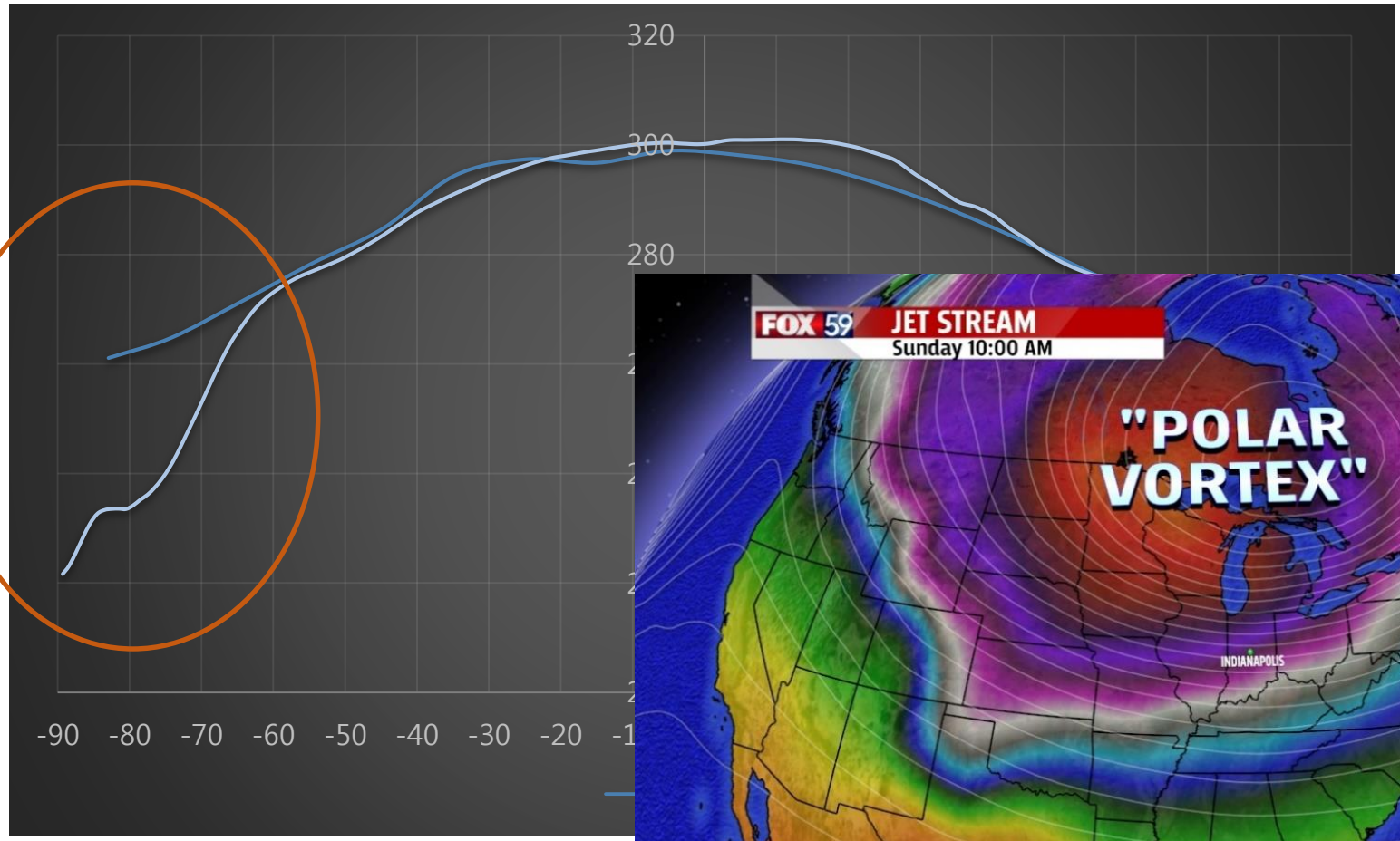


<Northern Hemisphere>



<Southern Hemisphere>

9 Box model

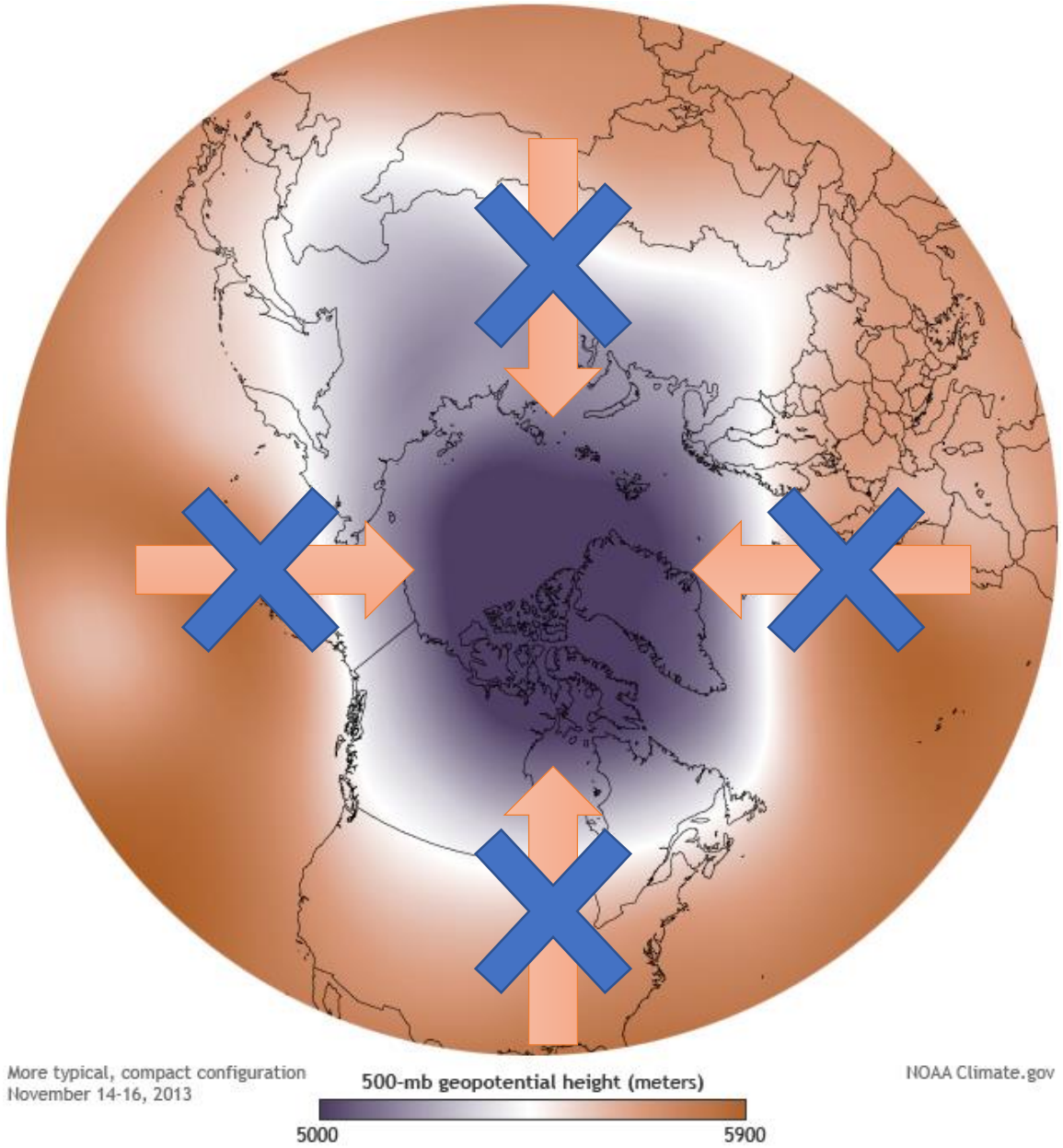

320
300
280
-90
-80
-70
-60
-50
-40
-30
-20

FOX 59
JET STREAM
Sunday 10:00 AM
"POLAR VORTEX"
INDIANAPOLIS

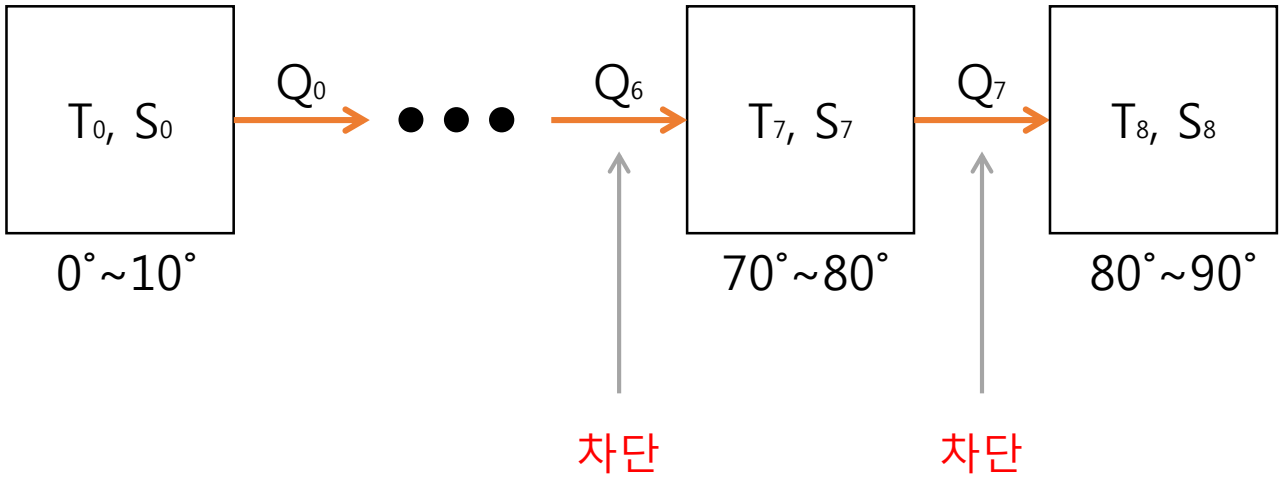
9 Box model

Polar Vortex

- 극지방에 저온, 저압의 소용돌이 존재.
- 열 전달 차단.
- 남극에서 더욱 현저하고 지속적.



Polar Vortex
- new assumption



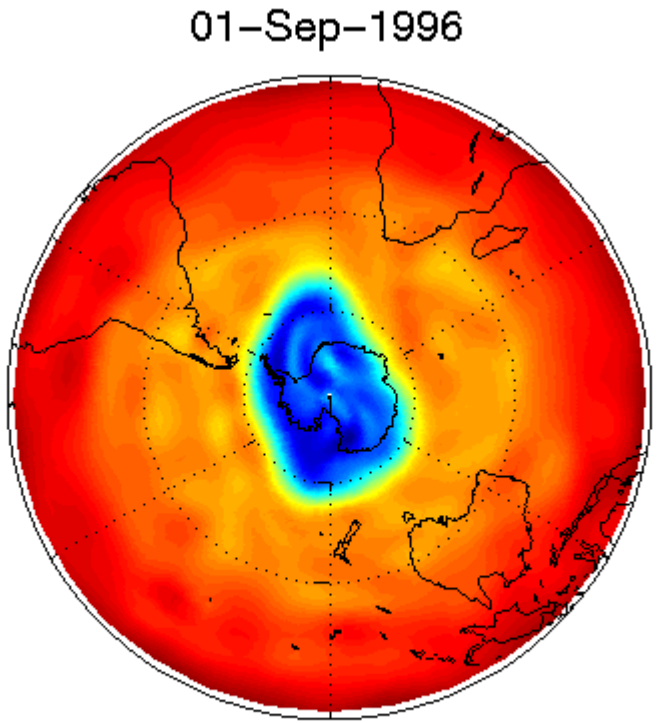
$$Q_n = k_n D_n (T_n - T_{n+1})$$

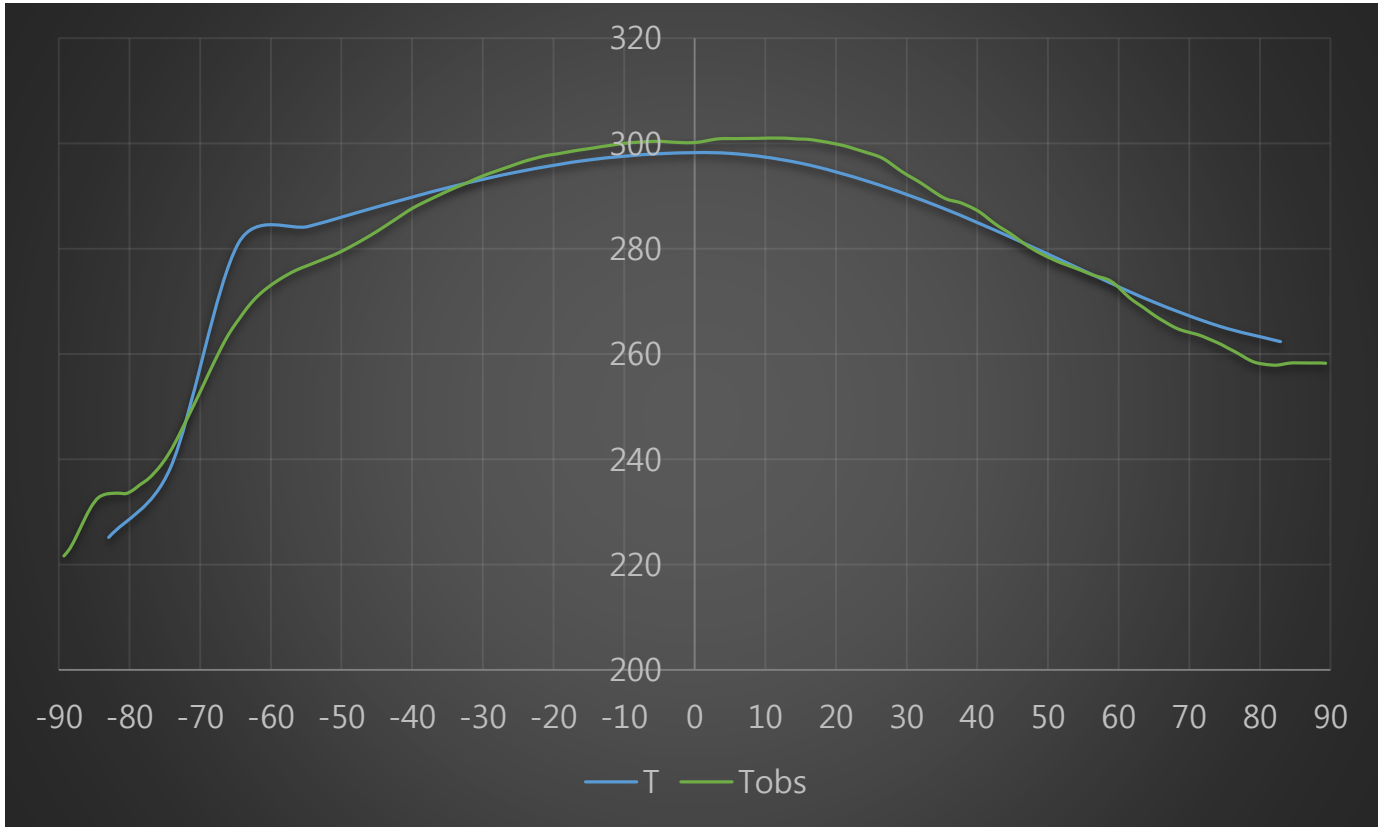
$$D_0 = D_1 = D_2 = D_3 = D_4 = D_5 = D_6 = D$$

$$D_6 = 0.01 \times D, \quad D_7 = 0.001 \times D$$

Polar Vortex 로 인한 효과 반영

$$D_6 = 0.01 \times D, \quad D_7 = 0.001 \times D$$

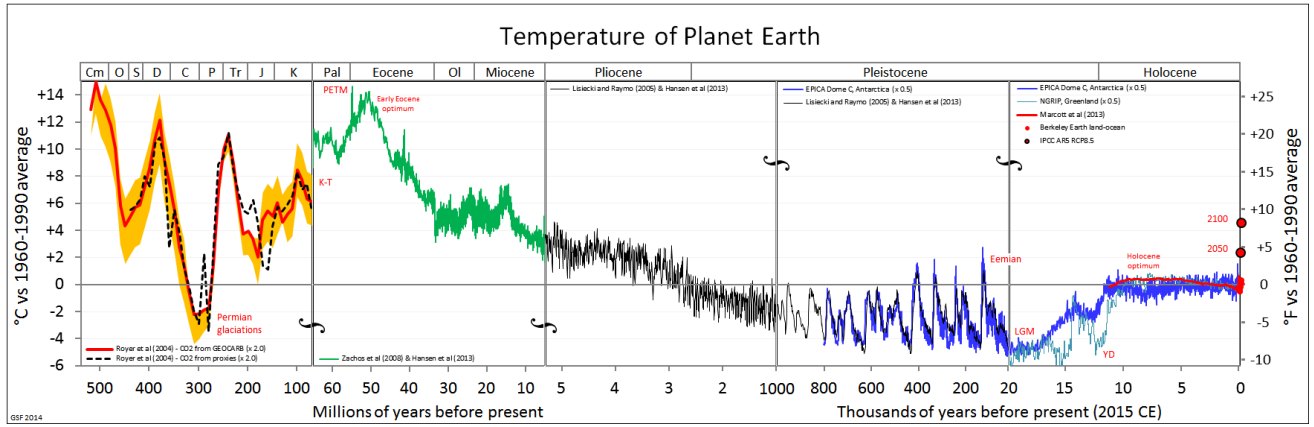


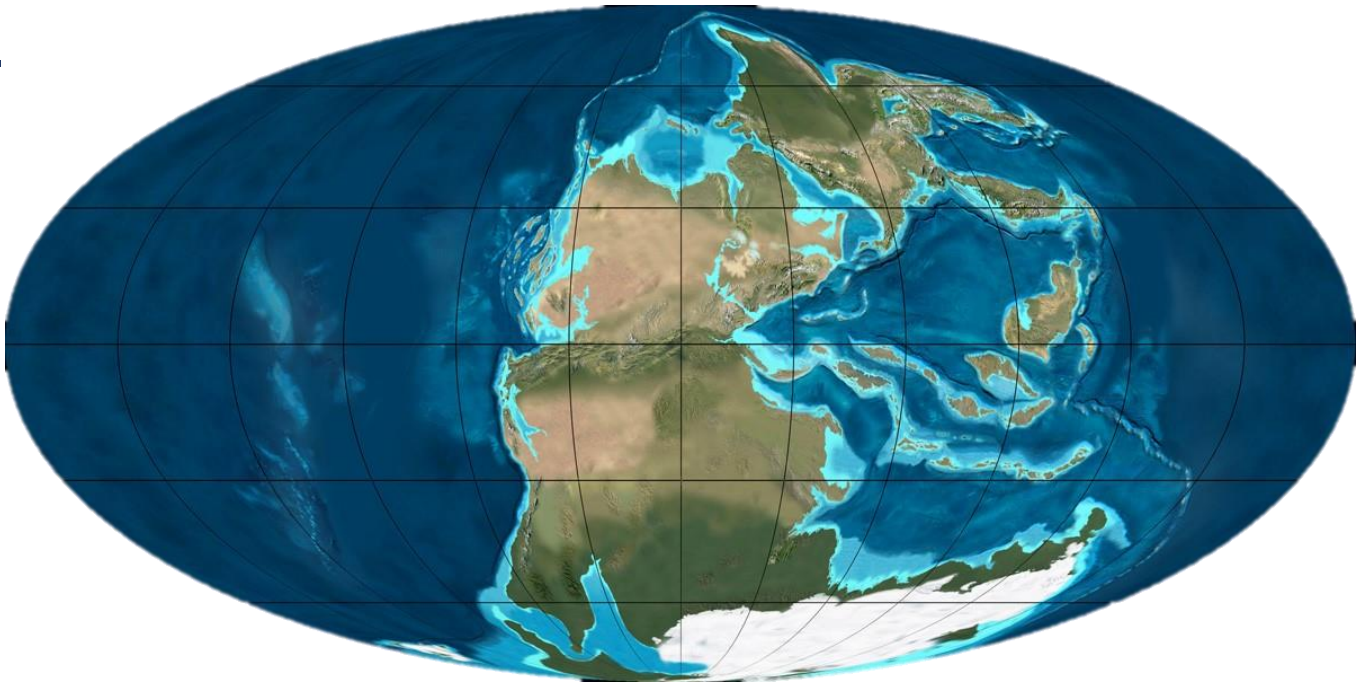


Pangaea

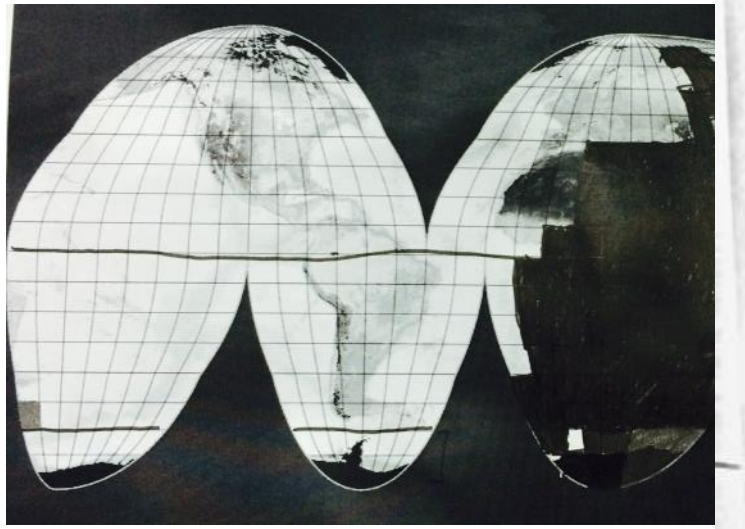
Assumptions

- 9-box model을 판게아에 적용.
- 대륙 분포가 다르기 때문에 알베도 다름.
- 평균 온도 지금보다 5도 정도 높다고 가정.





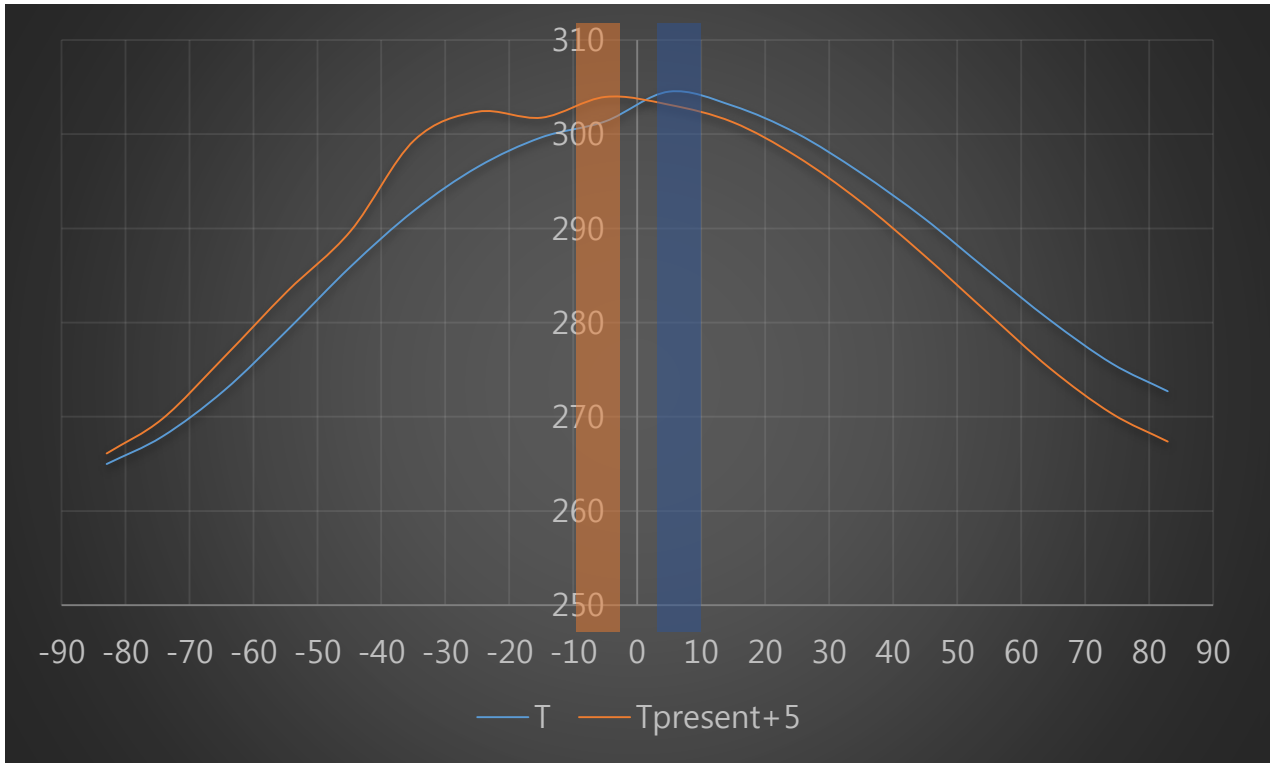
Pangaea



대륙일때도 0.06
등치일때도 0.5
육지 일때도 0.2

위도		현재지구	판게아
북반구	80-90	0.091	0.06
	70-80	0.147	0.076
	60-70	0.174	0.087
	50-60	0.138	0.099
	40-50	0.131	0.079
	30-40	0.118	0.103
	20-30	0.114	0.087
	10-20	0.091	0.087
	0-10	0.089	0.087
남반구	80-90	0.5	0.243
	70-80	0.372	0.239
	60-70	0.103	0.251
	50-60	0.062	0.18
	40-50	0.064	0.115
	30-40	0.076	0.095
	20-30	0.093	0.095
	10-20	0.091	0.103
	0-10	0.091	0.095

Pangaea



현재와 판게아 당시의 온도분포에 대한 차이

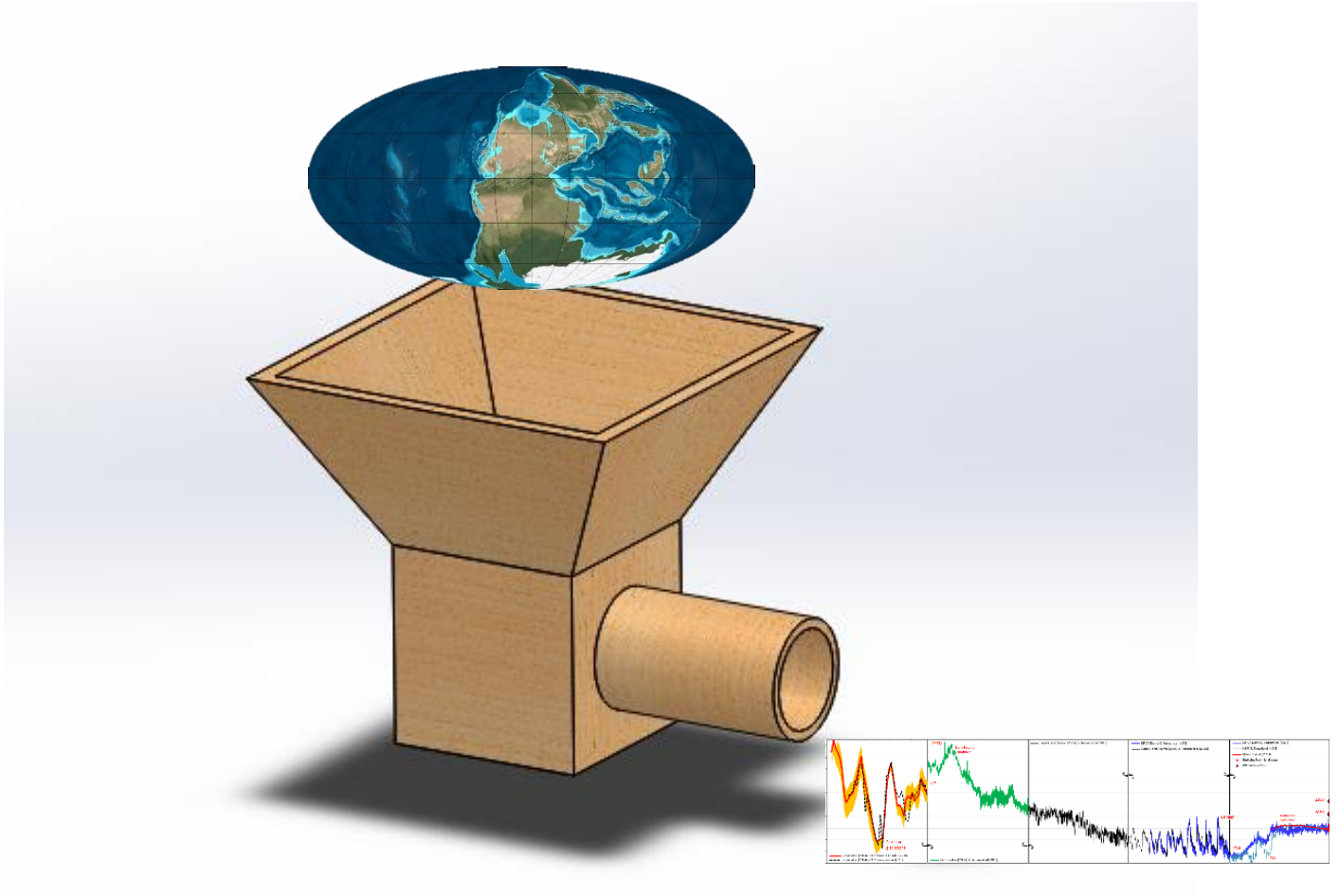
Conclusion

1. MEP 이론을 이용하여 위도별 지표면 온도를 추정하는 정교한 모델을 만들었다.

2. Pangaea 시기에 적용하여 모델의 유용성을 확인하였다.

3. 위도와 경도에 따른 지역별 온도 분포를 구하는 모델도 만들 수 있을 것이다.

Something More...?



[Appendix A] derivation of heat flux, F

$$\vec{q} = -k\nabla T$$

$$F = -\nabla \cdot \vec{q} = \nabla \cdot (k\nabla T)$$

$\nabla = \left(\frac{\partial}{\partial r}, \frac{\partial}{r\partial\varphi}, \frac{\partial}{rcos\varphi\partial\theta}\right)$ in spherical coordinates

$$T = T(\varphi),\ r = constant$$

$$k\nabla T = \left(0, \frac{k}{r}\frac{dT}{d\varphi}, 0\right)$$

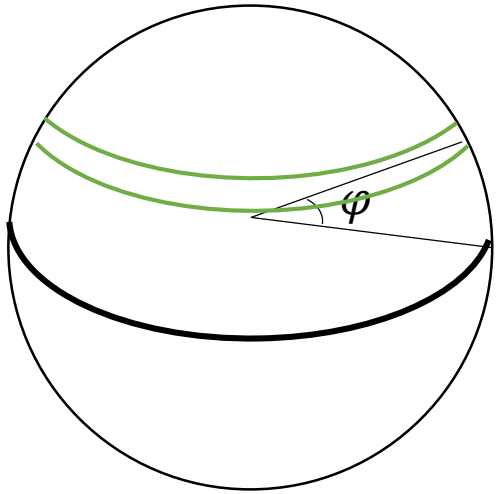
$$\nabla \cdot (k\nabla T) = \frac{1}{r^2\cos\varphi}\frac{\partial}{\partial\varphi}\left(rcos\varphi \cdot \frac{k}{r}\frac{dT}{d\varphi}\right) = \frac{1}{\cos\varphi}\frac{d}{d\varphi}\left(\frac{k}{r^2}\cos\varphi\frac{dT}{d\varphi}\right)$$

$$\frac{k}{r^2} = D,\ x = \sin\varphi$$

$$F = \frac{d}{dx}\left[D(1-x^2)\frac{dT}{dx}\right]$$

[Appendix B] division of two box regions

※위도에 따른 지표 면적 차이 고려



$$dS = 2\pi R cos\varphi \cdot R d\varphi$$

$$S(\varphi_1 \sim \varphi_2) = 2\pi R^2 \int_{\varphi_1}^{\varphi_2} cos\varphi \, d\varphi = 2\pi R^2 (\sin\varphi_2 - \sin\varphi_1)$$

$$\sin 30° - \sin 0° = \sin 90° - \sin 30° = \frac{1}{2}$$

[Appendix C] detailed equations of 2 boxes model

$$\begin{cases} I_0 = E_0 + F \\ I_1 + F = E_1 \end{cases} \qquad E_n = A + BT_n \ , \ B = \frac{4\sigma T^3}{1+0.75\tau} \ , \ \tau \approx 0.9$$

$$F = \frac{d}{dx}\left[D(1-x^2)\frac{dT}{dx}\right] \approx D(1-sin^2\,\varphi)\frac{\Delta T}{(\Delta \sin\varphi)^2}$$

$$\Delta \sin\varphi = 0.5,\ \sin\varphi \approx \sin 45° = \sqrt{0.5}$$

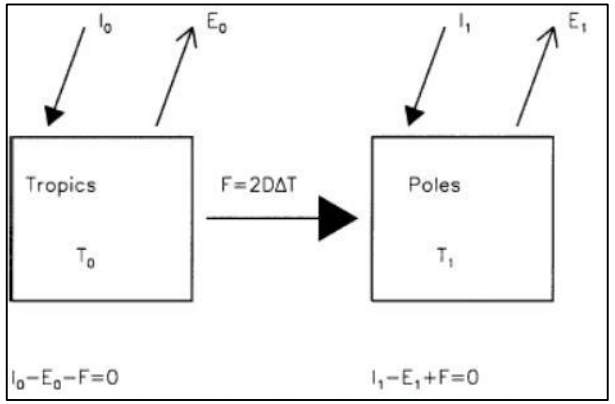
$$F \approx 2D\Delta T$$

$$\begin{cases} I_0 = A + BT_0 + 2D\Delta T \\ I_1 + 2D\Delta T = A + BT_1 \end{cases}$$

$$I_0 + I_1 = 2A + B(T_0 + T_1)$$

$$I_{avg} = A + BT_{avg}$$

$$A = I_{avg} - BT_{avg}, \quad I_{avg} = \frac{I_0+I_1}{2}, \quad T_{avg} = \frac{T_0+T_1}{2} = 288K$$



$$\begin{cases} I_0 - A = BT_0 + 2D\Delta T \\ I_1 - A = BT_1 - 2D\Delta T \end{cases}$$

$$\begin{bmatrix} B+2D & -2D \\ -2D & B+2D \end{bmatrix}\begin{bmatrix} T_0 \\ T_1 \end{bmatrix} = \begin{bmatrix} I_0 - A \\ I_1 - A \end{bmatrix}$$

$$\begin{bmatrix} T_0 \\ T_1 \end{bmatrix} = \begin{bmatrix} B+2D & -2D \\ -2D & B+2D \end{bmatrix}^{-1}\begin{bmatrix} I_0 - A \\ I_1 - A \end{bmatrix}$$

$$\begin{cases} T_0 = T_{avg} + \dfrac{I_0 - I_1}{2(B+4D)} \\ T_1 = T_{avg} - \dfrac{I_0 - I_1}{2(B+4D)} \end{cases}$$

$$\frac{dS_{turb}}{dt} = -\frac{F}{T_0} + \frac{F}{T_1}$$

[Appendix D] detailed equations of 9 boxes model

$$F = \frac{d}{dx}\left[D(1-x^2)\frac{dT}{dx}\right] = \frac{1}{\cos\varphi}\frac{d}{d\varphi}\left(D\cos\varphi\frac{dT}{d\varphi}\right) \approx \frac{1}{(\Delta\varphi)^2}D\Delta T$$

$$Q_n = F_n \cdot \overline{S_n} = \frac{S_n + S_{n+1}}{2(\Delta\varphi)^2}D\Delta T = k_n D(T_n - T_{n+1})$$

$$\begin{cases} I_0 = S_0(A + BT_0) + Q_0 \\ I_1 + Q_0 = S_1(A + BT_1) + Q_1 \\ \vdots \\ I_8 + Q_7 = S_8(A + BT_8) \end{cases}$$

$$I_{avg} = A + BT_{avg}$$

$$\begin{bmatrix} S_0B + k_0D & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_8B + k_7D \end{bmatrix}\begin{bmatrix} T_0 \\ \vdots \\ T_8 \end{bmatrix} = \begin{bmatrix} I_0 - S_0A \\ \vdots \\ I_8 - S_8A \end{bmatrix}$$

$$\begin{bmatrix} T_0 \\ \vdots \\ T_8 \end{bmatrix} = \begin{bmatrix} S_0B + k_0D & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_8B + k_7D \end{bmatrix}^{-1}\begin{bmatrix} I_0 - S_0A \\ \vdots \\ I_8 - S_8A \end{bmatrix}$$

$$\frac{dS_{turb}}{dt} = \frac{-Q_0}{T_0} + \frac{Q_0 - Q_1}{T_1} + \cdots + \frac{Q_7}{T_8}$$

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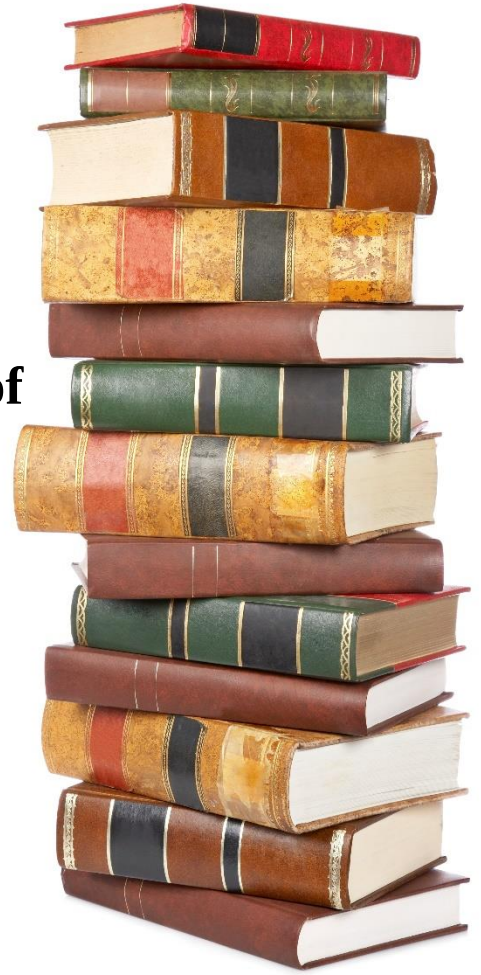

감사합니다.