Is it snowing in Titan?

Thermodynamic analysis of Titan's atmosphere

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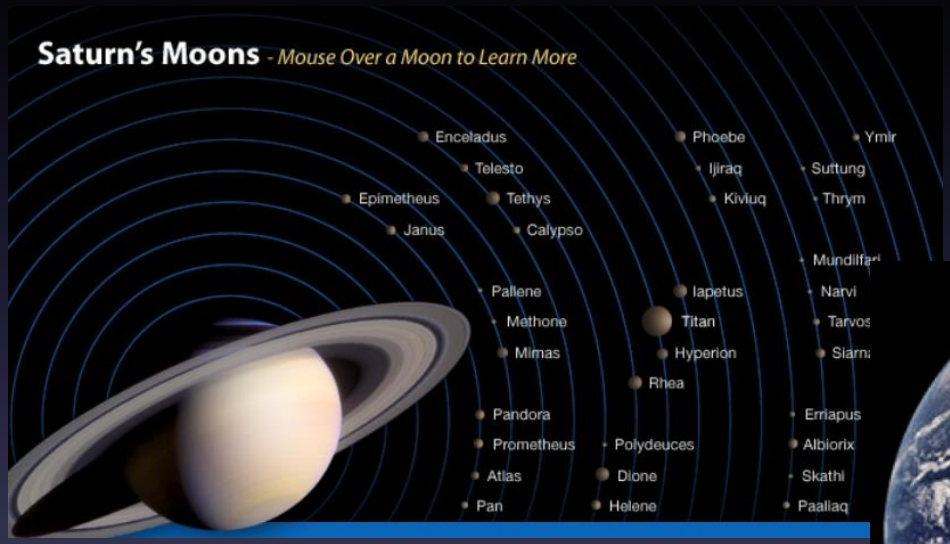
What is Titan?

토성의 위성 중 가장 큰 위성



Physical properties of Titan

- Atmospheric composition of Titan

	N_2	CH_4	H_2	CO	C_2H_6	C_2H_2	C_2H_4	HCN
(%)	93-98	7-2	0.1	$6*10^{-3}$	$2*10^{-3}$	$2*10^{-4}$	$4*10^{-5}$	$2*10^{-5}$

(Andreas, 2009)

Physical properties of Titan

- Properties

Physical Data on Titan	
Surface radius	2575.0 km
Mean density	1.881 g/cm^3
Rotation period (Davies, 1980)	15^{days} 22^{hr}
At the surface :	
Atmospheric pressure	1496 mbar
Atmospheric temperature	94.0 K
Acceleration of gravity	1.354 m/sec^2

(Lindal, 1982)

Raoult's ideal law

$$p \cdot y_{CH_4} = p_{CH_4}^{sat} \cdot x_{CH_4}$$

$$p \cdot y_{N_2} = p_{N_2}^{sat}(1 - x_{CH4})$$

x_i : The mole fractions in the liquid phase
y_i : The mole fractions in the gaseous atmosphere

Raoult's ideal law

$$p \cdot y_{CH_4} = p_{CH_4}^{sat} \cdot x_{CH_4}$$
$$p \cdot y_{N_2} = p_{N_2}^{sat}(1 - x_{CH4})$$

$$p_{N_2}^{sat}(T) = 10^{6.93878 - \frac{330.16}{277.196 + T(℃)}}\ (mmHg)$$
$$p_{CH_4}^{sat}(T) = 10^{6.61184 - \frac{389.93}{266 + T(℃)}}\ (mmHg)$$

At 93 K,

$$p_{CH_4}^{sat} = 0.1598 \text{ bar}\ ,\ p_{N_2}^{sat} = 4.625\ bar$$

Raoult's ideal law

Using these data, we obtain

$$x_{CH_4} = \frac{p - p_{N_2}^{sat}}{p_{CH_4}^{sat} - p_{N_2}^{sat}} = 0.698$$

$$x_{N_2} = 1 - x_{CH_4} = 0.302$$

The predicted composition of the atmosphere is therefore

$$y_{CH_4} = p_{CH_4}^{sat} \cdot \frac{x_{CH_4}}{p} = 0.074$$

$$y_{N_2} = p_{N_2}^{sat} \cdot \frac{x_{N_2}}{p} = 0.926$$

-> 실제 값과 거의 흡사한 값
(실제 값 : y_{N_2}:0.93-0.98 , y_{CH_4} : 0.02-0.07)

Cloud formation and rainfall

- *Polytropic transformation*
- $pV^{\eta} = constant$

$$\rightarrow Tp^{\frac{1-\eta}{\eta}} = constant$$

$$\rightarrow \frac{dT}{T} = \left(\frac{\eta-1}{\eta}\right)\frac{dp}{p} \qquad (1)$$

Cloud formation and rainfall

- Ideal gas law

$$p = \rho RT \quad \rightarrow \quad \rho = \frac{p}{RT} = \frac{\bar{M}p}{R^*T}$$

- Hydrostatic equilibrium

$$dp = -p\frac{\bar{M}g}{R^*T}dh \qquad (2)$$

Cloud formation and rainfall

- (1) and (2)

→ $\frac{dT}{T} = (\frac{\eta-1}{\eta})(-1)\frac{\bar{M}g}{R^*T}dh$

→ $\int_{T_0}^{T} dT = \int_0^h (\frac{1-\eta}{\eta})\frac{\bar{M}g}{R^*}dh$

→ $T-T_0 = (\frac{1-\eta}{\eta})\frac{\bar{M}g}{R^*}h$

→ $T(h) = T_0(1-\frac{\bar{M}g}{R^*}\frac{\eta-1}{\eta}\frac{h}{T_0})$ (3)

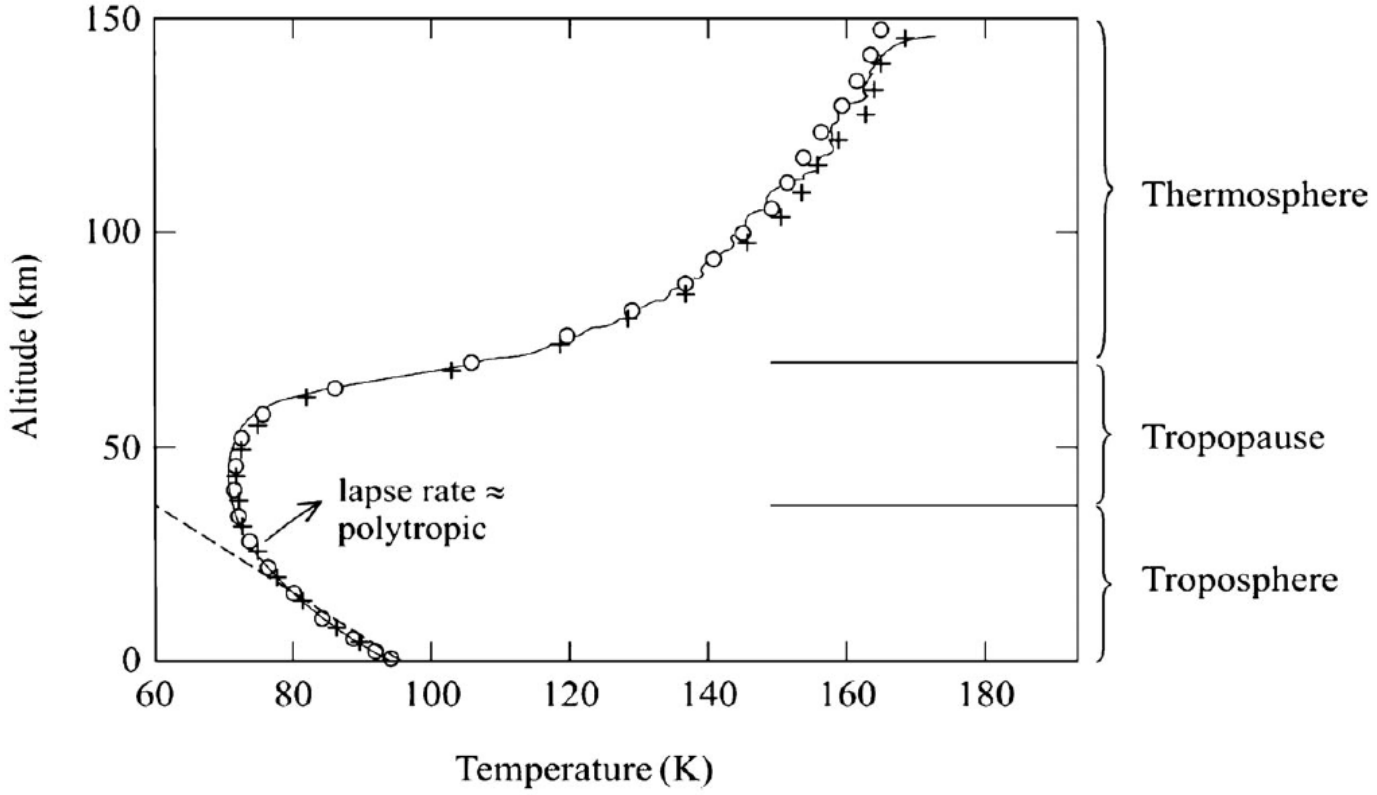


Fig. 2 Temperature profile in Titan's atmosphere (see text).

(Andreas, 2009)

Cloud formation and rainfall

- $Tp^{\frac{1-\eta}{\eta}} = constant$

→ $T_0(1-\frac{\bar{M}g}{R^*}\frac{\eta-1}{\eta}\frac{h}{T_0})\,p^{\frac{1-\eta}{\eta}}$ = constant

→ p(h) = $T_0^{\frac{\eta}{\eta-1}}(1-\frac{\bar{M}g}{R^*}\frac{\eta-1}{\eta}\frac{h}{T_0})^{\frac{\eta}{\eta-1}}$

= $p_0(\frac{T(h)}{T_0})^{\frac{\eta}{\eta-1}}$

Cloud formation and rainfall

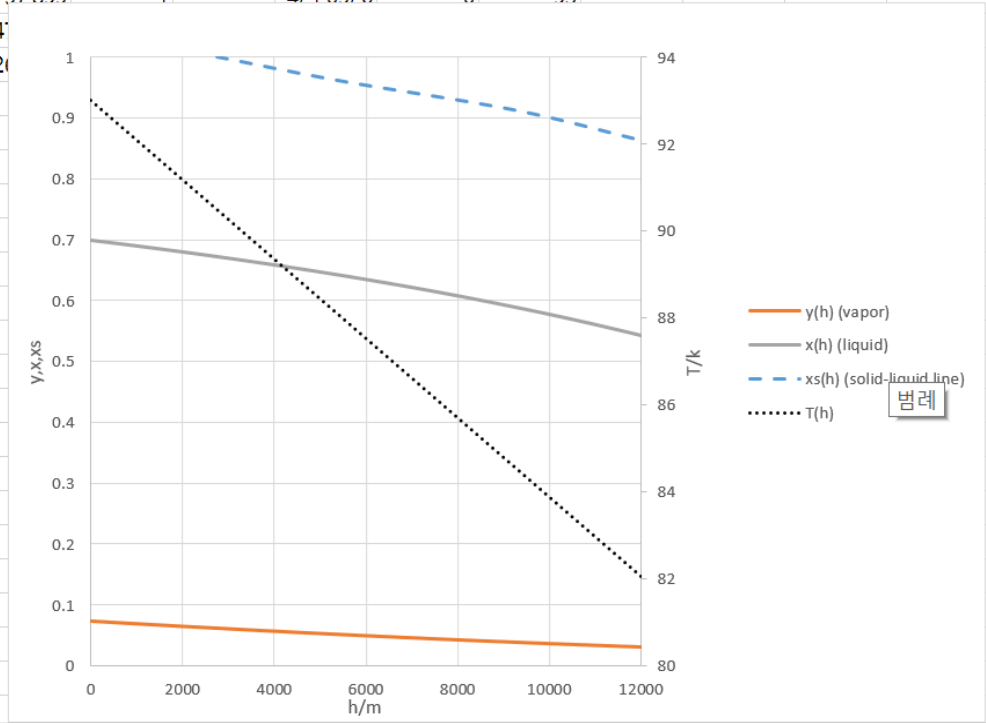
- $p(h) = p(y_{N_2} + y_{CH_4}) = p_{N_2}^{sat} x_{N_2} + p_{CH_4}^{sat} x_{CH_4}$

$$= p_0 \left(1 - \frac{\bar{M} g}{R^*} \frac{\eta - 1}{\eta} \frac{h}{T_0}\right)^{\frac{\eta}{\eta - 1}}$$

$$\rightarrow x_{CH_4}(h) = \frac{p(h) - p_{N_2}^{sat}}{p_{CH_4}^{sat} - p_{N_2}^{sat}}, \quad y_{CH_4}(h) = \frac{x_{CH_4}(h) \cdot p_{CH_4}^{sat}[T(h)]}{p(h)}$$

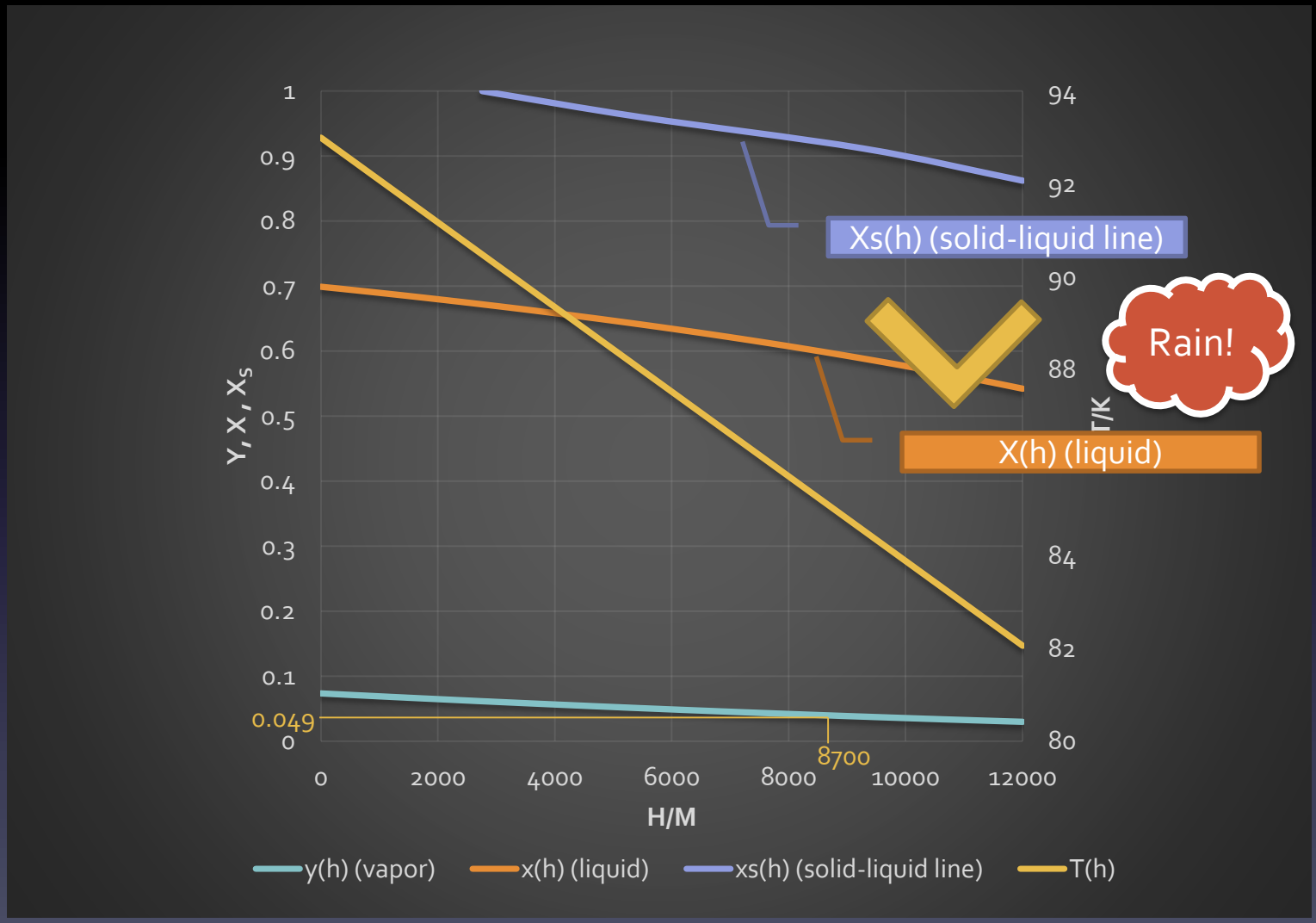
Is it snowing in Titan?

	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	h	y(h)	x(h)	p(h)	psat(T(h))	psatn2	T(h)	p(h)	h	xs			h	T(h)				
2	0	0.073785	0.699128	1.5083	0.159441	4.642614	93	1.5083	2757.895	1		421.0526	0	93				
3	150	0.073104	0.697755	1.497239	0.156811	4.591714	92.8632	1.497239	54									
4	300	0.072426	0.69637	1.486243	0.154216	4.541231	92.7264	1.486243	92									
5	450	0.071751	0.694975	1.475312	0.151656	4.491163	92.5896	1.475312										
6	600	0.07108	0.693569	1.464446	0.14913	4.441507	92.4528	1.464446										
7	750	0.070412	0.692153	1.453643	0.146639	4.392262	92.316	1.453643										
8	900	0.069747	0.690725	1.442904	0.144181	4.343426	92.1792	1.442904										
9	1050	0.069086	0.689285	1.432229	0.141757	4.294996	92.0424	1.432229										
10	1200	0.068427	0.687835	1.421617	0.139366	4.24697	91.9056	1.421617										
11	1350	0.067773	0.686373	1.411069	0.137008	4.199347	91.76879	1.411069										
12	1500	0.067121	0.684899	1.400582	0.134682	4.152124	91.63199	1.400582										
13	1650	0.066473	0.683414	1.390159	0.132389	4.1053	91.49519	1.390159										
14	1800	0.065828	0.681916	1.379797	0.130127	4.058872	91.35839	1.379797										
15	1950	0.065186	0.680407	1.369497	0.127897	4.012837	91.22159	1.369497										
16	2100	0.064548	0.678885	1.359259	0.125698	3.967196	91.08479	1.359259										
17	2250	0.063913	0.677351	1.349083	0.123529	3.921944	90.94799	1.349083										
18	2400	0.063281	0.675805	1.338967	0.121392	3.87708	90.81119	1.338967										
19	2550	0.062653	0.674246	1.328912	0.119284	3.832602	90.67439	1.328912										
20	2700	0.062028	0.672674	1.318917	0.117206	3.788508	90.53759	1.318917										
21	2850	0.061407	0.67109	1.308983	0.115158	3.744797	90.40079	1.308983										
22	3000	0.060789	0.669492	1.299109	0.113139	3.701465	90.26399	1.299109										
23	3150	0.060174	0.667881	1.289294	0.111149	3.658511	90.12719	1.289294										



Sheet1

Is it snowing in Titan?



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Thank you!
Q&A