Atmospheric Environment 42 (2008) 9063–9069

Contents lists available at ScienceDirect

Atmospheric Environment

journal homepage: www.elsevier.com/locate/atmosenv

Analytical solution of the advection-diffusion equation for a ground-level finite area source

Young-San Park [a], Jong-Jin Baik [b,*]

[a] Climate Environment System Research Center, Seoul National University, Seoul 151-742, Republic of Korea
[b] School of Earth and Environmental Sciences, Seoul National University, Seoul 151-742, Republic of Korea

ARTICLE INFO

Article history:
Received 14 April 2008
Received in revised form
12 September 2008
Accepted 15 September 2008

Keywords:
Advection-diffusion equation
Finite area source
Analytical solution

ABSTRACT

An advection-diffusion equation for a ground-level finite area source is solved analytically in a closed form using the superposition method. Power laws are assumed for height-dependent wind speed and vertical eddy diffusivity and for the downwind distance-dependent standard deviation of concentration distribution in the lateral direction. Results of the analysis show that the ground-level concentration increases with increasing downwind distance inside the source region and then decreases rapidly beyond the downwind edge of the source region. The ground-level concentration inside the source region is sensitive to exponents in the power laws for wind speed and vertical eddy diffusivity, while the ground-level concentration outside the source region is sensitive to the standard deviation of concentration distribution in the lateral direction. The solution for the ground-level finite area source is compared with solutions for a laterally infinite area source and a point source. The ratio of the ground-level concentration for the finite area source to that for the laterally infinite area source is highly dependent on lateral eddy diffusivity but almost independent of exponents in the power laws for wind speed and vertical eddy diffusivity. This is also true for the ratio of the ground-level concentration for the finite area source to that for the point source.

1. Introduction

The estimation of pollutant dispersion from area sources is useful in regulatory planning or risk assessment in urban areas or hazardous waste sites. Many realistic boundary layer parameterizations for eddy diffusivities are available and can be utilized in solving advection-diffusion equations, but the equations need to be solved numerically because of their complicated formulations (Degrazia et al., 1997; Tirabassi and Rizza, 1997; Ulke, 2000; Degrazia et al., 2002). Analytical solutions of advection-diffusion equations are of fundamental importance in describing and understanding dispersion phenomena, since all the parameters are expressed in a mathematically closed form and therefore the influence of individual parameters on pollutant concentration can be easily examined. Also, the analytical solutions make it easy to obtain asymptotic behaviors of the solutions, which are usually difficult to obtain through numerical calculations. The analytical solutions can be also used to improve the modeling of pollutant dispersion.

Analytical solutions of the advection-diffusion equation, with wind speed and vertical eddy diffusivity being expressed as a power function of height, are well known for point and line sources (Yeh and Huang, 1975; Huang, 1979; Seinfeld, 1986; Lin and Hildemann, 1996; Brown et al., 1997). Analytical solutions are also well known for laterally infinite area sources (Philip, 1959; Lebedeff and Hameed, 1975a,b; Mehta and Balasubramanyam, 1978; Singh and Srivastava, 1979; Wilson, 1982). An analytical solution for an area source that is finite in the along-wind (longitudinal)

* Corresponding author. Tel.: +82 2 880 6990; fax: +82 2 883 4972.
E-mail address: jjbaik@snu.ac.kr (J.-J. Baik).

1352-2310/$ – see front matter
doi:10.1016/j.atmosenv.2008.09.019

and vertical directions, like a plant canopy, is available for the Gaussian plume model (Park and Paw U, 2004). However, up to date no literature is found for the study of analytical solution in a closed form for an area source that is finite in the along-wind and crosswind (lateral) directions, even for a Gaussian plume model.

Since the analytical solution for a laterally finite area source is not available, analytical solutions for a laterally infinite area source and a point source have been applied instead, based on the assumption that a finite area source can be treated as a laterally infinite area source or a point source depending on the distance of the receptor from the source. At a downwind distance not far from a source compared with the lateral size of the source, the difference can be small even if one assumes that the source has a laterally infinite extent. This is because the portion of the source in an adjacent upwind sector makes an important contribution to the receptor (Dobbins, 1979; Pasquill and Smith, 1983). Another common approach is to model an area source as a virtual or imaginary point source located upwind of the actual source (Turner, 1970; Pasquill and Smith, 1983; Chitgopekar et al., 1990).

However, if an area source is distinctly different from neighboring sources or an area source contains isolated sources with high emission rates and therefore the effects of such isolated sources need to be considered separately, the approximation adopted to neglect lateral dispersion must be used cautiously (Rote, 1980; Pasquill and Smith, 1983). Furthermore, the virtual point or line source method for equivalent source strength leads to rather different concentration levels, especially at a short downwind distance from an area source (Rote, 1980; Chrysikopoulos et al., 1992).

Multiple sources like finite area sources can be treated as a set of point sources distributed uniformly over the region. The practical problem of pollutant concentration distribution for an array of sources may be dealt with by integrating specifiable distributions from individual point sources. This method is applicable for chemically inert pollutants under steady-state emission and meteorological conditions. This superposition method is reasonable and popularly used (Calder, 1977; Pasquill and Smith, 1983; Chrysikopoulos et al., 1992; Arya, 1999). If relevant integral formulas are available, the integration of a solution for a point source in the along-wind and crosswind directions is straightforward.

It is noteworthy to mention recent important progress in finding analytical solutions of the advection-diffusion equation for a continuous point source using integral transform techniques (Moreira et al., 2005; Moreira et al., 2006; Costa et al., 2006). Moreira et al. (2005) solved the steady two-dimensional advection-diffusion equation using the generalized integral Laplace transform technique (GILTT) and Moreira et al. (2006) solved the nonstationary two-dimensional advection-diffusion equation using the GILTT. Costa et al. (2006) presented semi-analytical solution of the steady three-dimensional advection-diffusion equation obtained using the generalized integral advection-diffusion multilayer technique. The solutions reported in those studies are analytical or semi-analytical in the sense that no approximation is made along the solution derivation (e.g., Moreira et al., 2006). The integral transform techniques provide a new insight into solutions of the advection-diffusion equation, although the solution is not in a complete-closed form.

In this study, we solve the advection-diffusion equation analytically for a continuous ground-level finite area source using the superposition method. The solved analytical solution is for ground-level concentration and is in a closed form. The dependence of ground-level concentration on important parameters is examined and then the solution for the finite area source is compared with solutions for a laterally infinite area source and a point source.

2. Analytical solutions of the advection-diffusion equation

The steady-state transport of a passive scalar (e.g., chemically inert pollutants) released from an elevated point source can be described by

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) + Q_p\delta(x)\delta(y-y_s)\delta(z-h), \tag{1}$$

where x, y, and z are coordinates in the along-wind, crosswind, and vertical directions, respectively, C is the mean concentration of pollutants, and u is the mean wind speed in the along-wind direction. K_y and K_z are eddy diffusivities of pollutants in the crosswind and vertical directions, respectively, Q_p is the source strength, and δ is the Dirac delta function. The point source is located at $(x, y, z) = (0, y_s, h)$. Transport by turbulent diffusion in the along-wind direction is neglected compared with that by advection in the along-wind direction. To obtain an analytical solution for the above advection-diffusion equation, height-dependent u and K_z are assumed to be given by

$$u(z) = az^p, \tag{2}$$

$$K_z(z) = bz^n, \tag{3}$$

where the parameters a, p, b, and n depend on atmospheric conditions. Using Taylor's hypothesis, the lateral eddy diffusivity can be represented by (Csanady, 1973; Huang, 1979; Brown et al., 1997)

$$K_y(x,z) = \frac{1}{2}u(z)\frac{d\sigma_y^2(x)}{dx}, \tag{4}$$

where σ_y is the standard deviation of concentration distribution in the crosswind direction. The boundary conditions applied are

$$K_z\frac{\partial C}{\partial z} = 0 \quad \text{at } z = 0, \tag{5a}$$

$$C \to 0 \quad \text{as } x, z \to \infty, \tag{5b}$$

$$C \to 0 \quad \text{as } y \to \pm\infty. \tag{5c}$$

The solution of Eq. (1) for an elevated point source satisfying Eqs. (2)–(5) is (Yeh and Huang, 1975; Huang, 1979; Lin and Hildemann, 1996; Brown et al., 1997)

$$C(x,y,z) = \frac{Q_p}{\sqrt{2\pi}\sigma_y}\frac{(zh)^{(1-n)/2}}{b\alpha x}\exp\left(-\frac{(y-y_s)^2}{2\sigma_y^2}\right) \times \exp\left(-\frac{a(z^\alpha+h^\alpha)}{b\alpha^2 x}\right)I_{-\nu}\left(\frac{2a(zh)^{\alpha/2}}{b\alpha^2 x}\right), \quad (6)$$

where

$$\alpha = 2+p-n>0, \quad (7)$$

$$\nu = \frac{1-n}{\alpha}, \quad (8)$$

and $I_{-\nu}$ is a modified Bessel function of the first kind of order $-\nu$.

For an elevated laterally infinite line source with strength Q_l, the solution can be obtained by integrating Eq. (6) from $y_s = -\infty$ to ∞ (Huang, 1979; Lin and Hildemann, 1996; Brown et al., 1997).

$$C(x,z) = \frac{Q_l(zh)^{(1-n)/2}}{b\alpha x}\exp\left(-\frac{a(z^\alpha+h^\alpha)}{b\alpha^2 x}\right)I_{-\nu}\left(\frac{2a(zh)^{\alpha/2}}{b\alpha^2 x}\right). \quad (9)$$

For a ground-level laterally infinite line source, Eq. (9) reduces to (Huang, 1979; Seinfeld, 1986; Lin and Hildemann, 1996)

$$C(x,z) = \frac{Q_l\alpha}{a\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2 x}\right)^{1-\nu}\exp\left(-\frac{az^\alpha}{b\alpha^2 x}\right), \quad (10)$$

since

$$I_\nu(h)\to\frac{h^\nu}{2^\nu\Gamma(1+\nu)} \quad \text{as } h\to 0. \quad (11)$$

Here, Γ is the complete gamma function defined by

$$\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}\,dt. \quad (12)$$

For a ground-level laterally infinite area source with uniform strength Q_a, noticing that

$$\int x^{-a}e^{-b/x}\,dx = x^{1-a}E_{2-a}\left(\frac{b}{x}\right), \quad (13)$$

the solution can be obtained using the superposition method. The solution is given by

$$C(x,z) = \int_0^x C(x-x_s,z)dx_s = \frac{Q_a\alpha}{a\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2}\right)^{1-\nu}x^\nu E_{\nu+1}\left(\frac{az^\alpha}{b\alpha^2 x}\right). \quad (14)$$

Here, E is the exponential integral defined by

$$E_a(x) = \int_0^1 t^{a-2}e^{-x/t}\,dt. \quad (15)$$

The use of symbols a and b in Eqs. (12), (13), (15), (17), (19), (20), (22), (27), and (29) should be distinguished from their use in Eqs. (2) and (3). Eq. (14) satisfies the assumption that pollutants are emitted as a steady flux on the ground, i.e.,

$$K_z\frac{\partial C}{\partial z} = -Q_a \quad \text{at } z = 0. \quad (16)$$

Noticing that

$$E_a(x) = x^{a-1}\Gamma(1-a,x), \quad (17)$$

Eq. (14) can be converted to

$$C(x,z) = \frac{Q_a z^{1-n}}{b\alpha\Gamma(1-\nu)}\Gamma\left(-\nu,\frac{az^\alpha}{b\alpha^2 x}\right), \quad (18)$$

where $\Gamma(a,x)$ is the incomplete gamma function defined by

$$\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt. \quad (19)$$

We have the following equation of

$$E_a(0) = \begin{cases} \infty & \text{for } a\le 1, \\ \dfrac{1}{a-1} & \text{for } a>1. \end{cases} \quad (20)$$

When the extent of an area source in the along-wind direction is from $x = x_1$ to x_2, the ground-level concentration for $\nu > 0$ is given by

$$C(x,0) = \frac{Q_a\alpha}{a\nu\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2}\right)^{1-\nu}\left[(x-x_1)^\nu - H(x_2,x)(x-x_2)^\nu\right], \quad (21)$$

where $H(a,b)$ is the Heaviside step function defined by

$$H(a,b) = \begin{cases} 0 & \text{for } a>b, \\ 1 & \text{for } a\le b. \end{cases} \quad (22)$$

Eq. (20) is applied to Eq. (14) to obtain Eq. (21). Eqs. (18) and (21) are identical to the analytical solutions obtained using the similarity theory by Lebedeff and Hameed (1975b). It has been mentioned in the literature that the solution for an infinite area source can be obtained by the superposition of the solution for a laterally infinite line source (Calder, 1977), but in fact it has not been presented to date.

To obtain an analytical solution for a ground-level finite area source, the solution for a ground-level point source is used by applying the approximation Eq. (11) to Eq. (6), giving (Yeh and Huang, 1975; Huang, 1979; Seinfeld, 1986; Lin and Hildemann, 1996)

$$C(x,y,z) = \frac{Q_p}{\sqrt{2\pi}\sigma_y}\frac{\alpha}{a\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2 x}\right)^{1-\nu}\exp\left(-\frac{(y-y_s)^2}{2\sigma_y^2}\right) \times \exp\left(-\frac{az^\alpha}{b\alpha^2 x}\right). \quad (23)$$

Eq. (23) can be integrated for a finite source region that extends from $x = x_1$ to x_2 and from $y = y_1$ to y_2 to give (Yeh and Huang, 1975; Chrysikopoulos et al., 1992)

$$C(x,y,z) = \int_{x_1}^{x_2}\int_{y_1}^{y_2} C(x-x_s, y-y_s, z)\mathrm{d}y_s\mathrm{d}x_s = \int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{Q_p}{\sqrt{2\pi}\sigma_y(x-x_s)}\frac{\alpha}{a\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2(x-x_s)}\right)^{1-\nu} \times \exp\left(-\frac{(y-y_s)^2}{2\sigma_y^2(x-x_s)}\right)\exp\left(-\frac{az^\alpha}{b\alpha^2(x-x_s)}\right)\mathrm{d}y_s\mathrm{d}x_s. \tag{24}$$

When the integration in Eq. (24) is performed in the crosswind direction first, it becomes

$$C(x,y,z) = \frac{Q_a\alpha}{2a\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2}\right)^{1-\nu}\int_{x_1}^{x_2}\frac{1}{(x-x_s)^{1-\nu}} \times \exp\left(-\frac{az^\alpha}{b\alpha^2(x-x_s)}\right)\Theta\,\mathrm{d}x_s, \tag{25}$$

where

$$\Theta = \mathrm{erf}\left(\frac{y-y_1}{\sqrt{2}\sigma_y(x-x_s)}\right) - \mathrm{erf}\left(\frac{y-y_2}{\sqrt{2}\sigma_y(x-x_s)}\right) \tag{26}$$

and erf is the error function defined by

$$\mathrm{erf}(a) = \frac{2}{\sqrt{\pi}}\int_0^a \mathrm{e}^{-t^2}\,\mathrm{d}t. \tag{27}$$

The analytical expression for the integral in Eq. (25) is not available at present. To get an analytical solution in a closed form, two constraints are imposed. First, the solution is limited to the ground-level. Second, the standard deviation of concentration distribution in the crosswind direction is represented by a power of downwind distance (Singer and Smith, 1966; Seinfeld, 1986), that is,

$$\sigma_y = Rx^r, \tag{28}$$

where R and r are constants depending on atmospheric stability. The exponent r tends to vary from 1 to 0.5 when the atmosphere changes from unstable to stable conditions (Singer and Smith, 1966; Koch and Thayer, 1971; Brown et al., 1997). We have the following equation of

$$\int x^{-a}\,\mathrm{erf}\left(\frac{b}{x^c}\right)\mathrm{d}x = \frac{1}{1-a}\left[\frac{1}{\sqrt{\pi}}b^{(1-a)/c}\Gamma\left(\frac{a+c-1}{2c},\left(\frac{b}{x^c}\right)^2\right) + x^{1-a}\,\mathrm{erf}\left(\frac{b}{x^c}\right)\right]. \tag{29}$$

Then, Eq. (25) becomes

$$C(x,y,0) = \frac{Q_a\alpha}{2a\nu\Gamma(1-\nu)}\left(\frac{a}{b\alpha^2}\right)^{1-\nu}\{[f(x_1,y_1)-f(x_1,y_2)] - H(x_2,x)[f(x_2,y_1)-f(x_2,y_2)]\}, \tag{30}$$

where $\nu > 0$ and

$$f(x_0,y_0) = \frac{1}{\sqrt{\pi}}\left(\frac{y-y_0}{\sqrt{2}R}\right)^{\nu/r}\Gamma\left(\frac{r-\nu}{2r},\frac{(y-y_0)^2}{2R^2(x-x_0)^{2r}}\right) + (x-x_0)^\nu\,\mathrm{erf}\left(\frac{y-y_0}{\sqrt{2}R(x-x_0)^r}\right). \tag{31}$$

Eq. (29) is used to obtain Eq. (30). Eq. (30) is the ground-level solution of the advection-diffusion equation for a ground-level finite area source.

3. Solution analysis and discussion

The solution for the ground-level concentration Eq. (30) is dependent on the parameters ν, R, and r. The solution is also dependent on the source size and parameters a, b, and α, but they are in a monotonic relation to the solution. The sensitivity of the ground-level concentration to each parameter ν, R, and r is tested. Parameter values in tested cases are given in Table 1. A typical range of ν is 0–0.5 (Arya, 1999). For all the three cases, the values of the parameters a, p, b, and n are chosen so that ν has values of 0.15, 0.25, and 0.35 (0.1 increments) and α/a and $a/b\alpha^2$ in Eq. (30) have values of 0.5 and 5, respectively. Thus, the ground-level concentration is a function of three parameters only: ν that is determined by the exponents in the power laws for wind speed and vertical eddy diffusivity [see Eqs. (2), (3), (7), and (8)], and R and r that determine the standard deviation of concentration distribution in the crosswind direction [see Eq. (28)]. The source region is set as a square with both the crosswind length and along-wind width being $L = 20$ m. The gamma function is calculated numerically by the approximation derived by Lanczos (1964) and the error function is calculated using the gamma function.

The ground-level concentration calculated using Eq. (30) represents the contributions of all the upwind sources to a given receptor point on the ground. Fig. 1 shows ground-level concentration as a function of downwind distance along the plume centerline for different values of ν, R, and r. The ground-level concentration is divided by source strength Q_a and the downwind distance is normalized by the width or length of the square source region ($L = 20$ m). Three cases with different values of $\nu (= 0.15$, 0.25, and 0.35) but with the same values of $R (= 0.4)$ and $r (= 0.8)$ are plotted in Fig. 1a and three cases with different values of R and r [$(R, r) = (0.2, 0.5), (0.4, 0.8)$, and $(0.5, 0.95)$] but with the same value of $\nu (= 0.25)$ are plotted in Fig. 1b. It can be seen from Fig. 1 that the concentration increases rapidly from the upwind edge ($x/L = 0$) to the downwind edge ($x/L = 1$) inside the source region and decreases abruptly beyond the downwind edge and that the concentration along the downwind distance depends on the values of ν, R, and r. The increase in the concentration

Table 1
Parameter values in tested cases.

	a	p	b	n	ν
Case 1	3.06	0.30	0.26	0.77	0.15
Case 2	4.00	0.50	0.20	0.50	0.25
Case 3	5.38	0.75	0.15	0.06	0.35

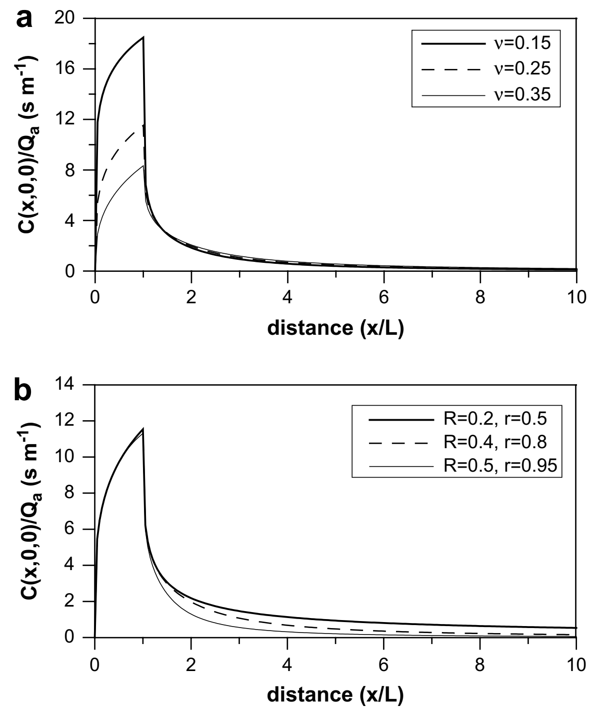

Fig. 1. Ground-level concentration $C(x, 0, 0)/Q_a$ as a function of normalized distance (a) for three cases with different values of ν but with the same values of $R\,(=0.4)$ and $r\,(=0.8)$ and (b) for three cases with different values of R and r but with the same value of $\nu\,(=0.25)$. Other parameter values are given in Table 1. The square-type source extends from $x/L = 0$ to 1.

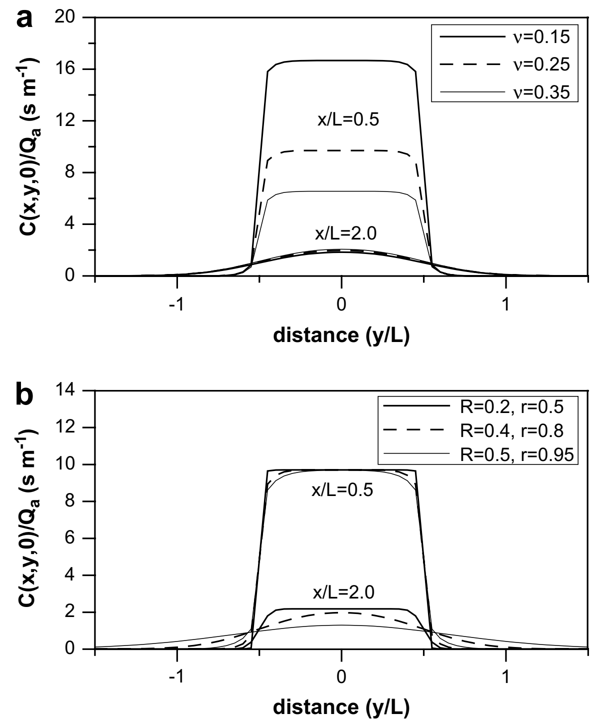

Fig. 2. Same as in Fig. 1 except for concentration $C(x, y, 0)/Q_a$ as a function of normalized lateral distance at locations of $x/L = 0.5$ (top three profiles) and 2 (bottom three profiles).

inside the source region ($x/L < 1$) is much larger when ν is smaller for fixed values of R and r (Fig. 1a). However, differences in the concentration outside the source region ($x/L > 1$) are very small for the three cases with different values of ν (Fig. 1a). These results are similar to those of Lebedeff and Hameed (1975b) for a laterally infinite area source. On the other hand, differences in the concentration inside the source region are indistinguishable for the three cases with different values of R and r (Fig. 1b). Outside the source region, the concentration at any downwind distance increases as R and r are small (Fig. 1b).

The ground-level concentration as a function of lateral distance for the cases corresponding to Fig. 1 is shown in Fig. 2. Two locations, $x/L = 0.5$ and 2, are selected. Fig. 2 clearly shows the differences stated above. The concentration profile in the crosswind direction is symmetric about the plume centerline. The concentration at $x/L = 0.5$ (a location inside the source region) is sensitive to ν for the fixed values R and r (top three profiles in Fig. 2a) but insensitive to R and r for the fixed value of ν (top three profiles in Fig. 2b). On the other hand, the concentration at $x/L = 2$ (a location outside the source region) is somewhat sensitive to R and r for the fixed value of ν (bottom three profiles in Fig. 2b) but insensitive to ν for the fixed values of R and r (bottom three profiles in Fig. 2a). Differences in the concentration from $y/L = -0.5$ to 0.5 between the three cases with different values of ν are laterally constant (Fig. 2a), but those between the three cases with different values of R and r are dependent on lateral location since σ_y determines the lateral spread of pollutants (Fig. 2b). Figs. 1 and 2 indicate that the ground-level concentration inside the source region is sensitive to the parameter ν, while the ground-level concentration outside the source region is sensitive to R and r (or σ_y).

The ground-level concentration for a finite area source obtained using Eq. (30) is compared with that for a laterally infinite area source obtained using Eq. (21). A square of 20 m by 20 m is considered for the finite area source and a downwind width of 20 m is considered for the laterally infinite area source. The ratio of the ground-level concentration for the finite area source to that for the laterally infinite area source at a certain location is not dependent on the parameters a, b, or α explicitly but dependent on ν, R, and r as well as the size of the area source. Fig. 3 shows the calculated ratio along the plume centerline $C(x, 0, 0)/C(x, 0)$ as a function of downwind distance. Four cases with different values of $\nu\,(=0.15, 0.25, 0.35, \text{and } 0.45)$ but with the same values of $R\,(=0.3)$ and $r\,(=0.7)$ are plotted in Fig. 3a and four cases with different values of R and r [$(R, r) = (0.2, 0.5), (0.3, 0.7), (0.4, 0.8), \text{and } (0.5, 0.95)$] but with the same value of $\nu\,(=0.25)$ are plotted in Fig. 3b. The ratio is 1 from the source up to some downwind distance and then decreases with increasing downwind distance. The profiles of the ratio along the downwind distance are the same for the four cases with different values of ν (Fig. 3a). On the other hand, the ratio is very sensitive to R and r (or σ_y) (Fig. 3b). At any downwind distance outside the source region, the larger the values of R and r, the smaller the ratio. This is because the diffusion in the crosswind direction

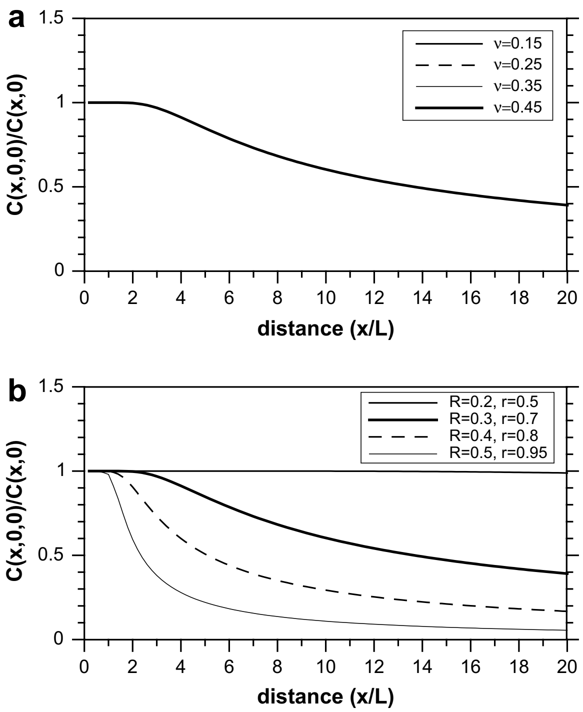

Fig. 3. Ratio of the ground-level concentration $C(x, 0, 0)$ for the finite area source to the ground-level concentration $C(x, 0)$ for the laterally infinite area source as a function of normalized distance. (a) shows four cases with different values of ν but with the same values of $R (= 0.3)$ and $r (= 0.7)$ and (b) shows four cases with different values of R and r but with the same value of $\nu (= 0.25)$. The source extends from $x/L = 0$ to 1.

from the finite area source is larger and the lateral diffusion from the infinite area source has no effect on the decrement of concentration at a distance downwind. When R and r are small (e.g., $R = 0.2$, $r = 0.5$) or the lateral diffusion is weak, the ratio is very close to 1 up to much greater downwind distance. When R and r are large (e.g., $R = 0.5$, $r = 0.95$) or the lateral diffusion is strong, the limiting location where the ratio is sufficiently close to one is at very short downwind distance. Hence, the downwind region where the finite area source can be treated as a laterally infinite area source is highly dependent on R and r.

The ground-level concentration along the plume centerline for the finite area source is compared with that for a ground-level point source obtained using Eq. (23). The point source is located at the center of the compared finite source region $(x, y, z) = (L/2, 0, 0)$, with equal total source strength. The ratio of the ground-level concentration for the finite area source to the ground-level concentration for the point source is also a function of ν, R, and r only at a certain location. The ratio is plotted in Fig. 4 as a function of downwind distance. The ratio increases gradually behind the downwind edge of the source and approaches 1 with increasing downwind distance and the ratio is irrelevant to ν except in the vicinity of the downwind edge of the finite area source (Fig. 4a). The ratio approaches 1 quickly with increasing downwind distance for large values of R and r (or σ_y) (Fig. 4b). The region (distance) where the two solutions are similar enough for the finite area source to be treated as

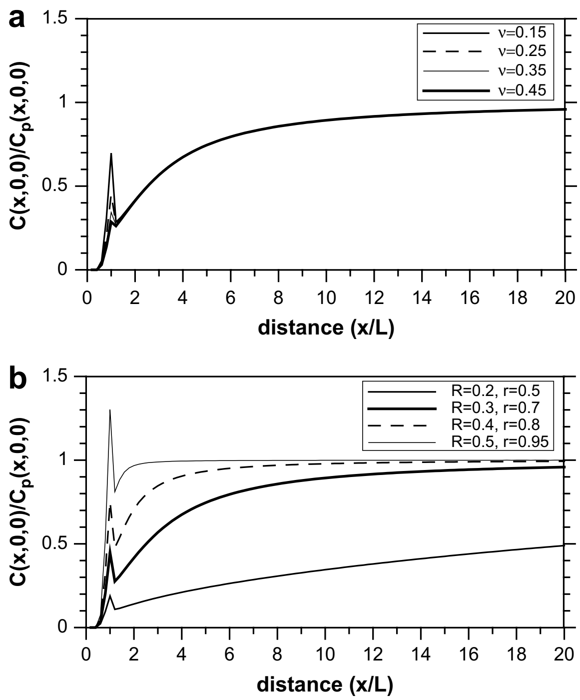

Fig. 4. Same as in Fig. 3 except for the ratio of the ground-level concentration $C(x, 0, 0)$ for the finite area source to the ground-level concentration $C_p(x, 0, 0)$ for the point source located at $(x, y, z) = (L/2, 0, 0)$.

a point source is highly dependent on σ_y but not related to ν. Since the ratio is much smaller than one near the source, the concentration level may be overestimated when the solution for an equivalent point source is applied instead of the solution for a finite area source (Chrysikopoulos et al., 1992).

The relations (ratios) between the analytical solutions for the finite area source, laterally infinite area source, and point source show very weak relations to the parameter ν responsible for vertical diffusion. On the other hand, the relations between the three analytical solutions are highly sensitive to the parameters R and r (or σ_y) responsible for lateral diffusion. Therefore, when we examine a range of distances from the source where the analytical solution for a finite area source can be replaced by that for a laterally infinite area source or a point source, the atmospheric conditions, especially the strength of diffusion in the crosswind direction, should be carefully considered.

4. Conclusions

We solved the advection-diffusion equation analytically for a continuous ground-level finite area source. Power laws were assumed for height-dependent wind speed and vertical eddy diffusivity and for the downwind distance-dependent standard deviation of concentration distribution in the crosswind direction. The analytical solution for the ground-level concentration was obtained by integrating the solution for a ground-level point source in the along-wind and crosswind directions. The analytical solution was analyzed by examining the influence of important parameters on ground-level concentration. Also, comparisons of the solution for the finite area source with solutions for a laterally infinite area source and a point source were made. Although the presented analytical solution is limited to the ground-level, the solution can be applied to any shape of area source by splitting the source region into rectangles and adding up the contributions of all upwind sources to a receptor. Along with the present line of research, a challenging problem would be to extend the ground-level solution to a solution in three dimensions.

Acknowledgments

The authors would like to thank two anonymous reviewers for providing valuable comments on this work. This work was funded by the Korea Meteorological Administration Research and Development Program under Grant CATER 2006–2202.

References

Arya, S.P., 1999. Air Pollution Meteorology and Dispersion. Oxford University Press, New York.

Brown, M.J., Arya, S.P., Snyder, W.H., 1997. Plume descriptors derived from a non-Gaussian concentration model. Atmospheric Environment 31, 183–189.

Calder, K.L., 1977. Multiple-source plume models of urban air pollution – their general structure. Atmospheric Environment 11, 403–414.

Chitgopekar, N.P., Reible, D.D., Thibodeaux, L.J., 1990. Modeling short-range air dispersion from area sources of nonbuoyant toxics. Journal of the Air & Waste Management Association 40, 1121–1128.

Chrysikopoulos, C.V., Hildemann, L.M., Roberts, P.V., 1992. A three-dimensional steady-state atmospheric dispersion–deposition model for emissions from a ground-level area source. Atmospheric Environment 26, 747–757.

Costa, C.P., Vilhena, M.T., Moreira, D.M., Tirabassi, T., 2006. Semi-analytical solution of the steady three-dimensional advection-diffusion equation in the planetary boundary layer. Atmospheric Environment 40, 5659–5669.

Csanady, G.T., 1973. Turbulent Diffusion in the Environment. D. Reidel, Holland.

Degrazia, G.A., Campos Velho, H.F., Carvalho, J.C., 1997. Nonlocal exchange coefficients for the convective boundary layer derived from spectral properties. Contributions to Atmospheric Physics 70, 57–64.

Degrazia, G.A., Moreira, D.M., Campos, C.R.J., Carvalho, J.C., Vilhena, M.T., 2002. Comparison between an integral and algebraic formulation for the eddy diffusivity using the Copenhagen experimental dataset. Nuovo Cimento della Societa Italiana di Fisica C 25, 207–218.

Dobbins, R.A., 1979. Atmospheric Motion and Air Pollution. John Wiley & Sons, New York.

Huang, C.H., 1979. A theory of dispersion in turbulent shear flow. Atmospheric Environment 13, 453–463.

Koch, R.C., Thayer, S.D., 1971. Validation and Sensitivity Analysis of the Gaussian Plume Multiple Source Urban Diffusion Model. National Technical Information Service, Virginia. No. NTIS-PB-206951.

Lanczos, C., 1964. A precision approximation of the gamma function. SIAM Journal on Numerical Analysis 1B, 86–96.

Lebedeff, S.A., Hameed, S., 1975a. Study of atmospheric transport over area sources by an integral method. Atmospheric Environment 9, 333–338.

Lebedeff, S.A., Hameed, S., 1975b. Steady-state solution of the semi-empirical diffusion equation for area sources. Journal of Applied Meteorology 14, 546–549.

Lin, J.S., Hildemann, L.M., 1996. Analytical solutions of the atmospheric diffusion equation with multiple sources and height-dependent wind speed and eddy diffusivities. Atmospheric Environment 30, 239–254.

Mehta, K.N., Balasubramanyam, R., 1978. Atmospheric transport of emissions over urban area sources. Atmospheric Environment 12, 1343–1347.

Moreira, D.M., Vilhena, M.T., Tirabassi, T., Buske, D., Cotta, R., 2005. Near-source atmospheric pollutant dispersion using the new GILTT method. Atmospheric Environment 39, 6289–6294.

Moreira, D.M., Vilhena, M.T., Buske, D., Tirabassi, T., 2006. The GILTT solution of the advection-diffusion equation for an inhomogeneous and nonstationary PBL. Atmospheric Environment 40, 3186–3194.

Park, Y.-S., Paw U, K.T., 2004. Numerical estimations of horizontal advection inside canopies. Journal of Applied Meteorology 43, 1530–1538.

Pasquill, F., Smith, F.B., 1983. Atmospheric Diffusion. Ellis Horwood Ltd., Great Britain.

Philip, J.R., 1959. The theory of local advection: I. Journal of Meteorology 16, 535–547.

Rote, D.M., 1980. Gaussian plume models – theoretical aspects. In: Longhetto, A. (Ed.), Atmospheric Planetary Boundary Layer Physics. Elsevier, Amsterdam, pp. 222–225.

Seinfeld, J.H., 1986. Atmospheric Chemistry and Physics of Air Pollution. Wiley-Interscience, New York.

Singer, I.A., Smith, M.E., 1966. Atmospheric dispersion at Brookhaven National Laboratory. International Journal of Air and Water Pollution 10, 125–135.

Singh, P., Srivastava, K.K., 1979. Non-equilibrium theory of atmospheric transport over area-source. International Journal of Engineering Science 17, 919–925.

Tirabassi, T., Rizza, U., 1997. Boundary layer parameterization for a non-Gaussian puff model. Journal of Applied Meteorology 36, 1031–1037.

Turner, D.B., 1970. Workbook of Atmospheric Dispersion Estimates, Office of Air Programs Pub. No. AP-26. Environmental Protection Agency, North Carolina.

Ulke, A.G., 2000. New turbulent parameterization for a dispersion model in the atmospheric boundary layer. Atmospheric Environment 34, 1029–1042.

Wilson, J.D., 1982. An approximate analytical solution to the diffusion equation for short-range dispersion from a continuous ground-level source. Boundary-Layer Meteorology 23, 85–103.

Yeh, G.T., Huang, C.H., 1975. Three-dimensional air pollutant modeling in the lower atmosphere. Boundary-Layer Meteorology 9, 381–390.