Dependency of the Horizontal Length of Cavity Region on Reynolds Number and Ridge Asymmetry

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Abstract

A physical modeling study with a circulating water channel is conducted to investigate the dependency of the horizontal length of the cavity region formed behind a two-dimensional, triangular ridge under a uniform, nonstratified incident flow on Reynolds number (Re) and ridge asymmetry. In all the experiments, the Reynolds number is smaller than the critical value above which the flow is Reynolds-number-independent. In the symmetric ridge cases, for a given Re the cavity length increases as the ridge slope increases. When the ridge slope is relatively steep, the cavity length increases as Re increases. When the ridge slope is relatively gentle, the cavity length slightly increases and then tends to slightly decrease as Re increases. In the asymmetric ridge cases, the cavity length increases as the ridge slope increases and Re increases. The cavity length is found to be much more sensitive to the upstream ridge slope than to the downstream ridge slope. This implies that the tilt degree of separated streamline near the ridge top, which contributes to the determination of the cavity length, is mainly dependent upon the upstream ridge slope and is little affected by the downstream ridge slope.

Key words: Circulating water channel, Reynolds number, ridge asymmetry, ridge slope, cavity region, horizontal length of cavity region

1. Introduction

When fluid flows past an obstacle, a cavity region can be formed behind the obstacle. This cavity region is characterized by much reduced but recirculating mean flow and intense turbulence (Arya and Shipman, 1981). A study of the cavity flow is interesting because of its practical implications/applications as well as its own fluid-dynamical aspects. Gaseous materials released inside a cavity region can stay there with little escape from the cavity. This has an important implication in air pollution problems.

There have been many wind tunnel and numerical modeling studies that examine factors influencing the size of the cavity region (Castro and Snyder, 1982; Zhang *et al.*, 1993). These factors include obstacle shape and slope, speed and vertical shear of incident flow, and so on. However, in previous studies, these factors were individually examined in small cases and very little attention was paid to obstacle asymmetry. Since the obstacle shape can play an important role in determining flow field formed around an obstacle, the cavity size can be strongly dependent upon the degree of obstacle asymmetry.

This study aims to investigate the dependency of the horizontal length of the cavity region formed behind a two-dimensional, triangular ridge on Reynolds number and obstacle asymmetry. For this, extensive fluid experiments are performed using a circulating water channel that has proven to be a useful tool in studying flow and dispersion in the presence of topographical features.

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2. Experimental design

The experimental apparatus used in this study is an Odell-Kovasznay-type circulating water channel, which was previously constructed to simulate and characterize urban street-canyon flows by Baik et al. (1999) and Kim and Baik (2005). Fig. 1 shows a sketch of the top view of the circulation water channel used in this study. The circulating water channel is 180 cm long, 100 cm wide, and 50 cm high. A motor rotates two vertical shafts with a stack of alternating large and small disks. A large disk on one shaft is arranged opposite a small disk on the other shaft. When the two vertical shafts rotate in opposite directions, the viscous drag of the large disks pulls water around the outside channels. This causes water to be ejected as horizontal jets emerging from gaps formed between the large disks (Odell and Kovasznay, 1971). To reduce abrupt changes in water flow, diffusers are placed downstream and upstream of the pump and a settling chamber is made upstream of the observation section. Turning vanes are placed at three corners. In the setting depicted in Fig. 1, water flows counterclockwise. Water depth is 20 cm and the density stratification is not considered. For more details of the circulating water channel, see Odell and Kovasznay (1971), Baik et al. (1999), and Kim and Baik (2005).

To systematically examine the dependency of the horizontal length of a cavity region on obstacle asym-

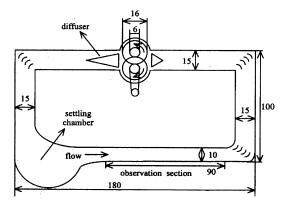


Fig. 1. Sketch of the circulating water channel (top view) used in the present study. All dimensions are in centimeters.

metry, each of two-dimensional, symmetric or asymmetric triangle-shaped ridges is mounted on the channel floor normal to the incident flow (Fig. 2). Neither the surface roughness elements nor the turbulence generator is placed upstream. Thus, the inflow turbulence level is very low. The incident flow is almost uniform in the vertical. There are inevitable shear layers very close to the walls and bottom, but these shear layers are very thin and their influence on cavity length might be negligible. The ridge height (h) is fixed at 4 cm in all the experiments. For symmetric ridges in which the upstream width of the ridge bottom (w_u) is equal to the downstream width of the ridge bottom (w_d) (see Fig. 1), the ridge bottom width varies (w/h = 0.83, 1.00, 1.25, 1.67, 2.00, and 2.50,where w is the half-width of the ridge bottom). The corresponding ridge slopes are 50.3°, 45.0°, 38.7°, 30.9°, 26.6°, and 21.8°, respectively. For asymmetric ridges in which the upstream width of the ridge bottom is different from its downstream width, experiments are performed for the cases of 1) $(w_u, w_d) = (6$ cm, 2 cm) and (2 cm, 6 cm) to examine the overall effect of ridge asymmetry on the horizontal length of the cavity region, 2) $(w_u, w_d) = (4 \text{ cm}, 3 \text{ cm}), (4 \text{ cm}, 3 \text{ cm})$ 5 cm), (4 cm, 7 cm), and (4 cm, 9 cm) to examine the effect of downstream ridge slope, and 3) (w_u, w_d) = (3 cm, 4 cm), (5 cm, 4 cm), (7 cm, 4 cm), and (9 cm, 4 cm) to examine the effect of upstream ridge slope.

The incident flow speed, denoted by U, varies with values of 1.8, 2.2, 2.8, 2.9, 3.2, and 3.3 cm s⁻¹. The corresponding Reynolds numbers (Re = Uh/ν , where ν is the kinematic viscosity of water) are 717, 891, 1103, 1147, 1261, and 1311, respectively. Dye is released at z = 0.5 cm above the downstream channel

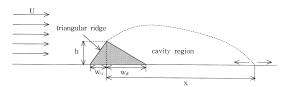


Fig. 2. Sketch of triangular ridge and cavity region. The triangular ridge height, the upstream and downstream widths of the ridge bottom, and the horizontal length of the cavity region are denoted by h, w_u , w_d , and x, respectively. The incident flow speed is U.

floor along the longitudinal direction to identify flow direction and measure the horizontal length of the cavity region. In the present study, the horizontal length of the cavity region is defined as a horizontal distance (x) from the ridge center to the downstream reattachment point, as depicted in Fig. 2. This might be different from the effective maximum horizontal length of the cavity region. This definition will be, however, taken since in this study measurements are made only near the channel floor. This study performs 96 different fluid experiments (16 different triangular ridge shapes times 6 different incident flow speeds), as described above. Each experiment is repeated three times and the values of three measurements are averaged to give the horizontal length of the cavity region for each ridge shape.

It is important to have an experimental setting in which the size of the cavity region formed behind a triangular ridge is not affected by walls or fluid depth. A wind tunnel study by Arya and Shipman (1981) shows that the cavity region behind a triangular ridge with a slope of 63.4° in an ambient wind speed of 8 m s⁻¹ extends to a distance of 13h downstream from the ridge top and to a maximum height of 2.5h. In this study, the fluid depth is 5h and the longitudinal distance of the observation section from the ridge top is 20h. The fluid depth of 5h is close to that set by Tampieri and Hunt (1985) for the higher obstacle in their water channel experiments. The largest horizontal length of the cavity region among all the experiments conducted in this study is 7.5h. In addition, as will be mentioned later, flows in the present fluid experiments are Reynolds-number-dependent. Therefore, it is believed that the size of the cavity region is only slightly affected by the front wall or fluid depth in the present experimental setting.

3. Results and discussion

Fig. 3 shows the normalized horizontal length of the cavity region as a function of Reynolds number (Re) in the symmetric ridge cases. The horizontal length of the cavity region is normalized by the ridge height. Note that since h is fixed and ν is a constant, Re varies only with U. For a given Re, the cavity

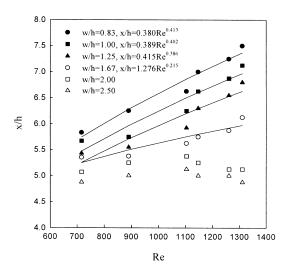


Fig. 3. The normalized horizontal length of the cavity region (x/h) as a function of Reynolds number (Re) in the symmetric ridge cases of w/h = 0.83, 1.00, 1.25, 1.67, 2.00, and 2.50. The regression equations for each symmetric ridge are given in the figure.

length increases as the ridge slope increases. This is related to the fact that the separated streamline near the edged ridge top, where flow separation occurs due to a discontinuity in the surface slope, becomes steeper and thus makes the cavity size wider as the ridge slope becomes larger. In each case of w/h = 0.83, 1.00, 1.25, and 1.67, the cavity length increases as Re increases. Using the data plotted in Fig. 3, a power-law form of regression equation relating the normalized horizontal length of the cavity region to Reynolds number is obtained.

$$x/h = a \operatorname{Re}^b$$
,

where a and b are regression coefficients. The determined regression equations are given in Fig. 3. In each case of w/h = 2.00 and 2.50, the cavity length slightly increases with increasing Re up to Re = 1103 but tends to decrease slightly with increasing Re beyond that value. That is, when the ridge is relatively gentle (w/h > 2) and the incident flow speed is relatively high $(U > 2.8 \text{ cm s}^{-1})$, the cavity length tends to slightly decrease up to Re = 1311 as Re increases.

The present result that for a given Re the cavity length increases with increasing ridge slope is con-

sistent with the numerical modeling result of Kim *et al.* (2001) that the size of the reciculation (cavity) zone behind a single hill becomes large as the hill slope increases. Their study also shows that the horizontal length of the recirculation zone is much smaller than that in a study using a triangular ridge by Arya and Shipman (1981). This indicates that obstacle shape is an important factor determining the cavity length.

Next, the influence of ridge asymmetry on the horizontal length of the cavity region is examined. Fig. 4 shows the normalized horizontal length of the cavity region as a function of Re in the experiments with asymmetric triangular ridges of $(w_u, w_d) = (6 \text{ cm}, 2 \text{ cm})$ [denoted by asymmetric_1] and (2 cm, 6 cm) [asymmetric_2]. Also, the experimental data with $(w_u, w_d) = (4 \text{ cm}, 4 \text{ cm})$ [symmetric] are plotted in Fig. 4. The asymmetric_1, symmetric, and asymmetric_2 cases have upstream ridge slopes of 33.7° , 45.0° , and 63.4° , respectively, and downstream ridge slopes of 63.4° , 45.0° , and 33.7° , respectively. Fig. 4 indicates

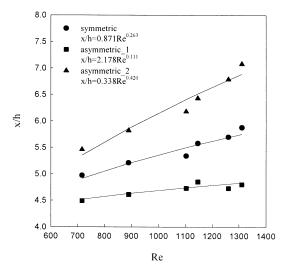


Fig. 4. The normalized horizontal length of the cavity region (x/h) as a function of Reynolds number (Re) in the symmetric and asymmetric ridge cases. The symmetric case has a ridge-bottom upstream width (w_u) of 4 cm and a ridge-bottom downstream width (w_d) of 4 cm, the asymmetric_1 case has $w_u = 6$ cm and $w_d = 2$ cm, and the asymmetric_2 case has $w_u = 2$ cm and $w_d = 6$ cm. The regression equations for each ridge are given in the figure.

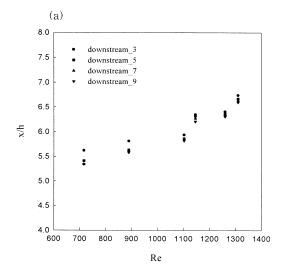
that the cavity length in the asymmetric ridge cases increases with increasing Re and depends more on the upstream ridge slope in relatively high Re than in relatively low Re. Fig. 4 also indicates that the cavity length increases as the upstream ridge slope increases. For Re = 1147, the horizontal lengths of the cavity region in the asymmetric_1, symmetric, and asymmetric_2 cases are 4.9h, 5.6h, and 6.4h, respectively.

Fig. 5a shows the variation of the normalized horizontal length of the cavity region with Re in the asymmetric ridge cases in which the upstream width of the ridge bottom is fixed at 4 cm, but its downstream width varies with 3, 5, 7, and 9 cm. The downstream ridge slopes with w_d = 3, 5, 7, and 9 cm are 53.1°, 38.7°, 29.7°, and 24.0°, respectively. For a given Re, the cavity length slightly increases with increasing downstream ridge slope, but the variation of the cavity length with downstream ridge slope is very small. For Re = 1147, the horizontal length of the cavity region ranges from x = 6.2h in the downstream_9 case to x = 6.4h in the downstream_3 case. The difference between the two experiments is only 0.2h.

Fig. 5b is for the asymmetric ridge cases in which the downstream width of the ridge bottom is fixed at 4 cm, but its upstream width varies with 3, 5, 7, and 9 cm. The upstream ridge slopes with $w_u = 3, 5, 7$, and 9 cm are 53.1°, 38.7°, 29.7°, and 24.0°, respectively. For a given Re, the variation of the cavity length with upstream ridge slope is very large compared with that in Fig. 5a. For Re = 1147, the cavity length ranges from x = 4.9h in the upstream_9 case to x = 6.9h in the upstream_3 case. The difference is 2.0h, thus much larger than the difference in Fig. 5a. Fig. 5 clearly shows that the horizontal length of the cavity region is much more sensitive to the upstream ridge slope than to the downstream ridge slope. This implies that the tilt degree of separated streamline near the ridge top, which contributes to the determination of the cavity length, is mainly dependent upon the upstream ridge slope and is little affected by the downstream ridge slope.

The above results suggest that the upstream obstacle slope rather than the downstream obstacle slope is an important factor in determining the hori-

zontal size of the cavity region formed behind an obstacle. The above results also suggest that even for the same Reynolds-number flow the horizontal size of the cavity region can vary substantially due to a



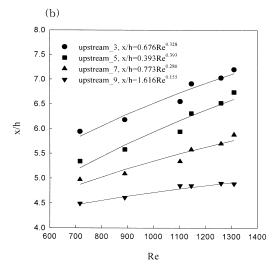


Fig. 5. The normalized horizontal length of the cavity region (x/h) as a function of Reynolds number (Re) in the asymmetric ridge cases. In (a), the upstream width of the ridge bottom (w_u) is fixed at 4 cm, but its downstream width (w_d) varies with 3 cm (downstream_3), 5 cm (downstream_5), 7 cm (downstream_7), and 9 cm (downstream_9). In (b), the downstream width of the ridge bottom (w_d) is fixed at 4 cm, but its upstream width (w_u) varies with 3 cm (upstream_3), 5 cm (upstream_5), 7 cm (upstream_7), and 9 cm (upstream_9). The regression equations for each asymmetric ridge are given in Fig. 5b.

change in upstream obstacle slope.

4. Summary and conclusions

The dependency of the cavity region formed behind a two-dimensional, triangular ridge on Reynolds number (Re) and ridge asymmetry was investigated using a circulating water channel. In the present experiments, Re is smaller than the critical value above which the flow is Reynolds-number-independent. The incident flow speed was set to vary with the ridge height being fixed. That is, Re varies only with incident flow speed, but the dynamic similarity is maintained. As expected, in the symmetric ridge cases, for a given Re the cavity length increases with increasing ridge slope. For relatively steep ridge slopes, the cavity length increases with increasing Re. For relatively gentle ridge slopes, as Re increases, the cavity length slightly increases and then tends to slightly decrease. In the asymmetric ridge cases, the cavity length increases as the ridge slope increases and Re increases. It was found that the cavity length is much more sensitive to the upstream ridge slope than to the downstream ridge slope.

It is generally known that the flow around an obstacle is Reynolds-number-independent if Re exceeds a certain critical value. The towing water-tank experiments (Snyder, 1994) show that the critical Re required for Reynolds-number-independent flow over a cubical building under a uniform, nonstratified, low-turbulence approach flow is about 11000. The largest Re in the present study is 1311. Hence, flows exhibited in the present fluid experiments are expected to be Reynolds-number-dependent. Indeed, the experimental results presented in Figs. 3, 4, and 5 show an apparently strong dependency of the horizontal length of the cavity region on Re. However, no attempts were made to find the critical Re required for Reynolds-number-independent flow over a triangular ridge. An interesting extension of this study would be to find the critical Re and examine its dependency on the ridge asymmetry.

The flow stratification affects the size of the cavity formed behind an obstacle (Snyder, 1994; Robins, 1994; Smith *et al.*, 2001). For a stratified flow, two

relevant nondimensional parameters to the problem are the Reynolds number and the Froude number (Fr), defined by U/Nh, where N is the buoyancy frequency. The towing water-tank experiments with a cubical building (Snyder, 1994) reveal that the flow is insensitive to Fr for Re exceeding 11000, provided that Fr is greater than 2.5. In this study, Fr is infinity because of nonstratification, but the maximum Re (1311) is much smaller than the critical Re for a cubical building (11000). An interesting investigation would be to find Fr beyond which the flow over a triangular ridge is insensitive to Fr in the Reynoldsnumber-independent flow regime and examine its dependency on the ridge asymmetry. Also, the influence of incident flow shear on the cavity size deserves further study. These investigations will provide deeper insights into cavity fluid dynamics and also enable us to apply fluid modeling results to the full-scale atmospheric flow and dispersion problem.

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공동 영역 수평 길이의 레이놀즈 수와 봉우리 비대칭 의존

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요 약

균일하고 비성층화된 유입류 조건하에서 이차원, 삼각형 봉우리 후면부에서 형성되는 공동 영역의 수평 길이의 레이놀즈 수 (Re)와 봉우리 비대칭 의존을 조사하기 위하여 순환 수로 장치를 이용한 물리 모델링 연구를 수행하였다. 모든 실험에서 레이놀즈 수는 그 값 보다 클 경우 흐름이 레이놀즈 수에 독립적인 임계 값보다 작다. 대칭 봉우리 경우에 주어진 Re에 대해서 봉우리 경사가 증가함에 따라 공동 길이는 증가한다. 봉우리 경사가 상대적으로 가파를 때 Re가 증가함에 따라 공동 길이는 증가한다. 봉우리 경사가 상대적으로 완만할 때 공동 길이는 Re가 증가함에 따라 약간 증가하다가 그 후 약간 감소하는 경향을 보인다. 비대칭 봉우리 경우에 봉우리 경사가 증가하고 Re가 증가함에 따라 공동 길이는 증가한다. 공동 길이는 하류쪽 봉우리 경사 보다 상류쪽 봉우리 경사에 훨씬 더 민감하다. 이는 공동 길이를 결정하는데 기여하는 봉우리 정상 부근에서 분리된 유선의 기울기 정도가 상류쪽 봉우리 경사에 주로 의존하며 하류쪽 봉우리 경사에는 거의 영향을 받지 않음을 의미한다.

Key words: 순환 수로, 레이놀즈 수, 봉우리 비대칭, 봉우리 경사, 공동 영역, 공동 영역의 수평 길이