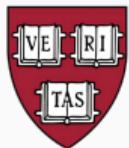


# Fractionalized Fermi Liquids: Mean-Field Theories, Instabilities, and Variational Wavefunctions

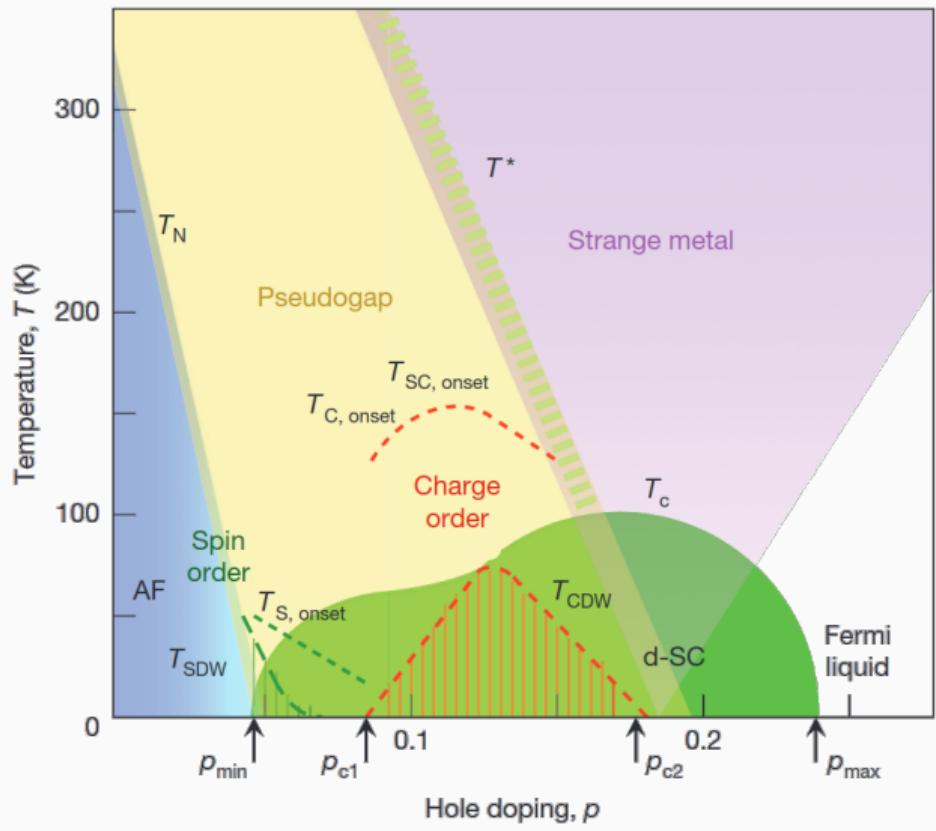
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Henry Shackleton

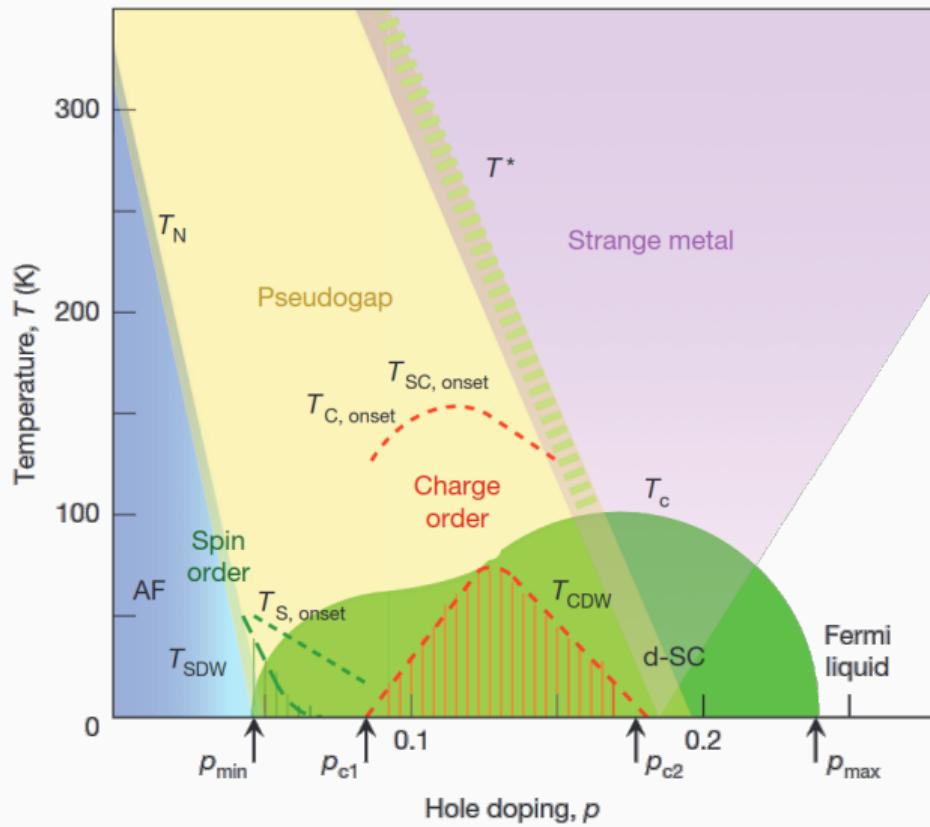
June 18, 2024



# Cuprate phase diagram as an inspiration for correlated physics

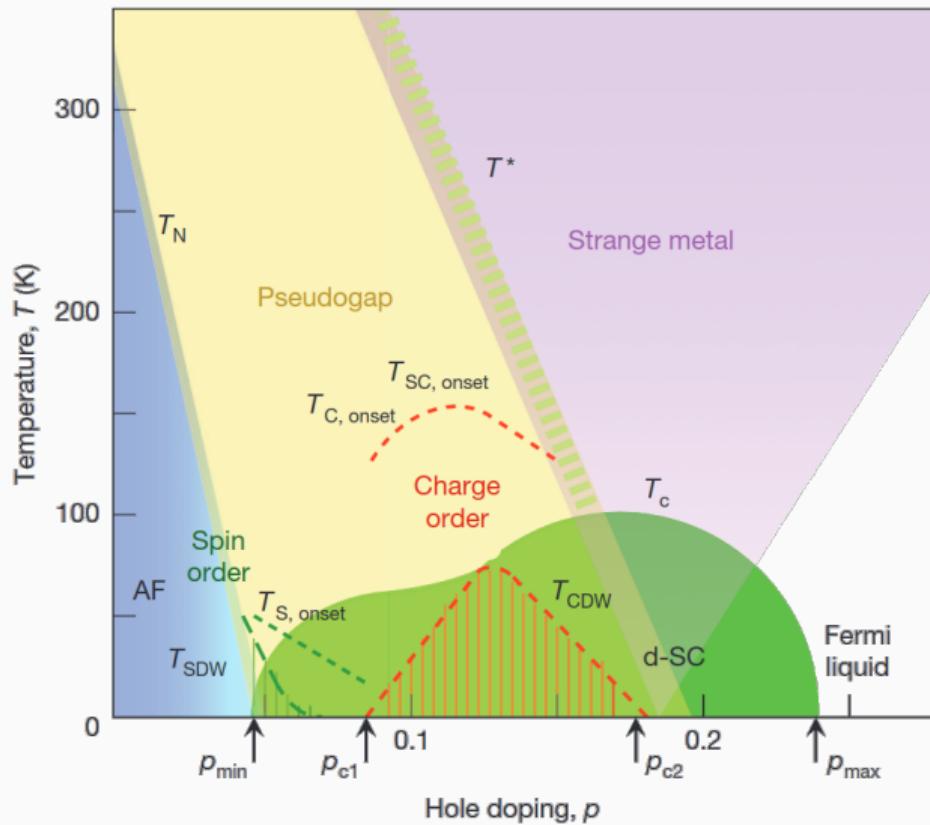


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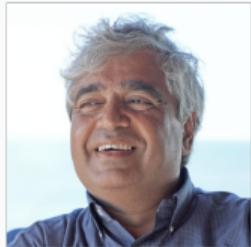
- “Strange metal” phase -  $T$  linear resistivity from a theory of a quantum critical metal

# Cuprate phase diagram as an inspiration for correlated physics



- “Strange metal” phase -  $T$  linear resistivity from a theory of a quantum critical metal
- Pseudogap metal and proximate ordered phases from a theory of fractionalized Fermi liquids

## (Partial) acknowledgements



Subir Sachdev



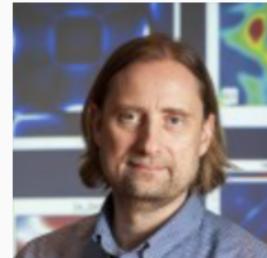
Yahui Zhang



Maria  
Tikhonovskaya



Jonas von  
Milczewski



Dirk Morr



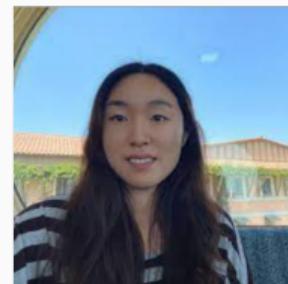
Darshan Joshi



Maine Christos



Alexander  
Nikolaenko



Zhu-Xi Luo



Eric Mascot

# Parton construction for fractionalized insulators: quantum spin liquids

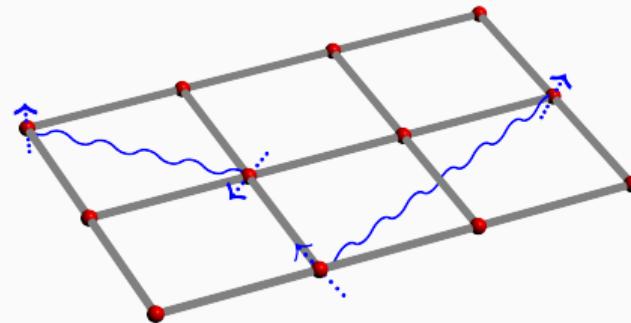
$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \rightarrow f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$ , emergent gauge fluctuations

$$H \rightarrow \sum_{ij} t_{ij} f_{i\sigma}^\dagger f_{j\sigma} + \dots$$

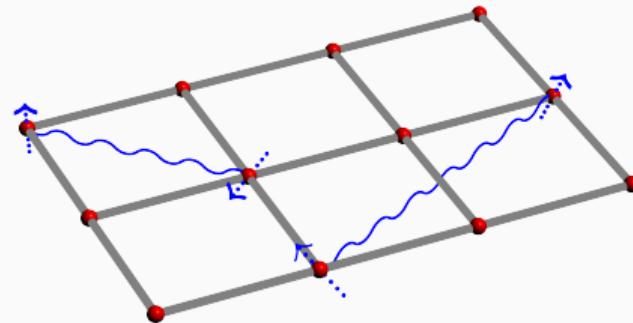


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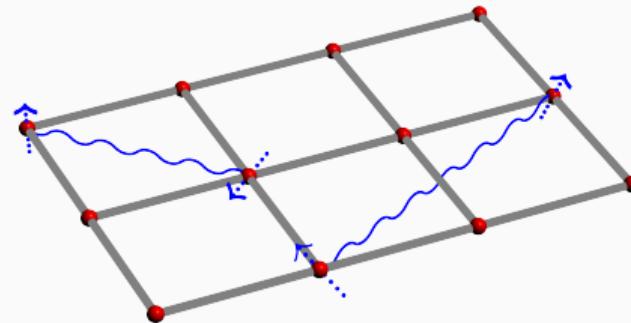
- Bosonic/fermionic theories, classification with projective symmetry groups

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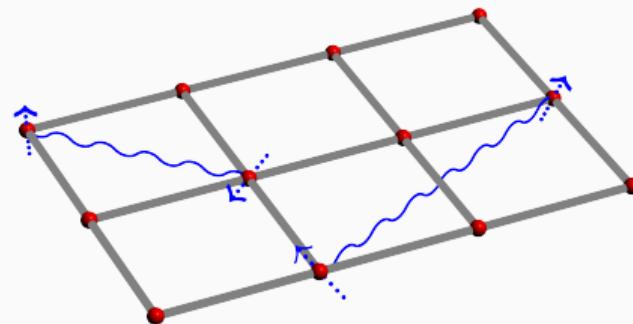
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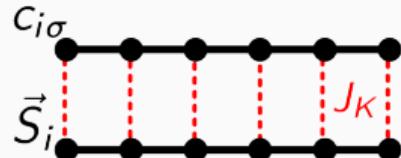
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- Bosonic/fermionic theories, classification with projective symmetry groups
- Instabilities to ordered phases (spinon condensation, confining instabilities)
- Numerical evaluation of correlated wavefunctions,  $\mathcal{P}_G |\psi_0\rangle$  - important for quantitative predictions

# Fractionalized Fermi liquids in Kondo lattices<sup>1</sup>

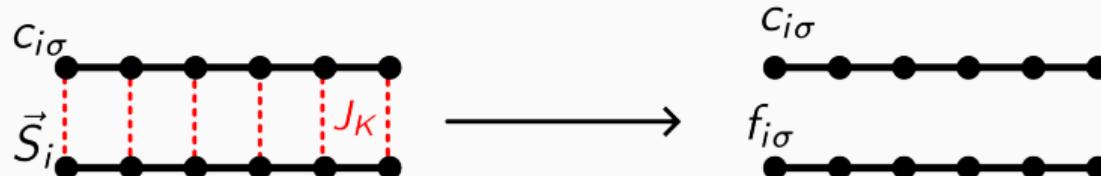
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## Fractionalized Fermi Liquid

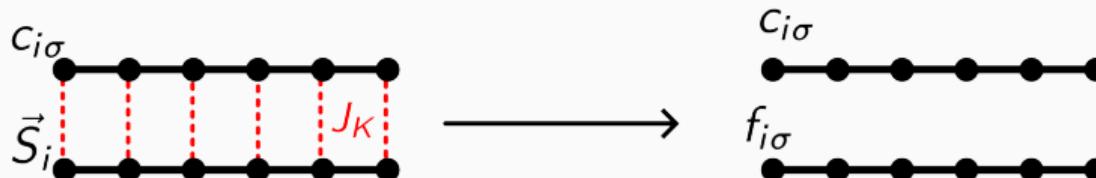
Electrons,  
filling  $1 + p$

Spin liquid

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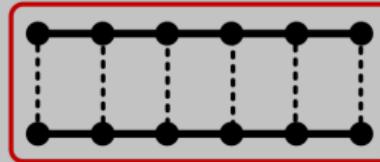


Fractionalized Fermi Liquid



Electrons,  
filling  $1 + p$   
Spin liquid

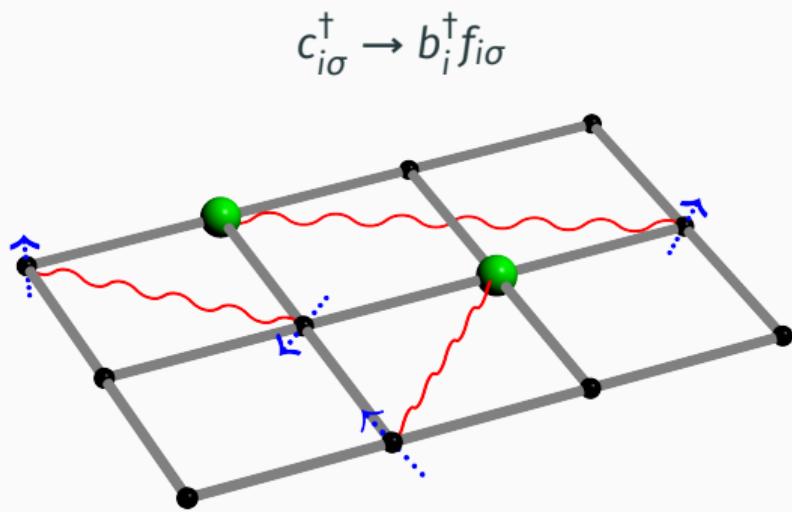
Heavy Fermi Liquid



Electrons, filling  $p$

<sup>1</sup>Senthil, Sachdev, and Vojta, *Physical Review Letters*, 2003.

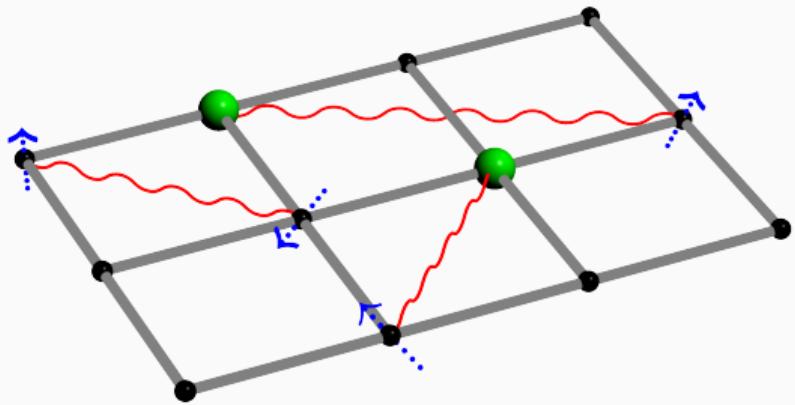
## Conventional electron fractionalization for single-band models<sup>2</sup>



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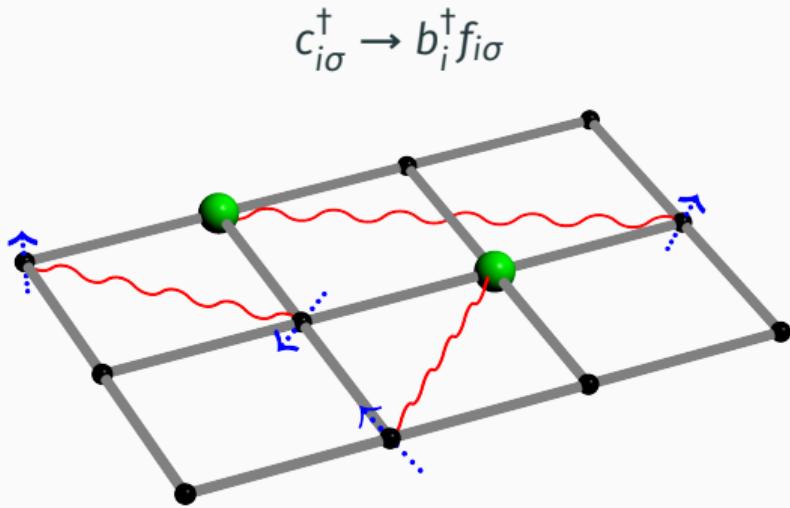
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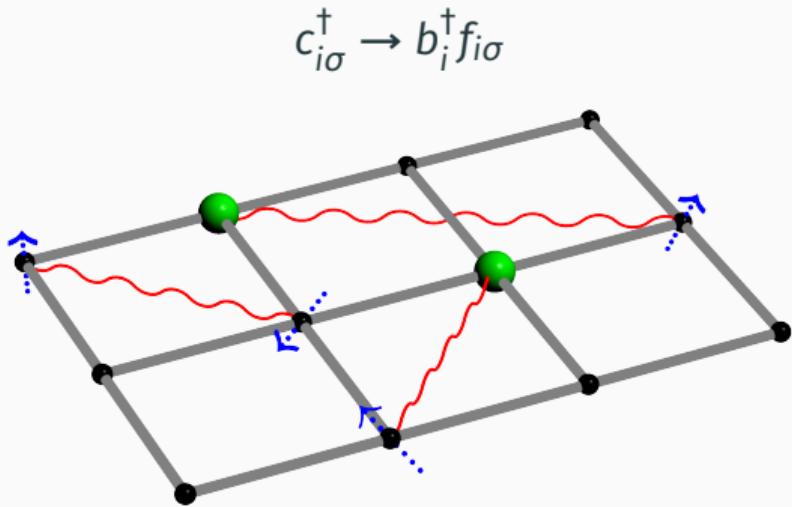
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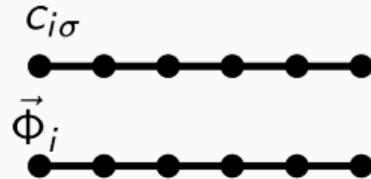
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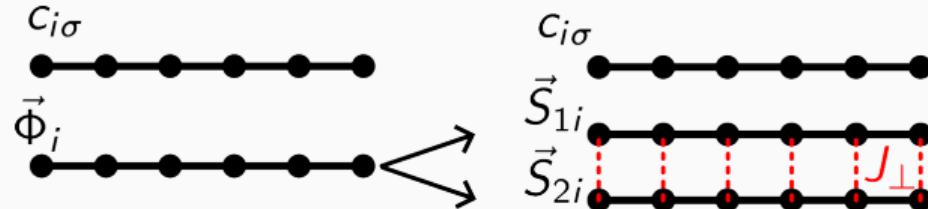
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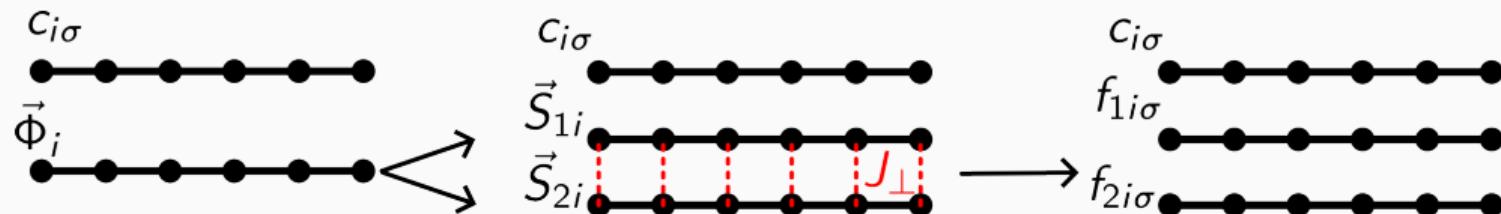
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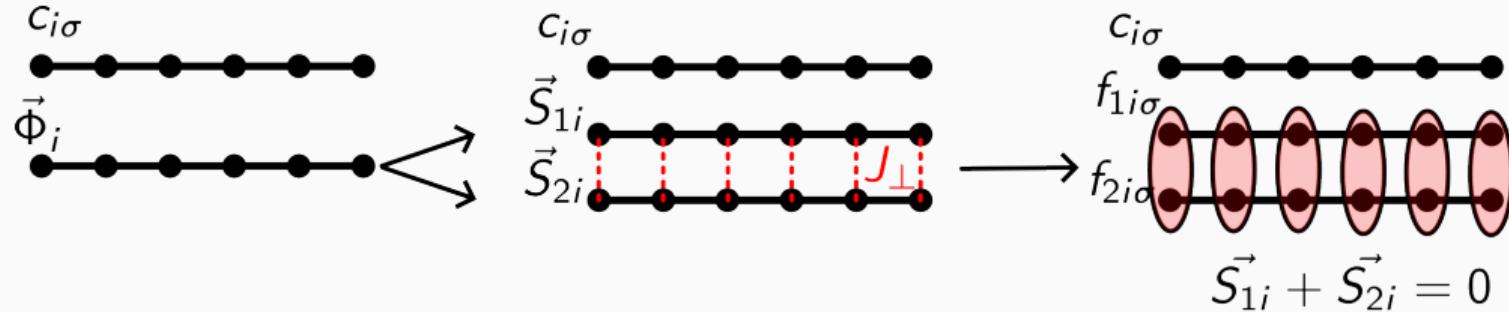
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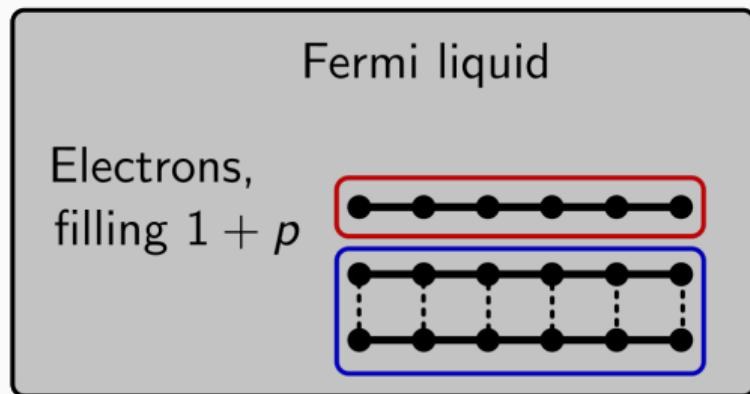
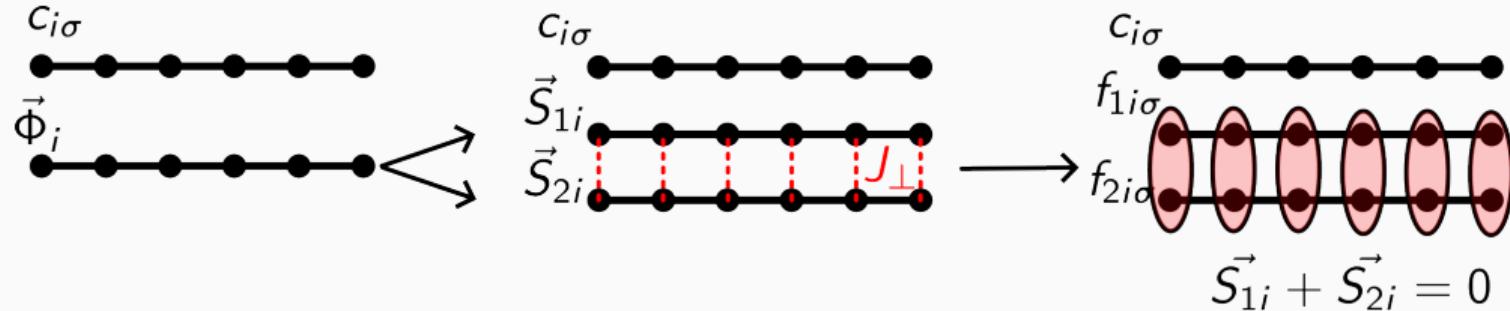
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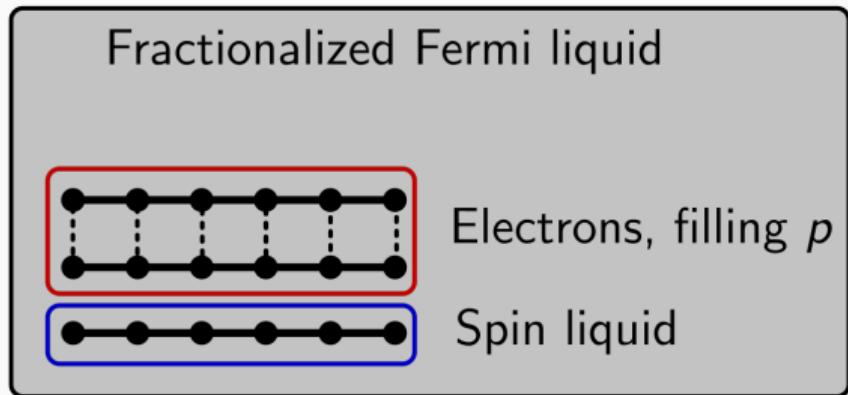
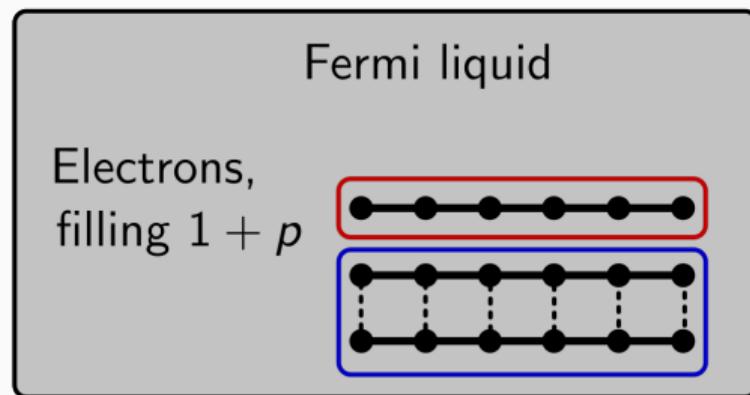
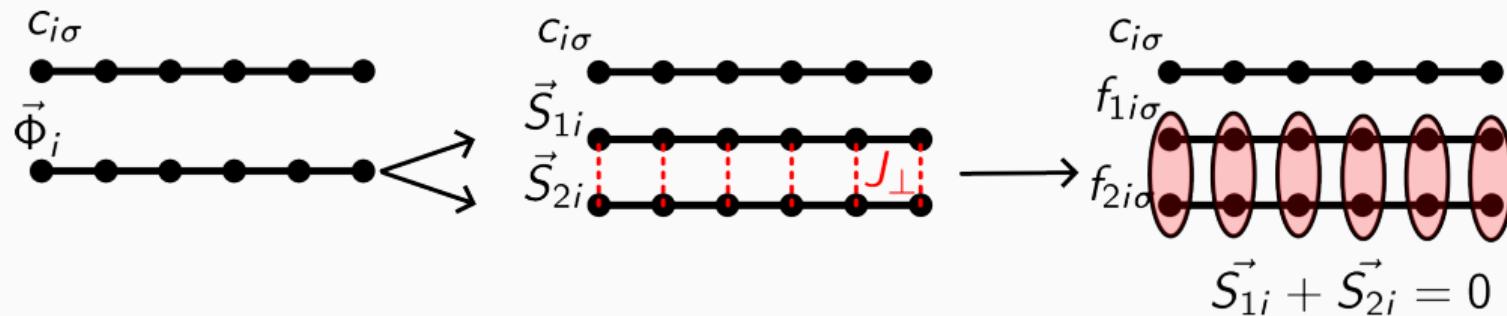
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## Mean-field analysis on square lattice yields pseudogap-like features

Mean-field picture: electron-like quasiparticles + decoupled spin liquid <sup>4</sup>

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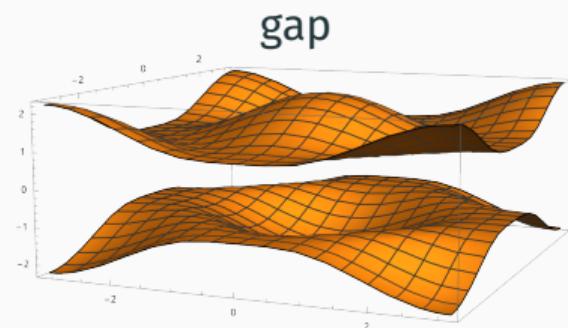
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Half filling: Mott insulator with charge



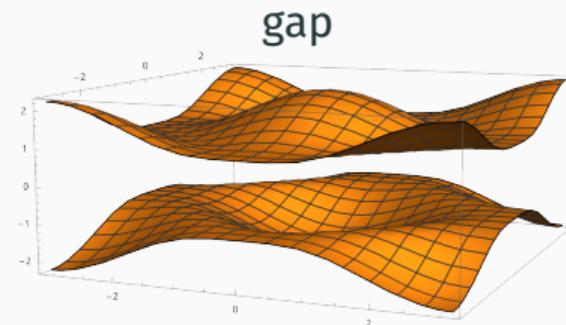
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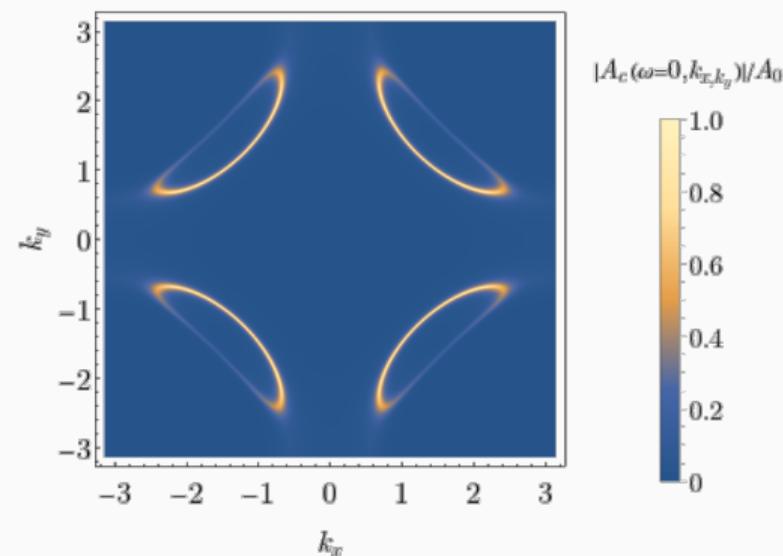
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Hole doping similar to YRZ ansatz for

Green's function<sup>5</sup>



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## Choice of spin liquid dictates proximate phases

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<sup>6</sup>Tanaka and Hu, *Phys. Rev. Lett.*, 2005; Wang et al., *Phys. Rev. X*, 2017

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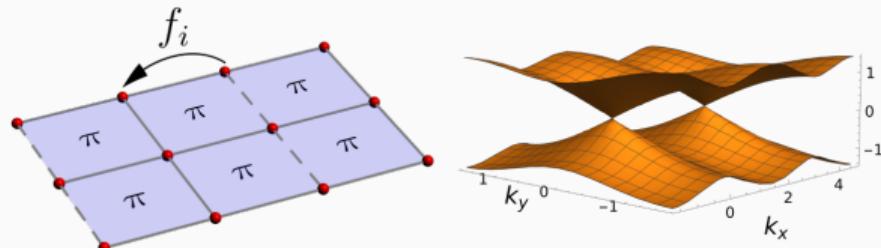
- Intrinsic instabilities in spin liquid phase give one route to ordered phases

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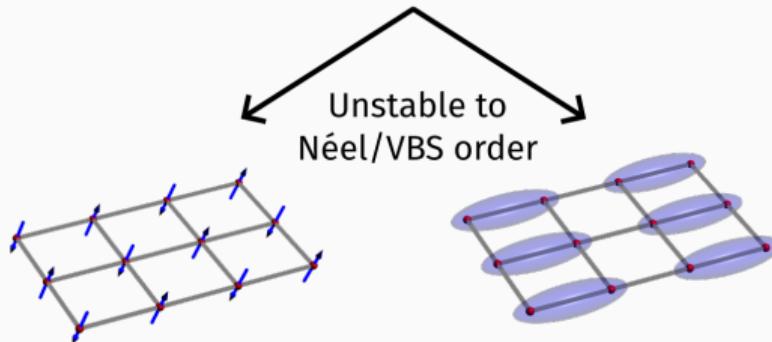
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# Choice of spin liquid dictates proximate phases

- Intrinsic instabilities in spin liquid phase give one route to ordered phases
- Fermionic theory of a  $\pi$ -flux spin liquid leads to Néel/VBS order<sup>6</sup>



$N_f = 2$  QCD<sub>3</sub>, emergent SO(5) symmetry

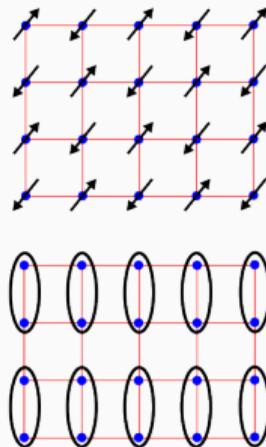


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# Charge instabilities arise from chargon condensation

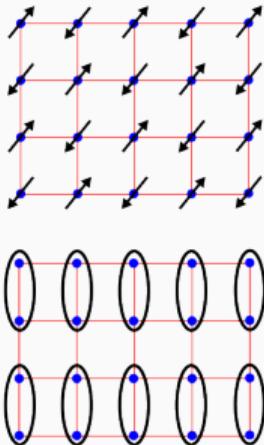
$\pi$ -flux confinement

"Pseudogap metal"  
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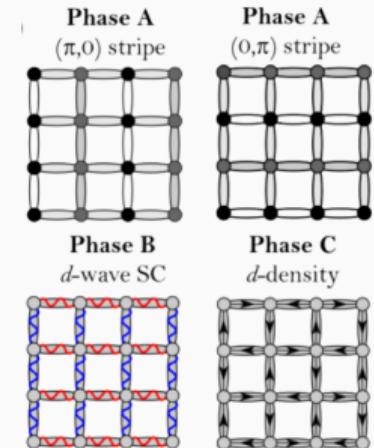
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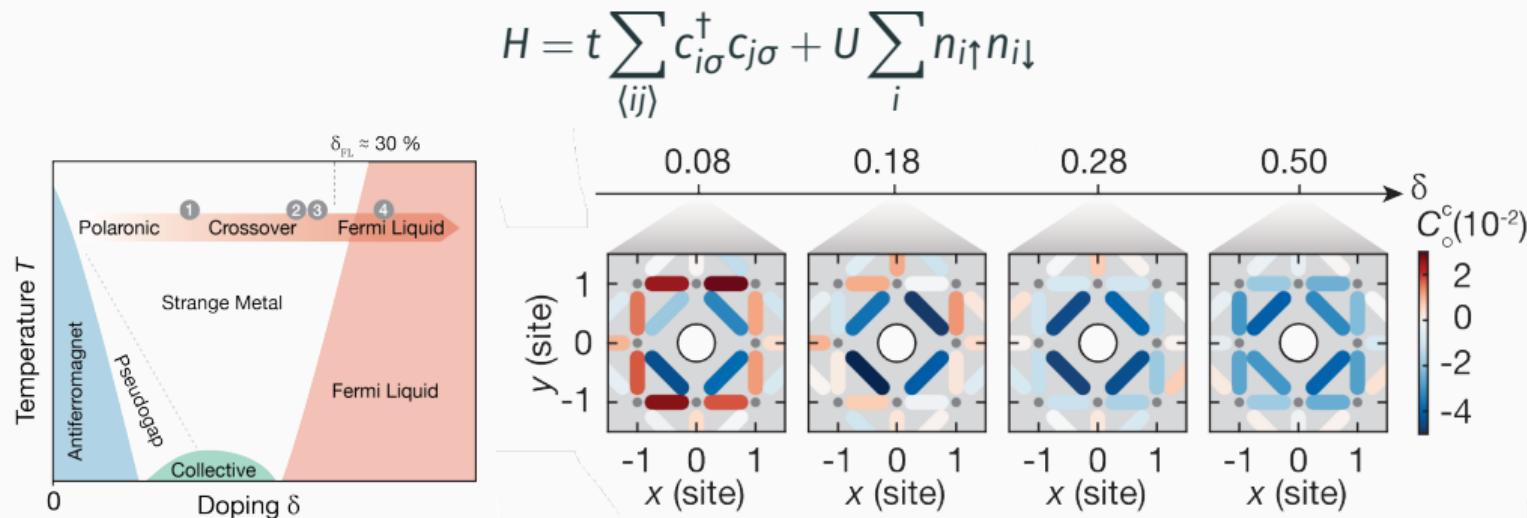
"Pseudogap metal"  
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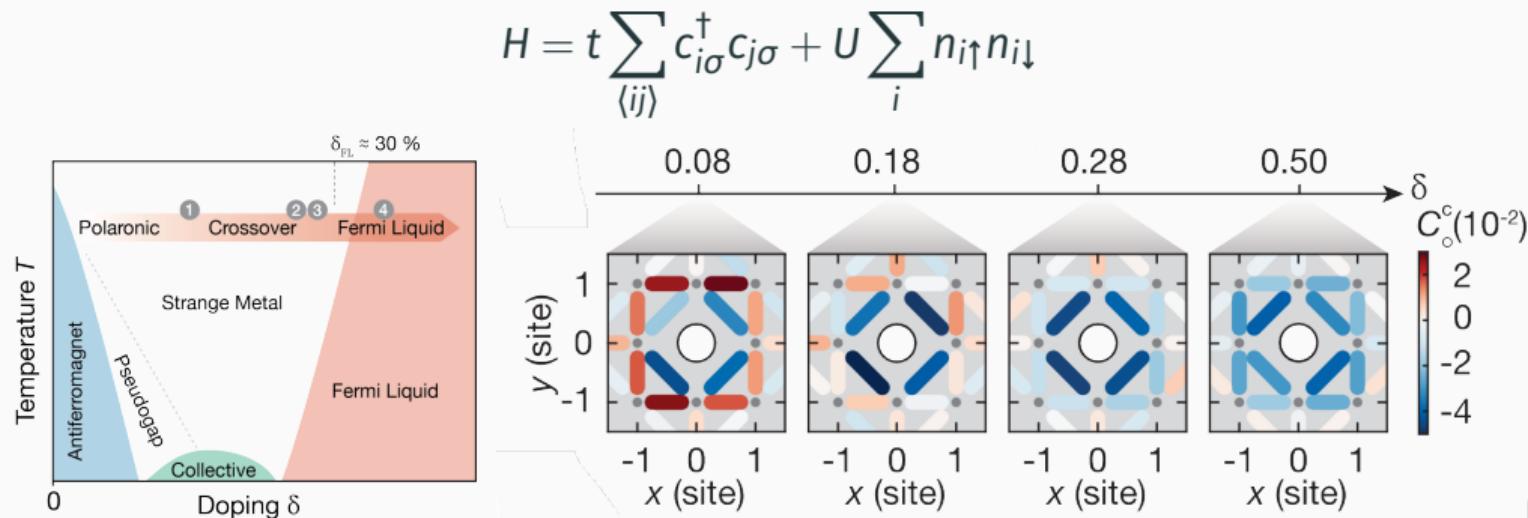
M. Christos, Z.-X. Luo, H.  
**Shackleton**, Y.-H. Zhang, M. S.  
Scheurer, and S. Sachdev,  
PNAS 120, e2302701120 (2023)



# Polaronic correlations central for capturing doped Mott insulators



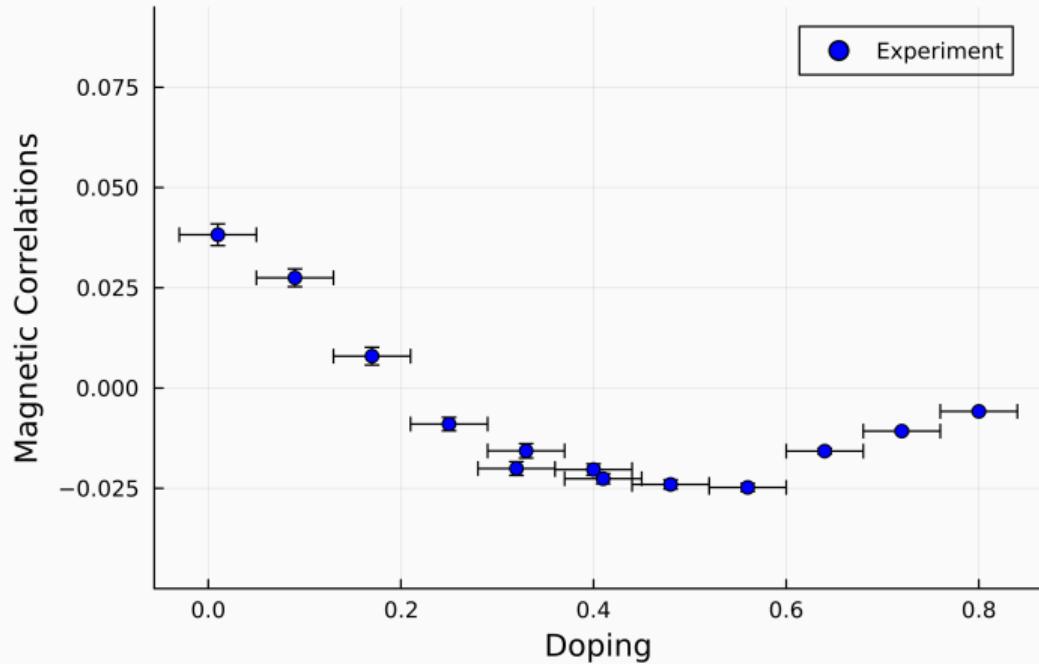
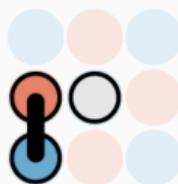
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Do these wavefunctions support polaronic correlations?

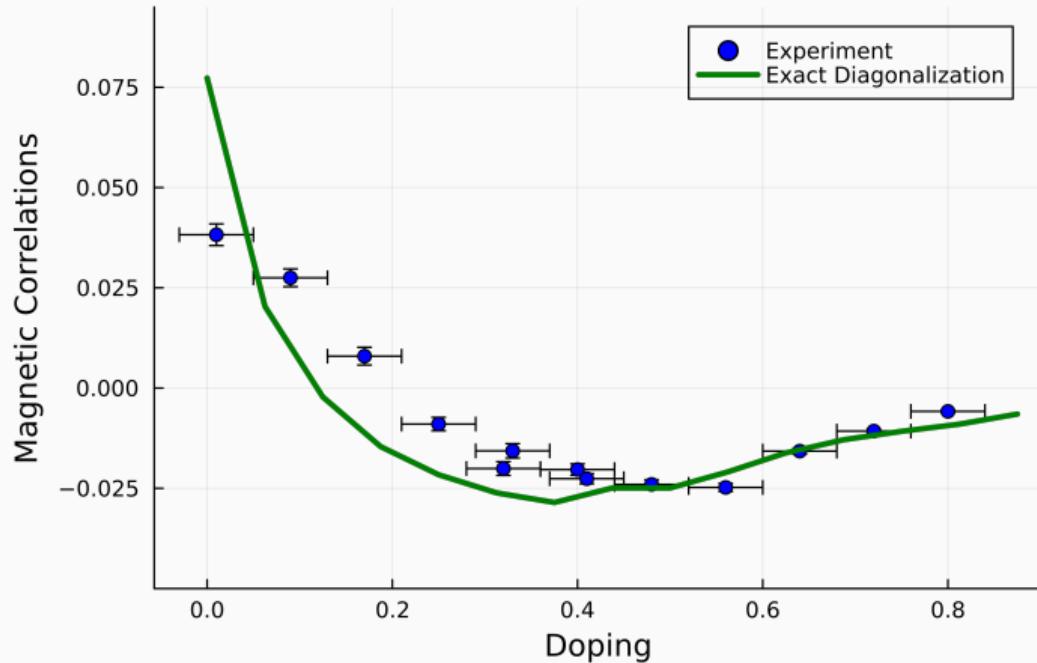
# Nearest neighbor magnetic correlations ( $U/t = 7.4$ )

Polaronic correlations probed by multipoint correlator  $\langle h_i S_j^z S_k^z \rangle$



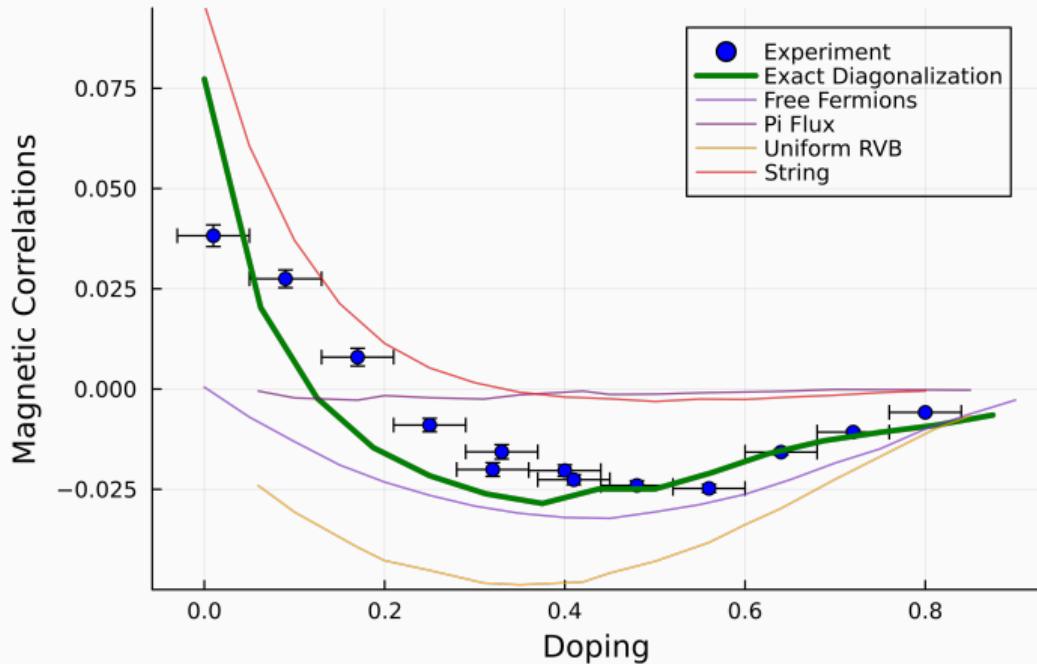
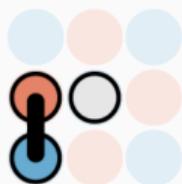
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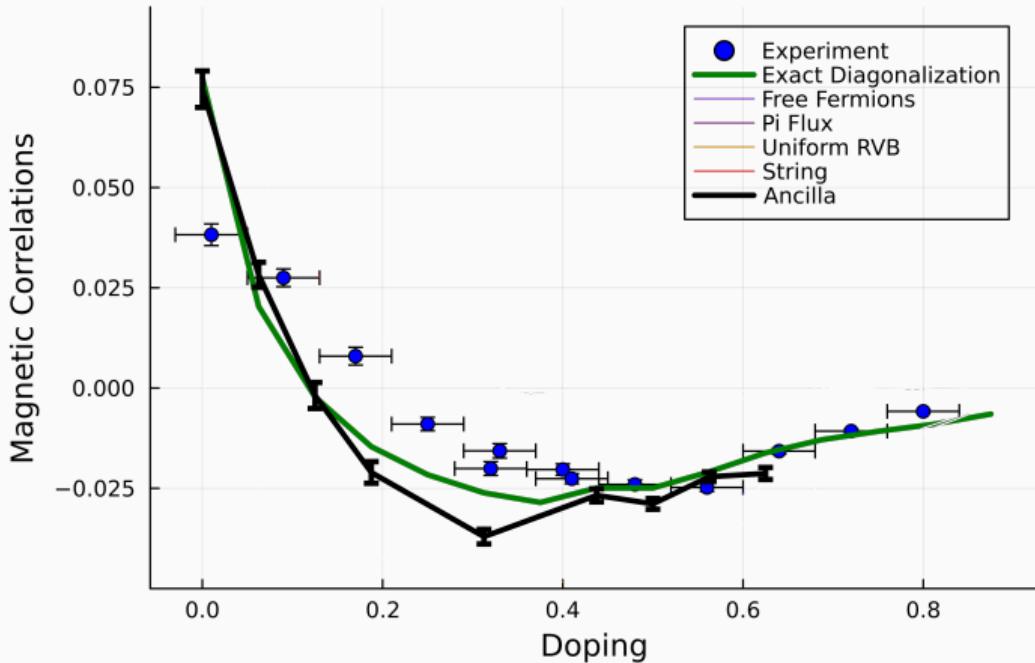
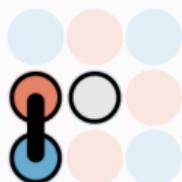
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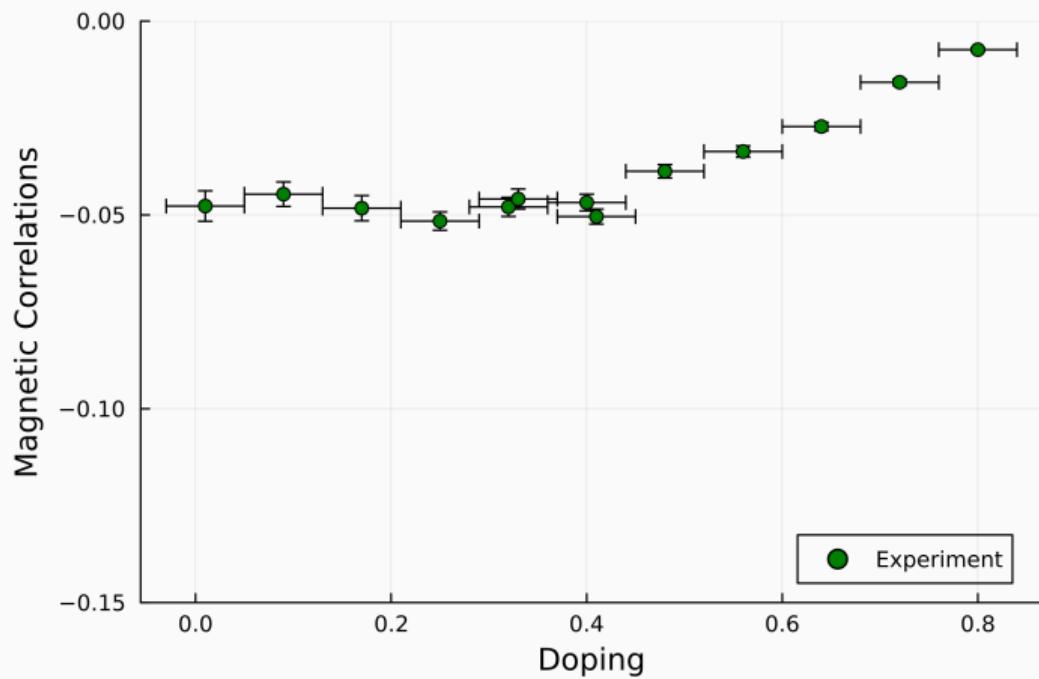


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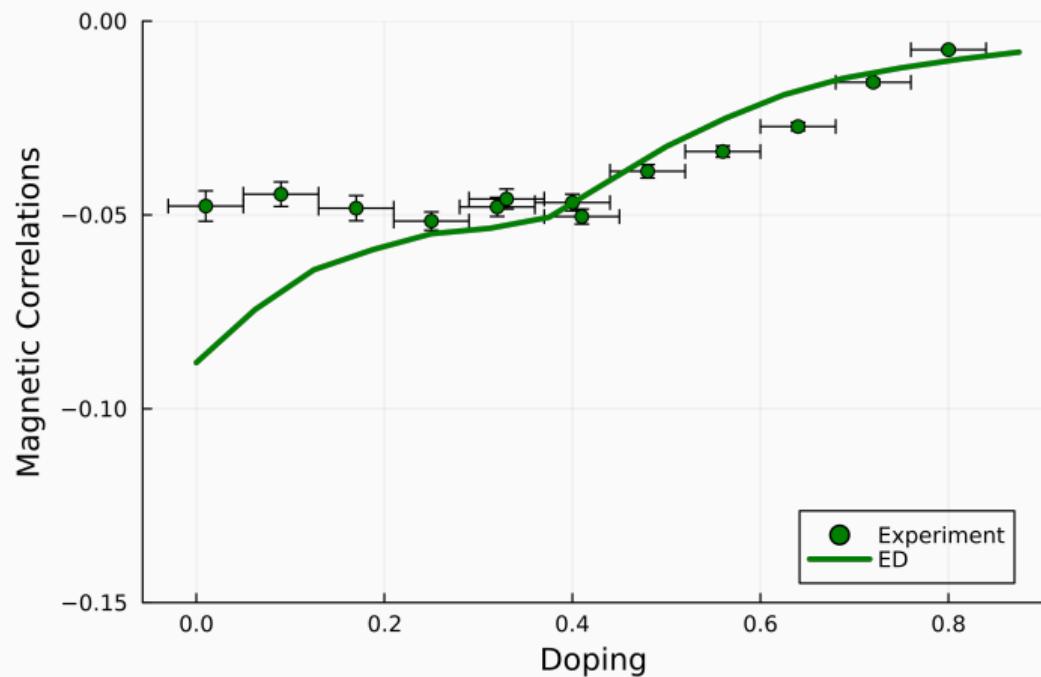
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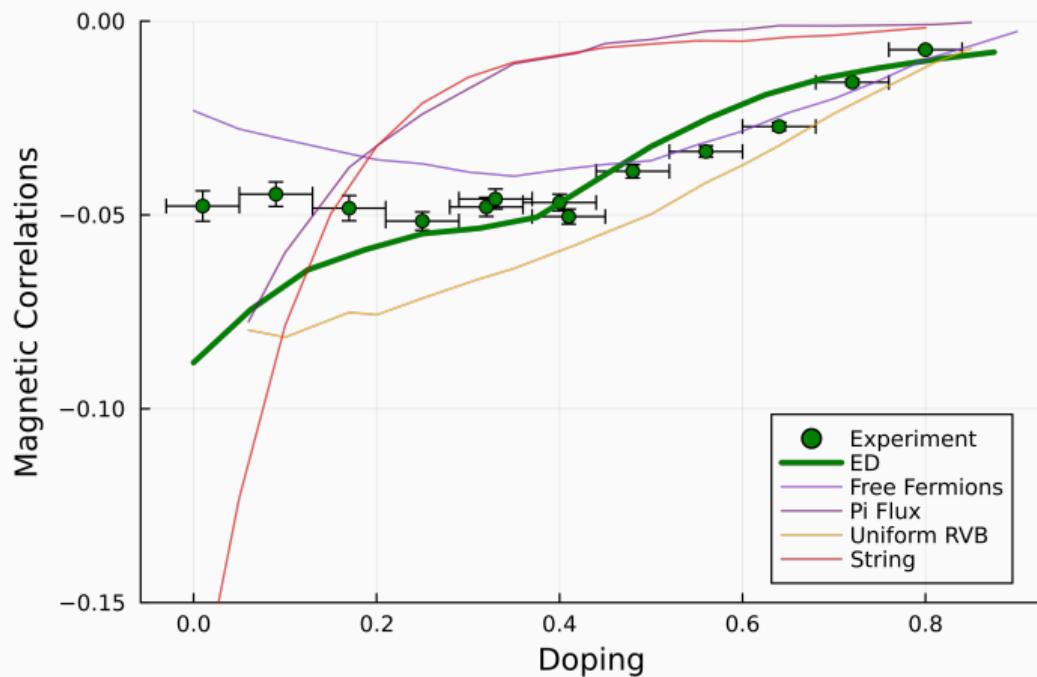
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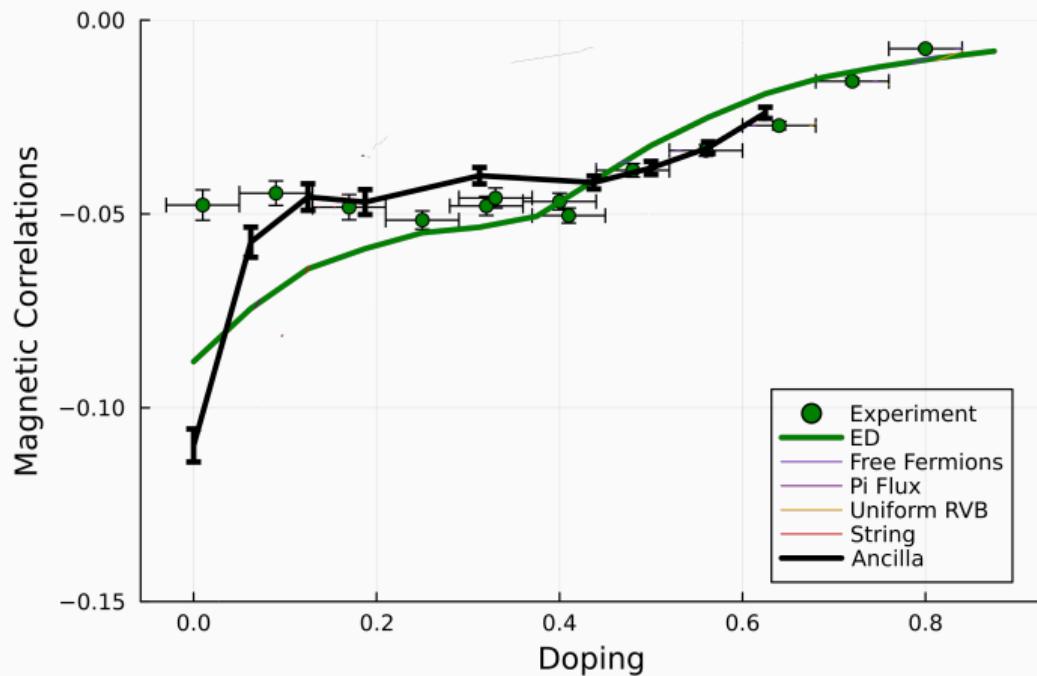
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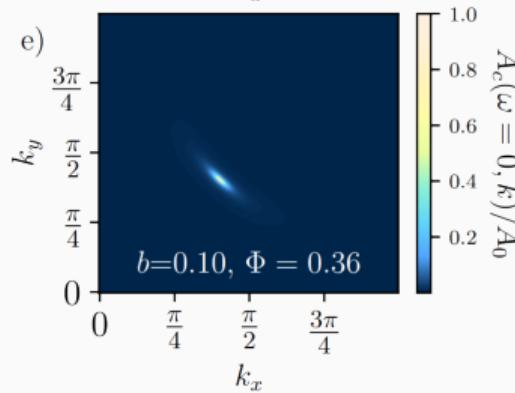


# Next nearest neighbor magnetic correlations ( $U/t = 7.4$ )



## Related work and future directions

Nodal anisotropic  
quasiparticles in  
superconducting state<sup>7</sup>



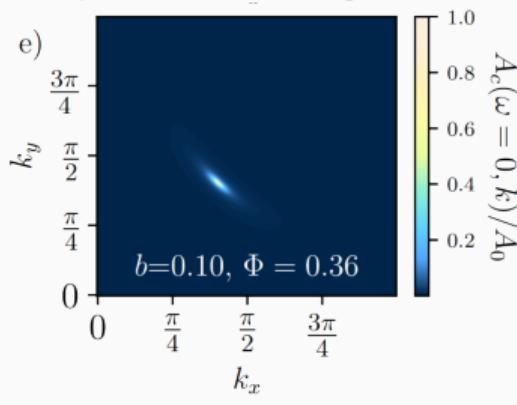
<sup>7</sup>Christos and Sachdev, *npj Quantum Materials*, 2024

<sup>8</sup>Bonetti et al., arXiv:2405.08817

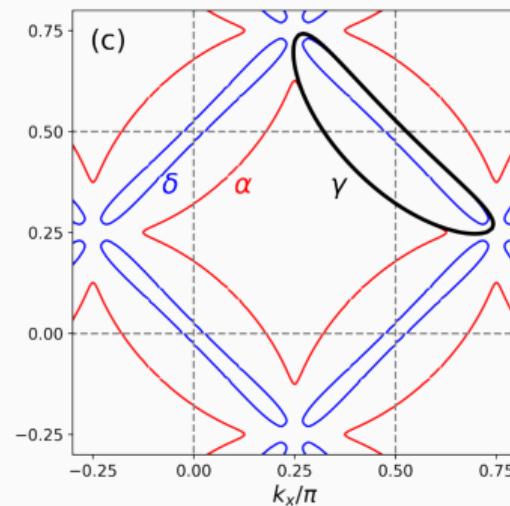
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FS reconstruction in CDW<sup>8</sup>  
 $FL^* \rightarrow CDW$



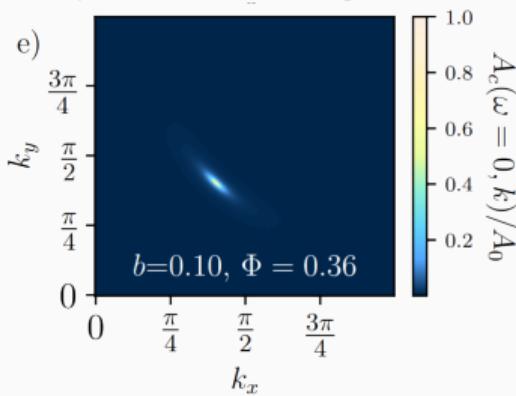
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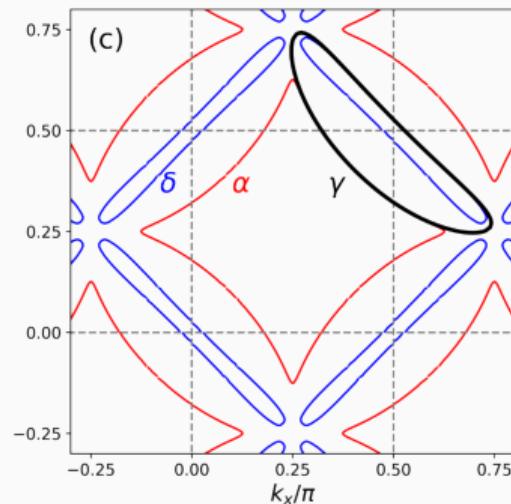
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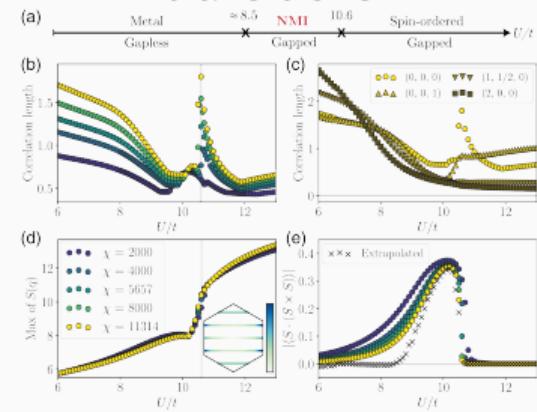
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Variational wavefunctions for spin liquids emerging at metal/insulator transitions<sup>9</sup>



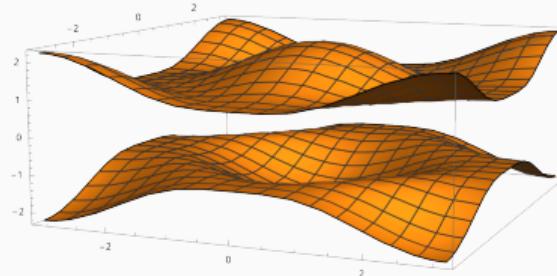
<sup>7</sup>Christos and Sachdev, *npj Quantum Materials*, 2024

<sup>8</sup>Bonetti et al., arXiv:2405.08817

<sup>9</sup>Szasz et al., *Physical Review X*, 2020

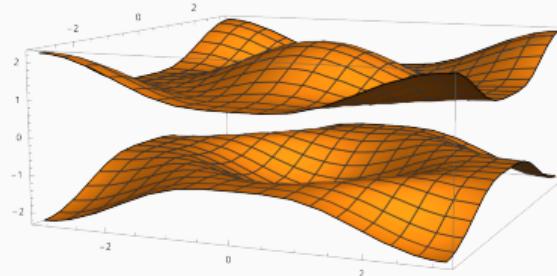
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- Actual ground state: AF insulator for  $U/t > 0$
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- Simple ansatz gives mean-field charge gap  $2\Phi$ , which we fix to be  $U$



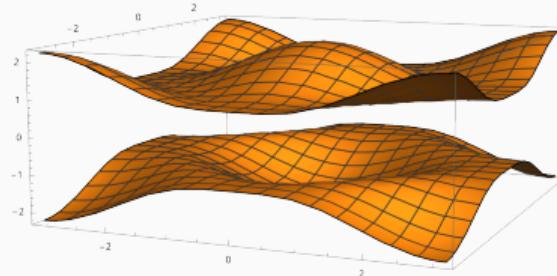
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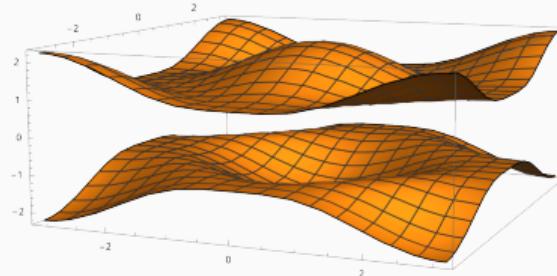
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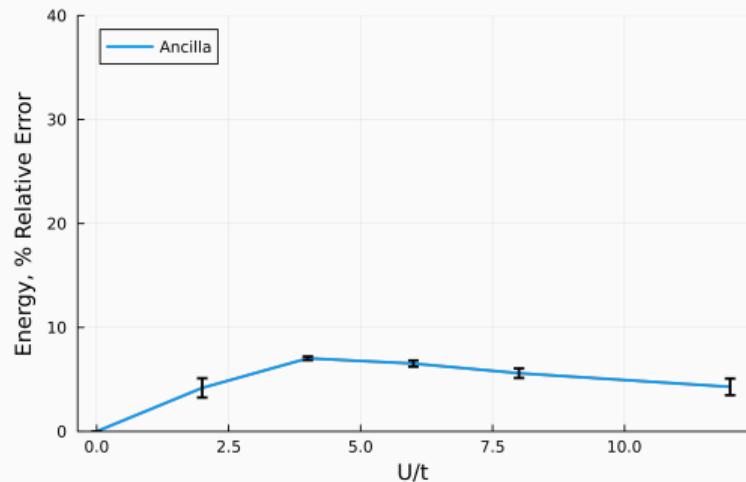
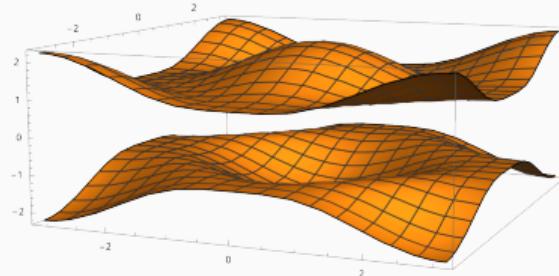
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