Relativistic Behavior Detection through Electron Acceleration

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April 5, 2017

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Special Relativity

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- The speed of light, c, is constant in all reference frames
- The velocity of any particle is capped at c

Classical and Relativistic Kinetic Energies are Different

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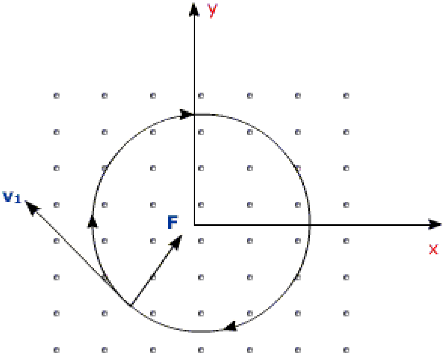
Relativistic Kinetic Energy

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2$$

Electrons in Magnetic Fields are Accelerated in Circular Orbits

- $\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = e\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$
- Electrons follow a circular orbit with radii ρ proportional to their momentum, $p = \frac{\rho e}{c} B$



Lorentz Law Yields Kinetic Energy vs. Magnetic Field Relations

Classically,

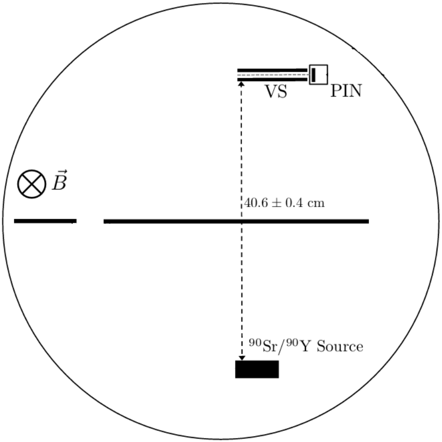
$$K = \frac{e^2\rho^2}{2mc^2}B^2$$

Relativistically,

$$K = \sqrt{e^2\rho^2B^2 + m^2c^4} - mc^2$$

Experimentally measuring K vs. B will let us pick the more likely model

Experimental Setup Constrains Radius of Electron Orbit



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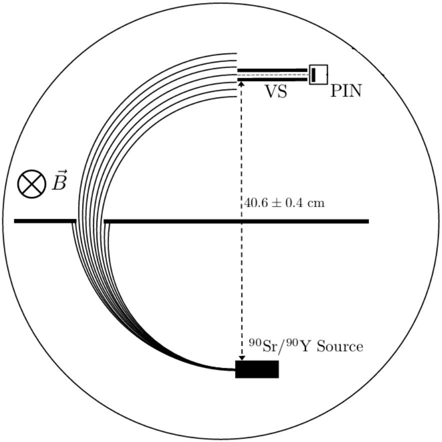

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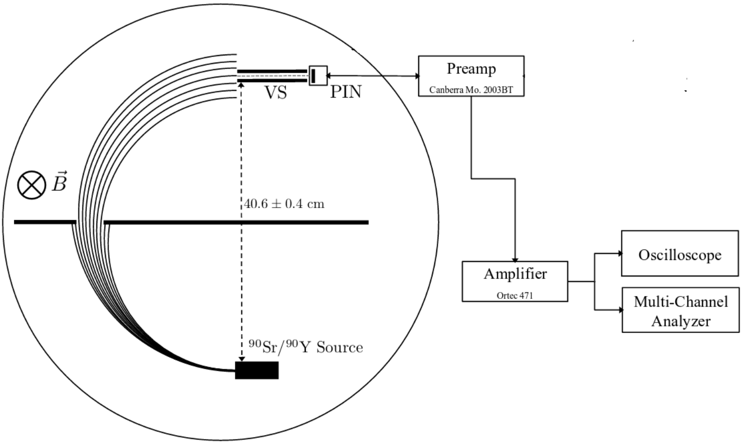
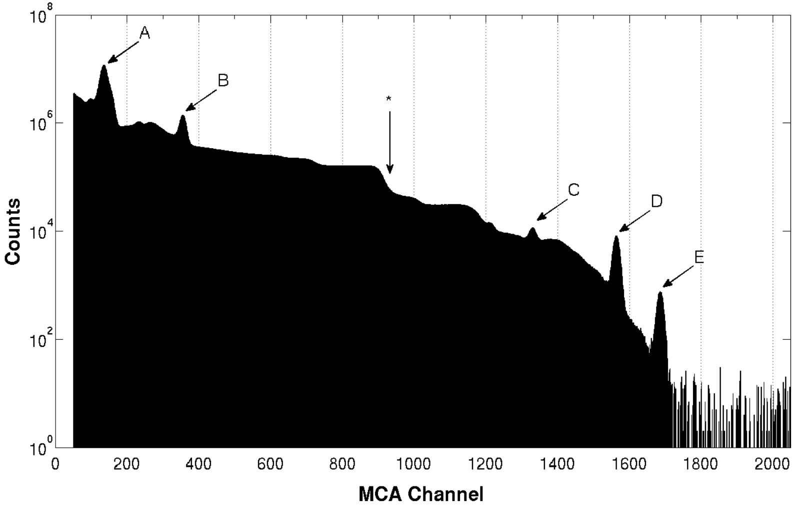
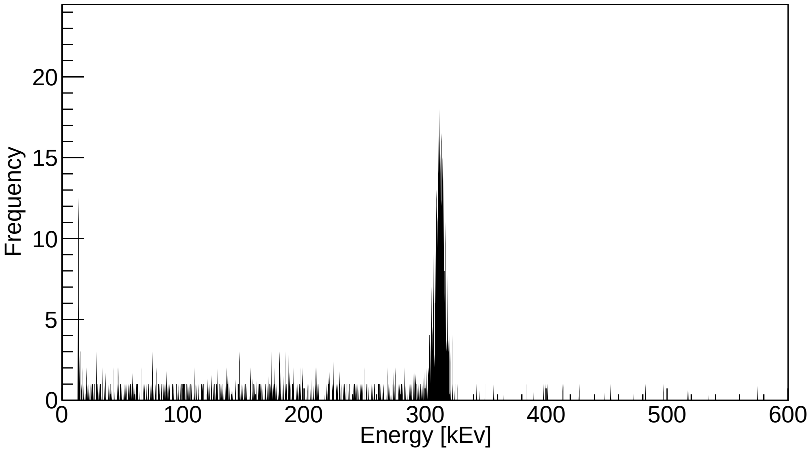


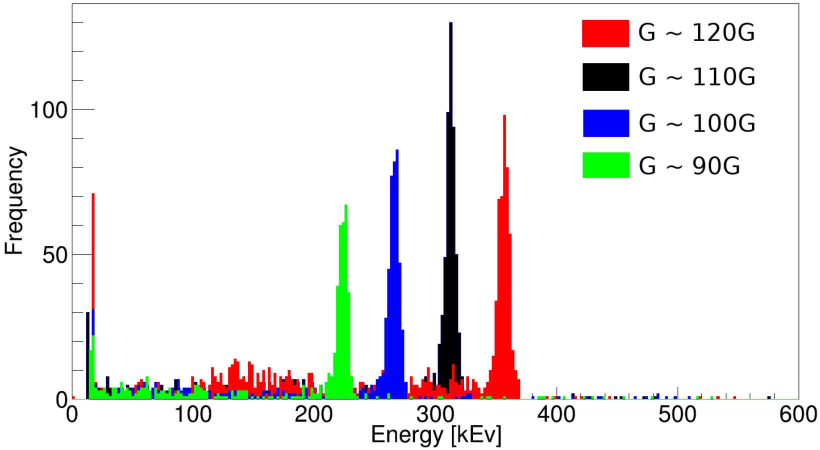
Barium-133 Produces MCA Peaks at Known Energies



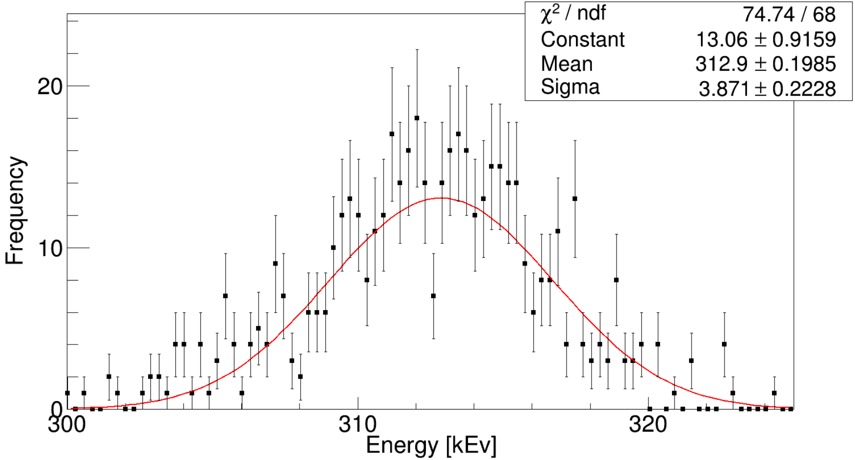
MCA Readout for Sr-90/Y-90 Sharply Peaked around Energy Range



Magnetic Field Affects Peak Energy Range



Kinetic Energy Determined through Gaussian Fitting



Gaussian Fits Bring Uncertainty in Kinetic Energy

B_{approx} (G)	K (kEv)	σ_K (kEv)
90	222	1.47
100	265	1.08
110	312	.79
120	355	.66

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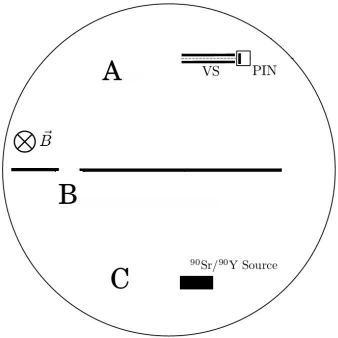
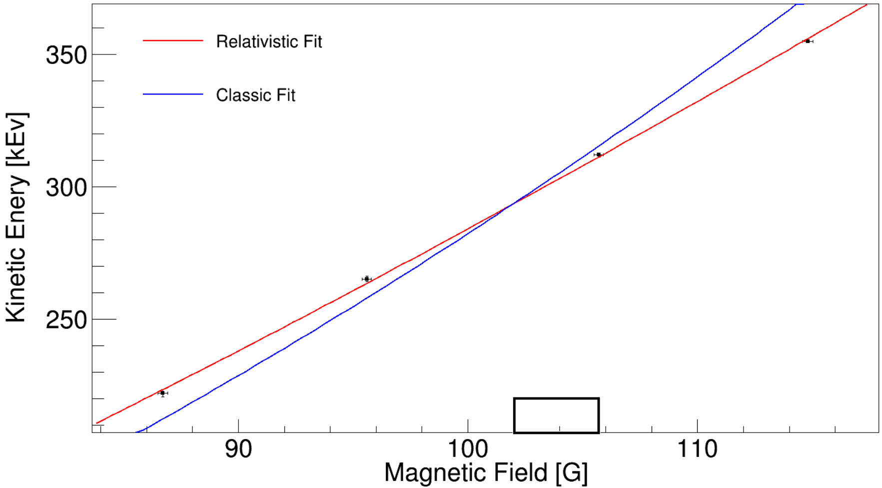
- Systematic uncertainty in magnetometer

Inhomogeneity Addressed by Averaging over Multiple Points

- Measured at point C during experimental runs
- Determined correspondance between magnetic field at point C and the average magnetic field over the path of the electron



Data Follows Relativistic Trend



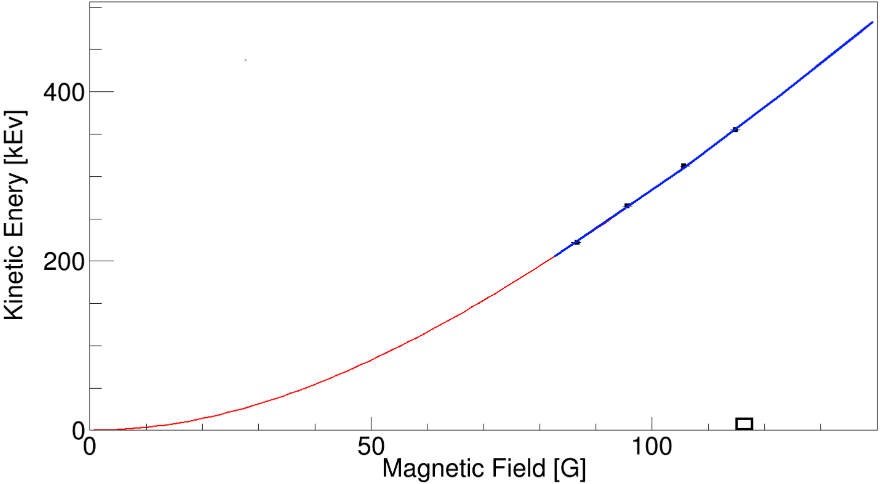
Relativistic Fit Predicts Realistic Constants

	Measured Values	Expected Values
e (esu)	$5.02 \pm .03$	4.8
mc^2 (kEv)	505 ± 76	511

Correlated fit pays off - uncertainty in mc^2 reduces from 200 kEv to 76 kEv.

Rest Energy Is Not Dominant In Our Regime

$$K = \sqrt{\rho^2 e^2 B^2 - m^2 c^4} - mc^2$$



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- Special Relativity and Classical Mechanics make different predictions for $K(p)$
- Using Lorentz's Law, we can determine electron momentum by the magnetic field supplied
- Experimentally determined relationship of K and B follow relativistic trends and predict electron charge and rest energy within expected values

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- For two plates separated by a distance d, the voltage required is $V = \frac{dvB}{c}$

Calibration Fit for Magnetometer Results in Large Errors