Relativistic Dynamic Detection through Electron Acceleration

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Classical and relativsitic mechanics differ in their predictions of how the momentum and energy of particles depend on their velocity. By accelerating electrons emitted from a ^{90}Sr source through a magnetic field, we are able to measure these momentum and energy relations for the electrons. Using this, we are able to conclude that the relativistic theory more accurately predicts the trends measured. Fitting this relation also allows us to calculate the elctron charge to mass ratio, e/m.

I. INTRODUCTION

Albert Einstein's 1905 paper, "On the Electrodynamics of Moving Bodies," was one of the foundational papers to developing the theory of special relativity. This theory was intended to reconcile inconsistencies with classical mechanics, electrodynamics, and observed phenomena. In his paper, Einstein postulated that the speed of light is a constant value in all reference frames, contradicting the classical prediction that light travels faster or slower depending on one's speed relaive to the light. This postulate, along with the additional assumption that the laws of physics are the same in all reference frames, leads to a new theory of dynamics, with different predictions than classical mechanics. These discrepencies allow us to compare the two theories against experimental data, and thus determine which theory yields more accurate predictions. We test this theory by accelerating electrons to speeds close to the speed of light - the regime where classical and relativistic theories differ the most.

In classical mechanics, the kinetic energy K of a particle as a function of momentum p is given by

$$K = \frac{p^2}{2m} \tag{1}$$

where m is the mass of the particle. Most notably, the kinetic energy as a function of momentum is quadratic.

In special relativity, the kinetic energy of a particle as a function of momentum is instead given by

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2 \tag{2}$$

where c is the speed of light - approximately 3.0×10^8 km/s. The value mc^2 is known as the *rest energy* of a particle. For particles with a small rest energy, K becomes approximately a linear function of p. For massless particles like photons, $K(p)$ is exactly linear.

These predictions from classical mechanics and special relativity gives us relations that can be used to fit an experimentally-measured graph of $K(p)$. The remainder of our analysis of the physical theory will be used to dermine how the kinetic energy and momentum of a particle relate to our setup.

The Lorentz law is an equation that holds in both classical mechanics and special relativity. This law determines how electric and magnetic fields affect the motion of a charged particle. In the presence of an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$, the change in momentum is related to the two fields, as well as the charge of the particle e, the velocity of the particle $\mathbf{v}$, and the speed of light c by

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = e\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \tag{3}$$

Equation (3) implies that charged particles subject to a constant magnetic field will move in orbits in the plane perpendicular to the magnetic field, with a radius of rotation proportional to the momentum of the particle. Specifically, a particle subject to a constant magnetic field of magnitude B that has a radius of rotation ρ will have momentum

$$p = \left(\frac{e\rho}{c}\right) B \tag{4}$$

II. EXPERIMENTAL SETUP

Our experimental setup consists of a large, spherical shell with coils wrapped around it. When these coils are fed a current, it generates an approximately uniform magnetic field across the inside of the shell, oriented vertically. By varying the current supplied to the coils, we can control the magnetic field inside the shell. The top and bottom hemispheres of the shell are disconnected, to allow for inspection of the internal components of the shell.

In the bottom hemisphere of the shell, a vacuum chamber is maintained through a mechanical pump and monitored by a pressure gauge. On one side of the chamber, a ^{90}Sr/^{90}Y beta source is placed. This beta source emits electrons with energies up to 2.28 MeV. When electrons are emitted in the prsence of the magnetic field, their trajectories are curved into orbits relative to their momentum.

On the other side of the chamber, placed at a distance of 40.6 ± 0.4 cm away from the source, is a parallel-plate velocity selector and a PIN diode detector. When the electrons are accelerated in a helical orbit, only electrons of a certain momentum will pass between the two plates.

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If these electrons continue on their helical orbit, they will hit the side of the plates and not be detected by the PIN diode. By supplying a constant electric field E between the two plates , the trajectory of the elctrons can be straightened out and detected by the PIN diode at some E proportional to the velocity of the electrons. When the electrons come in contact with the PIN diode, the kinetic energy is converted into an electric signal, which is fed through an amplifier and read by a multi-channel analyzer (MCA). This configuration allows us to detect the velocity, momentum, and kinetic energy of incoming electrons by looking at the supplied electric field, magnetic field, and MCA readout respectively.

Located halfway between the helical trajectory between the beta source and the detector is a metal divider with a small slit in it. In the absence of a perfect vacuum, our vacuum chamber will be filled with other particles, which the electrons from our beta source can scatter off of. To minimize the effects of this, we place a slit to attempt to screen our particles and only allow particles to pass through the slit whose trajectories have not been altered by scattering.

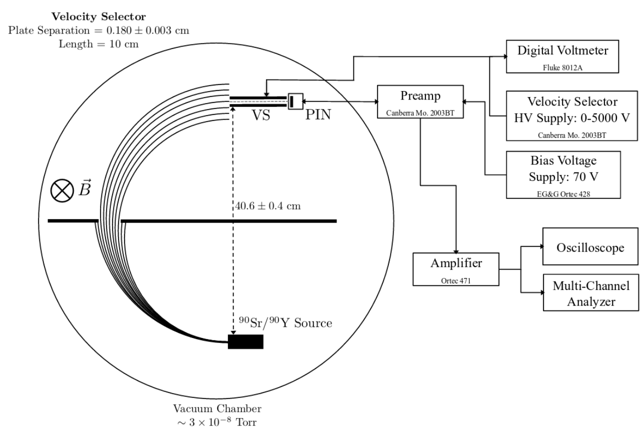

FIG. 1. A block diagram of our experimental setup. The circular lines drawn depict the expected trajectories from the beta source to the PIN diode detector.

III. EXPERIMENTAL CALIBRATION

There are two calibration procedures necessary for this experiment - the MCA calibration and the magnetometer calibration. The MCA detects the strength of incoming electric signals and bins them accordingly, but the correspondance between this measurement and the kinetic energy of the detected electron is, by default, unknown. To determine this relation, we place a ^{1}Ba source next to the PIN diode. The spectrum of ^{1}Ba is well known and can be used to calibrate our MCA. Displayed in Figure 2 is an example of an MCA readout of a ^{1}Ba source, with various peaks labeled. In Table 1, the energy known to correspond to these peaks is listed. Using this data, we can calibrate our MCA to display the kinetic energy of incoming electrons.

To measure the magnetic field supplied to our electrons, we use a Hall effect magnetometer.