title: "Homework - 2" author: "Group 7" date: "10/18/2020" output: pdf_document: default html_document: default —

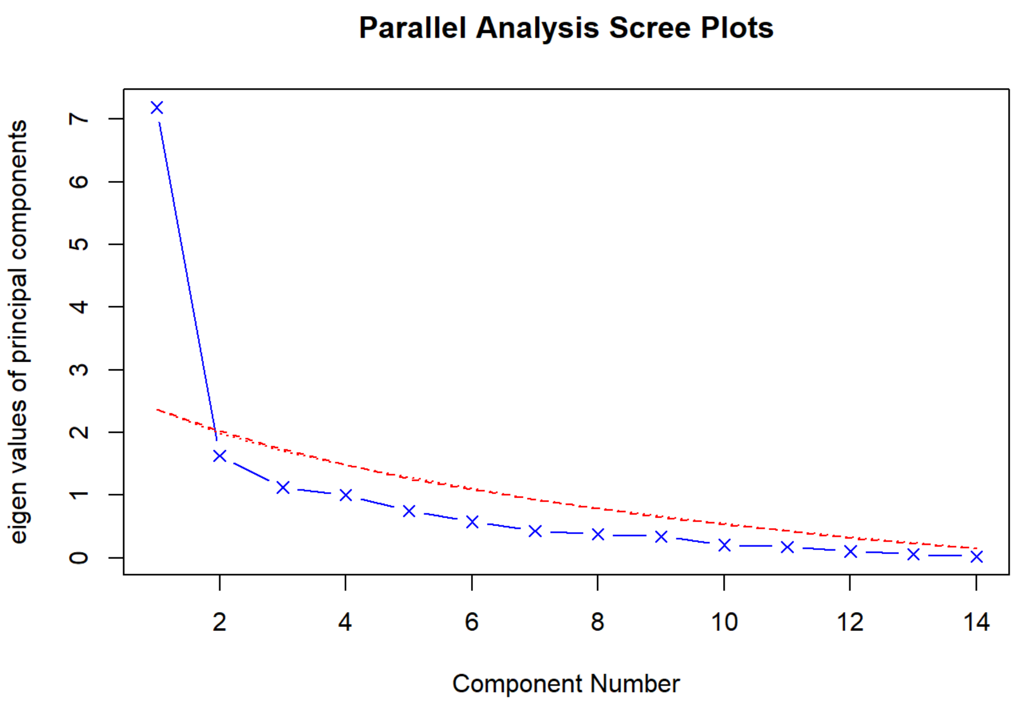
Importing the libraries

```
library(dplyr)
library(readxl)
library(psych)
```

```
## Warning: package 'psych' was built under R version 3.6.3
```

Problem 1a

```
# Import the NHL excel file as a dataframe
NHL_data <- data.frame(read_xlsx("NHL.xlsx", sheet = "Data"))
# Select the columns 13-26, the 1st column is the index column from excel
df <- NHL_data[, 13:26]
# Use Parallel Analysis Scree Plots to figure out the number of factors to extract
fa.parallel(df, fa = "pc", n.iter = 100, show.legend = FALSE)
```




```
## Parallel analysis suggests that the number of factors =  NA  and the number of components =  1
```

#It is appropriate to use 1 factor # 1b

```
# Performing PCA with varimax orthogonal rotation
pc <- principal(df, nfactors = 1, rotate = "none", scores = TRUE)
pc
```

```
## Principal Components Analysis
## Call: principal(r = df, nfactors = 1, rotate = "none", scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1    h2   u2 com
## gg     0.83 0.682 0.32   1
## gag   -0.82 0.671 0.33   1
## five   0.90 0.818 0.18   1
## PPP    0.16 0.026 0.97   1
## PKP    0.70 0.490 0.51   1
## shots  0.61 0.367 0.63   1
## sag   -0.63 0.400 0.60   1
## sc1    0.82 0.666 0.33   1
## tr1    0.74 0.552 0.45   1
## lead1  0.82 0.667 0.33   1
## lead2  0.75 0.564 0.44   1
## wop    0.71 0.500 0.50   1
## wosp   0.84 0.710 0.29   1
## face   0.28 0.078 0.92   1
##
##                 PC1
## SS loadings    7.19
## Proportion Var 0.51
##
## Mean item complexity =  1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.11
##  with the empirical chi square  70.28  with prob <  0.69
##
## Fit based upon off diagonal values = 0.95
```

#Rotating the components # Problem 1c

```
# PCA with varimax rotation
pc <- principal(df, nfactors = 1, rotate = "varimax", scores = TRUE)
pc
```

```
## Principal Components Analysis
## Call: principal(r = df, nfactors = 1, rotate = "varimax", scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1    h2   u2 com
## gg     0.83 0.682 0.32   1
## gag   -0.82 0.671 0.33   1
## five   0.90 0.818 0.18   1
## PPP    0.16 0.026 0.97   1
## PKP    0.70 0.490 0.51   1
## shots  0.61 0.367 0.63   1
## sag   -0.63 0.400 0.60   1
## sc1    0.82 0.666 0.33   1
## tr1    0.74 0.552 0.45   1
## lead1  0.82 0.667 0.33   1
## lead2  0.75 0.564 0.44   1
## wop    0.71 0.500 0.50   1
## wosp   0.84 0.710 0.29   1
## face   0.28 0.078 0.92   1
##
##                 PC1
## SS loadings    7.19
## Proportion Var 0.51
##
## Mean item complexity =  1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.11
##  with the empirical chi square  70.28  with prob <  0.69
##
## Fit based upon off diagonal values = 0.95
```

Problem 1d

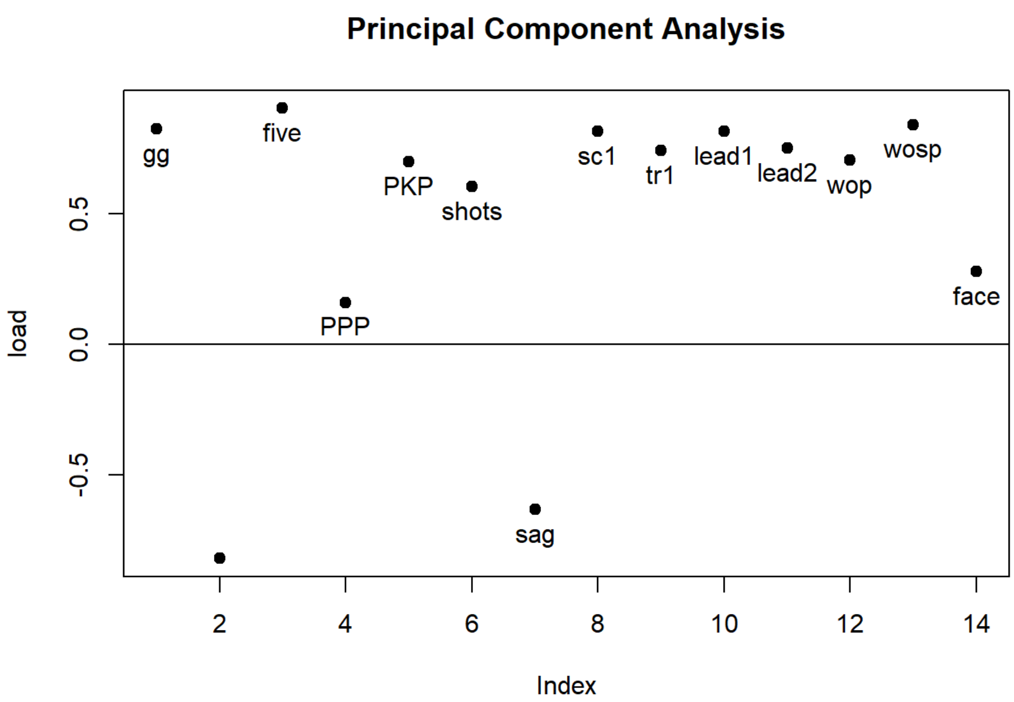
#Displaying the components score

```
head(pc$scores)
```

```
##            PC1
## [1,] 1.3503592
## [2,] 1.1106603
## [3,] 0.7062801
## [4,] 0.7251884
## [5,] 1.1956402
## [6,] 1.1813631
```

Problem 1e

```
# Plotting the components and analyze them
factor.plot(pc, labels = colnames(df))
```




```
rm(list = ls())
```

Problem 1f

#Interpretations - # - The PCA loads all the components except face and PPP.

- It is unclear as to what the component signifies without a deeper analysis

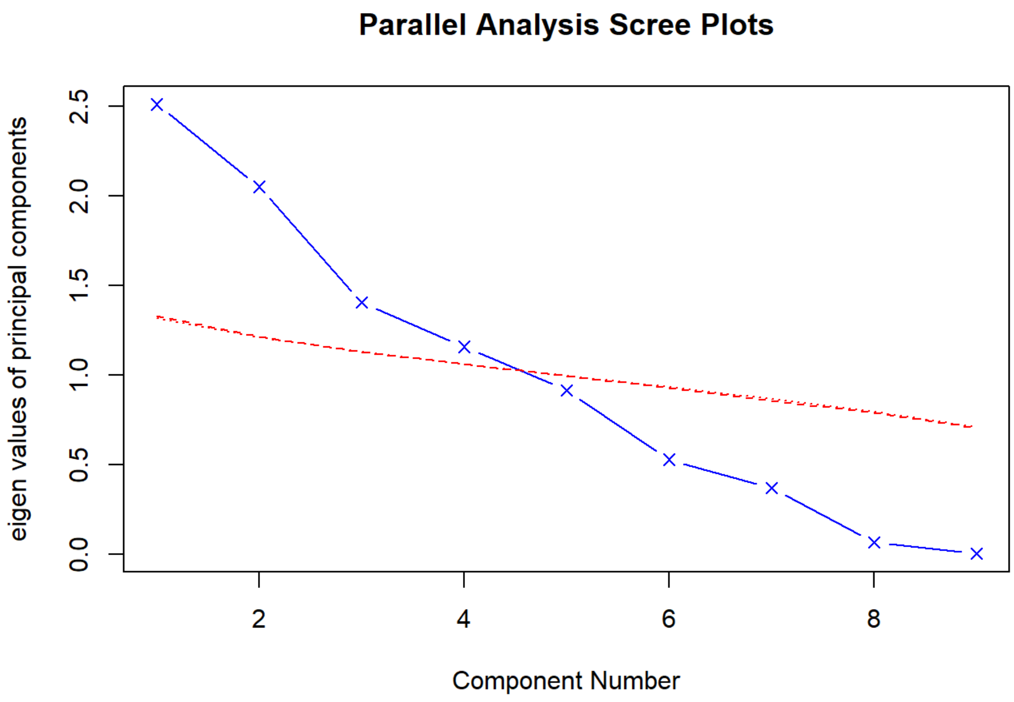
---

Problem 2a

```
glass_data <- data.frame(read_xlsx("Glass Identification Data.xlsx",
  sheet = "Glass Data"
))
fa.parallel(glass_data[, 2:10], fa = "pc", n.iter = 100, show.legend = FALSE)
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```




```
## Parallel analysis suggests that the number of factors =  NA  and the number of components =  4
```

#The Skree Plot and the fa.parallel() function overlaps and shows us that the nfactors as 4.

Problem 2b

```
# Perform PCA without rotation
pc <- principal(glass_data[, 2:10], nfactors = 4, rotate = "none", scores = TRUE)
pc
```

```
## Principal Components Analysis
## Call: principal(r = glass_data[, 2:10], nfactors = 4, rotate = "none",
##     scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      PC1   PC2   PC3   PC4   h2    u2 com
## RI -0.86  0.41  0.10 -0.16 0.95 0.051 1.5
## Na  0.41  0.39 -0.46 -0.53 0.80 0.195 3.8
## Mg -0.18 -0.85  0.01 -0.41 0.92 0.081 1.5
## Al  0.68  0.42  0.39  0.15 0.81 0.186 2.5
## Si  0.36 -0.22 -0.54  0.70 0.97 0.031 2.7
## K   0.35 -0.22  0.79  0.04 0.79 0.212 1.6
## CA -0.78  0.49  0.00  0.30 0.94 0.058 2.0
## Ba  0.40  0.69  0.09 -0.14 0.67 0.333 1.7
## Fe -0.29 -0.09  0.34  0.25 0.27 0.730 3.0
##
##                        PC1  PC2  PC3  PC4
## SS loadings           2.51 2.05 1.40 1.16
## Proportion Var        0.28 0.23 0.16 0.13
## Cumulative Var        0.28 0.51 0.66 0.79
## Proportion Explained  0.35 0.29 0.20 0.16
## Cumulative Proportion 0.35 0.64 0.84 1.00
##
## Mean item complexity =  2.3
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.08
##  with the empirical chi square  102.53  with prob <  7.4e-20
##
## Fit based upon off diagonal values = 0.92
```

Problem 2c

#Rotating the components

```
# Performing PCA with rotation
pc <- principal(glass_data[, 2:10], nfactors = 4, rotate = "varimax", scores = TRUE)
pc
```

```
## Principal Components Analysis
## Call: principal(r = glass_data[, 2:10], nfactors = 4, rotate = "varimax",
##     scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC1   RC2   RC3   RC4   h2    u2 com
## RI  0.84 -0.07  0.15  0.47 0.95 0.051 1.7
## Na -0.06  0.22 -0.86  0.09 0.80 0.195 1.2
## Mg -0.35 -0.86  0.04  0.21 0.92 0.081 1.5
## Al -0.42  0.80  0.03  0.01 0.81 0.186 1.5
## Si -0.13  0.00 -0.02 -0.98 0.97 0.031 1.0
## K  -0.62  0.22  0.51  0.30 0.79 0.212 2.7
## CA  0.91  0.12  0.30  0.06 0.94 0.058 1.3
## Ba -0.01  0.72 -0.33  0.17 0.67 0.333 1.5
## Fe  0.12 -0.04  0.50  0.07 0.27 0.730 1.2
##
##                        RC1  RC2  RC3  RC4
## SS loadings           2.26 2.03 1.48 1.36
## Proportion Var        0.25 0.23 0.16 0.15
## Cumulative Var        0.25 0.48 0.64 0.79
## Proportion Explained  0.32 0.28 0.21 0.19
## Cumulative Proportion 0.32 0.60 0.81 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.08
##  with the empirical chi square  102.53  with prob <  7.4e-20
##
## Fit based upon off diagonal values = 0.92
```

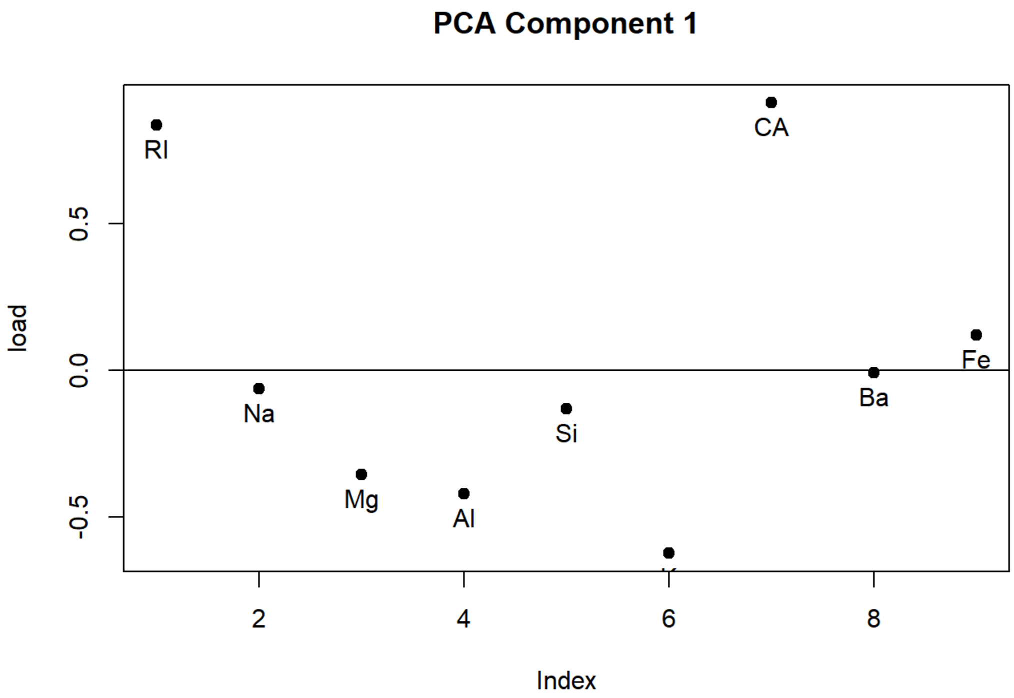
Problem 2d

```
head(pc$scores)
```

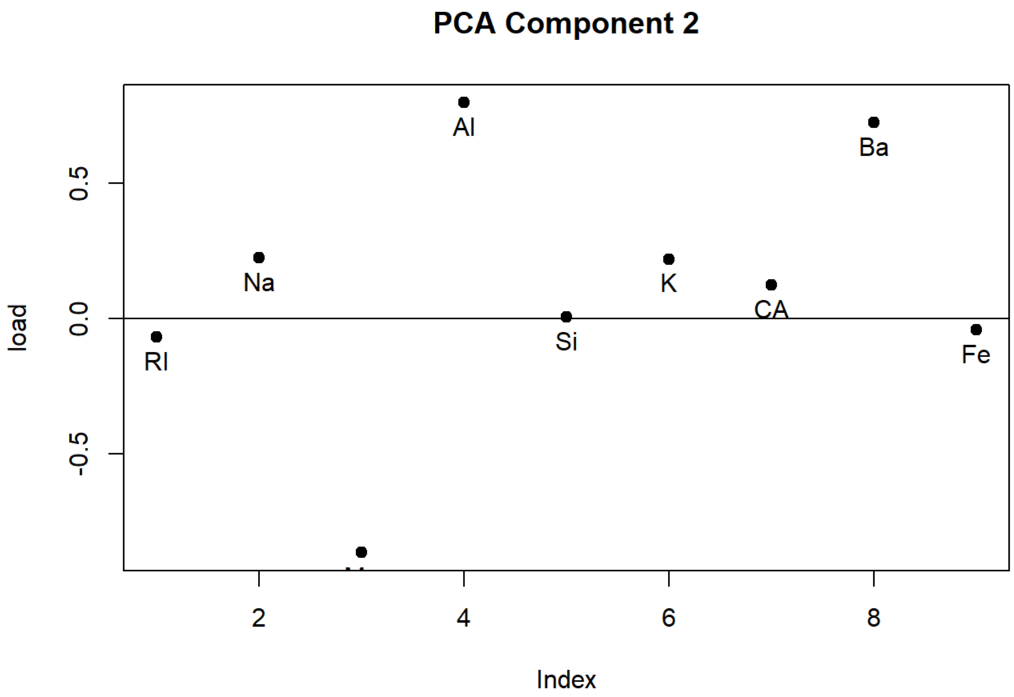
```
##              RC1        RC2        RC3         RC4
## [1,]  0.2516834 -1.1257154 -0.8331376  1.14203433
## [2,] -0.5120556 -0.5823124 -0.7217195  0.07184681
## [3,] -0.6811108 -0.4417522 -0.4610237 -0.39146231
## [4,] -0.4363986 -0.6266048 -0.1520952  0.09532063
## [5,] -0.4446499 -0.6485935 -0.1947898 -0.37616223
## [6,] -0.7149524 -0.2237372  1.1926990 -0.41874608
```

Problem 2e

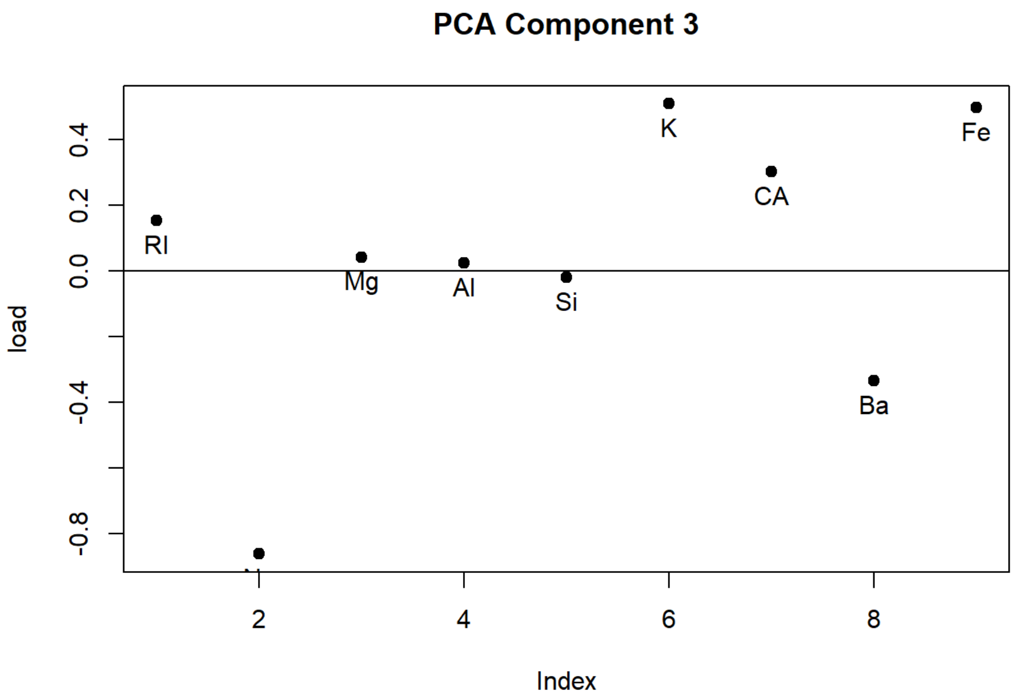
```
# Plotting the components to interpret the results
factor.plot(pc,
  choose = c(1),
  labels = colnames(glass_data[, 2:10]),
  title = "PCA Component 1"
)
```



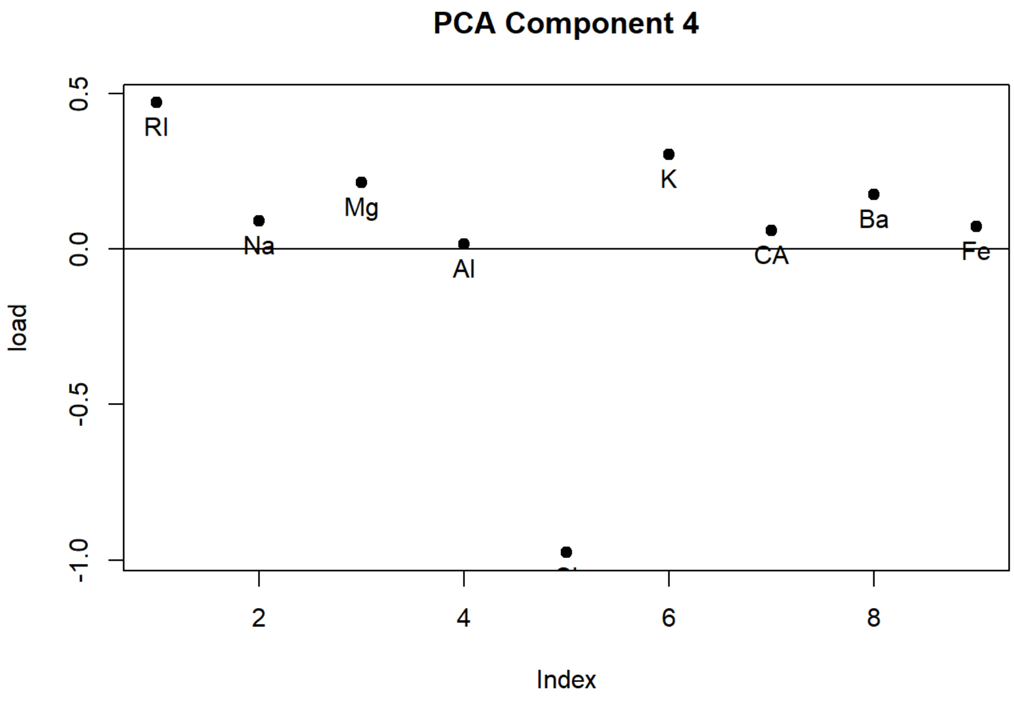

```
factor.plot(pc,
  choose = c(2),
  labels = colnames(glass_data[, 2:10]),
  title = "PCA Component 2"
)
```



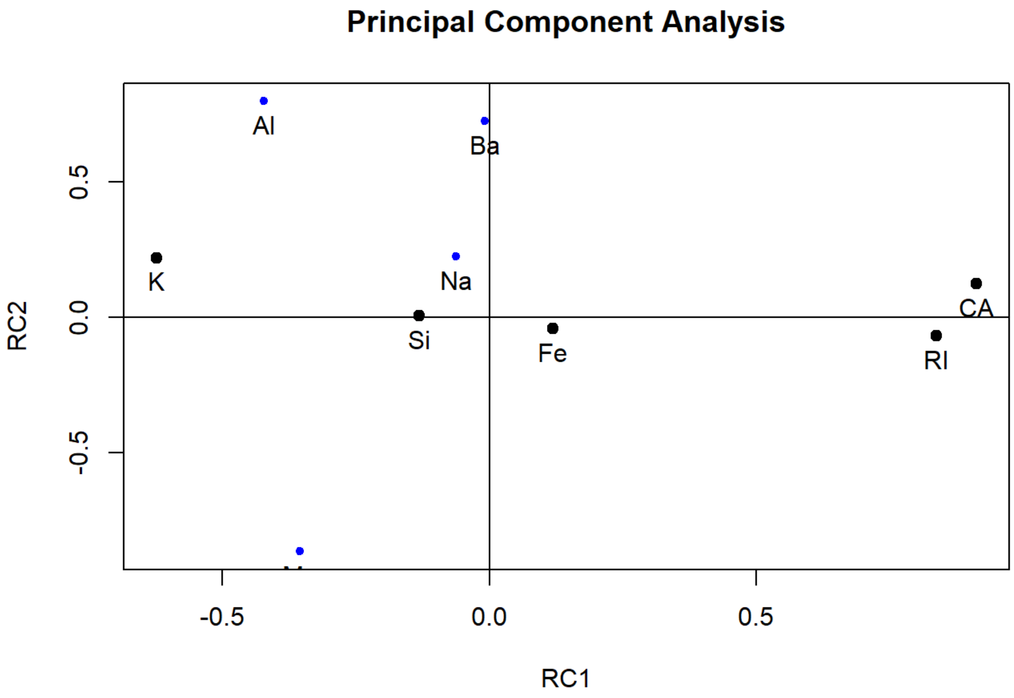

```
factor.plot(pc,
  choose = c(3),
  labels = colnames(glass_data[, 2:10]),
  title = "PCA Component 3"
)
```



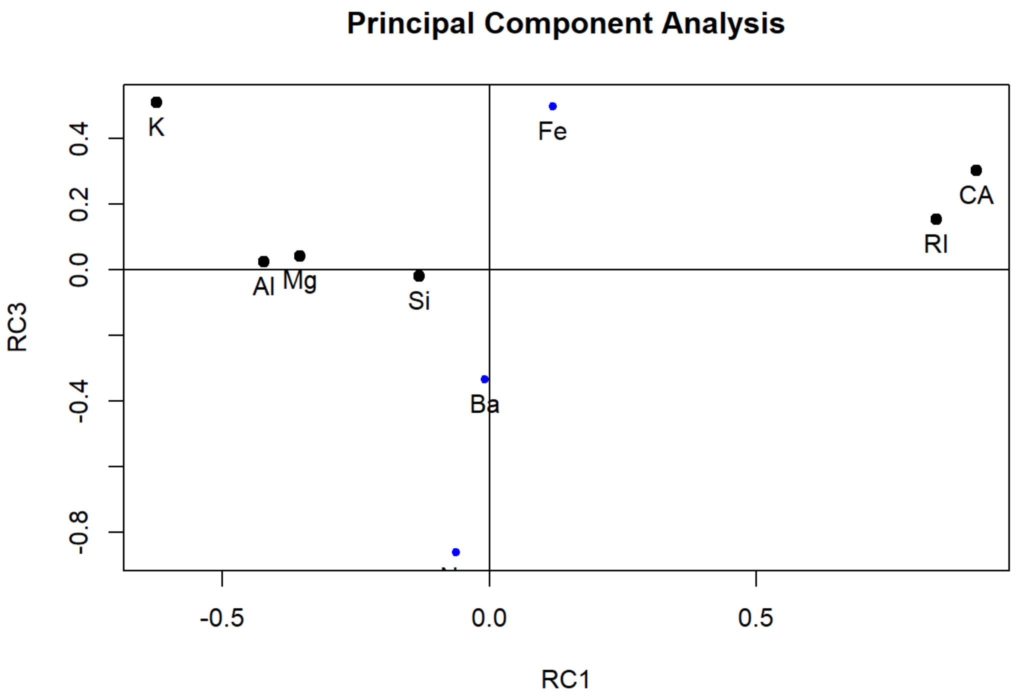

```
factor.plot(pc,
  choose = c(4),
  labels = colnames(glass_data[, 2:10]),
  title = "PCA Component 4"
)
```



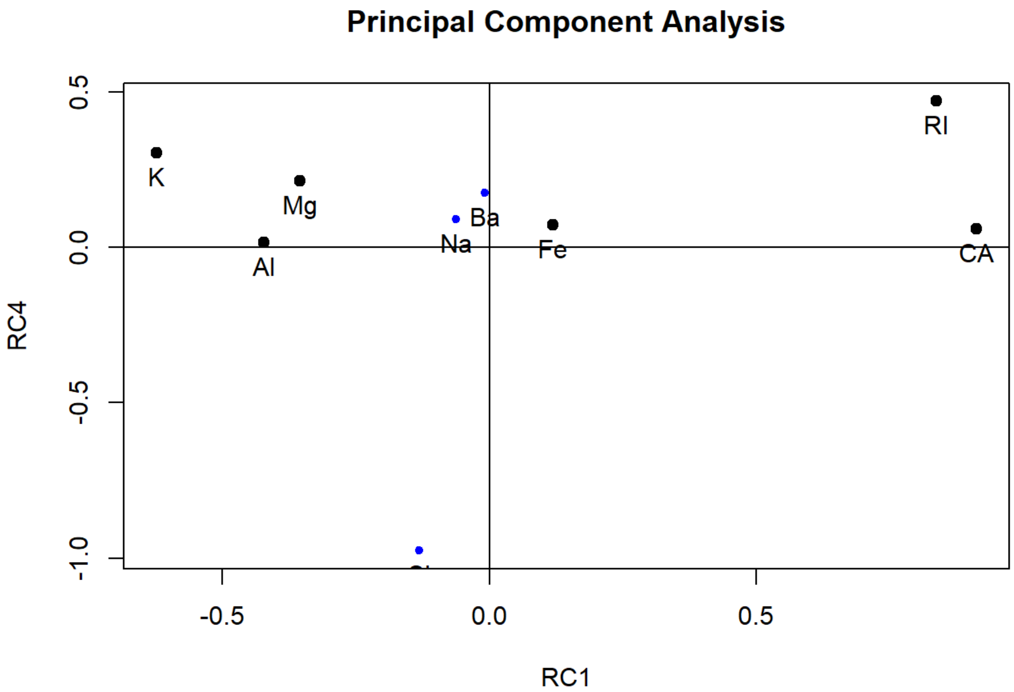

```
factor.plot(pc, choose = c(1, 2), labels = colnames(glass_data[, 2:10]))
```



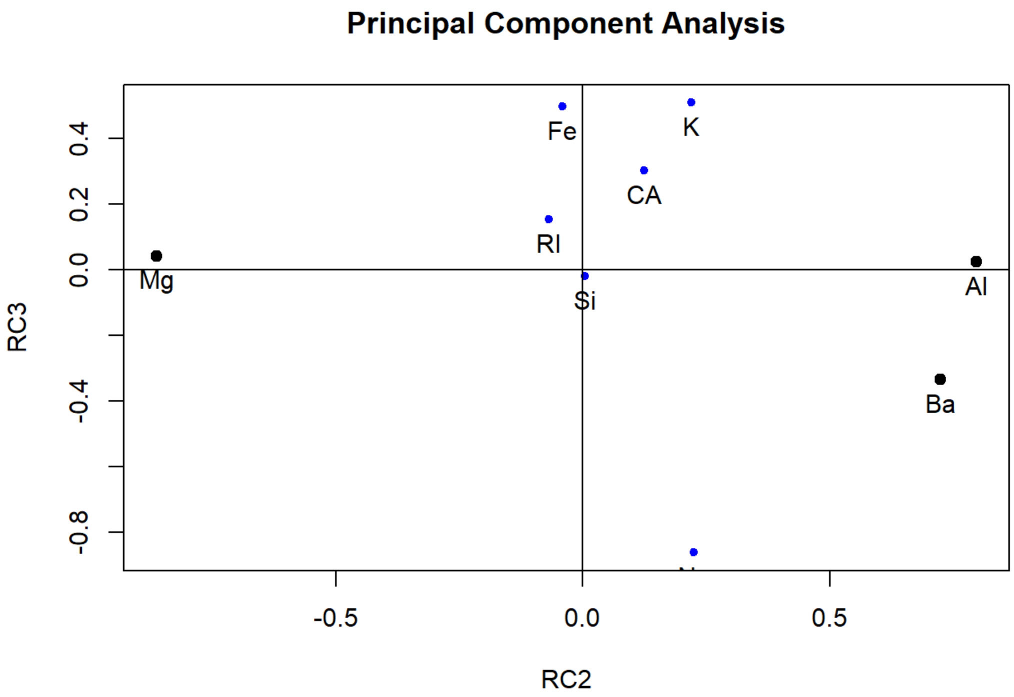

```
factor.plot(pc, choose = c(1, 3), labels = colnames(glass_data[, 2:10]))
```



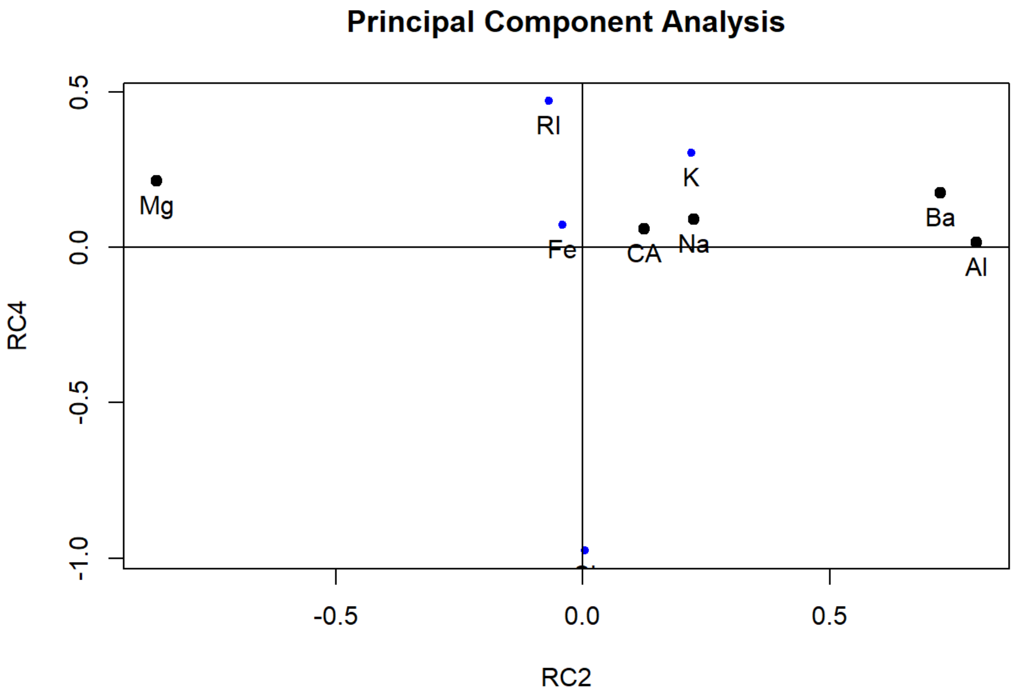

```
factor.plot(pc, choose = c(1, 4), labels = colnames(glass_data[, 2:10]))
```



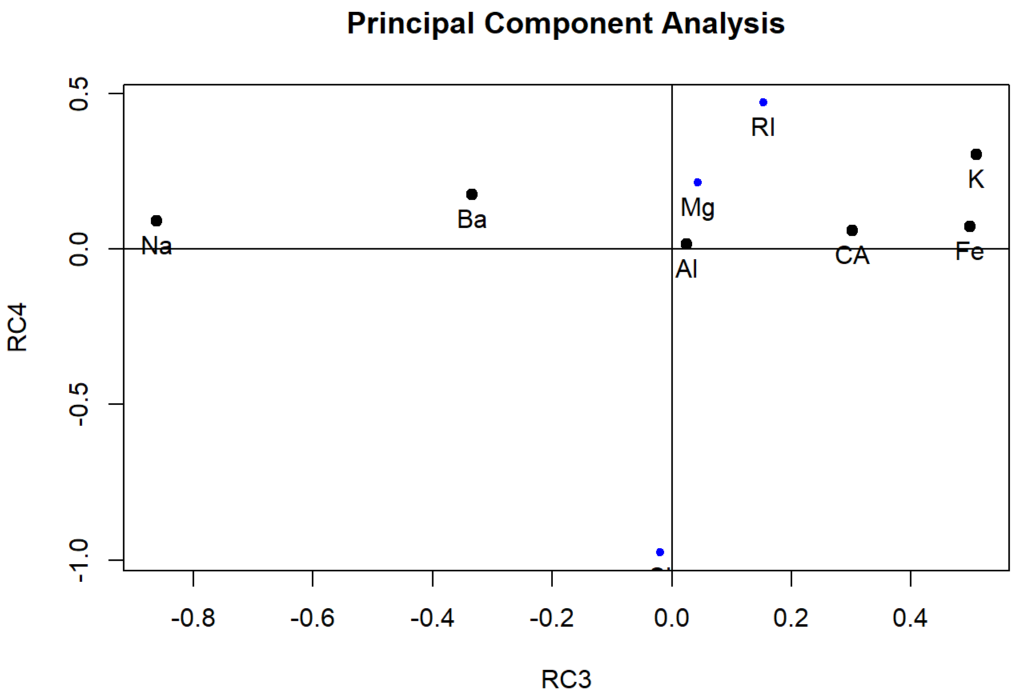

```
factor.plot(pc, choose = c(2, 3), labels = colnames(glass_data[, 2:10]))
```



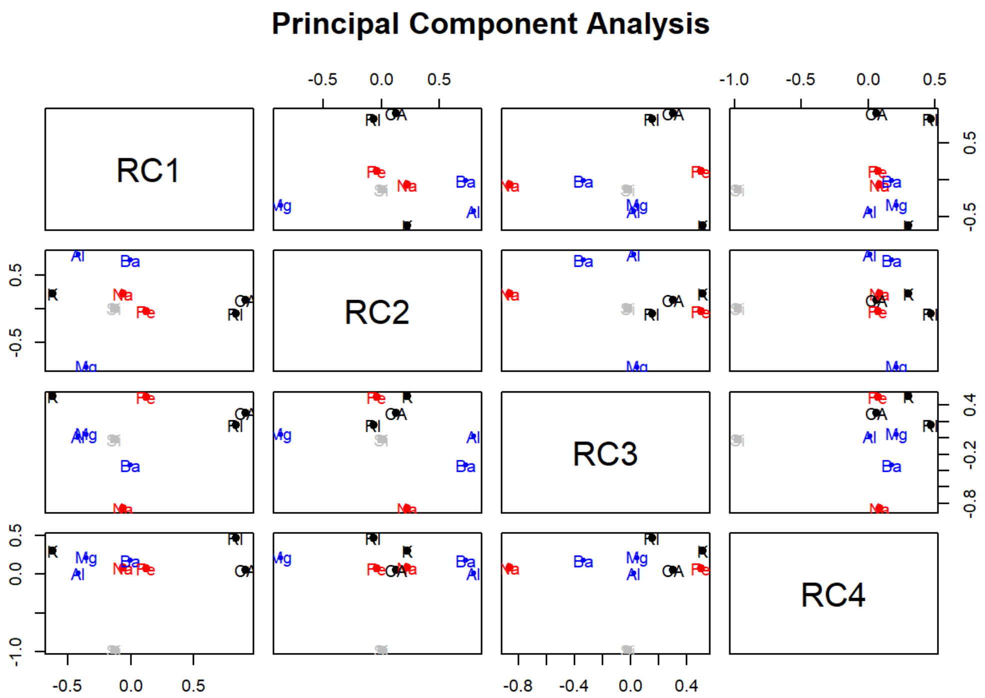

```
factor.plot(pc, choose = c(2, 4), labels = colnames(glass_data[, 2:10]))
```




```
factor.plot(pc, choose = c(3, 4), labels = colnames(glass_data[, 2:10]))
```




```r
factor.plot(pc, labels = colnames(glass_data[, 2:10]))
```




```r
rm(list = ls())
```

#Interpretations from the plot - #- PC1 - Indicates RI, CA, Mg, Al and K. It also signifies glass with high refractive index. #- PC2 - Indicates Mg, Al and Ba. Signifies glass with high Magnesium, Aluminum and Barium concentration. #- PC3 - Indicates Na, K, Ba and Fe. Signifies glass with high Sodium, Potassium, Barium and Iron concentration. #- PC4 - Indicates RI and Si. Signifies glass with high Refractive Index and Silicon concentration.

Problem 3a

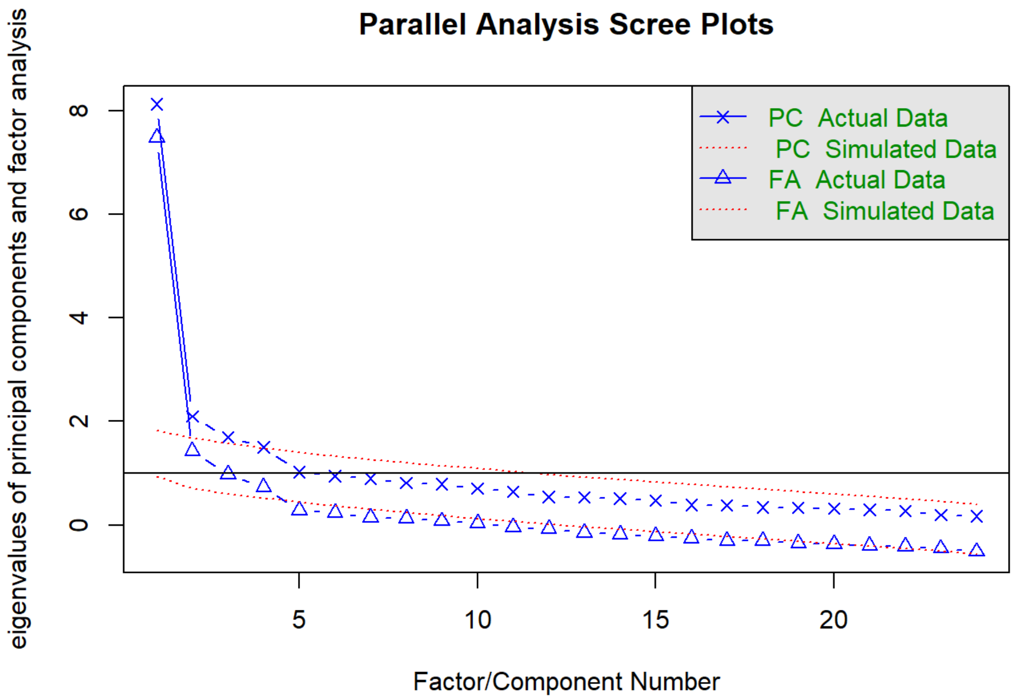
```r
"Harman75.cor" <-
  structure(list(cov = structure(c(
    1, 0.318, 0.403, 0.468, 0.321,
```

```
    0.335, 0.304, 0.332, 0.326, 0.116, 0.308, 0.314, 0.489, 0.125,
    0.238, 0.414, 0.176, 0.368, 0.27, 0.365, 0.369, 0.413, 0.474,
    0.282, 0.318, 1, 0.317, 0.23, 0.285, 0.234, 0.157, 0.157, 0.195,
    0.057, 0.15, 0.145, 0.239, 0.103, 0.131, 0.272, 0.005, 0.255,
    0.112, 0.292, 0.306, 0.232, 0.348, 0.211, 0.403, 0.317, 1, 0.305,
    0.247, 0.268, 0.223, 0.382, 0.184, -0.075, 0.091, 0.14, 0.321,
    0.177, 0.065, 0.263, 0.177, 0.211, 0.312, 0.297, 0.165, 0.25,
    0.383, 0.203, 0.468, 0.23, 0.305, 1, 0.227, 0.327, 0.335, 0.391,
    0.325, 0.099, 0.11, 0.16, 0.327, 0.066, 0.127, 0.322, 0.187,
    0.251, 0.137, 0.339, 0.349, 0.38, 0.335, 0.248, 0.321, 0.285,
    0.247, 0.227, 1, 0.622, 0.656, 0.578, 0.723, 0.311, 0.344, 0.215,
    0.344, 0.28, 0.229, 0.187, 0.208, 0.263, 0.19, 0.398, 0.318,
    0.441, 0.435, 0.42, 0.335, 0.234, 0.268, 0.327, 0.622, 1, 0.722,
    0.527, 0.714, 0.203, 0.353, 0.095, 0.309, 0.292, 0.251, 0.291,
    0.273, 0.167, 0.251, 0.435, 0.263, 0.386, 0.431, 0.433, 0.304,
    0.157, 0.223, 0.335, 0.656, 0.722, 1, 0.619, 0.685, 0.246, 0.232,
    0.181, 0.345, 0.236, 0.172, 0.18, 0.228, 0.159, 0.226, 0.451,
    0.314, 0.396, 0.405, 0.437, 0.332, 0.157, 0.382, 0.391, 0.578,
    0.527, 0.619, 1, 0.532, 0.285, 0.3, 0.271, 0.395, 0.252, 0.175,
    0.296, 0.255, 0.25, 0.274, 0.427, 0.362, 0.357, 0.501, 0.388,
    0.326, 0.195, 0.184, 0.325, 0.723, 0.714, 0.685, 0.532, 1, 0.17,
    0.28, 0.113, 0.28, 0.26, 0.248, 0.242, 0.274, 0.208, 0.274, 0.446,
    0.266, 0.483, 0.504, 0.424, 0.116, 0.057, -0.075, 0.099, 0.311,
    0.203, 0.246, 0.285, 0.17, 1, 0.484, 0.585, 0.408, 0.172, 0.154,
    0.124, 0.289, 0.317, 0.19, 0.173, 0.405, 0.16, 0.262, 0.531,
    0.308, 0.15, 0.091, 0.11, 0.344, 0.353, 0.232, 0.3, 0.28, 0.484,
    1, 0.428, 0.535, 0.35, 0.24, 0.314, 0.362, 0.35, 0.29, 0.202,
    0.399, 0.304, 0.251, 0.412, 0.314, 0.145, 0.14, 0.16, 0.215,
    0.095, 0.181, 0.271, 0.113, 0.585, 0.428, 1, 0.512, 0.131, 0.173,
    0.119, 0.278, 0.349, 0.11, 0.246, 0.355, 0.193, 0.35, 0.414,
    0.489, 0.239, 0.321, 0.327, 0.344, 0.309, 0.345, 0.395, 0.28,
    0.408, 0.535, 0.512, 1, 0.195, 0.139, 0.281, 0.194, 0.323, 0.263,
    0.241, 0.425, 0.279, 0.382, 0.358, 0.125, 0.103, 0.177, 0.066,
    0.28, 0.292, 0.236, 0.252, 0.26, 0.172, 0.35, 0.131, 0.195, 1,
    0.37, 0.412, 0.341, 0.201, 0.206, 0.302, 0.183, 0.243, 0.242,
    0.304, 0.238, 0.131, 0.065, 0.127, 0.229, 0.251, 0.172, 0.175,
    0.248, 0.154, 0.24, 0.173, 0.139, 0.37, 1, 0.325, 0.345, 0.334,
    0.192, 0.272, 0.232, 0.246, 0.256, 0.165, 0.414, 0.272, 0.263,
    0.322, 0.187, 0.291, 0.18, 0.296, 0.242, 0.124, 0.314, 0.119,
    0.281, 0.412, 0.325, 1, 0.324, 0.344, 0.258, 0.388, 0.348, 0.283,
    0.36, 0.262, 0.176, 0.005, 0.177, 0.187, 0.208, 0.273, 0.228,
    0.255, 0.274, 0.289, 0.362, 0.278, 0.194, 0.341, 0.345, 0.324,
    1, 0.448, 0.324, 0.262, 0.173, 0.273, 0.287, 0.326, 0.368, 0.255,
    0.211, 0.251, 0.263, 0.167, 0.159, 0.25, 0.208, 0.317, 0.35,
    0.349, 0.323, 0.201, 0.334, 0.344, 0.448, 1, 0.358, 0.301, 0.357,
    0.317, 0.272, 0.405, 0.27, 0.112, 0.312, 0.137, 0.19, 0.251,
    0.226, 0.274, 0.274, 0.19, 0.29, 0.11, 0.263, 0.206, 0.192, 0.258,
    0.324, 0.358, 1, 0.167, 0.331, 0.342, 0.303, 0.374, 0.365, 0.292,
    0.297, 0.339, 0.398, 0.435, 0.451, 0.427, 0.446, 0.173, 0.202,
    0.246, 0.241, 0.302, 0.272, 0.388, 0.262, 0.301, 0.167, 1, 0.413,
    0.463, 0.509, 0.366, 0.369, 0.306, 0.165, 0.349, 0.318, 0.263,
    0.314, 0.362, 0.266, 0.405, 0.399, 0.355, 0.425, 0.183, 0.232,
    0.348, 0.173, 0.357, 0.331, 0.413, 1, 0.374, 0.451, 0.448, 0.413,
    0.232, 0.25, 0.38, 0.441, 0.386, 0.396, 0.357, 0.483, 0.16, 0.304,
    0.193, 0.279, 0.243, 0.246, 0.283, 0.273, 0.317, 0.342, 0.463,
    0.374, 1, 0.503, 0.375, 0.474, 0.348, 0.383, 0.335, 0.435, 0.431,
    0.405, 0.501, 0.504, 0.262, 0.251, 0.35, 0.382, 0.242, 0.256,
    0.36, 0.287, 0.272, 0.303, 0.509, 0.451, 0.503, 1, 0.434, 0.282,
    0.211, 0.203, 0.248, 0.42, 0.433, 0.437, 0.388, 0.424, 0.531,
    0.412, 0.414, 0.358, 0.304, 0.165, 0.262, 0.326, 0.405, 0.374,
    0.366, 0.448, 0.375, 0.434, 1
  ), .Dim = c(24, 24), .Dimnames = list(
    c(
      "VisualPerception", "Cubes", "PaperFormBoard", "Flags",
      "GeneralInformation", "PargraphComprehension", "SentenceCompletion",
      "WordClassification", "WordMeaning", "Addition", "Code",
      "CountingDots", "StraightCurvedCapitals", "WordRecognition",
      "NumberRecognition", "FigureRecognition", "ObjectNumber",
      "NumberFigure", "FigureWord", "Deduction", "NumericalPuzzles",
      "ProblemReasoning", "SeriesCompletion", "ArithmeticProblems"
    ), c(
      "VisualPerception", "Cubes", "PaperFormBoard", "Flags",
      "GeneralInformation", "PargraphComprehension", "SentenceCompletion",
```

```
       "WordClassification", "WordMeaning", "Addition", "Code",
       "CountingDots", "StraightCurvedCapitals", "WordRecognition",
       "NumberRecognition", "FigureRecognition", "ObjectNumber",
       "NumberFigure", "FigureWord", "Deduction", "NumericalPuzzles",
       "ProblemReasoning", "SeriesCompletion", "ArithmeticProblems"
     )
  )), center = c(
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0
  ), n.obs = 145), .Names = c(
     "cov",
     "center", "n.obs"
  ))
fa.parallel(Harman74.cor$cov,
  n.obs = Harman74.cor$n.obs,
  n.iter = 100
)
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```




```
## Parallel analysis suggests that the number of factors =  4  and the number of components =  3
```

#The Screeplot indicates nfactor of 4

Problem 3b

```
# Performing Factor Analysis
correlations <- cov2cor(Harman74.cor$cov)
fa <- fa(correlations, nfactors = 4, rotate = "none", scores = TRUE)
fa
```

```
## Factor Analysis using method =  minres
## Call: fa(r = correlations, nfactors = 4, rotate = "none", scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##                          MR1   MR2   MR3   MR4   h2   u2  com
## VisualPerception        0.60  0.03  0.38 -0.22 0.55 0.45 2.0
## Cubes                   0.37 -0.03  0.26 -0.15 0.23 0.77 2.2
## PaperFormBoard          0.42 -0.12  0.36 -0.13 0.34 0.66 2.3
## Flags                   0.48 -0.11  0.26 -0.19 0.35 0.65 2.0
## GeneralInformation      0.69 -0.30 -0.27 -0.04 0.64 0.36 1.7
## PargraphComprehension   0.69 -0.40 -0.20  0.08 0.68 0.32 1.8
## SentenceCompletion      0.68 -0.41 -0.30 -0.08 0.73 0.27 2.1
## WordClassification      0.67 -0.19 -0.09 -0.11 0.51 0.49 1.3
## WordMeaning             0.70 -0.45 -0.23  0.08 0.74 0.26 2.0
## Addition                0.47  0.53 -0.48 -0.10 0.74 0.26 3.1
## Code                    0.56  0.36 -0.16  0.09 0.47 0.53 2.0
## CountingDots            0.47  0.50 -0.14 -0.24 0.55 0.45 2.6
## StraightCurvedCapitals  0.60  0.26  0.01 -0.29 0.51 0.49 1.9
## WordRecognition         0.43  0.06  0.01  0.42 0.36 0.64 2.0
## NumberRecognition       0.39  0.10  0.09  0.37 0.31 0.69 2.2
## FigureRecognition       0.51  0.09  0.35  0.25 0.45 0.55 2.3
## ObjectNumber            0.47  0.21 -0.01  0.39 0.41 0.59 2.4
## NumberFigure            0.52  0.32  0.16  0.14 0.41 0.59 2.1
## FigureWord              0.44  0.10  0.10  0.13 0.23 0.77 1.4
## Deduction               0.62 -0.13  0.14  0.04 0.42 0.58 1.2
## NumericalPuzzles        0.59  0.21  0.07 -0.14 0.42 0.58 1.4
## ProblemReasoning        0.61 -0.10  0.12  0.03 0.40 0.60 1.1
## SeriesCompletion        0.69 -0.06  0.15 -0.10 0.51 0.49 1.2
## ArithmeticProblems      0.65  0.17 -0.19  0.00 0.49 0.51 1.3
##
##                         MR1  MR2  MR3  MR4
## SS loadings            7.65 1.69 1.22 0.92
## Proportion Var         0.32 0.07 0.05 0.04
## Cumulative Var         0.32 0.39 0.44 0.48
## Proportion Explained   0.67 0.15 0.11 0.08
## Cumulative Proportion  0.67 0.81 0.92 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective function was  11.44
## The degrees of freedom for the model are 186  and the objective function was  1.72
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                    MR1  MR2  MR3  MR4
## Correlation of (regression) scores with factors   0.97 0.91 0.87 0.79
## Multiple R square of scores with factors          0.94 0.82 0.75 0.62
## Minimum correlation of possible factor scores     0.89 0.65 0.50 0.24
```

Problem 3c

#Rotating the factors

```r
fa_orthogonal <- fa(correlations, nfactors = 4, rotate = "varimax", scores = TRUE)
fa_orthogonal
```

```
## Factor Analysis using method =  minres
## Call: fa(r = correlations, nfactors = 4, rotate = "varimax", scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##                          MR1   MR3   MR2  MR4   h2   u2 com
## VisualPerception        0.15  0.68  0.20 0.15 0.55 0.45 1.4
## Cubes                   0.11  0.45  0.08 0.08 0.23 0.77 1.3
## PaperFormBoard          0.15  0.55 -0.01 0.11 0.34 0.66 1.2
## Flags                   0.23  0.53  0.09 0.07 0.35 0.65 1.5
## GeneralInformation      0.73  0.19  0.22 0.14 0.64 0.36 1.4
## PargraphComprehension   0.76  0.21  0.07 0.23 0.68 0.32 1.4
## SentenceCompletion      0.81  0.19  0.15 0.07 0.73 0.27 1.2
## WordClassification      0.57  0.34  0.23 0.14 0.51 0.49 2.2
## WordMeaning             0.81  0.20  0.05 0.22 0.74 0.26 1.3
## Addition                0.17 -0.11  0.82 0.16 0.74 0.26 1.2
## Code                    0.18  0.11  0.54 0.37 0.47 0.53 2.1
## CountingDots            0.02  0.20  0.71 0.09 0.55 0.45 1.2
## StraightCurvedCapitals 0.18  0.42  0.54 0.08 0.51 0.49 2.2
## WordRecognition         0.21  0.05  0.08 0.56 0.36 0.64 1.3
## NumberRecognition       0.12  0.12  0.08 0.52 0.31 0.69 1.3
## FigureRecognition       0.07  0.42  0.06 0.52 0.45 0.55 2.0
## ObjectNumber            0.14  0.06  0.22 0.58 0.41 0.59 1.4
## NumberFigure            0.02  0.31  0.34 0.45 0.41 0.59 2.7
## FigureWord              0.15  0.25  0.18 0.35 0.23 0.77 2.8
## Deduction               0.38  0.42  0.10 0.29 0.42 0.58 2.9
## NumericalPuzzles        0.18  0.40  0.43 0.21 0.42 0.58 2.8
## ProblemReasoning        0.37  0.41  0.13 0.29 0.40 0.60 3.0
## SeriesCompletion        0.37  0.52  0.23 0.22 0.51 0.49 2.7
## ArithmeticProblems      0.36  0.19  0.49 0.29 0.49 0.51 2.9
##
##                         MR1  MR3  MR2  MR4
## SS loadings            3.64 2.93 2.67 2.23
## Proportion Var         0.15 0.12 0.11 0.09
## Cumulative Var         0.15 0.27 0.38 0.48
## Proportion Explained  0.32 0.26 0.23 0.19
## Cumulative Proportion 0.32 0.57 0.81 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective function was  11.44
## The degrees of freedom for the model are 186  and the objective function was  1.72
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                    MR1  MR3  MR2  MR4
## Correlation of (regression) scores with factors   0.93 0.87 0.91 0.82
## Multiple R square of scores with factors          0.87 0.76 0.83 0.68
## Minimum correlation of possible factor scores     0.74 0.52 0.65 0.36
```

Problem 3d

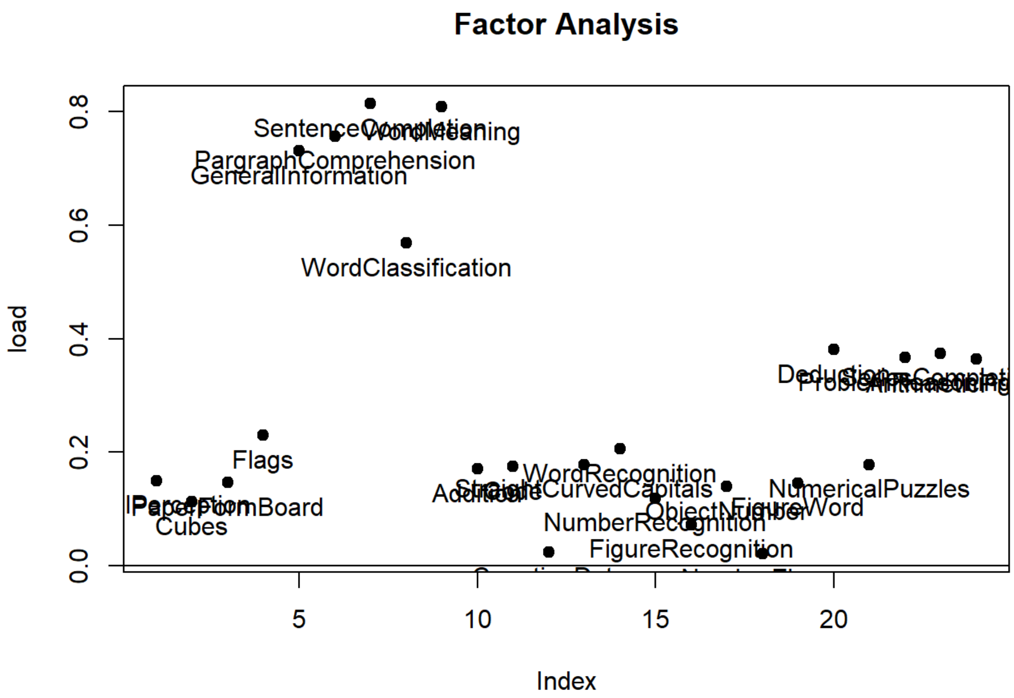
#Let’s see the factor scores

```
fa_orthogonal$weights
```

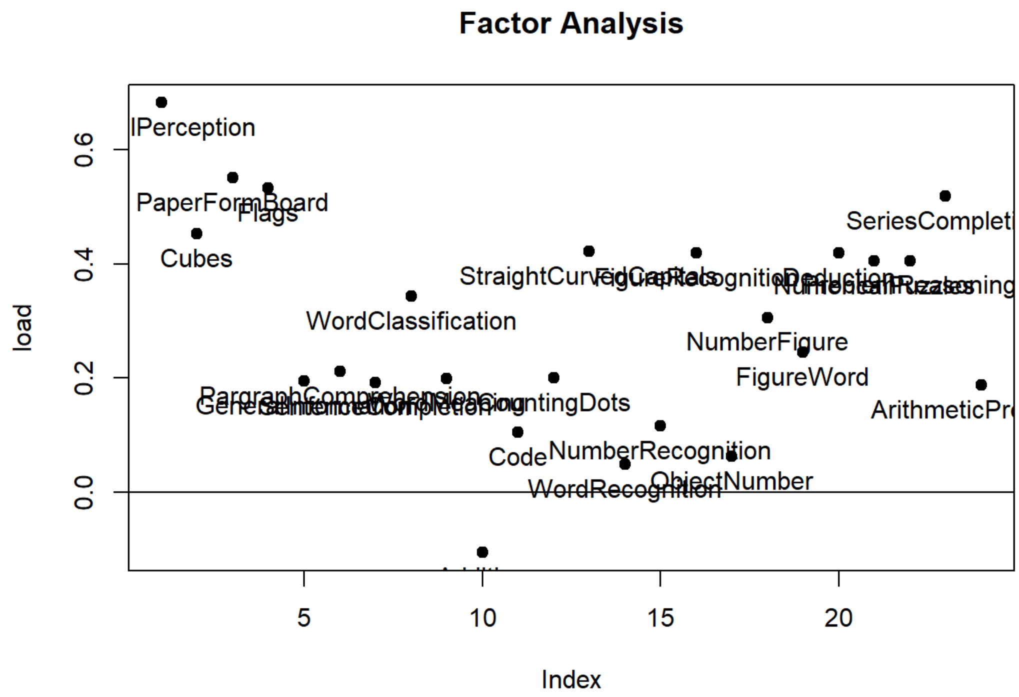
```
##                                  MR1          MR3          MR2          MR4
## VisualPerception       -0.0912528883  0.30864122  0.019076056 -0.059475135
## Cubes                  -0.0273878717  0.12425214 -0.003998151 -0.036046103
## PaperFormBoard         -0.0002664514  0.14435978 -0.021455124 -0.013910017
## Flags                  -0.0158992046  0.16513630 -0.018187085 -0.067188757
## GeneralInformation      0.1791509833 -0.01576399  0.006192041 -0.064803732
## PargraphComprehension   0.2087548894 -0.02967329 -0.089257772  0.053969292
## SentenceCompletion      0.3560671670 -0.07802378  0.009878325 -0.152657827
## WordClassification      0.0744517660  0.07646981  0.004918710 -0.042449858
## WordMeaning             0.3541762245 -0.12345571 -0.083274087  0.047708409
## Addition                0.0370184288 -0.28588788  0.530044962 -0.033006173
## Code                   -0.0152781994 -0.07727463  0.134310508  0.115144252
## CountingDots           -0.0568764212  0.04221931  0.241300880 -0.083059603
## StraightCurvedCapitals -0.0599618479  0.14994774  0.162459388 -0.110602297
## WordRecognition         0.0022332713 -0.07875443 -0.049514025  0.253344226
## NumberRecognition      -0.0305344160 -0.04164026 -0.040528450  0.215786525
## FigureRecognition      -0.0678653395  0.09799311 -0.077886130  0.223301180
## ObjectNumber           -0.0435663449 -0.07170630 -0.015111696  0.278035153
## NumberFigure           -0.0730633557  0.05023790  0.036233555  0.166892502
## FigureWord             -0.0300328554  0.02668857  0.002475198  0.083460720
## Deduction               0.0074291922  0.09386230 -0.040441038  0.055443652
## NumericalPuzzles       -0.0250346901  0.11130425  0.071040209 -0.001990422
## ProblemReasoning        0.0145135443  0.07473804 -0.014585388  0.057911235
## SeriesCompletion        0.0124897301  0.17157853 -0.007344283 -0.017896058
## ArithmeticProblems      0.0062265043  0.00390309  0.085098413  0.047726702
```

#Plotting an orthogonal solution # Problem 3e

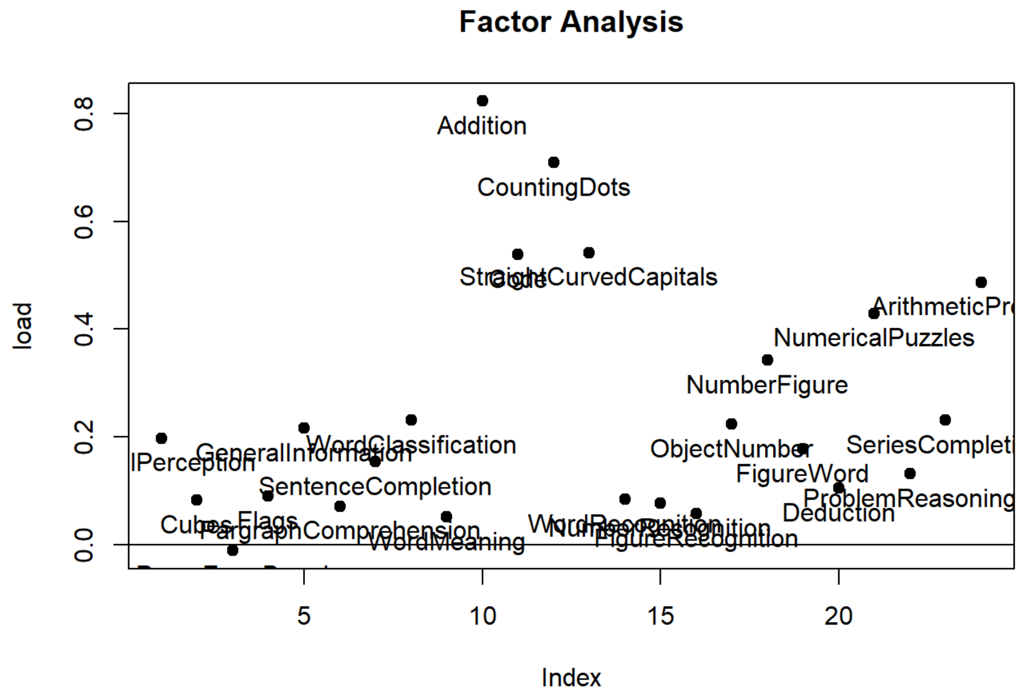
```
factor.plot(fa_orthogonal, choose = c(1), labels = rownames(fa$loadings))
```



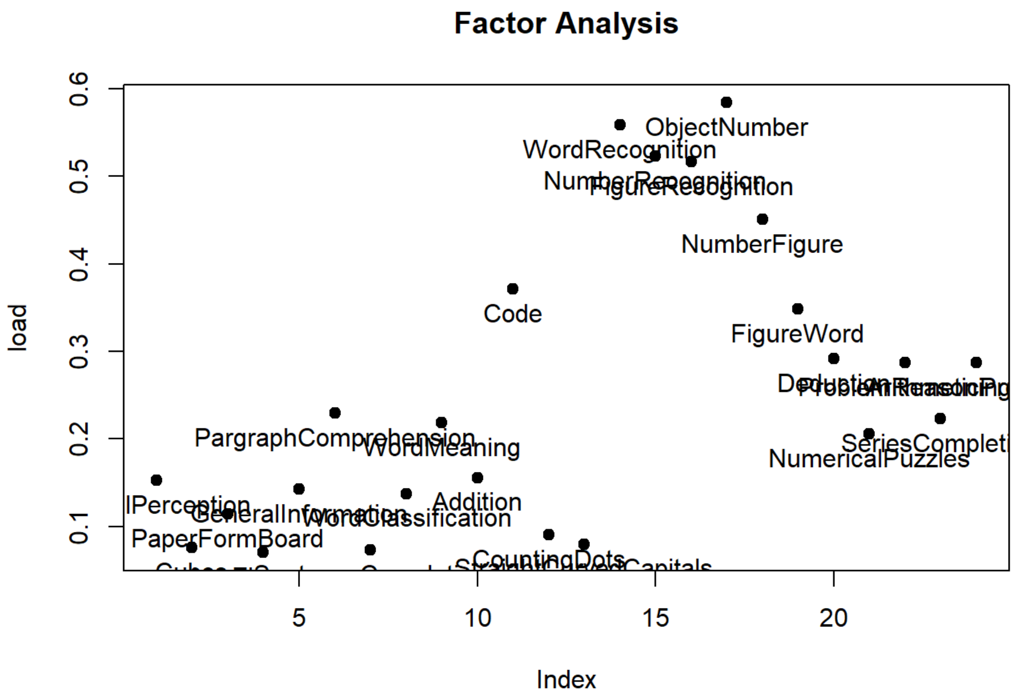

```
factor.plot(fa_orthogonal, choose = c(2), labels = rownames(fa$loadings))
```



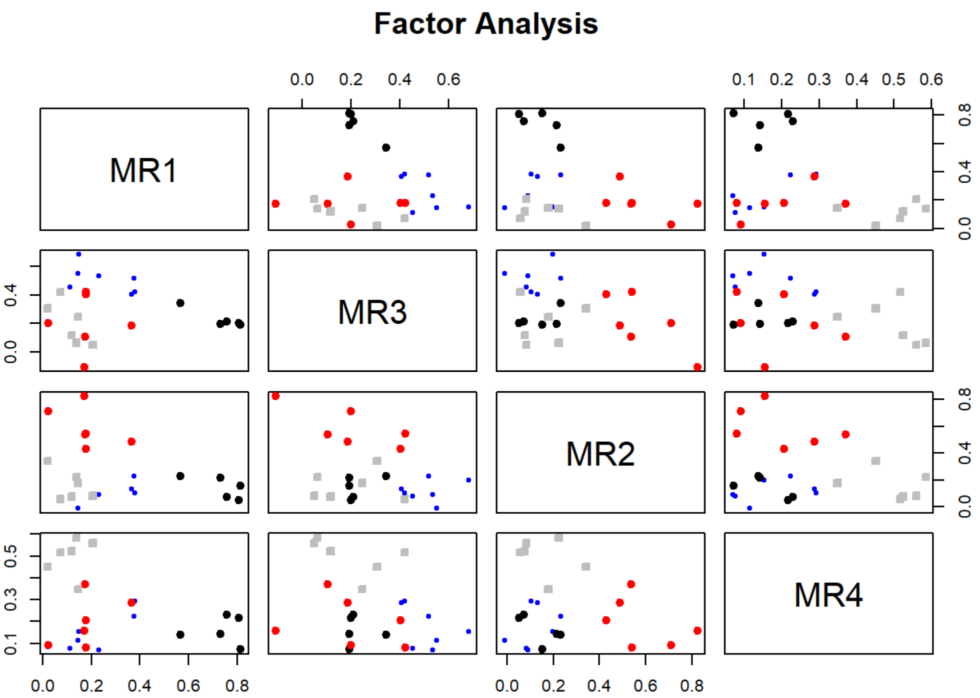

```
factor.plot(fa_orthogonal, choose = c(3), labels = rownames(fa$loadings))
```




```
factor.plot(fa_orthogonal, choose = c(4), labels = rownames(fa$loadings))
```




```
factor.plot(fa_orthogonal)
```

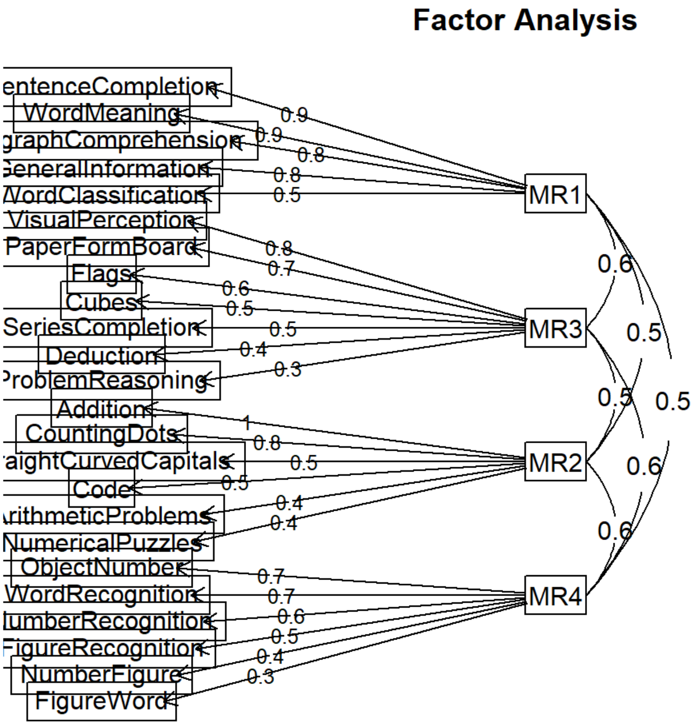


Now let's plot an oblique solution

```
fa_oblique <- fa(Harman74.cor$cov, nfactors = 4, rotate = "promax", scores = TRUE)
```

```
## Loading required namespace: GPArotation
```

```
fa.diagram(fa_oblique)
```




```
rm(list = ls())
```

Problem 3e

#We can make the following observations -

#- Factor 1 indicates that it is related to language related attributes like Sentence Completion, WordMeaning, Word Classification etc.

#- Factor 2 looks to be related to more general problem solving attributes like Deduction, Problem Reasoning, Cubes, Flage etc.

#- Factor 3 shows that it is related to mathematical attributes like Code, Counting Data, Addition, Number Recognition etc.

#- Factor 4 indicates that it is related to Word and Number recognition.

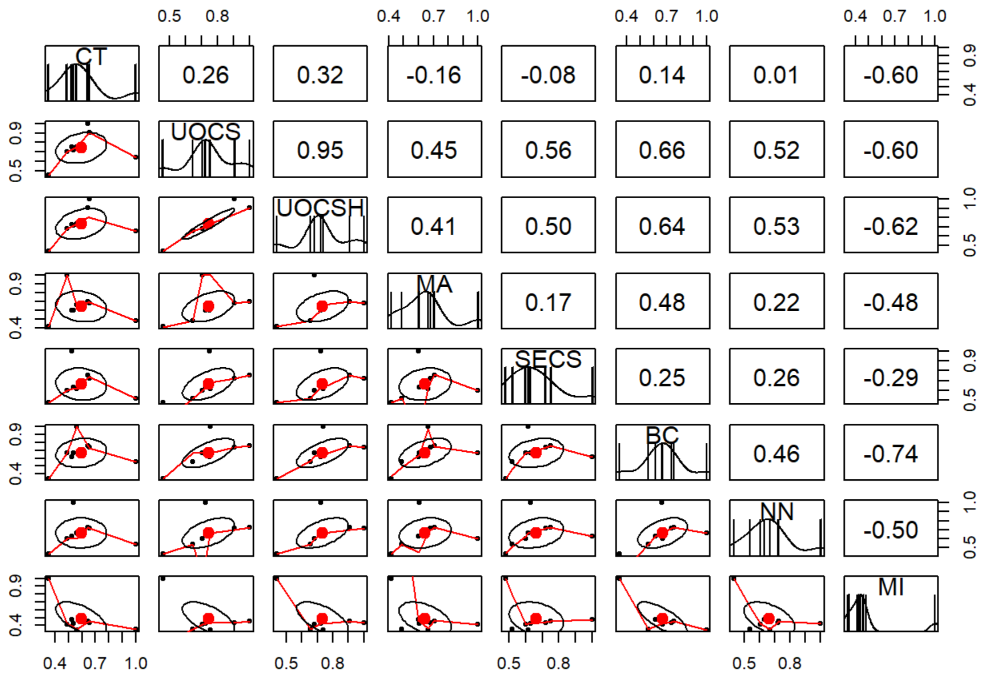
Problem 4a

```
breast_cancer <- read_xlsx("breast-cancer-wisconsin.xlsx",
  sheet = "breast-cancer-wisconsin.csv"
)
breast_cancer <- select(breast_cancer, c(1, 2, 3, 4, 5, 6, 8, 9, 10, 11))
bc.cor <- cor(breast_cancer[, 2:9])
```

Problem 4b

#Visualizing the correlation

```
pairs.panels(bc.cor)
```



#Interpretations from the plot -

- The following variable pairs did not show any coorelation:

1. CT and NN

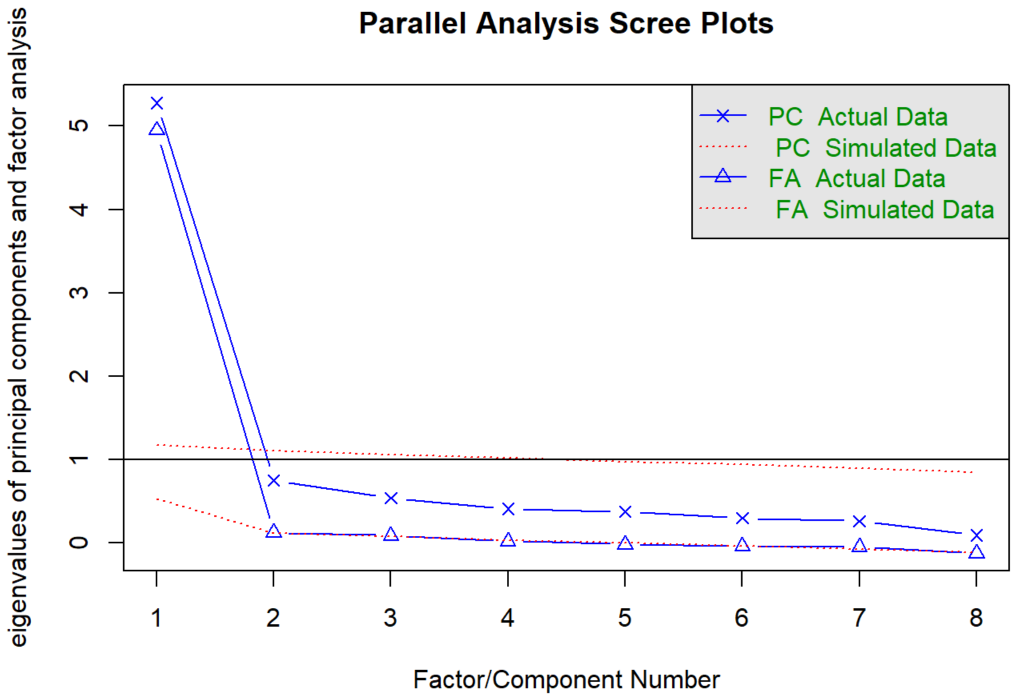
2. CT and MA

3. CT and SECS

4. CT and BC

#Analyzing the scree plot

```
fa.parallel(bc.cor, n.obs = 699)
```




```
## Parallel analysis suggests that the number of factors =  1  and the number of components =  1
```

Problem 4c

From the scree plot the number of factors is 1

```
fa <- fa(bc.cor, nfactors = 1, rotate = "none", n.obs = 699, fm = "pa")
fa
```

```
## Factor Analysis using method =  pa
## Call: fa(r = bc.cor, nfactors = 1, n.obs = 699, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##        PA1   h2   u2 com
## CT    0.68 0.46 0.54   1
## UOCS  0.94 0.89 0.11   1
## UOCSH 0.92 0.85 0.15   1
## MA    0.76 0.58 0.42   1
## SECS  0.79 0.63 0.37   1
## BC    0.81 0.65 0.35   1
## NN    0.79 0.63 0.37   1
## MI    0.51 0.26 0.74   1
##
##                 PA1
## SS loadings    4.95
## Proportion Var 0.62
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  6.07 with Chi Square o
f  4216.08
## The degrees of freedom for the model are 20  and the objective function was  0.19
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.03
##
## The harmonic number of observations is  699 with the empirical chi square  30.89  with prob <  0.057
## The total number of observations was  699  with Likelihood Chi Square =  132.85  with prob <  1.1e-18
##
## Tucker Lewis Index of factoring reliability =  0.962
## RMSEA index =  0.09  and the 90 % confidence intervals are  0.076 0.105
## BIC =  1.86
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1
## Correlation of (regression) scores with factors   0.98
## Multiple R square of scores with factors          0.95
## Minimum correlation of possible factor scores     0.91
```

Problem 4d

Rotate the factors now

```
fa_orthogonal <- fa(bc.cor, nfactors = 1, rotate = "varimax", fm = "pa", scores = TRUE)
fa_orthogonal
```

```
## Factor Analysis using method =  pa
## Call: fa(r = bc.cor, nfactors = 1, rotate = "varimax", scores = TRUE,
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##        PA1   h2   u2 com
## CT    0.68 0.46 0.54   1
## UOCS  0.94 0.89 0.11   1
## UOCSH 0.92 0.85 0.15   1
## MA    0.76 0.58 0.42   1
## SECS  0.79 0.63 0.37   1
## BC    0.81 0.65 0.35   1
## NN    0.79 0.63 0.37   1
## MI    0.51 0.26 0.74   1
##
##                 PA1
## SS loadings    4.95
## Proportion Var 0.62
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  6.07
## The degrees of freedom for the model are 20  and the objective function was  0.19
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.03
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1
## Correlation of (regression) scores with factors   0.98
## Multiple R square of scores with factors          0.95
## Minimum correlation of possible factor scores     0.91
```

Problem 4e

#Displaying the factors :

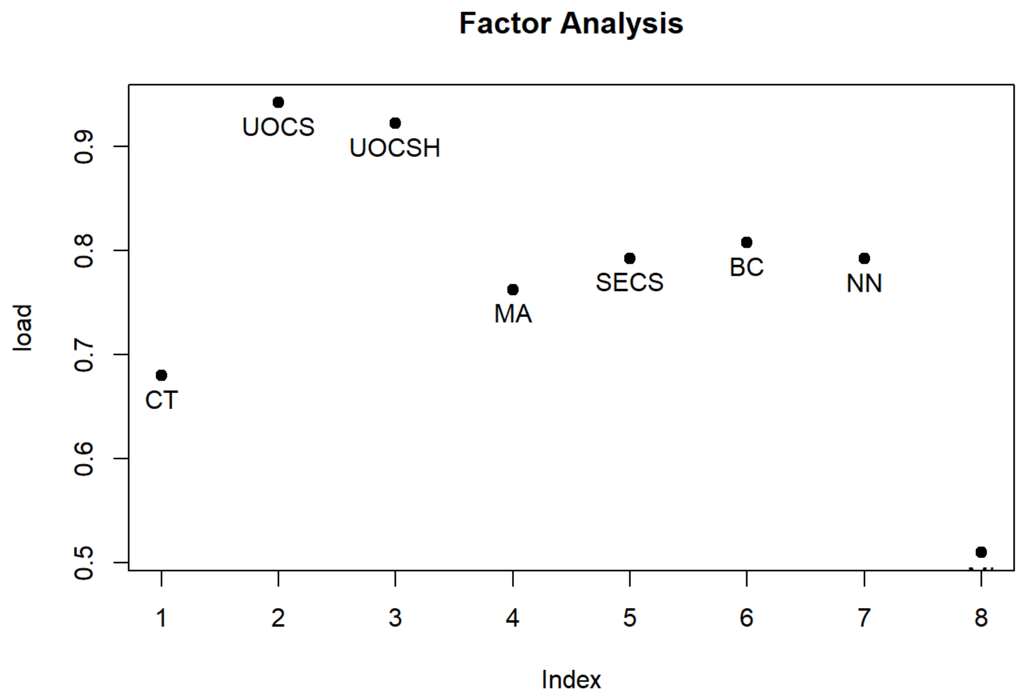
```
fa_orthogonal$weights
```

```
##              PA1
## CT    0.05188612
## UOCS  0.35646040
## UOCSH 0.23549867
## MA    0.09673322
## SECS  0.10413278
## BC    0.11236394
## NN    0.11950954
## MI    0.04460901
```

Problem 4f

Plotting the orthogonal solution

```
factor.plot(fa_orthogonal, labels = colnames(bc.cor))
```

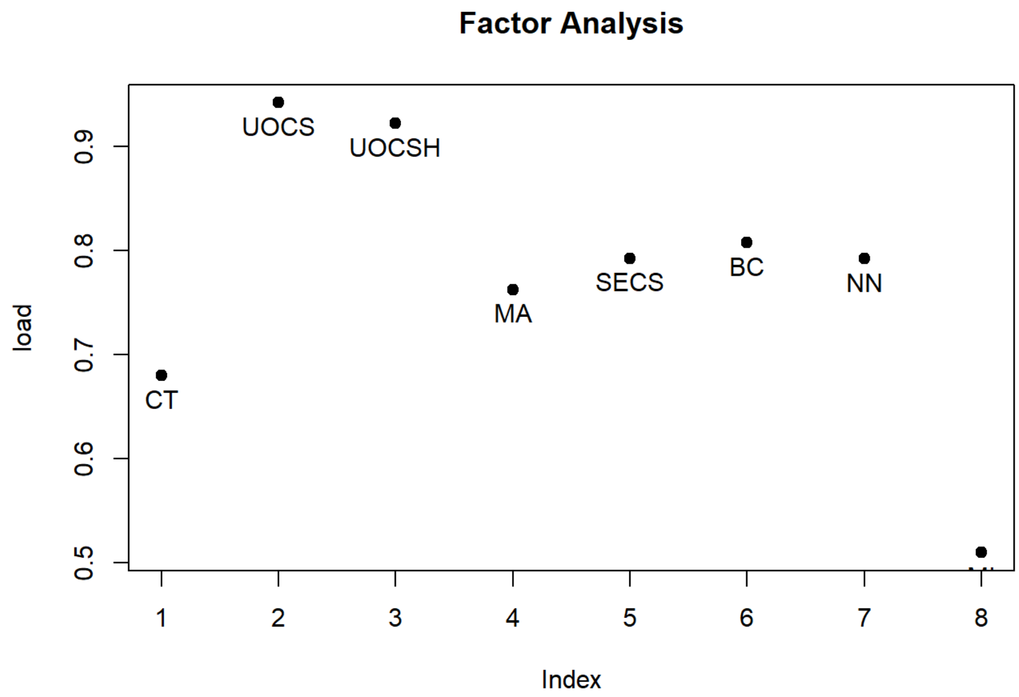


Plotting the oblique solution

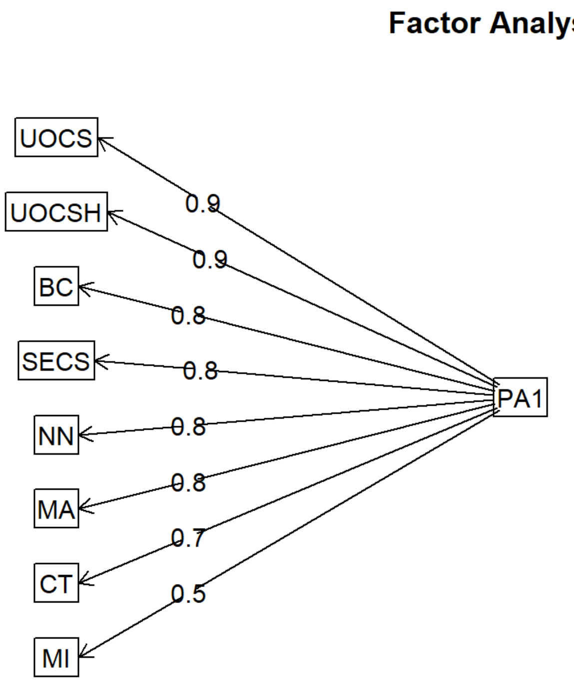
```
fa_oblique <- fa(bc.cor, nfactors = 1, rotate = "promax", fm = "pa", scores = TRUE)
fa_oblique
```

```
## Factor Analysis using method =  pa
## Call: fa(r = bc.cor, nfactors = 1, rotate = "promax", scores = TRUE,
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##        PA1   h2   u2 com
## CT     0.68 0.46 0.54   1
## UOCS   0.94 0.89 0.11   1
## UOCSH  0.92 0.85 0.15   1
## MA     0.76 0.58 0.42   1
## SECS   0.79 0.63 0.37   1
## BC     0.81 0.65 0.35   1
## NN     0.79 0.63 0.37   1
## MI     0.51 0.26 0.74   1
##
##                 PA1
## SS loadings    4.95
## Proportion Var 0.62
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  28  and the objective function was  6.07
## The degrees of freedom for the model are 20  and the objective function was  0.19
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.03
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                   PA1
## Correlation of (regression) scores with factors   0.98
## Multiple R square of scores with factors          0.95
## Minimum correlation of possible factor scores     0.91
```

```
factor.plot(fa_oblique, labels = colnames(bc.cor))
```




```
fa.diagram(fa_oblique)
```




```
rm(list = ls())
```

Interpretations from the plots -

#The high loading scores and the test of hypothesis of the #factor analysis indicate that one factor is sufficient to explain all the variables owing to their #high loading scores, indicating high values for communality. #both the orthoganal and oblique plot reinforce the same.

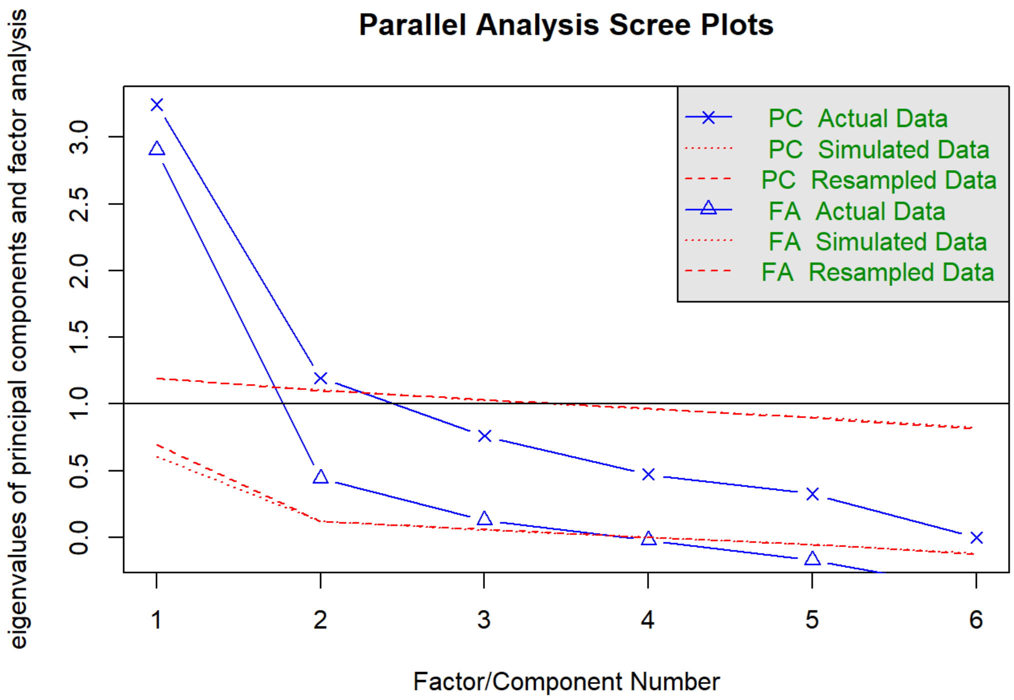
Problem 5a

```
vertebral_column_data <- data.frame(read_xlsx("Vertebral Column Data.xlsx",
  sheet = "column_3C"
))
fa.parallel(vertebral_column_data[, 1:6], n.iter = 100)
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```




```
## Parallel analysis suggests that the number of factors =  3  and the number of components =  2
```

#Scree Plots denotes factor 3

Problem 5b

```
# Calculate the distance matrix
d <- dist(vertebral_column_data[, 1:6])
# Perform Multidimensional Scaling
fit <- cmdscale(d, k = 3, eig = TRUE)
```

#The MDS has reduced the data to 3 dims

```
head(fit$points)
```

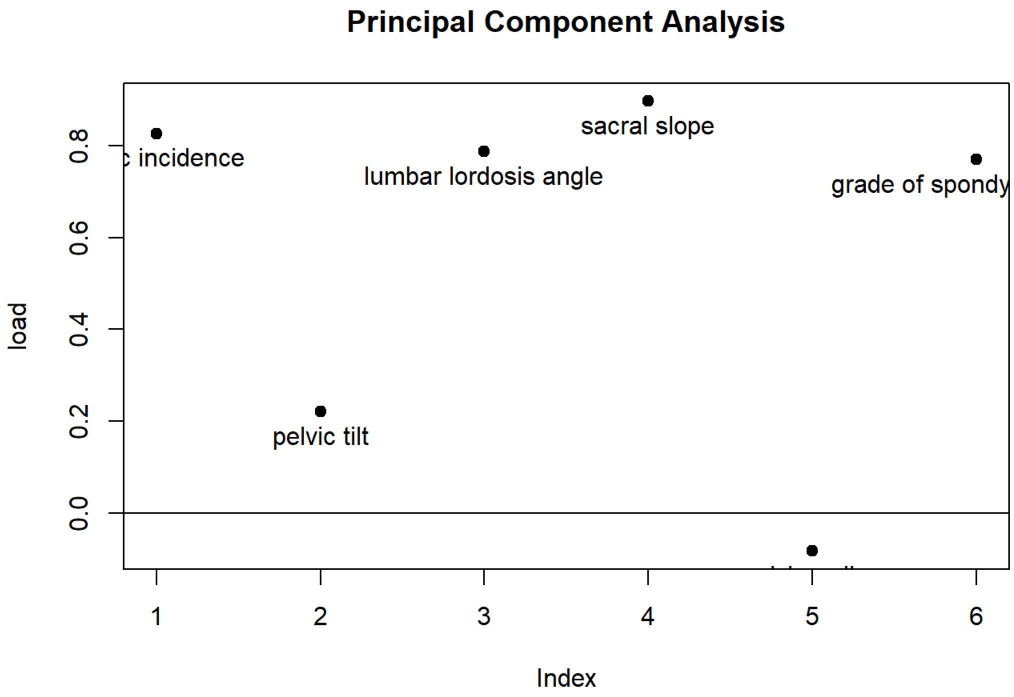
```
##           [,1]       [,2]       [,3]
## [1,] -25.21264  13.204206 -15.891671
## [2,] -37.55028 -18.951621 -11.839171
## [3,] -21.95087  23.063614  -6.318516
## [4,] -10.84709  13.917984 -12.971068
## [5,] -27.73305  -7.589005 -18.435332
## [6,] -39.74800 -22.959841   2.545529
```

#This is the same as calculating 3 factors.

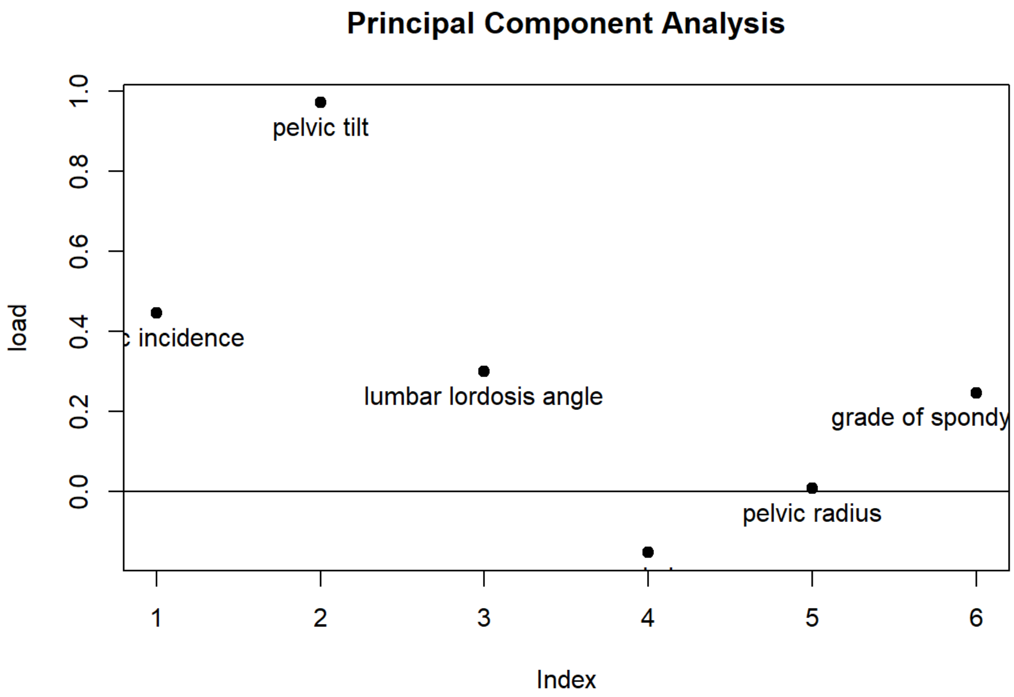
Problem 5c

#Plotting the orthogonal solution

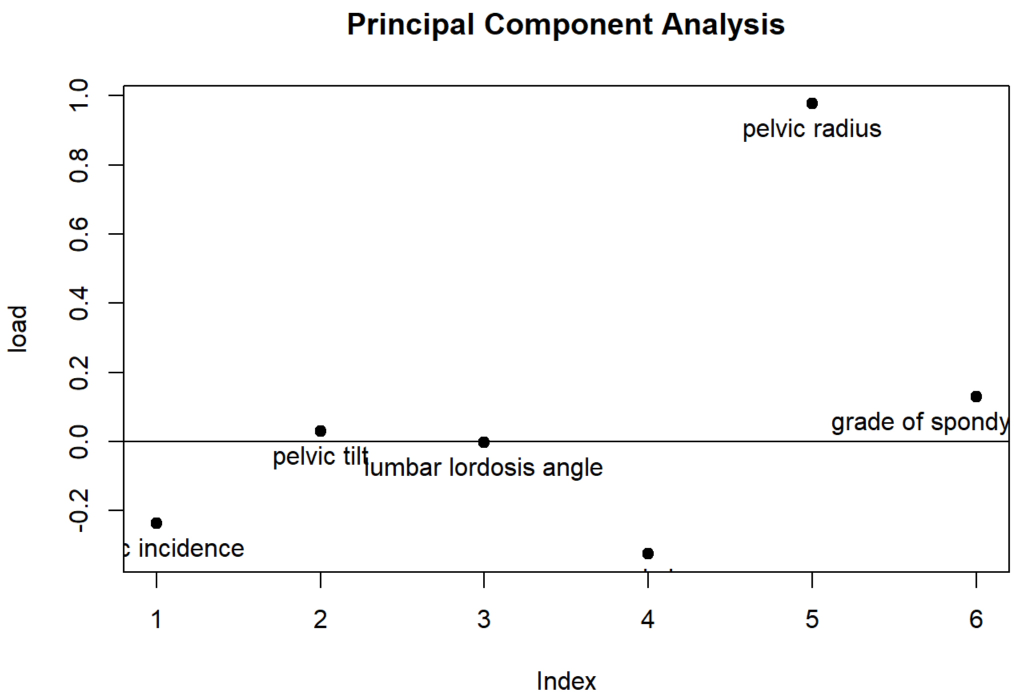
```
pc <- principal(vertebral_column_data[, 1:6], nfactors = 3, rotate = "varimax")
factor.plot(pc, choose = c(1), labels = c(
  "pelvic incidence", "pelvic tilt",
  "lumbar lordosis angle", "sacral slope",
  "pelvic radius", " grade of spondylolisthesis."
))
```



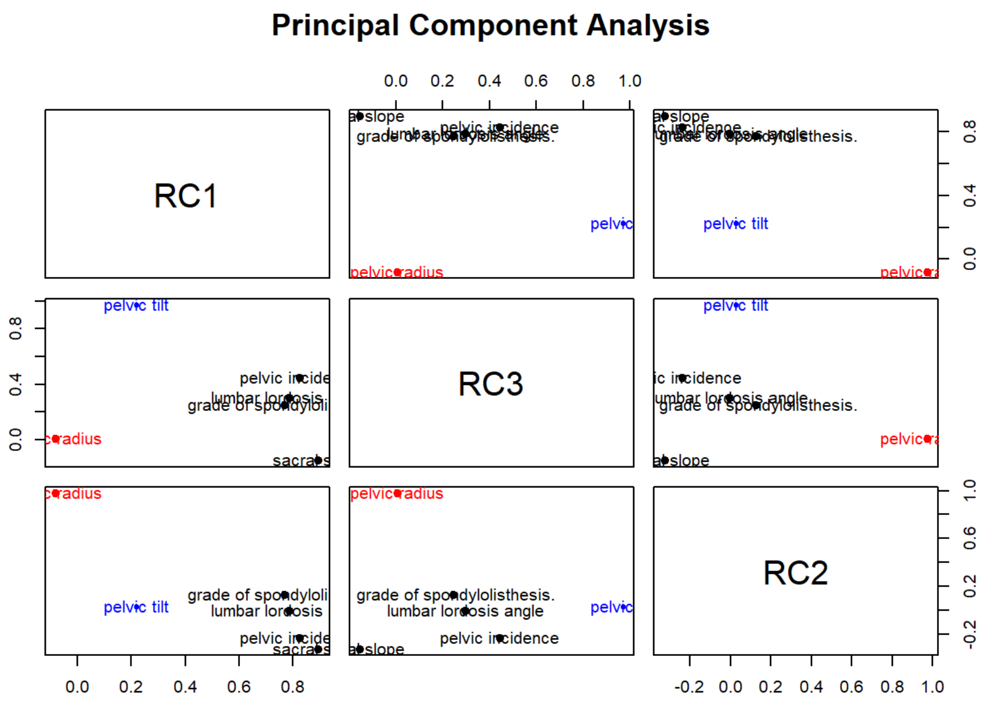

```
factor.plot(pc, choose = c(2), labels = c(
  "pelvic incidence", "pelvic tilt",
  "lumbar lordosis angle", "sacral slope",
  "pelvic radius", " grade of spondylolisthesis."
))
```




```
factor.plot(pc, choose = c(3), labels = c(
  "pelvic incidence", "pelvic tilt",
  "lumbar lordosis angle", "sacral slope",
  "pelvic radius", " grade of spondylolisthesis."
))
```




```
factor.plot(pc, labels = c(
  "pelvic incidence", "pelvic tilt",
  "lumbar lordosis angle", "sacral slope",
  "pelvic radius", " grade of spondylolisthesis."
))
```




```
rm(list = ls())
```

#Interpretations-

#Component 1 loads pelvic incidence, lumbar lordosis angle, sacral slope and grade of spondylolisthesis.

#Component 2 loads pelvic incidence and pelvic tilt.

#Component 3 loads sacral slope and pelvic radius.