Catastrophe theory and pattern formation:
Comparing two approaches for understanding hysteretic transitions

Henry S. Harrison, Till D. Frank
University of Connecticut

Introduction

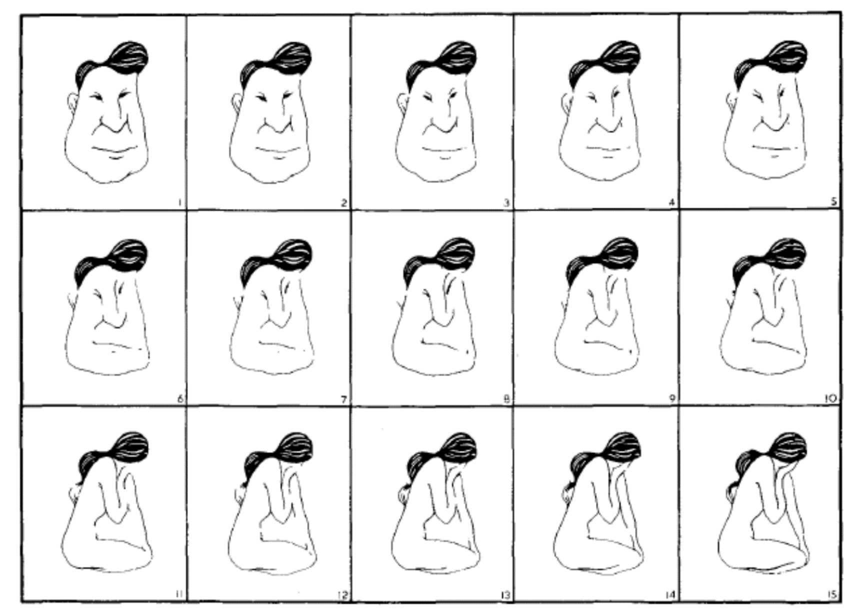
Hysteresis is a form of time-dependence in which a system's state depends on its history.

For example, whether an object is grasped with one or two hands depends on the size of the object. As object size is increased, a participant transitions from one- to two-handed grasping at a different object size than the reverse transition as object size is decreased (Lopresti-Goodman et al., 2011).

Positive hysteresis occurs when the transition is delayed; negative hysteresis occurs when the transition is anticipated.

Two mathematical approaches that predict hysteretic transitions are catastrophe theory (Guastello, 1995) and the pattern-formation model (Lopresti-Goodman et al., 2011). We compared these two models analytically.

In order to illustrate the analytical considerations, we also conducted two pilot experiments, and fit the models to sample participant data.



(Fisher, 1967)

Methods

Two groups of University of Connecticut students (each N = 6) participated in the experiments.

Experiment 1 was adapted from Stewart and Peregoy (1983). Participants viewed a series of images (from Fisher, 1967; shown to the left) of varying ambiguity and judged each along a continuous scale between "looks more like a man's face" and "looks more like a woman's figure".

Each participant experienced three blocks of fifteen trials: ascending order, descending order, and randomized. We counterbalanced the order of the ascending and descending blocks; the random block was always in-between.

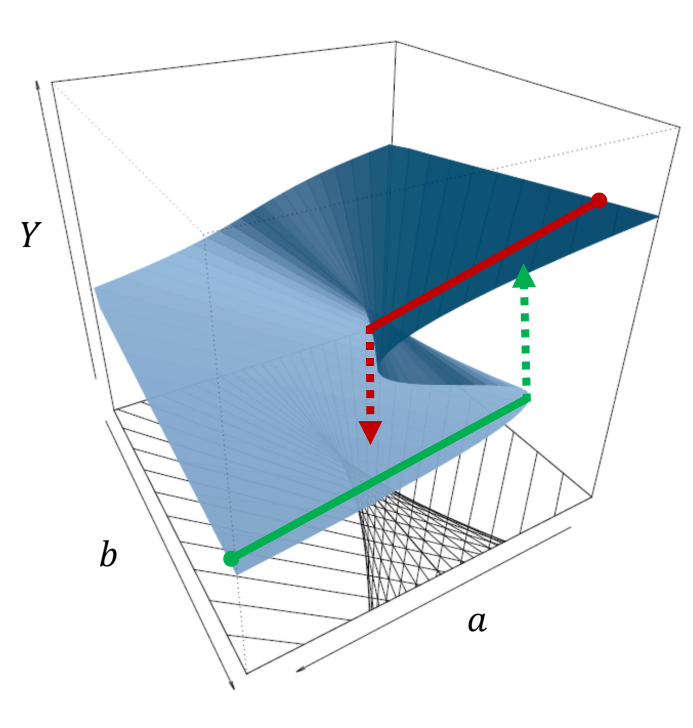
Experiment 2 used a two-choice forced alternative version of the task. We replaced the continuous scale with a categorical scale. Participants were forced to choose either "looks more like a man's face" or "looks more like a woman's figure".

Catastrophe theory

Geometric analysis of degenerate critical points has revealed a small number of standard forms. Catastrophe theory is the study of these forms (Guastello, 1995). The cusp catastrophe is a standard form in two-dimensional parameter space. As a result of its bistability, it describes hysteretic jumps in a continuous state parameter. The cusp catastrophe is modeled with the equation

$$\frac{d}{dt}Y = a + bY - Y^3$$

where Y is the state variable, a is an asymmetry parameter, and b is a bifurcation parameter. Hysteresis is observed as a is varied, at values of b resulting in bistablity. The system is stable on the surface pictured below, with example hysteretic



(Grasman et al., 2009)

Synergetics

The pattern-formation model was developed to understand hysteretic transitions between one- and two-handed grasping (Lopresti-Goodman et al., 2011) from the perspective of synergetics. Coupled pattern amplitudes ζ_1, ζ_2 representing behavioral modes evolve according to

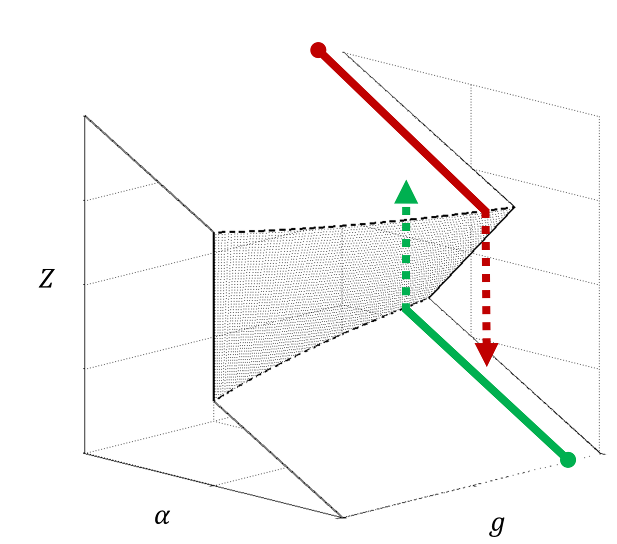
$$\frac{d}{dt} = (1-\alpha)\zeta_1 - g\zeta_1\zeta_2^2 - \zeta_1^3$$
$$\frac{d}{dt} = \alpha\zeta_2 - g\zeta_2\zeta_1^2 - \zeta_2^3$$

where α is the primary control parameter (e.g., it corresponds to object size scaled by hand size in grasping experiments), and the parameter g describes the level of interaction between the pattern amplitudes.

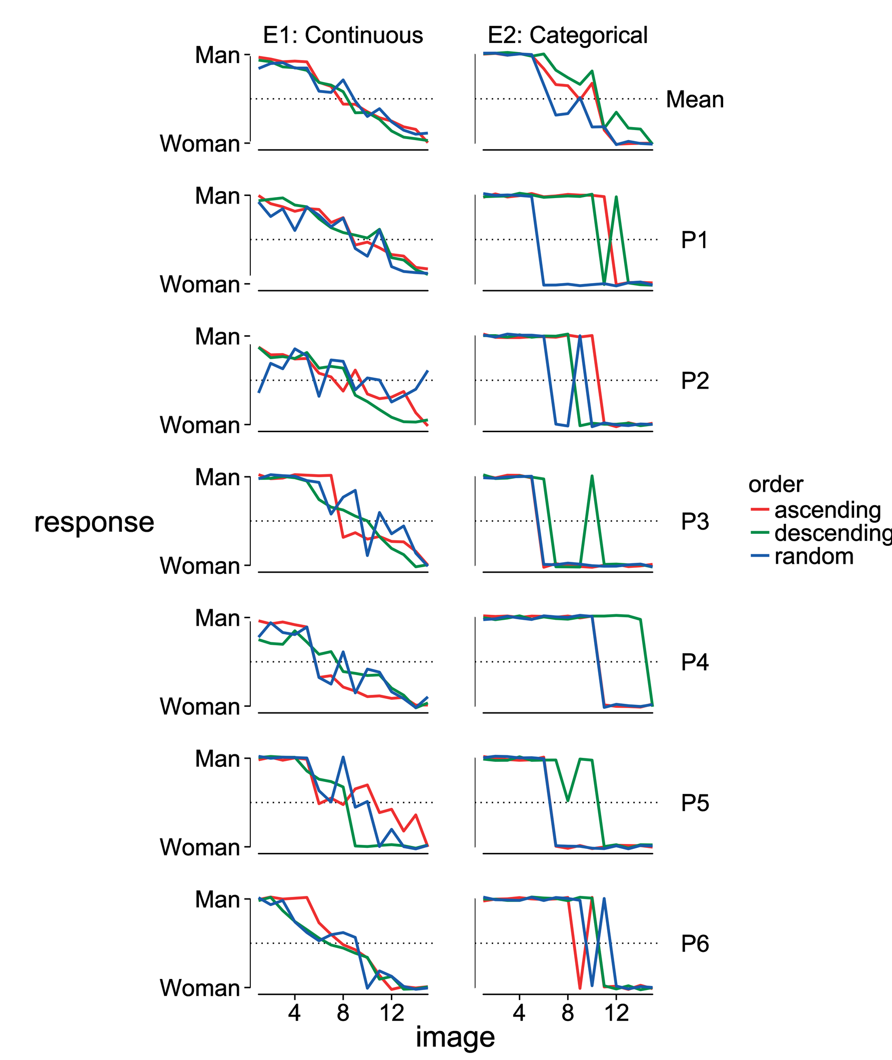
To compare the pattern-formation model with the cusp model, we introduce a new scalar state variable

$$Z = \zeta_1^2 - \zeta_2^2$$

We calculated a stability surface for Z, pictured below.



Results



Hysteresis was determined using the heuristic method of Lopresti-Goodman et al. (2011). A similar pattern of qualitative results was found across the two experiments.

Hysteresis	Exp. 1: Continuous response	Exp. 2: Categorical response
Positive	1	1
Negative	1	1
None	4	4

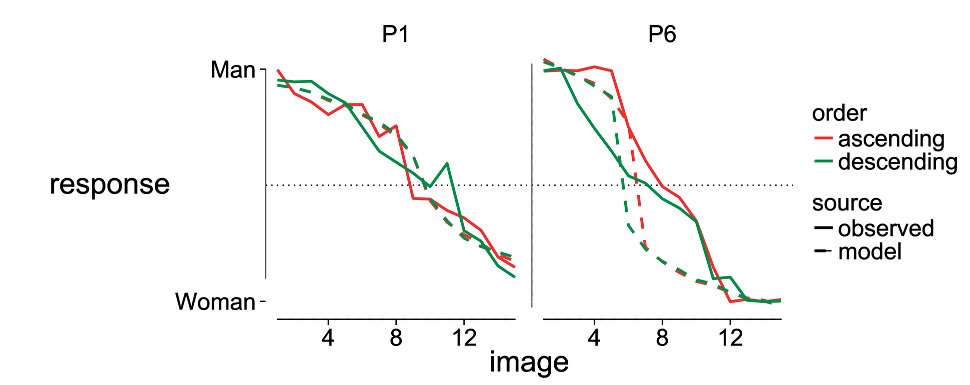
Modeling

To illustrate the two mathematical approaches, we fit the models to the data from selective participants. Both models can exhibit positive hysteresis and no hysteresis. Neither model, as described here, can exhibit negative hysteresis.

Cusp model: Continuous responses (Exp. 1)

To fit the cusp model, we allowed a and b to vary as linear functions of the image number and Y as a linear function of the participant response (see Grasman et al., 2009).

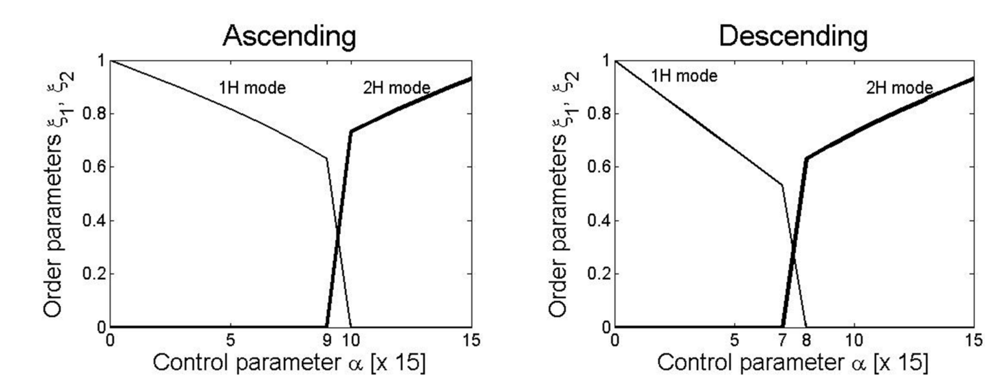
In this way, we fit the model to the data of participant 1, who exhibited no hysteresis, and participant 6, who exhibited positive hysteresis. The results are shown below.



Pattern-formation model: Categorical responses (Exp. 2)

We used the procedure of Lopresti-Goodman et al. (2011) to fit the data from participant #2.

Model parameters were determined to reproduce an ascending transition between images 9 and 10, and a descending transition between images 7 and 8. A simulation with the resulting parameters is shown below.



The pattern-formation model can also capture the qualitative results from the participant exhibiting no hysteresis (not shown).

Further information

Presented July 17, 2015 at the 18th International Conference on Perception & Action, University of Minnesota.

Correspondence:
henry.schafer.harrison@gmail.com

Permalink:
http://hsharrison.github.io/publications/harrison_frank_icpa2015

Discussion

Analytical findings

In terms of a scalar state variable Z, the pattern-formation model exhibits a stability surface similar to the cusp model.

Catastrophe theory describes transitions with a scalar variable, whereas the pattern-formation approach describes transitions using winner-take-all pattern amplitudes. That is, they focus on different levels of description.

Unlike the cusp model, the pattern-formation model predicts jump-like transitions even when no hysteresis is present.

Empirical findings

The pattern-formation model can capture data from participants showing perceptual switches between two categorical responses with positive hysteresis.

The cusp model can capture data from participants showing changes in continuous perceptual judgments with positive hysteresis.

Negative hysteresis was observed experimentally. Neither model, as described here, can explain these results. The pattern-formation model has been extended with a parameter-dynamics level to address negative hysteresis (Lopresti-Goodman et al., 2013). Likewise for the cusp model (Tuller et al., 1994).

Both of these extensions involve an additional parameter that varies during the course of the experiment. This additional variable may correspond to an internal dynamic such as perceptual adaptation.

Negative hysteresis may be related to presentation speed. Faster presentation speed is associated with negative hysteresis (Lopresti-Goodman et al., 2011).

Future studies should examine to what extent continuous response data can be fitted to the Z response variable of the pattern-formation model.

References

Fisher, G. H. (1967). Preparation of ambiguous stimulus materials. *Perception & Psychophysics, 2*(9), 421-422.

Grasman, R. P., van der Maas, H. L., & Wagenmakers, E. (2009). Fitting the cusp catastrophe in R: A cusp package primer. *Journal of Statistical Software, 32*(8), 1-27.

Guastello, S. J. (1995). *Chaos, catastrophe, and human affairs.* Mahwah, NJ: Lawrence Erlbaum.

Lopresti-Goodman, S. M., Turvey, M. T., & Frank, T. D. (2011). Behavioral dynamics of the affordance "graspable". *Attention, Perception, and Psychophysics, 73*, 1948-1965.

Lopresti-Goodman, S. M., Turvey, M. T., & Frank, T. D. (2013). Negative hysteresis in the behavioral dynamics of the affordance "graspable". *Attention, Perception, and Psychophysics, 75*, 1075-1091.

Stewart, I. N., & Peregoy, P. L. (1983). Catastrophe theory modeling in psychology. *Psychological Bulletin, 94*(2), 336-362.

Tuller, B., Case, P., Ding, M., & Kelso, J. A. (1994). The nonlinear dynamics of speech categorization. *Journal of Experimental Psychology: Human Perception and Performance, 20*, 3–16.