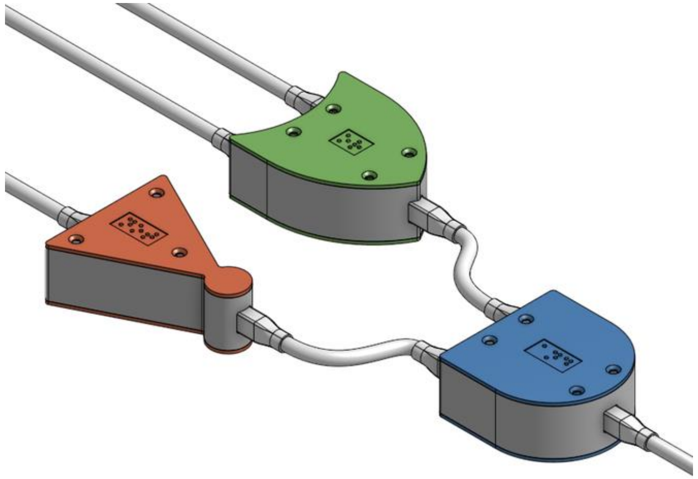

CS 211 - Digital Logic Design
211 عال - تصميم المنطق الرقمي

First Term - 1439/1440

Lecture #3

Dr. Hazem Ibrahim Shehata

Assistant Professor

College of Computing and Information Technology

Administrivia

- Course website:
  - http://hshehata.github.io/courses/su/cs211/

Complements of Binary Numbers

- **1's complement:**
  - Definition: 1's complement of $xxx...xxx_2 = 111...111_2 - xxx...xxx_2$.
    $= (2^n - 1)_{10} - xxx...xxx_2$.
  - Meaning: Value that complements $xxx...xxx_2$ to $111...111_2$.
  - Method: Flip each bit in the binary number, i.e., (1 → 0, 0 → 1).
  - Example: What is the 1's complement of 11010001_2?
  - Solution:

1	1	0	1	0	0	0	1
↓	↓	↓	↓	↓	↓	↓	↓
0	0	1	0	1	1	1	0

Complements of Binary Numbers

- 2's complement:
  - Definition: 2's complement of $xxx...xxx_2 = 1000...000_2 - xxx...xxx_2$.
    $= (2^n)_{10} - xxx...xxx_2$
    $=$ 1's complement + 1.
  - Meaning: Value that complements $xxx...xxx_2$ to $1000...000_2$.

Complements of Binary Numbers

➢ 2's complement (Cont.):

- ...
- Method #1: (1) Find 1's complement, and then add 1 to it!
- Example: What is the 2's complement of 10011000_2?
- Solution:

$$
\begin{array}{cccccccc}
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\
 & & & & & & & 1 \\
\hline
0 & 1 & 1 & 0 & 1 & 0 & 0 & 0
\end{array}
\; +
$$

Complements of Binary Numbers

- **2's complement (Cont.):**
  - ...
  - Method #2: Flip all bits to the left of the least significant 1.
  - Example: What is the 2's complement of 10011000_2?
  - Solution:

$$1\ 0\ 0\ 1\ \boxed{1}\ 0\ 0\ 0$$

$$0\ 1\ 1\ 0\ 1\ 0\ 0\ 0$$

Signed Numbers

- Signed numbers refer to: positive and negative integers.
- Represented in digital systems (using bits) in 1 of 3 forms:
  - Sign-magnitude, 1's complement, or 2's complement.
  - Most significant (i.e., left-most) bit represents the sign → sign bit.
    - Sign bit = 0 → positive number
    - Sign bit = 1 → negative number

Signed Number Representation

	Sign-magnitude	1's Complement	2's Complement
Positive Numbers $+xxx...xxx_2$	0 for sign bit followed by magnitude $0xxx...xxx_2$		
Negative Numbers $-xxx...xxx_2$	1 for sign bit followed by magnitude $1xxx...xxx_2$	1's complement of corresponding +ve number 1's of $0xxx...xxx_2$	2's complement of corresponding +ve number 2's of $0xxx...xxx_2$
Range n bits	$-(2^{n-1}-1) \rightarrow 2^{n-1}-1$	$-(2^{n-1}-1) \rightarrow 2^{n-1}-1$	$-(2^{n-1}-1) \rightarrow 2^{n-1}$

Conversion: Decimal ➔ Signed (Ex.)

- **Example**: Represent $+12_{10}$ and -12_{10} using 6 bits in the sign-magnitude, 1's complement, 2's complement forms.
- **Solution**:
  - In binary 12_{10} ➔ 001100

	S-M	1's	2's
+12	001100	001100	001100
-12	101100	110011	110100

Conversion: Signed ➔ Decimal (Ex.)

- **Example**: Which decimal number is represented by the signed value 101110_2?
- **Solution**:
  - Sign bit = 1 ➔ decimal number is negative!
    - S-M Form ➔ Number = - 01110_2 = -14_{10}
    - 1's Comp. Form ➔ Number = - (1's of 101110_2) = - 010001 = -17_{10}
    - 2's Comp. Form➔ Number = - (2's of 101110_2) = - 010010 = -18_{10}

Representations of 4-Bit Signed Integers

Range of integers that can be represented by 4 bits:

- S-M → -7 : +7
  → $-(2^{4-1}-1) : 2^{4-1}-1$
- 1's → -7 : +7
  → $-(2^{4-1}-1) : 2^{4-1}-1$
- 2's → -8 : +7
  → $-2^{4-1} : 2^{4-1}-1$

Decimal	S-M	1's	2's
+8	-	-	-
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	1000	1111	-
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001
-8	-	-	1000

Signed Addition (2’s Complement)

- Addition of 2’complement signed numbers is similar to addition in unsigned binary with two exceptions:
  1. Final carry bit must be discarded!
  2. Overflow can happen ➔ Result is incorrect, when:
     a. Added numbers are positive yet result is negative!!
     b. Added numbers are negative yet result is positive!!

Signed Addition (2’s Complement) (Ex.)

- **Example:**

00000111	7	00001111	15	01111101	125
+ 00000100	+ 4	+ 11111010	+ -6	+ 00111010	+ 58
00001011	11	~~1~~ 00001011	9	10110111	183
		Discard!		Overflow!	
00010000	16	11111011	-5	10110000	-80
+ 11101000	+ -24	+ 11110111	+ -9	+ 11001001	+ -55
11111000	-8	~~1~~ 11110010	-14	~~1~~ 01111001	-135
		Discard!		Overflow!	

Signed Subtraction (2's Complement)

- Transformed into addition by the following rule:
  - x – y = x + (-y) = x + 2's complement of y
- Example:

	11100111		-25
−	11110100	−	-12
	11100111		-25
+	00001100	+	12
	11110011		-13

Signed Multiplication (2's Complement)

- Performed in three steps:
  - Transform multiplicand and multiplier into positive numbers.
  - Perform unsigned multiplication.
  - Adjust the sign of product if needed!
    - Leave product positive if multiplicand and multiplier signs are similar.
    - Negate product (taking its 2's complement) if multiplicand and multiplier signs are different.

Signed Division (2's Complement)

- Performed in three steps:
  - Transform dividend and divisor into positive numbers.
  - Perform unsigned division.
  - Adjust the sign of quotient and remainder if needed!
    - Leave quotient positive if dividend and divisor signs are similar.
    - Negate quotient (taking its 2's complement) if dividend and divisor signs are different.
    - Leave remainder positive if dividend is positive.
    - Negate remainder (taking its 2's complement) if dividend is negative.

Reading Material

- Floyd, Chapter 2:
  - Pages 58 - 63
  - Pages 66 - 72