CSE 411: Artificial Intelligence (Elective Course #6)

400 Level, Mechatronics Engineering
2nd Term 2016/2017, Lecture #8

Hazem Shehata

Dept. of Computer & Systems Engineering
Zagazig University

April 24^{th}, 2017

Credits to Dr. Mohamed El Abd for the slides

Adminstrivia

Notes

- Midterm:
  - Date: Wednesday, Apr. 26, 2017.
  - Time: 1:00pm - 2:00pm.
  - Scope: lectures $1 \rightarrow 6$.

Course Info:

- Website: http://hshehata.github.io/courses/zu/cse411/
- Office hours: Sunday 11:30am - 12:30pm

Outline

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1. **Logical Agents**
   - Knowledge-based agents
   - Wumpus world
   - Logic
   - Propositional logic

2. **Requirements & Reading Material**

Knowledge-based agents

Introduction

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- They are specially useful since they:
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- They are specially useful since they:
  - Infer hidden information, needed in partially-observable environments.
  - Flexible:
    - Can learn new knowledge about the environment.
    - Adapt to environmental changes by updating relevant knowledge.

Knowledge-based agents

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  - Deducing new facts or ***reasoning*** about the possible actions.
- ***Reasoning*** is also known as ***inferencing***.

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- Sentences are expressed using a ***knowledge representation language***.

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- When the agent ***ASKs*** a question, the answer should follow from what the KB has been ***TOLD*** (or rather ***TELLed***) before.
- Inference might be used in both ***ASK*** and ***TELL*** operations.

Knowledge-based agents

Algorithm

An algorithm for a generic knowledge-based agent:

funcion KB-AGENT (*percept*) **returns** an *action*
  persistent: *KB*, a knowledge base
    t, a counter, initially 0, indicating time

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TELLs the KB what it senses

ASKs the KB about which actions to take

TELLs the KB the action taken

Knowledge-based agents

Knowledge base

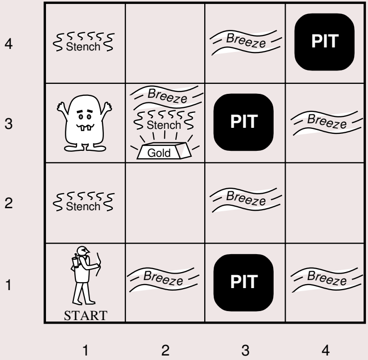
In general, an agent must be able to:

- Represent states, actions, *etc*.
- Incorporate new percepts.
- Update internal representations of the world.
- Deduce hidden properties of the world.
- Deduce appropriate actions.

Wumpus world

Example: wumpus world - PEAS

Performance measure:



Wumpus world

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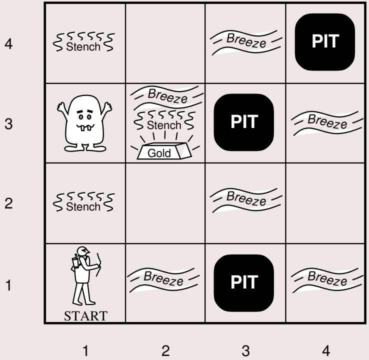
Performance measure:

- gold +1000.
- death -1000
- -1 per step.
- -10 for using the arrow.



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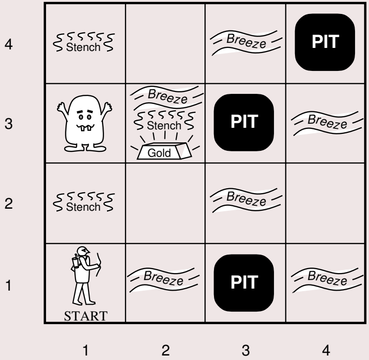
Environment:



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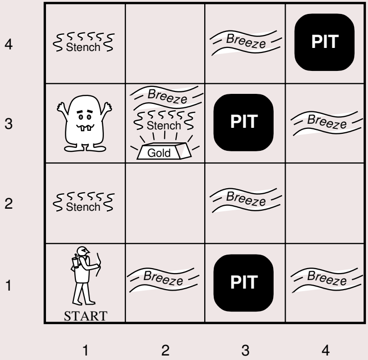
- Squares adjacent to wumpus are smelly.



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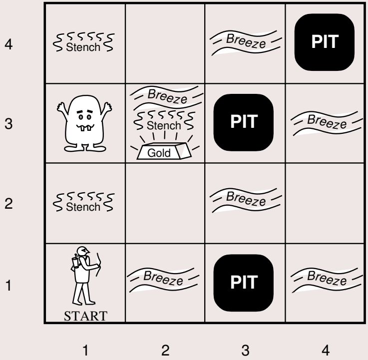
Environment:

- Squares adjacent to wumpus are smelly.
- Squares adjacent to pit are breezy.



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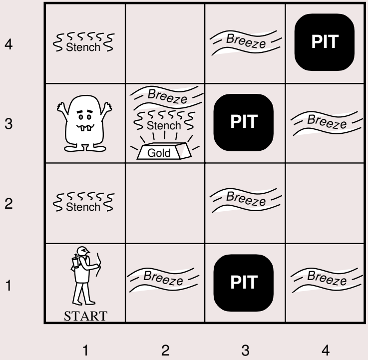
- Squares adjacent to wumpus are smelly.
- Squares adjacent to pit are breezy.
- Glitter iff gold is in the same square.



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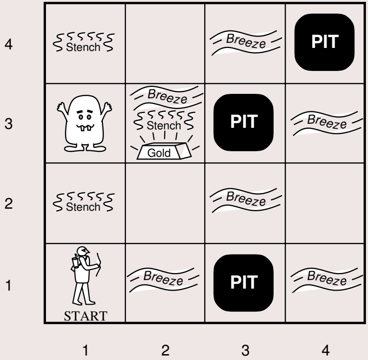
Environment:

- Squares adjacent to wumpus are smelly.
- Squares adjacent to pit are breezy.
- Glitter iff gold is in the same square.
- Shooting kills wumpus if you are facing it.



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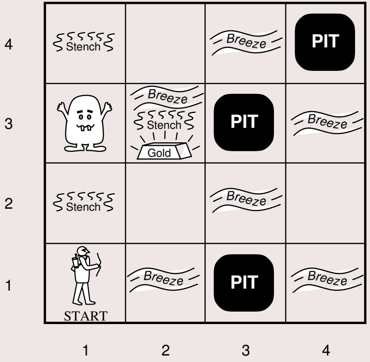
Environment (Cont.):



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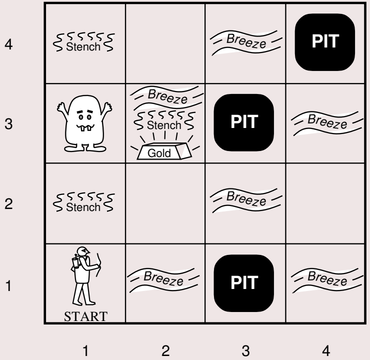
Environment (Cont.):

- Shooting uses up the only arrow.



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Environment (Cont.):

- Shooting uses up the only arrow.
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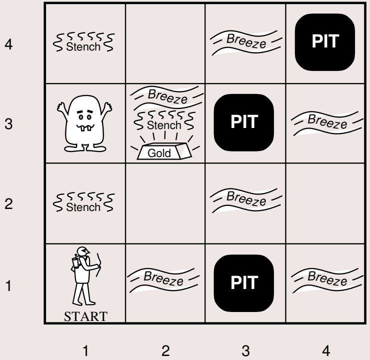

Wumpus world

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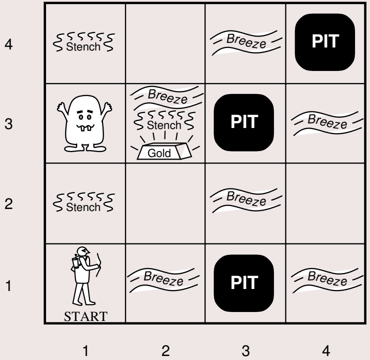
Environment (Cont.):

- Shooting uses up the only arrow.
- Grabbing picks up gold if in same square.
- Game ends:
  - Entering square w/t live wumpus or pit.
  - Climbing out of square (1,1).



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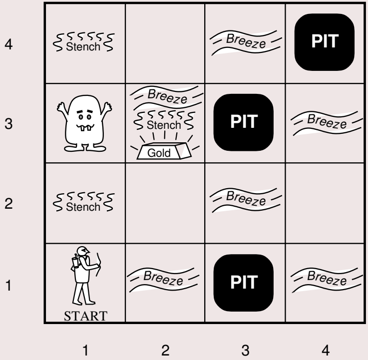
Sensors:

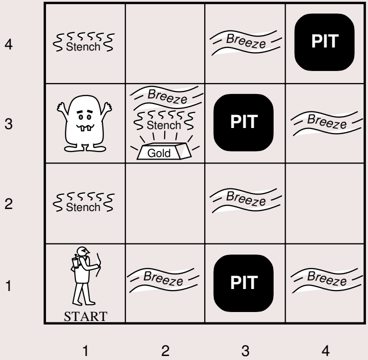


Wumpus world

Example: wumpus world - PEAS

Sensors:

- Stench.
- Breeze.
- Glitter.
- Bump.
- Scream.



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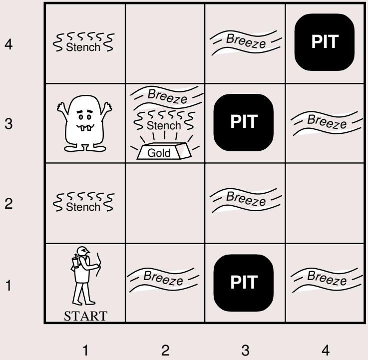
Actuators:



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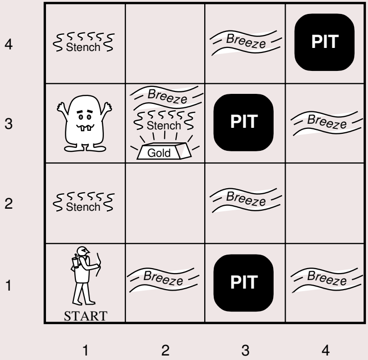
Example: wumpus world - PEAS

Actuators:

- Left turn.
- Right turn.
- Forward.
- Grab.
- Shoot.
- Climb.



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Example: wumpus world - properties of environment

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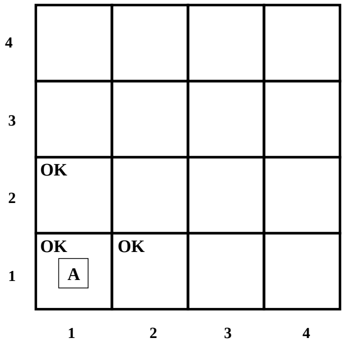
Example: wumpus world - agent in action

A sequence of actions leading to gold:



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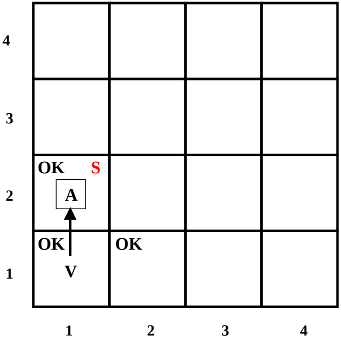

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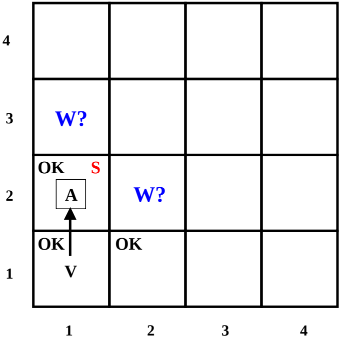

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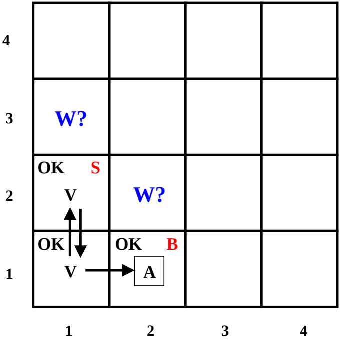

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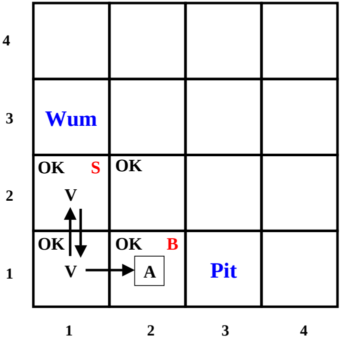

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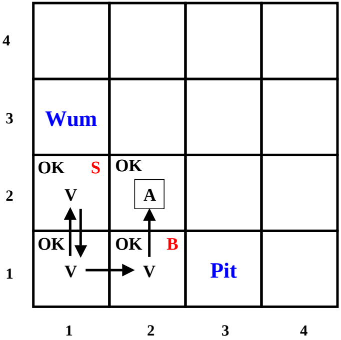

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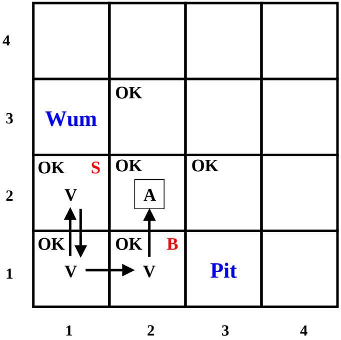

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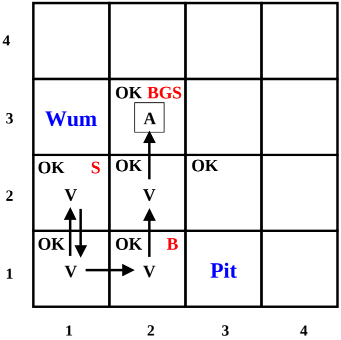

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Example: wumpus world - KB agent

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- ***Logical reasoning*** is required to overcome this ignorance.

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  - Relying on the lack of a percept.

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- The sentences are expressed using a certain **syntax** specifying all well formed sentences.
- For example, in a mathematical syntax:
  - "X + Y = 2" is a well formed sentence.
  - "X 2 Y + =" is not.

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- For example, ”X + Y = 4” is:
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- A ***possible world*** is referred to as a ***model***.

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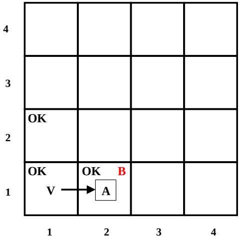
- This means that in **every model** where α is **TRUE**, β is also **TRUE**.
- If an inference algorithm i can derive β from α:
  $\alpha \vdash_i \beta$
  which is read as: "sentence β is **derived from** sentence α by i".

Logic

Example: wumpus world - logical reasoning

So, consider the shown case, after one step, the KB consists of:



Logic

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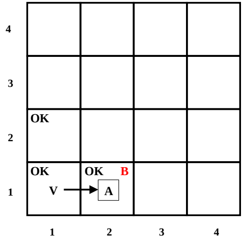

Logic

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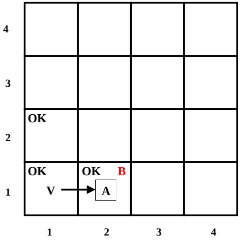
- ***FACT:*** nothing is detected in [1,1].



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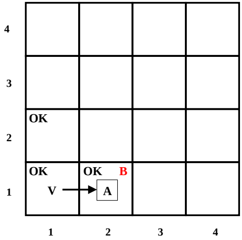
- ***RULES:*** The set of rules governing the world.
- ***FACT:*** nothing is detected in [1,1].
- ***FACT:*** a breeze is detected in [2,1].



Logic

Example: wumpus world - logical reasoning

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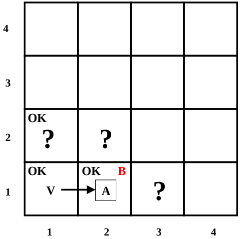

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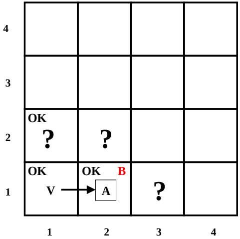
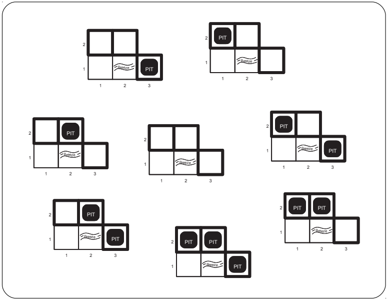
- Let's say the agent wants to know if any of the squares [1,2], [2,2] or [3,1] contain a pit.
- We have $2^3 = 8$ possible models.



Logic

Example: wumpus world - logical reasoning

Logical reasoning by model checking:



We have 8 different possible worlds, (8 different models)

Logic

Example: wumpus world - logical reasoning

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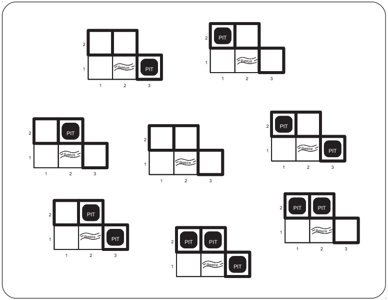

We have 8 different possible worlds, (8 different models)

KB is not TRUE in all of the possible models

Logic

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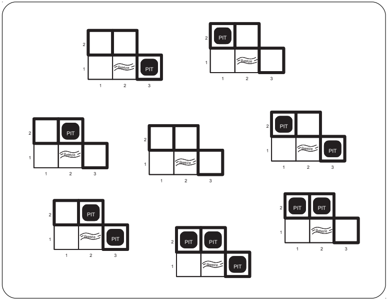
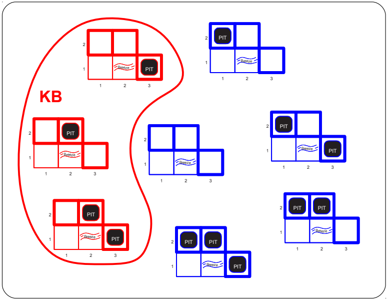


We have 8 different possible worlds, (8 different models)
KB is not TRUE in all of the possible models
For example, there cannot be a pit in [1,2]

Example: wumpus world - logical reasoning

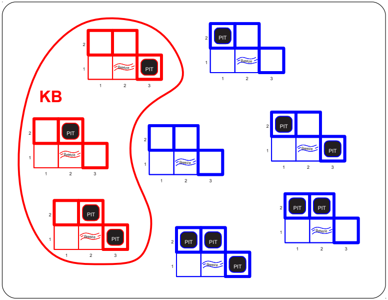
Logical reasoning by model checking:



In fact, KB is only TRUE in three of them

Example: wumpus world - logical reasoning

Logical reasoning by model checking:



Now, let's consider:

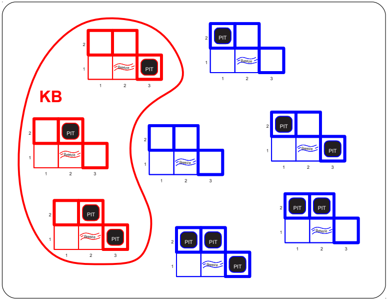
β_1 = "There's no pit in [1,2]"

β_2 = "There's no pit in [2,2]"

Logic

Example: wumpus world - logical reasoning

Logical reasoning by model checking:

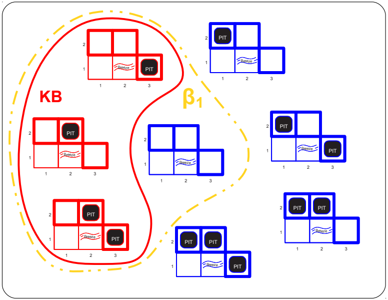


β_1 = "There's no pit in [1,2]"
β_2 = "There's no pit in [2,2]"
Can we prove that: $KB \models \beta_1$ and $KB \models \beta_2$?

Logic

Example: wumpus world - logical reasoning

Logical reasoning by model checking:

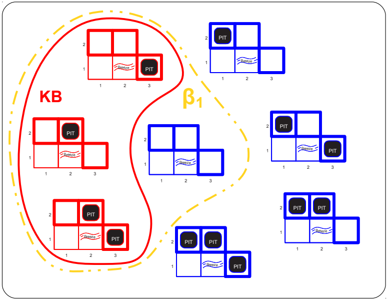


Shown are models in which β_1 is TRUE

Logic

Example: wumpus world - logical reasoning

Logical reasoning by model checking:



Shown are models in which β_1 is TRUE

In all models in which KB is TRUE, β_1 is also TRUE

Logic

Example: wumpus world - logical reasoning

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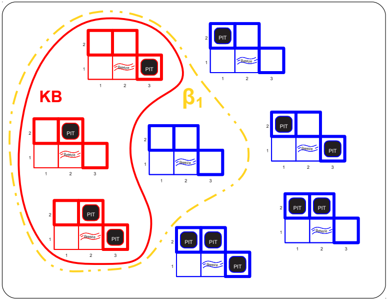


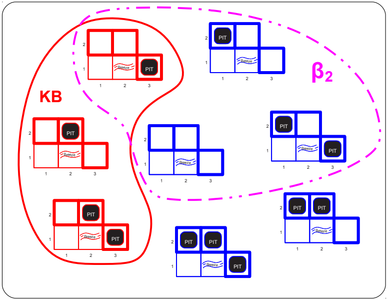
Shown are models in which β_1 is TRUE

In all models in which KB is TRUE, β_1 is also TRUE

Hence, $KB \models \beta_1$, no pit in [1,2]

Example: wumpus world - logical reasoning

Logical reasoning by model checking:

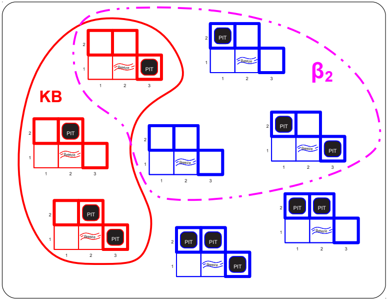


Shown are models in which β_2 is TRUE

Logic

Example: wumpus world - logical reasoning

Logical reasoning by model checking:



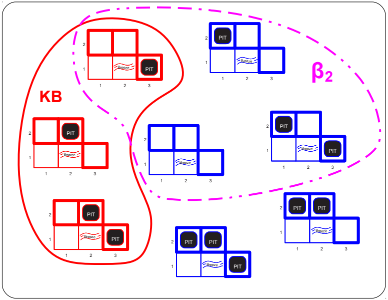
Shown are models in which β_2 is TRUE

In some models in which KB is TRUE, β_2 is FALSE

Logic

Example: wumpus world - logical reasoning

Logical reasoning by model checking:



Shown are models in which β_2 is TRUE
In some models in which KB is TRUE, β_2 is FALSE
Hence, $KB \not\models \beta_2$, cannot conclude there's no pit in [2,2]

Logic

Logical reasoning

- The previous approach is known as ***Model checking***.

Logic

Logical reasoning

- The previous approach is known as ***Model checking***.
- It enumerates all possible models to check whether α is TRUE in all models in which KB is TRUE.

Logic

Logical reasoning

- The previous approach is known as ***Model checking***.
- It enumerates all possible models to check whether α is TRUE in all models in which KB is TRUE.
- This approach is:
  - ***Sound:*** derive entailed sentences only, never makes things up.
  - ***Complete:*** can derive any sentence that is entailed.

Propositional logic

Introduction

- ***Propositional logic*** is a very simple logic approach that could be used by an agent for traveling through the wumpus world.

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- Also known as ***Boolean logic***.
- We will take a look at:
  - Syntax.
  - Semantics.
  - Entailment.

Propositional logic

Introduction

- ***Propositional logic*** is a very simple logic approach that could be used by an agent for traveling through the wumpus world.
- Also known as ***Boolean logic***.
- We will take a look at:
  - Syntax.
  - Semantics.
  - Entailment.
- This will lead for a very simple algorithm for logical inference.

Propositional logic

Syntax

- Two types of sentences occur:

Propositional logic

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  - ***Atomic sentences:*** a single ***proposition symbol*** which are basically any variable name, TRUE or FALSE.

Propositional logic

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Propositional logic

Syntax

- Two types of sentences occur:
  - ***Atomic sentences:*** a single ***proposition symbol*** which are basically any variable name, TRUE or FALSE.
  - ***Complex sentences:*** constructed from simpler sentences using ***logical connectives***.
- Logical connectives:

$\neg$	negation.
$\wedge$	conjunction (and).
$\vee$	disjunction (or).
$\Rightarrow$	implication (if-then).
$\Leftrightarrow$	biconditional (if and only if).

Propositional logic

Syntax

- Sentences formed in this way can be called ***Well-Formed Formula*** (***WFF***).

Propositional logic

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Propositional logic

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- Parentheses can be used to indicate precedence.
- Complex sentences evaluate to TRUE or FALSE.
- The symbols and the connectives together define the syntax of the language.

Propositional logic

Semantics

- Semantics define rules for determining the truth of a sentence with respect to a particular model.

Propositional logic

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Propositional logic

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Propositional logic

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Propositional logic

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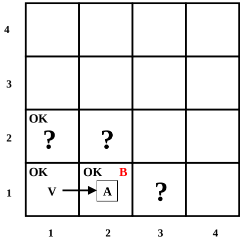
P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Propositional logic

Example: wumpus world - a simple KB in PL

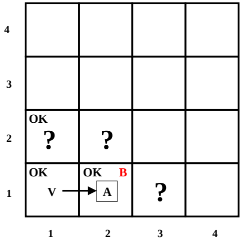
- We want to represent what we know about the wumpus world in a simple KB using propositional logic.



Propositional logic

Example: wumpus world - a simple KB in PL

- We want to represent what we know about the wumpus world in a simple KB using propositional logic.
- Proposition symbols:
  - Let P_{ij} be TRUE is there's a pit in location [i,j].
  - Let B_{ij} be TRUE is there's a breeze in location [i,j].



Propositional logic

Example: wumpus world - a simple KB in PL

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Propositional logic

Example: wumpus world - a simple KB in PL

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Propositional logic

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    R1: $\neg P_{11}$.

Propositional logic

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    R2: $B_{11} \Leftrightarrow (P_{12} \lor P_{21})$.
    R3: $B_{21} \Leftrightarrow (P_{11} \lor P_{22} \lor P_{31})$.

Propositional logic

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Propositional logic

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    R4: $\neg B_{11}$.
    R5: B_{21}.

Propositional logic

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    R2: $B_{11} \Leftrightarrow (P_{12} \vee P_{21})$.
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  - ***FACTS:*** from sensors:

    R4: $\neg B_{11}$.
    R5: B_{21}.
- The KB can be represented as the conjunction of all sentences:

  $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	false	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	true	true	true	true	true	true	<u>true</u>
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

Now, let's consider:

β_1 = "There's no pit in [1,2]" = $\neg P_{12}$

Can we prove that $KB \models \beta_1$?

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

We have seven proposition symbols in the KB:

$B_{11}, B_{21}, P_{11}, P_{12}, P_{21}, P_{22}, P_{31}$

We have $2^7 = 128$ possible models

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

There is only 3 models in which the KB is TRUE

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

There is only 3 models in which the KB is TRUE

In all three models, $\beta_1 = \neg P_{12}$ is also TRUE

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

There is only 3 models in which the KB is TRUE

In all three models, $\beta_1 = \neg P_{12}$ is also TRUE

Hence, $KB \models \beta_1$.

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

On the other hand, if β_2 = “There’s no pit in [2,2]” = $\neg P_{22}$

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

On the other hand, if β_2 = "There's no pit in [2,2]" = $\neg P_{22}$

$\beta_2 = \neg P_{22}$ is TRUE in only one of these models

Propositional logic

Example: wumpus world - reasoning in PL

Reasoning in propositional logic by model checking:

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

On the other hand, if β_2 = "There's no pit in [2,2]" = $\neg P_{22}$

$\beta_2 = \neg P_{22}$ is TRUE in only one of these models

Hence, $KB \not\models \beta_2$.

Propositional logic

Logical reasoning

- The previous approach is known as ***Model checking***.

Propositional logic

Logical reasoning

- The previous approach is known as ***Model checking***.
- Also known as ***Inferencing with Truth Tables***,

Propositional logic

Logical reasoning

- The previous approach is known as ***Model checking***.
- Also known as ***Inferencing with Truth Tables***,
- Sound and complete.

Propositional logic

Logical reasoning

- The previous approach is known as ***Model checking***.
- Also known as ***Inferencing with Truth Tables***,
- Sound and complete.
- However, Time complexity = $O(2^n)$.

Outline

1. Logical Agents
   - Knowledge-based agents
   - Wumpus world
   - Logic
   - Propositional logic

2. Requirements & Reading Material

Requirements

What do I need from you

- When given a certain problem you should be able to:
  - Express the problem in terms of propositional logic (i.e. write the KB in form of rules).
  - Inference certain conclusions using:
    - Truth tables.
- Answer descriptive questions.

Reading Material

Which parts of the textbook are covered

- Russell-Norvig, Chapters 7:
  - Pages 234 - 248.