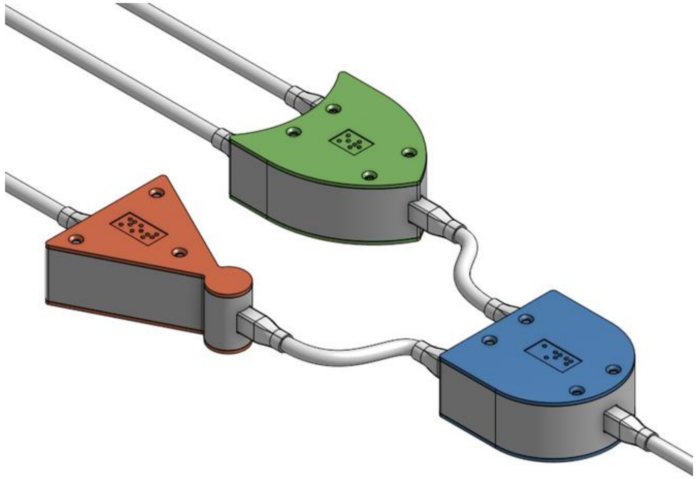

CS 211 - Digital Logic Design
211 عال - تصميم المنطق الرقمي

First Term - 1439/1440
Lecture #7

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Administrivia

- Midterm #1:
  - Date: **Wednesday, October 24, 2018**.
  - Time: 8:30am - 9:30am
  - Scope: Chapters 2 and 3 (Lectures 1, 2, 3, 4, 5, and first part of 6).
- Tutorial:
  - There will be no tutorial this Sunday!
  - Instead, there will be a review tutorial on Monday at 12:30pm.

Website: http://hshehata.github.io/courses/su/cs211

Chapter 4: Boolean Algebra … (… Continuing …)

Laws and Rules of Boolean Algebra

Five Laws

- **CL1:** $A + B = B + A$
- **CL2:** $AB = BA$
- **AL1:** $A + (B + C) = (A + B) + C$
- **AL2**: $A(BC) = (AB)C$
- **DL:** $A(B + C) = AB + AC$

Twelve Rules

- **R1:** $A + 0 = A$
- **R2:** $A + 1 = 1$
- **R3:** $A \cdot 0 = 0$
- **R4:** $A \cdot 1 = A$
- **R5:** $A + A = A$
- **R6:** $A + \bar{A} = 1$
- **R7:** $A \cdot A = A$
- **R8:** $A \cdot \bar{A} = 0$
- **R9:** $\bar{\bar{A}} = A$
- **R10:** $A + AB = A$
- **R11:** $A + \bar{A}B = A + B$
- **R12:** $(A + B)(A + C) = A + BC$

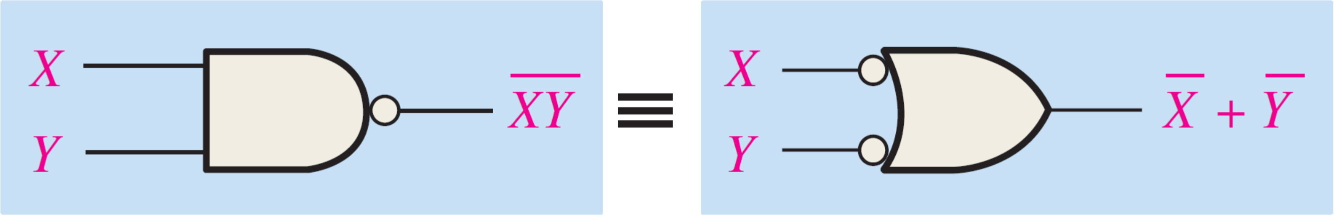
DeMorgan's Theorems

➢ DeMorgan's First Theorem:

- States that: "The complement of a product of variables is equal to the sum of the complements of the variables".

$$\overline{XY} = \overline{X} + \overline{Y}$$

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



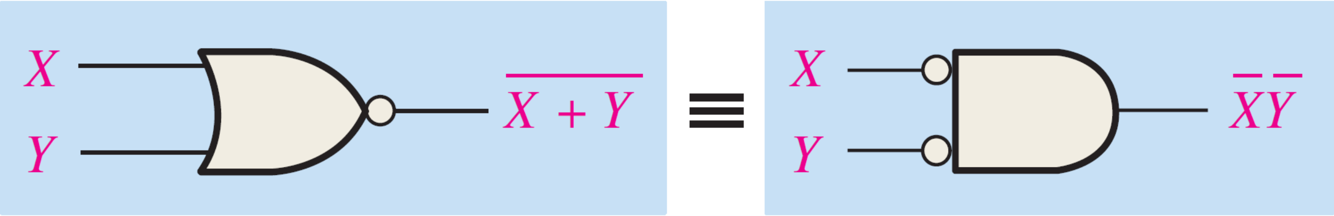
NAND Negative-OR

DeMorgan's Theorems (DMT)

➢ DeMorgan's Second Theorem:

- States that: "The complement of a sum of variables is equal to the product of the complements of the variables".

$$\overline{X+Y} = \overline{X}\,\overline{Y}$$



NOR   Negative-AND

Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Applying DeMorgan's Theorems

- **Example**: Apply DeMorgan's theorems to the expressions: $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

- **Solution**:
  - $\overline{WXYZ} = \bar{W} + \bar{X} + \bar{Y} + \bar{Z}$
  - $\overline{W + X + Y + Z} = \bar{W}\bar{X}\bar{Y}\bar{Z}$

Applying DeMorgan’s Theorems

- **Example**: Develop Boolean expression for XNOR gate given that Boolean expression for XOR gate is: $A\bar{B} + \bar{A}B$.

- **Solution**:
  - XNOR Output = $\overline{A\bar{B} + \bar{A}B}$
  - = $(\overline{A\bar{B}})(\overline{\bar{A}B})$ [DeMorgan’s Theorem]
  - = $(\bar{A} + \bar{\bar{B}})(\bar{\bar{A}} + \bar{B})$ [DeMorgan’s Theorem]
  - = $(\bar{A} + B)(A + \bar{B})$ [Rule 9]
  - = $\bar{A}A + \bar{A}\bar{B} + AB + B\bar{B}$ [Distributive Law]
  - = $\bar{A}\bar{B} + AB$ [Rule 8]

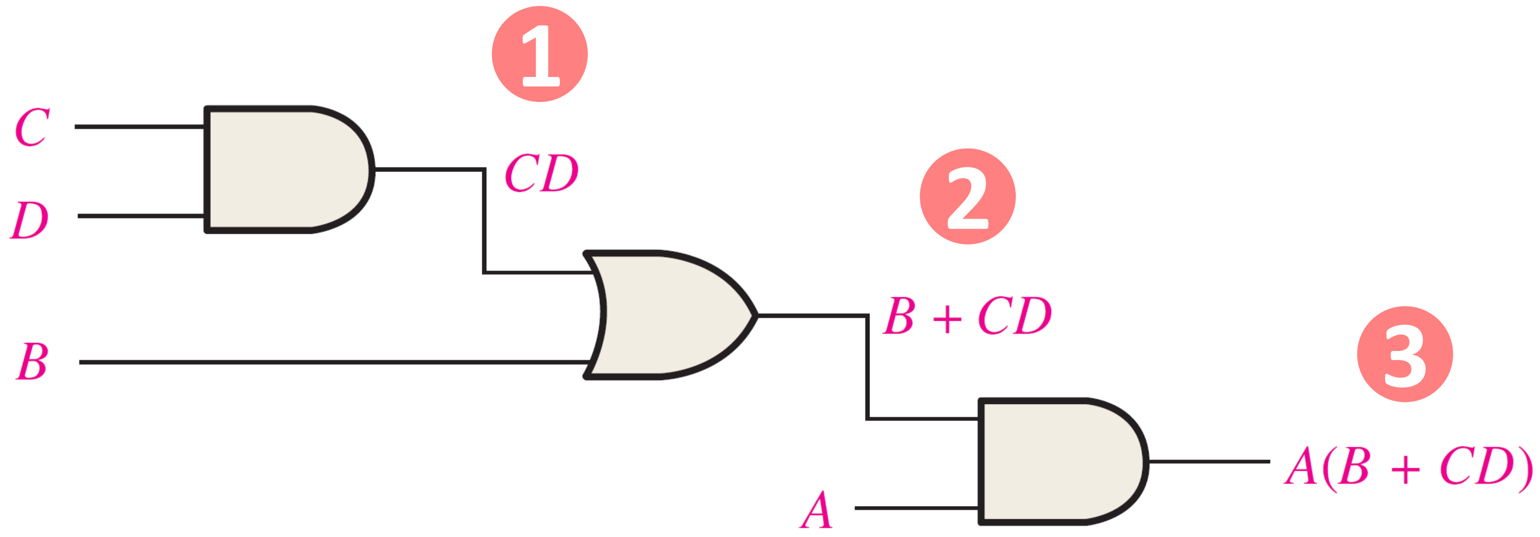
Boolean Analysis of Logic Circuits

- **Goal**: Given a logic circuit, can we construct a truth table for its output?!
- **Method**:
  1. Derive Boolean expression for the logic circuit (i.e., output).
  2. Evaluate Boolean expression (know what inputs make it 1).
  3. Put evaluation results in a truth table format.

Boolean Analysis of Logic Circuits

1. Derive Boolean Expression for Logic Circuit
   - Begin at left-most inputs and work toward final output, writing expression for each gate.

➢Example:



Boolean Analysis of Logic Circuits

2. Evaluate Expression
   - Find the values of the variables that make the expression equal to 1!

Example:

- Expression = $A(B + CD)$

  To make $A(B + CD) = 1$:

  ➔ $A = 1$ and $B + CD = 1$

  ➔ $A = 1$ and
  $(B = 1$ or $CD = 1)$

  ➔ $A = 1$ and
  $(B = 1$ or $(C = 1$ and $D = 1))$

  ➔ $A = B = 1$ or $A = C = D = 1$

Boolean Analysis of Logic Circuits

3. Put Results in Truth Table Format

- Place a 1 in output column for each combination of input variables determined in the evaluation.

Example:

	Inputs				Output
	A	B	C	D	$A(B + CD)$
	0	0	0	0	0
	0	0	0	1	0
	0	0	1	0	0
	0	0	1	1	0
	0	1	0	0	0
	0	1	0	1	0
	0	1	1	0	0
	0	1	1	1	0
	1	0	0	0	0
	1	0	0	1	0
	1	0	1	0	0
$A = C = D = 1$	1	0	1	1	1
$A = B = 1$	1	1	0	0	1
	1	1	0	1	1
	1	1	1	0	1
	1	1	1	1	1

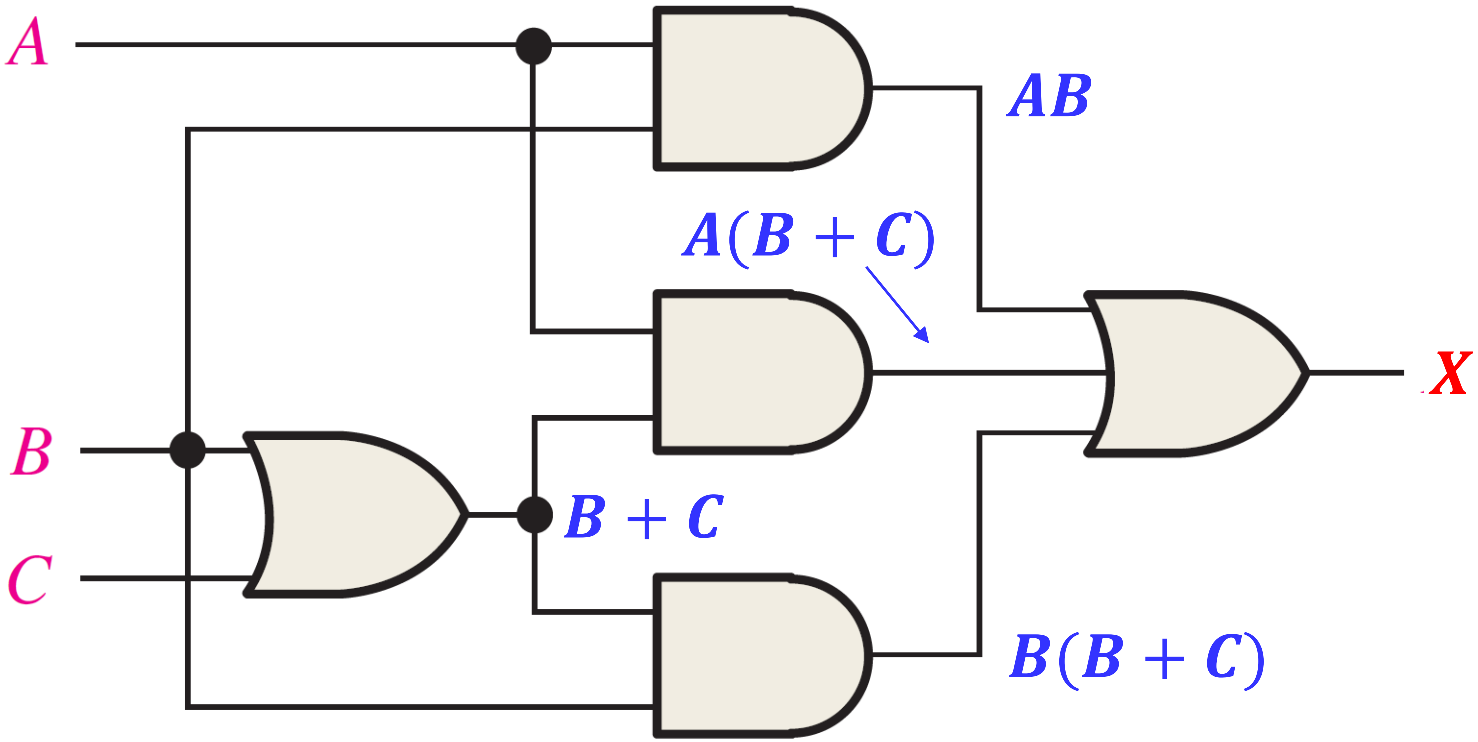
Logic Simplification Using Boolean Algebra

- **Goal**: Given a logic circuit, can we construct an equivalent circuit with fewer number of gates?!
  - “Equivalent” means “has the same Truth Table”.
- **Method**:
  1. Derive Boolean expression for the logic circuit (i.e., output).
  2. Simplify Boolean expression (using laws, rules, and theorems).
  3. Construct logic circuit from the simplified Boolean Expression.

Logic Simplification Using Boolean Algebra

1. Derive Boolean expression for logic circuit.

➤ Example:



$$X = AB + A(B + C) + B(B + C)$$

Logic Simplification Using Boolean Algebra

2. Simplify Boolean expression.

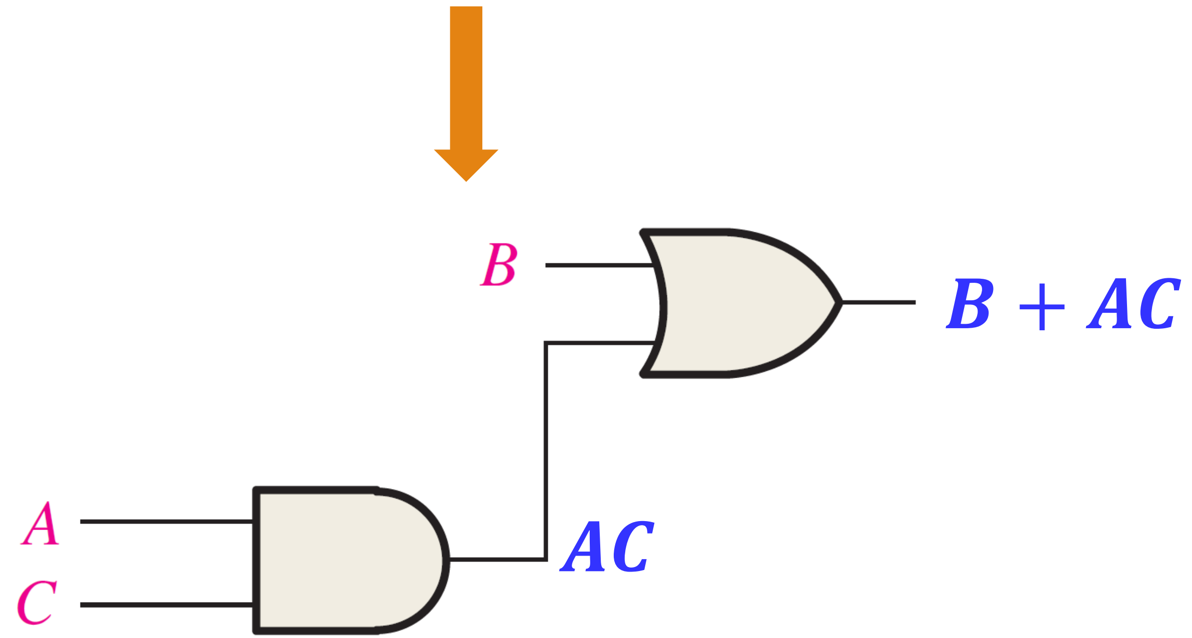
- Example:
  - $X = AB + A(B+C) + B(B+C)$ [DL]
  - $= AB + AB + AC + BB + BC$ [R7]
  - $= AB + AB + AC + B + BC$ [R5]
  - $= AB + AC + B + BC$ [R10]
  - $= AB + AC + B$ [R10]
  - $= B + AC$

Logic Simplification Using Boolean Algebra

3. Construct logic circuit from simplified Boolean Expression

➢ Example:

$$X = B + AC$$



Standard Forms of Boolean Expressions

- All Boolean expressions, regardless of their form, can be converted into either of two standard forms:

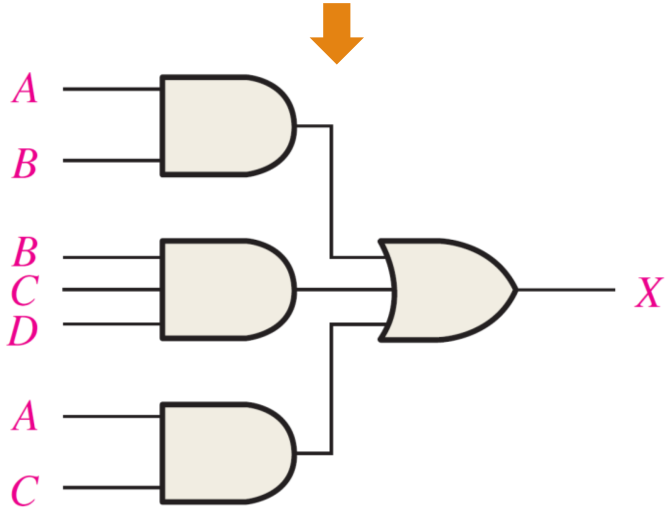
1. Sum-of-products (SOP): 2^+ product terms added together.
   - Examples: "$\bar{A}B + A\bar{B}C + \bar{C}$" and "$\bar{A}\bar{B}\bar{C}\bar{D} + AC + \bar{C}D$".
   - Note: $\bar{A}\bar{B}\overline{CD} + AC$ is not in SOP form, because first term is not product term!!
2. Product-of-sums (POS): 2^+ sum terms multiplied together.
   - Examples: "$(\bar{A} + B + C)(A + C)$" and "$(\bar{A} + \bar{B} + \bar{C} + D)(\bar{B} + C)(A + D)$".
   - Note: $(\overline{A + B} + \bar{C} + D)(\bar{B} + C)$ is not in POS form, because first term is not sum term!!

Implementing SOP & POS Expressions

IMPLEMENTING SOP EXPRESSIONS

- ORing outputs of 2^{+} AND gates
- Example:

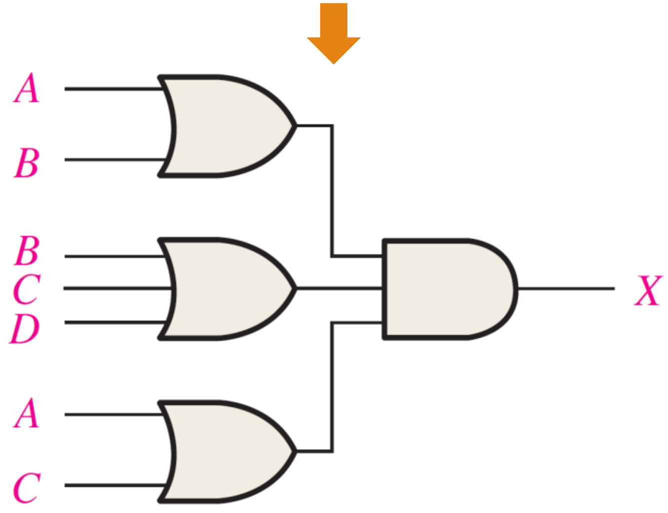
$$X = AB + BCD + AC$$



IMPLEMENTING POS EXPRESSIONS

- ANDing outputs of 2^{+} OR gates
- Example:

$$X = (A + B)(B + C + D)(A + C)$$



Standard SOP & POS Expression

STANDARD SOP EXPRESSION

- Every product term must contain all variables!
- Example:
  - Non-standard SOP
    - $A\bar{B} + \bar{A}BC$
    - "C" is missing in 1st term!
  - Standard SOP
    - $A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$

STANDARD POS EXPRESSION

- Every sum term must contain all variables!
- Example:
  - Non-standard POS
    - $(\bar{A} + C)(A + B + \bar{C})$
    - "B" is missing in first term!
  - Standard POS
    - $(\bar{A} + \bar{B} + C)(\bar{A} + B + C)(A + B + \bar{C})$

Conversion: General Expression ➔ SOP

- **Method**: Apply Boolean laws, rules, and theorems!
  - Use DMT and R9 to get rid of term negations!
  - Use DL to distribute a term over a sum of terms!

- **Example**: Convert $\overline{\left(\overline{(A + B)} + C\right)}$ to SOP form.

- **Solution**:
  - $\overline{\left(\overline{(A + B)} + C\right)} = \left(\overline{\overline{A + B}}\right)\bar{C}$ [DMT]
  - $= (A + B)\bar{C}$ [R9]
  - $= A\bar{C} + B\bar{C}$ [DL]

Conversion: SOP ➔ Standard SOP

- **Method**: For each product term that misses a variable X, multiply that term by $X + \bar{X}$ (R4 & R6), and then apply DL!
- **Example**: Convert to $A\bar{B} + \bar{A}BC$ to standard SOP.
- **Solution**:
  - $A\bar{B} + \bar{A}BC = A\bar{B}(C + \bar{C}) + \bar{A}BC$ [R4 & R6]
  - $= A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$ [DL]

Conversion: POS ➔ Standard POS

- **Method**: For each sum term that misses a variable X, Add $X\bar{X}$ to that term (R1 & R8), and then apply R12!
- **Example**: Convert $(\bar{A} + C)(A + B + \bar{C})$ to standard POS.
- **Solution**:
  - $(\bar{A} + C)(A + B + \bar{C}) = (\bar{A} + C + B\bar{B})(A + B + \bar{C})$ [R4 & R6]
  - $= (\bar{A} + \bar{B} + C)(\bar{A} + B + C)(A + B + \bar{C})$ [R12]

Conversion: Standard SOP → Truth Table

Method: For each product term, determine binary value of inputs that makes it 1, and then place a 1 in output column at corresponding row. Place 0's in all remaining rows.

➢ Example: Develop truth table for the standard SOP expression: $\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

Inputs			Output	
A	B	C	X	Product Term
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

Conversion: Standard POS → Truth Table

Method: For each sum term, determine binary value of inputs that makes it 0, and then place a 0 in output column at corresponding row. Place 1's in all remaining rows.

- Example: Develop truth table for the standard POS expression: $(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

Inputs			Output	
A	**B**	**C**	**X**	**Sum Term**
0	0	0	0	$(A+B+C)$
0	0	1	1	
0	1	0	0	$(A+\bar{B}+C)$
0	1	1	0	$(A+\bar{B}+\bar{C})$
1	0	0	1	
1	0	1	0	$(\bar{A}+B+\bar{C})$
1	1	0	0	$(\bar{A}+\bar{B}+C)$
1	1	1	1	

Conversion: Truth Table → Standard SOP

Method: Identify binary values of inputs that make output = 1. Convert each binary value to a product term replacing each 1 with corresponding var. and 0 with corresponding var. complement

➢ Example: Construct standard SOP expression from following truth table:

Inputs			Output	
A	*B*	*C*	*X*	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\bar{A}BC$
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

$$X = \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

Conversion: Truth Table → Standard POS

Method: Identify binary values of inputs that make output = 0. Convert each binary value to a sum term replacing each 0 with corresponding var. and 1 with corresponding var. complement.

➢ Example: Construct standard POS expression from following truth table:

Inputs			Output	
A	B	C	X	
0	0	0	0	$A + B + C$
0	0	1	0	$A + B + \bar{C}$
0	1	0	0	$A + \bar{B} + C$
0	1	1	1	
1	0	0	1	
1	0	1	0	$\bar{A} + B + \bar{C}$
1	1	0	1	
1	1	1	1	

$$X = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

Conversion: Stand. POS ←→ Stand. SOP

- Standard SOP → Standard POS:
  - Method:
    1. Standard SOP → truth table
    2. Truth table → standard POS
- Standard POS → Standard SOP:
  - Method:
    1. Standard POS → truth table
    2. Truth table → standard SOP

Reading Material

- Floyd, Chapter 4:
  - Pages 179 - 199