1. Simplify the following expressions using Boolean algebra:

a. $(\boldsymbol{B}+\overline{\boldsymbol{C}})(\overline{\boldsymbol{B}}+\boldsymbol{C})+\overline{\overline{\boldsymbol{A}}+\boldsymbol{B}+\overline{\boldsymbol{C}}}$

Solution #1:

$= B\bar{B} + BC + \bar{B}\bar{C} + C\bar{C} + \bar{\bar{A}}\bar{B}\bar{\bar{C}}$ [Dist. + DM]
$= 0 + BC + \bar{B}\bar{C} + 0 + A\bar{B}C$ [R8, R9]
$= BC + \bar{B}(\bar{C} + AC)$ [R1, Dist.]
$= BC + \bar{B}(\bar{C} + A)$ [R11]
$= BC + \bar{B}\bar{C} + A\bar{B}$ [Dist.]

Solution #2:

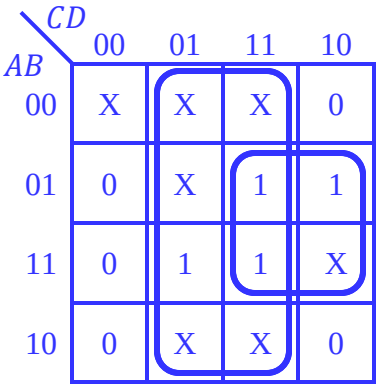
$= B\bar{B} + BC + \bar{B}\bar{C} + C\bar{C} + \bar{\bar{A}}\bar{B}\bar{\bar{C}}$ [Dist. + DM]
$= 0 + BC + \bar{B}\bar{C} + 0 + A\bar{B}C$ [R8, R9]
$= \bar{B}\bar{C} + C(B + A\bar{B})$ [R1, Dist.]
$= \bar{B}\bar{C} + C(B + A)$ [R11]
$= \bar{B}\bar{C} + BC + AC$ [Dist.]

b. $\overline{(\overline{\boldsymbol{A}}+\boldsymbol{B}+\overline{\boldsymbol{C}})(\boldsymbol{A}+\boldsymbol{B}+\overline{\boldsymbol{C}})(\overline{\boldsymbol{A}}+\boldsymbol{B}+\boldsymbol{C})(\overline{\boldsymbol{BC}+\overline{\boldsymbol{A}}\boldsymbol{B}\overline{\boldsymbol{C}}})}$

$= \overline{(\bar{A} + B + \bar{C})} + \overline{(A + B + \bar{C})} + \overline{(\bar{A} + B + C)} + \overline{(\overline{BC + \bar{A}B\bar{C}})}$ [DM]
$= \bar{\bar{A}}\bar{B}\bar{\bar{C}} + \bar{A}\bar{B}\bar{\bar{C}} + \bar{\bar{A}}\bar{B}\bar{C} + BC + \bar{A}\bar{B}\bar{C}$ [DM, R9]
$= A\bar{B}C + \bar{A}\bar{B}C + A\bar{B}\bar{C} + BC + \bar{A}\bar{B}\bar{C}$ [R9]
$= A\bar{B}C + \bar{A}\bar{B}C + A\bar{B}\bar{C} + BC + \bar{A}\bar{B}\bar{C}$ [R9]
$= A\bar{B}(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + BC$ [Dist.]
$= A\bar{B} + \bar{A}\bar{B} + BC$ [R6, R4]
$= \bar{B}(A + \bar{A}) + BC$ [Dist.]
$= \bar{B} + BC$ [R6, R4]
$= \bar{B} + C$ [R11]

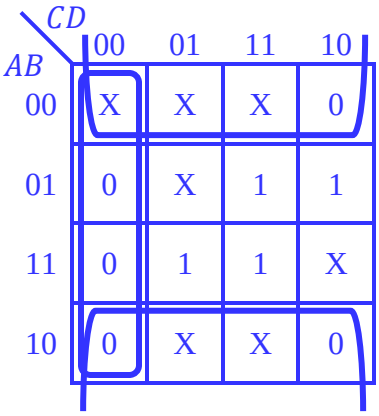
2. Suppose a Boolean variable *Z* is described by the following Karnaugh map

a. Construct a minimum SOP expression for *Z*.



$Z = D + BC$

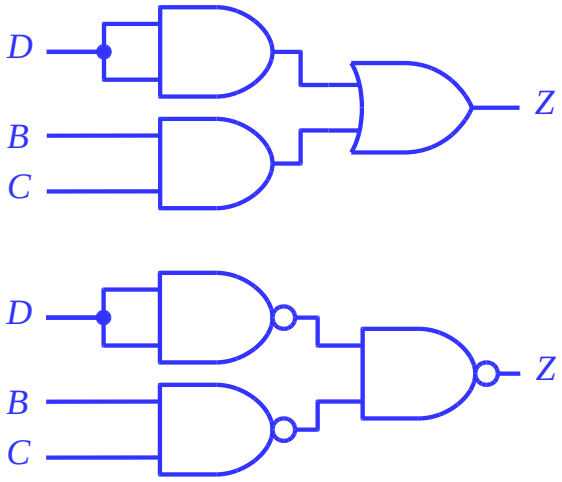
b. Construct a minimum POS expression for *Z*.



$Z = B(\mathrm{C} + \mathrm{D})$

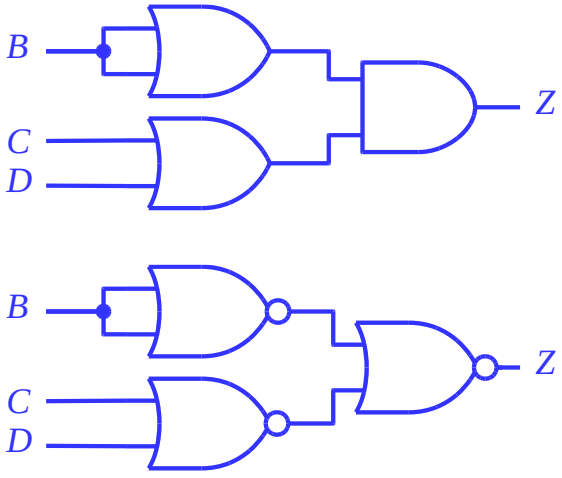
c. Implement *Z* using NAND gates.

$$Z = D + BC$$

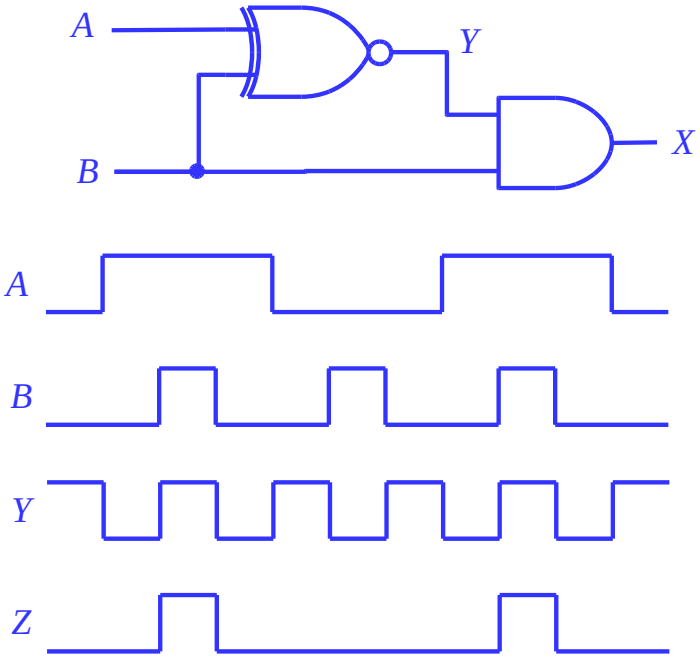


d. Implement *Z* using NOR gates.

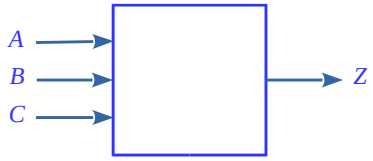
$$Z = B(\mathrm{C} + \mathrm{D})$$



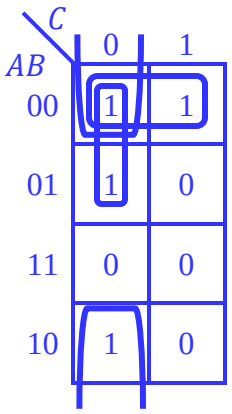
3. For the following logic circuit, draw the output waveform …



4. Design a logic circuit whose output Z is HIGH only when a majority of its inputs A, B, and C are LOW.



Inputs			Output
A	B	C	Z
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0



$$Z = \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$$