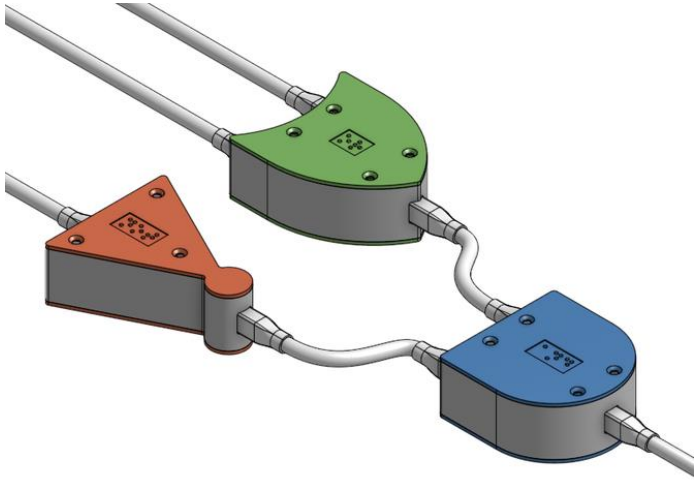
جامعة شقراء
Shaqra University



CSC 211 - Digital Logic Design
211 عال - تصميم المنطق الرقمي

First Term - 1439/1440
Lecture #8

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Assistant Professor

College of Computing and Information Technology

Administrivia

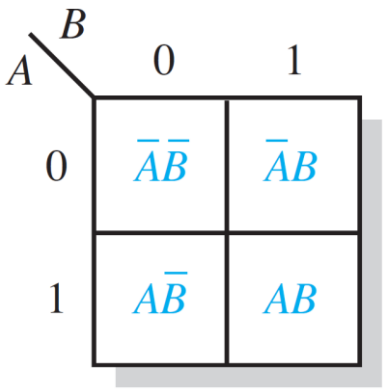
- Midterm #1:
  - Exam/Solution is available on the website.
  - Marks were posted to the website.
- ILMS:
  - Will replace the website.
  - Please contact Mr. Nayaar to activate your accounts.

Website: http://hshehata.github.io/courses/su/cs211

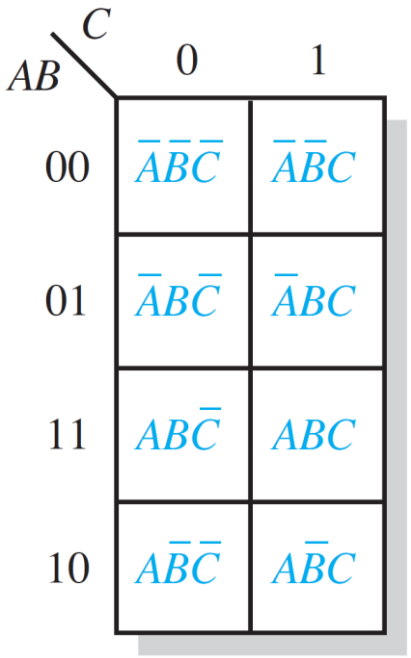
Chapter 4: Boolean Algebra … (… Continuing …)

Karnaugh Map

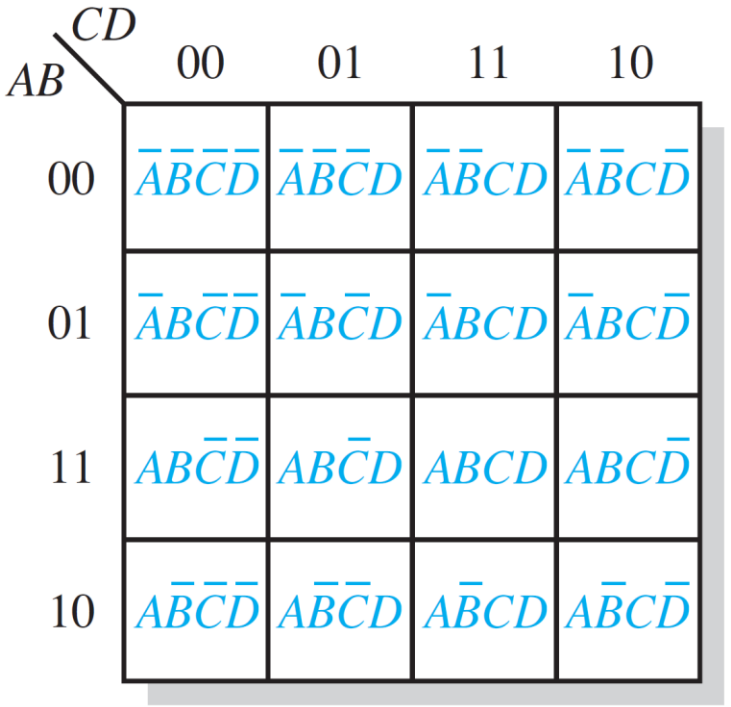
- **Purpose**:
  - Systematic method for simplifying Boolean expressions.
  - Produce simplest SOP/POS expression ➔ minimum SOP/POS.
  - Suitable for expressions containing 5 variables at most!
- **Properties**:
  - Array of cells arranged in a way so simplification becomes a matter of grouping these cells!
  - Each cell represents one row in truth table.
    - Number of cells = 2^n (where n is the number of variables).



2-Variable (A,B)



3-Variable (A,B,C)

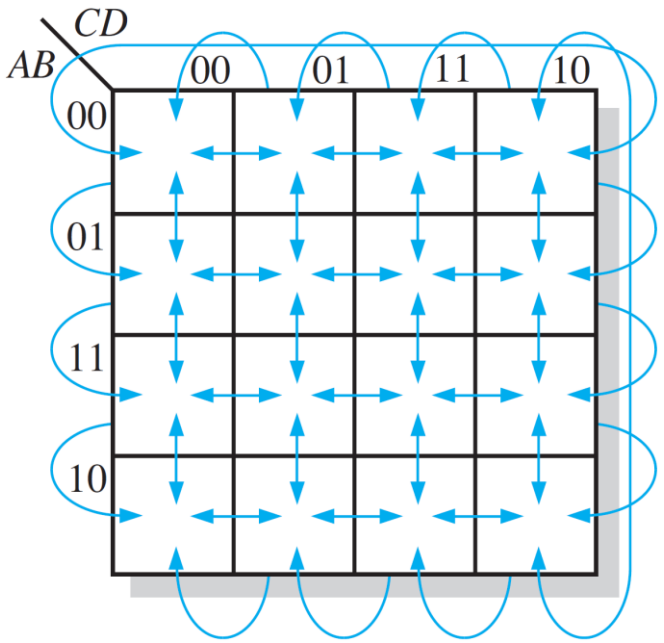


4-Variable (A,B,C,D)

2-, 3-, and 4-Variable Karnaugh Maps

Cell Adjacency

- Cells in a Karnaugh map are arranged so that there is only a single-variable change between adjacent cells.
  - Cells that differ by 1 variable are adjacent.
    - Example: 0011 & 0111, 0100 & 0110
  - Cells with values that differ by 2^+ variables are not adjacent.
    - Example: 1101 & 0111



SOP Minimization Using Karnaugh Maps

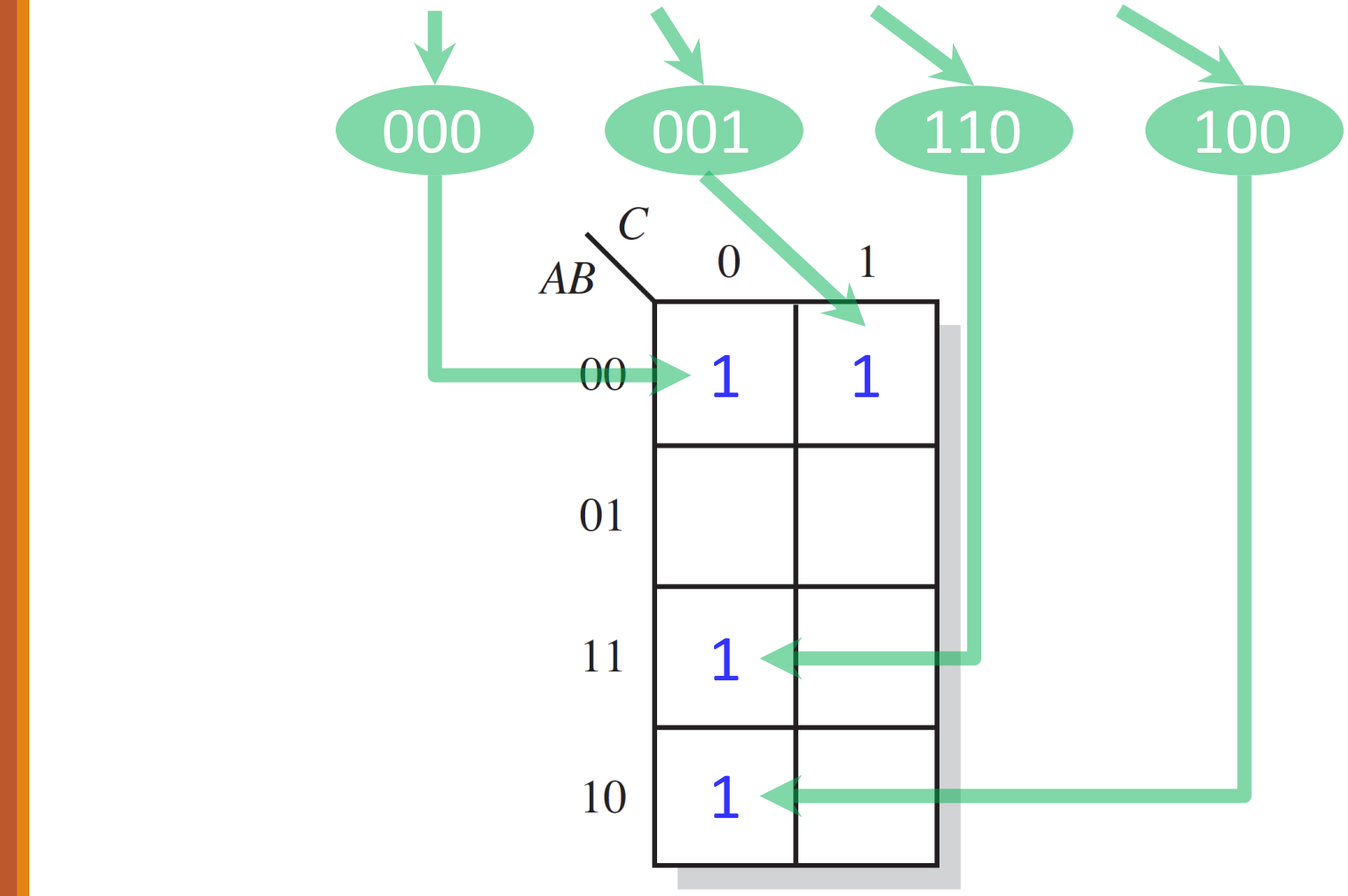
- Goal: Simplify a SOP expression (standard or non-standard) to its minimum form ➔ Minimum SOP!
  - Minimum SOP expression contains the fewest possible product terms with the fewest possible variables per term!!
- Method:
  1. Map SOP expression on a Karnaugh map.
  2. Combine the 1's on the map into maximum groups.
  3. Determine minimum product term for each group, and combine minimum product terms to form a minimum SOP.

SOP Minimization Using Karnaugh Maps

1. Map SOP expression on a Karnaugh map.
   - Determine binary values of product terms.
   - Place 1's in corresponding cells.

➢ Example (1): Standard-SOP ➔ K-Map

◦ $X = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$

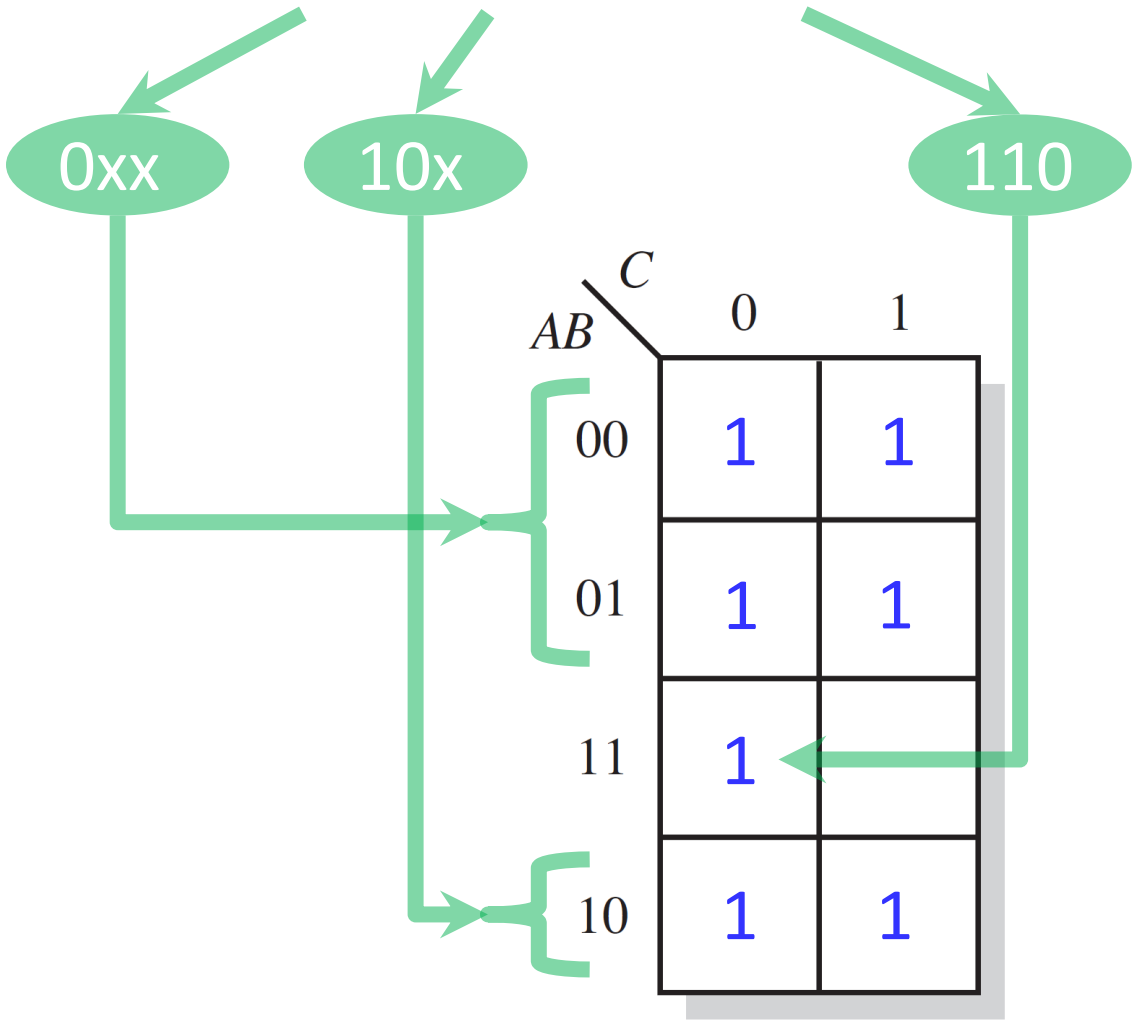


SOP Minimization Using Karnaugh Maps

1. Map SOP expression on a Karnaugh map.
   - Determine binary values of product terms.
   - Place 1's in corresponding cells.

➢ Example (1): Non-Standard-SOP ➔ K-Map

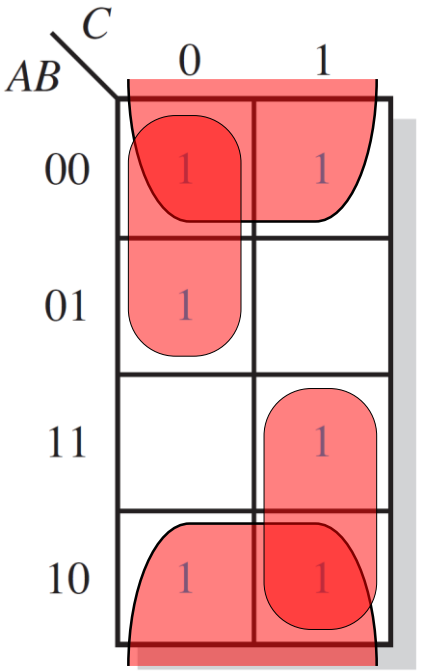
◦ $X = \bar{A} + A\bar{B} + AB\bar{C}$

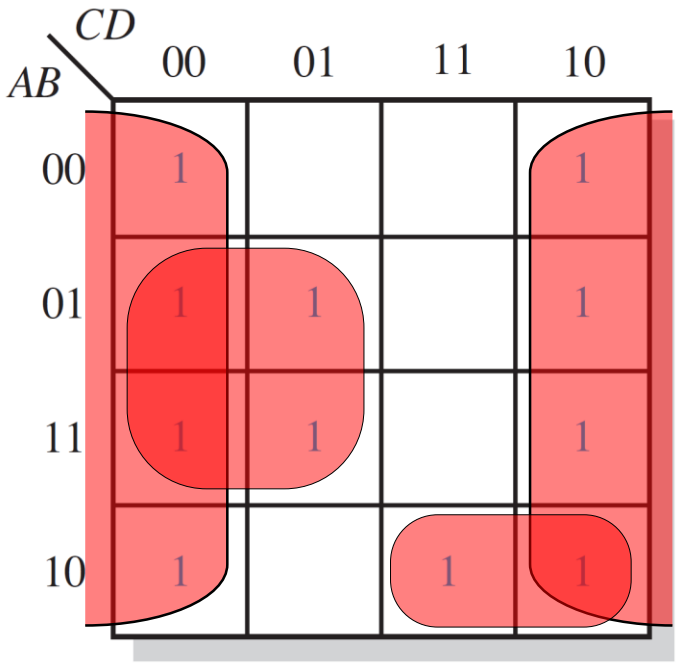


SOP Minimization Using Karnaugh Maps

2. Combine the 1's into max. groups.
   - Each group contains 2^x adjacent cells.
   - Every 1 must belong to at least 1 group.

➢ Examples:

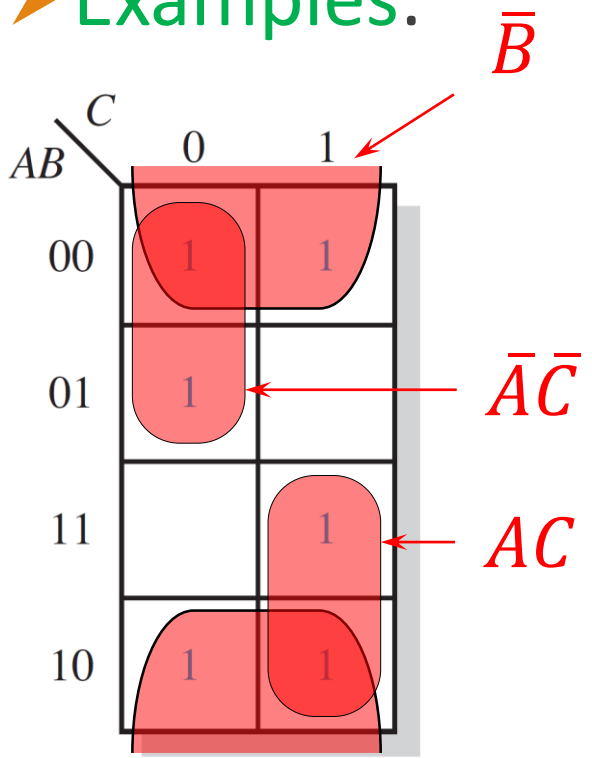




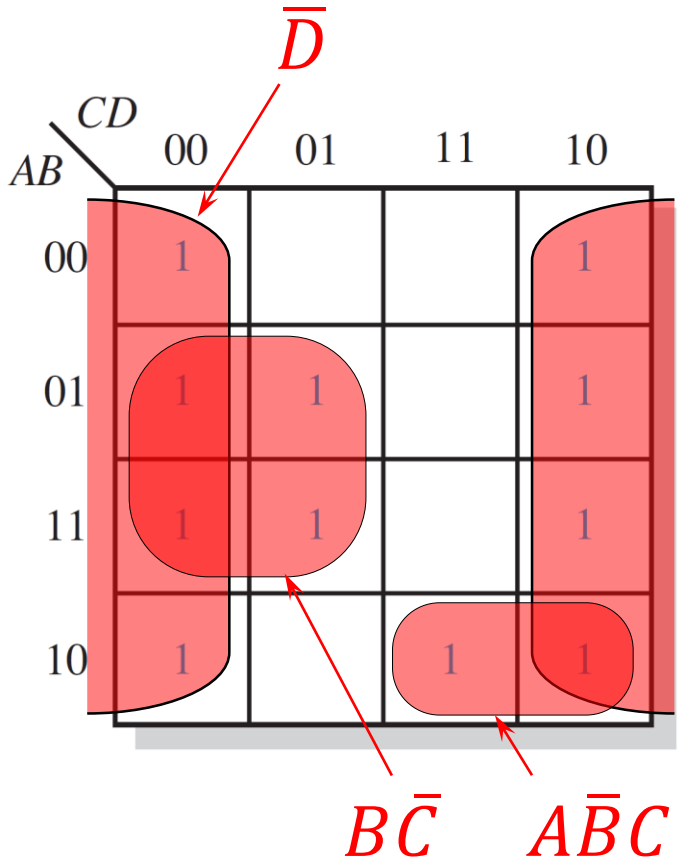
SOP Minimization Using Karnaugh Maps

3. Determine minimum product terms (1 term per group) and combine them to form a minimum SOP.

➢ Examples:



$$X = \bar{B} + \bar{A}\bar{C} + AC$$



$$X = \bar{D} + B\bar{C} + A\bar{B}C$$

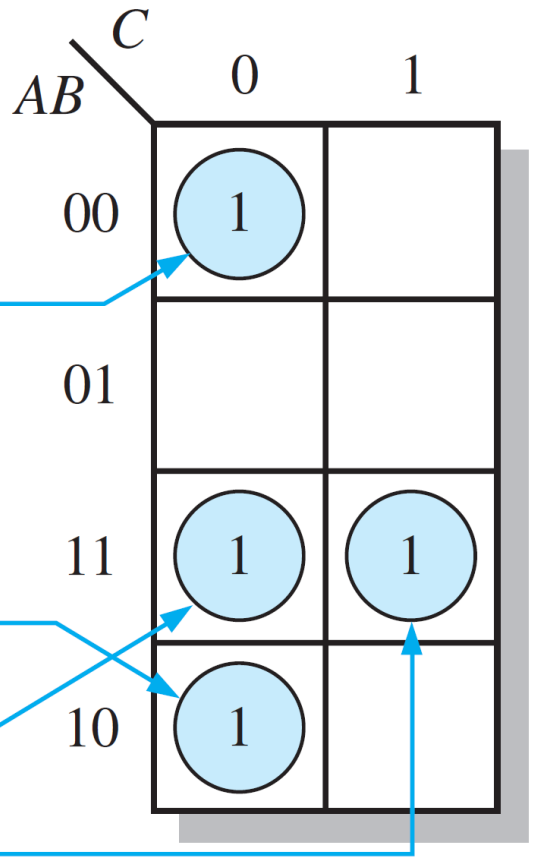
Mapping Directly from a Truth Table

- The 1's in the output column are mapped directly into the cells corresponding to the values of the associated input variable combinations.

➢Example: Truth Table ➔ K-Map

$$X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

Inputs			Output
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



"Don't Care" Conditions

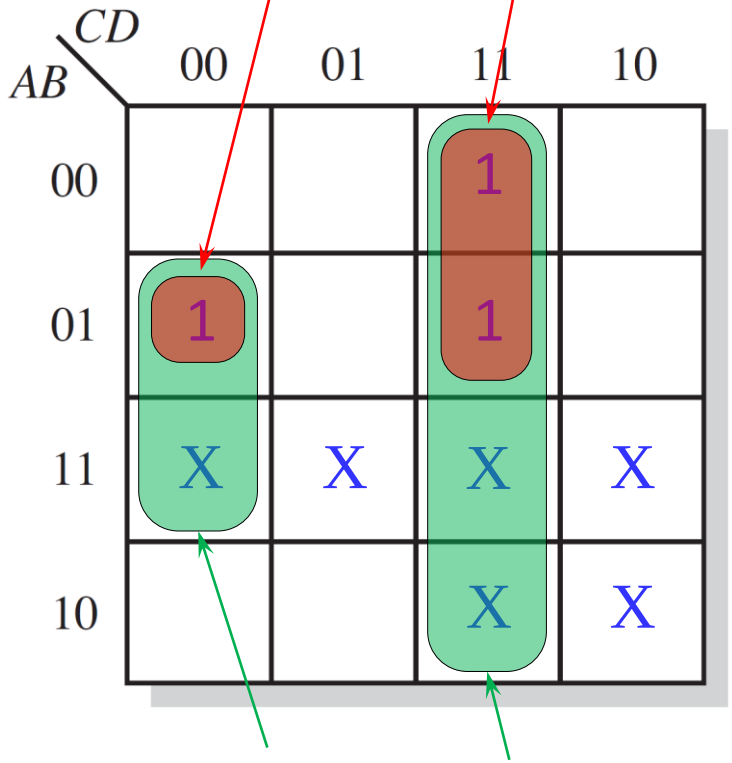
- If some input combinations are not allowed, we mark their corresponding output value by X's.
- "X" means we "don't care" about the output value in this case, and hence it can be set to 0 or 1.

Example:

Inputs $ABCD$	Output Y
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	1
0 1 0 0	1
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	0
1 0 0 1	0
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

Without Don't cares

$$Y = \bar{A}B\bar{C}\bar{D} + \bar{A}CD$$



$$Y = B\bar{C}\bar{D} + CD$$

With Don't cares

POS Minimization Using Karnaugh Maps

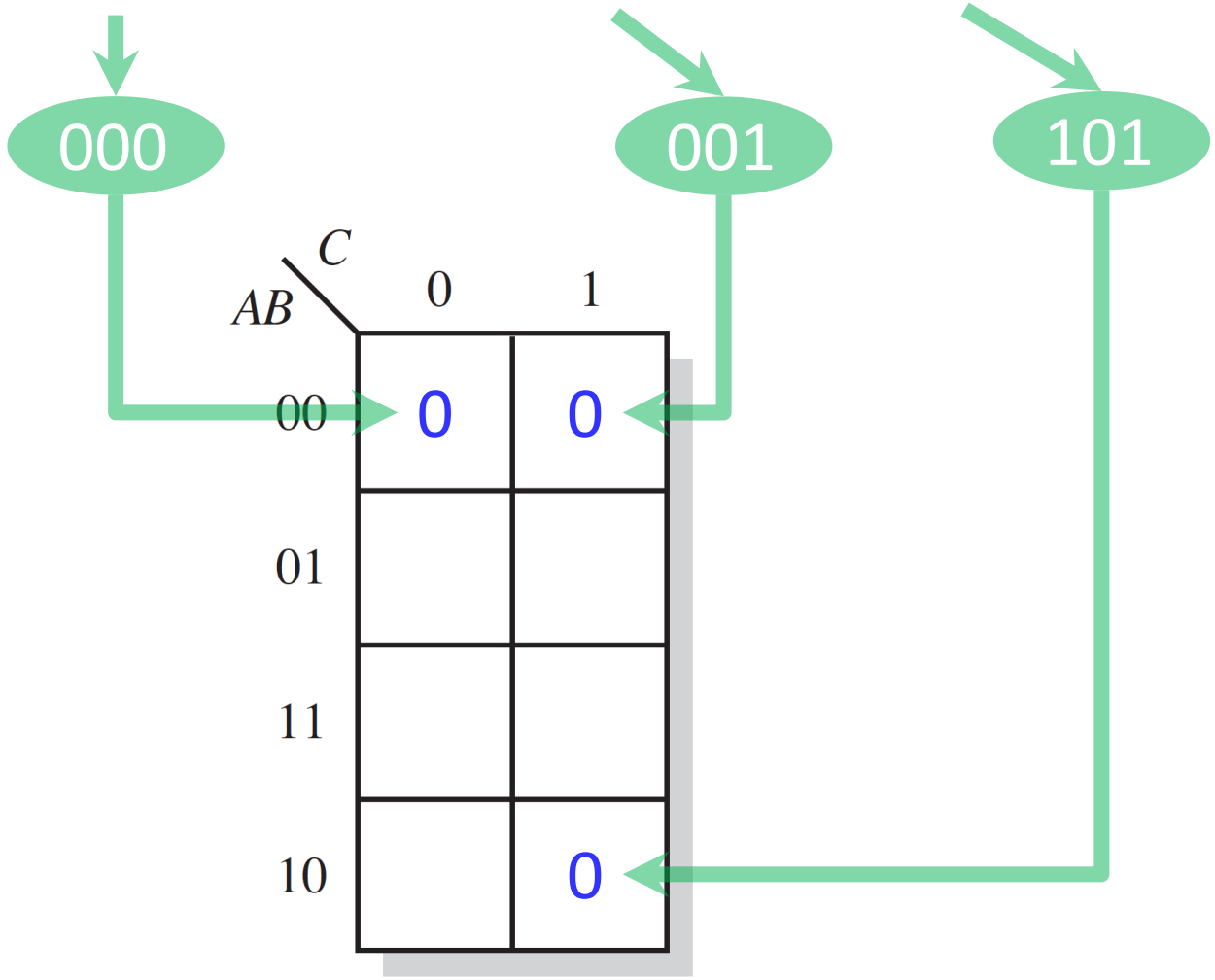
- Goal: Simplify a POS expression (standard or non-standard) to its minimum form ➔ Minimum POS!
  - Minimum POS expression contains the fewest possible product terms with the fewest possible variables per term!!
- Method:
  1. Map POS expression on a Karnaugh map.
  2. Combine the 0's on the map into maximum groups.
  3. Determine minimum sum term for each group, and combine minimum sum terms to form a minimum POS.

POS Minimization Using Karnaugh Maps

1. Map POS expression on a Karnaugh map.
   - Determine binary values of sum terms.
   - Place 0's in corresponding cells.

➢ Example: POS ➔ K-Map

◦ $X = (A + B + C)(A + B + \bar{C})(\bar{A} + B + \bar{C})$

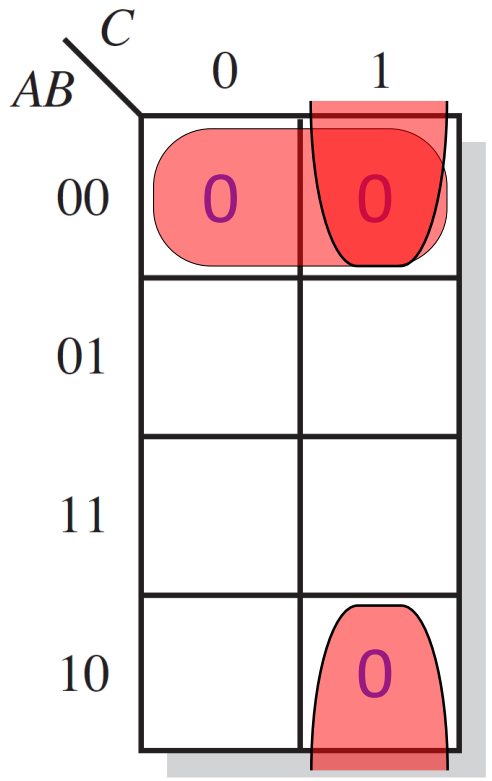


POS Minimization Using Karnaugh Maps

2. Combine the 0's into max. groups.
   - Each group contains 2^x adjacent cells.
   - Every 0 must belong to at least 1 group.

➢ Example:

◦ $X = (A + B + C)(A + B + \bar{C})(\bar{A} + B + \bar{C})$



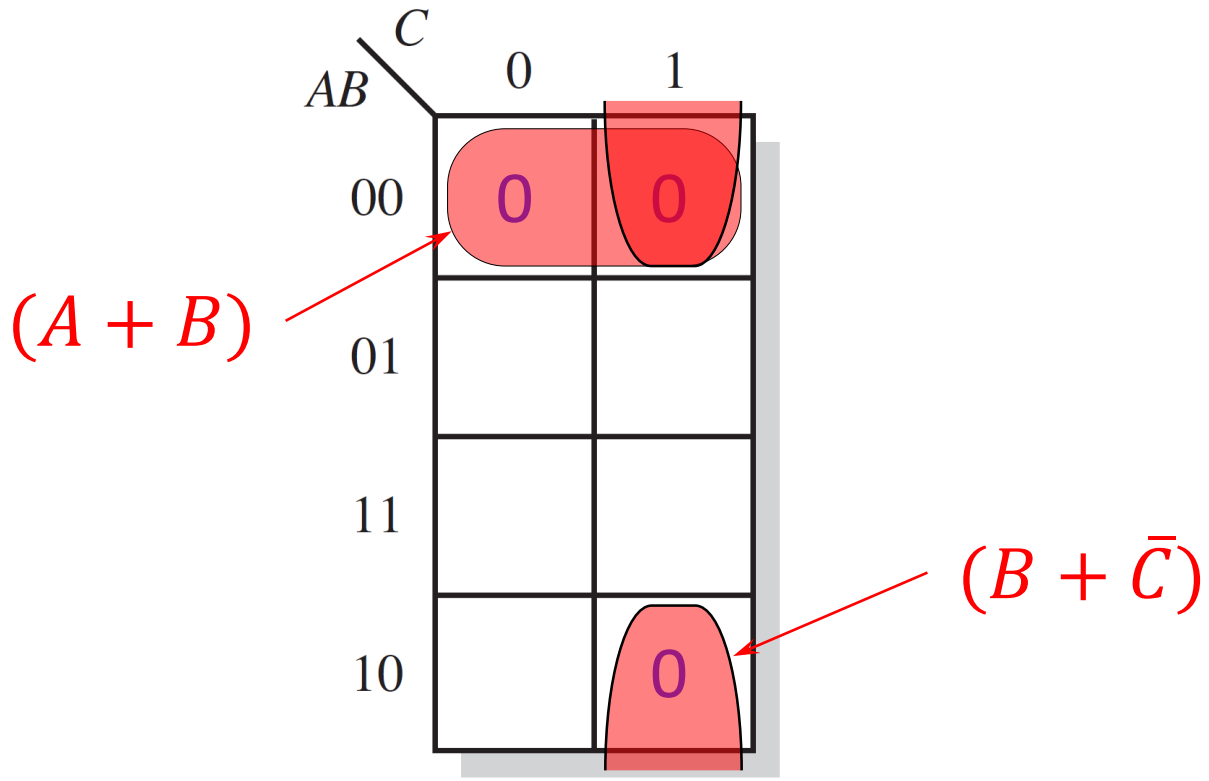
POS Minimization Using Karnaugh Maps

3. Determine minimum sum terms (1 term per group) and combine them to form a minimum POS.

➢ Example:

◦ $X = (A + B + C)(A + B + \bar{C})(\bar{A} + B + \bar{C})$

➔ $X = (A + B)(B + \bar{C})$



Conversion: POS ➔ SOP Using Karnaugh Maps

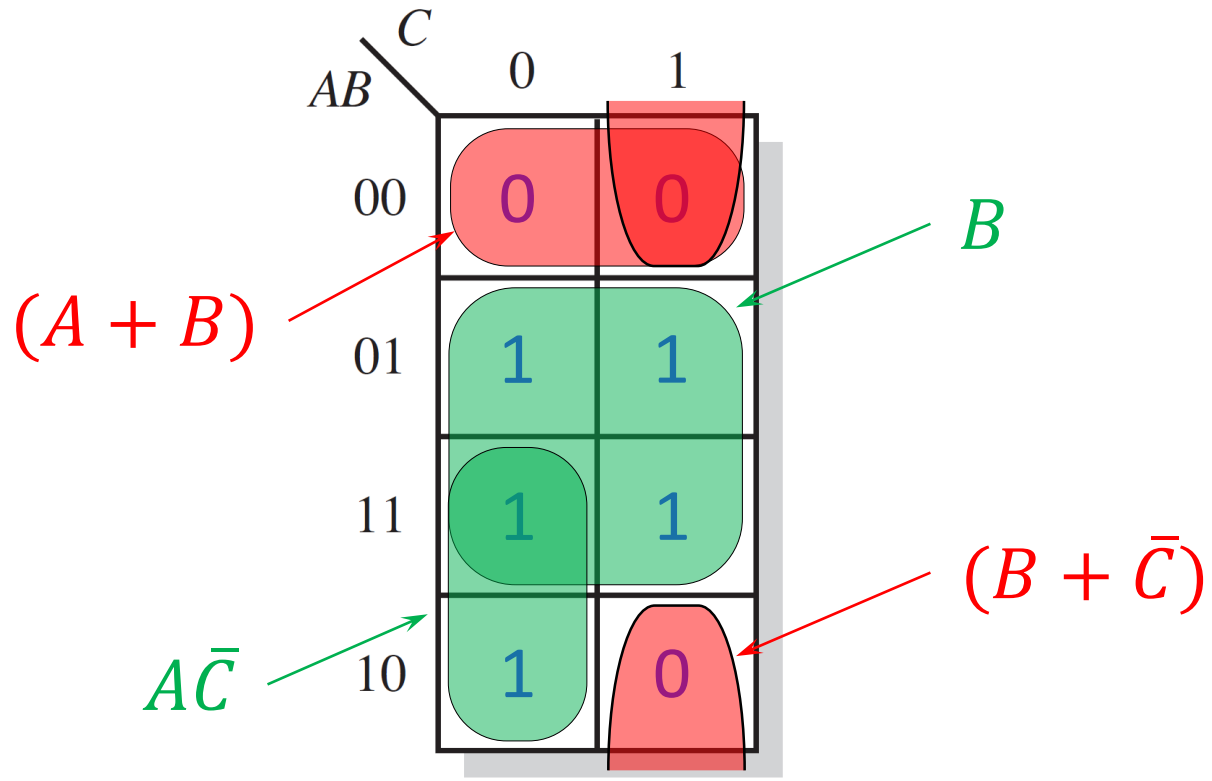
- After placing the 0's in the map, fill the rest of the cells with 1's and follow the steps to construct minimum SOP from K-maps.

➢ Example: POS ➔ SOP

- $X = (A + B + C)(A + B + \bar{C})(\bar{A} + B + \bar{C})$

➔ $X = (A + B)(B + \bar{C})$

➔ $X = B + A\bar{C}$



Reading Material

- Floyd, Chapter 4:
  - Pages 199 - 212