CSE 411: Artificial Intelligence (Elective Course #6)

400 Level, Mechatronics Engineering
2nd Term 2016/2017, Lecture #9

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Credits to Dr. Mohamed El Abd for the slides

Adminstrivia

Notes

- Midterm:
  - Done marking!
  - Solution was posted!

Course Info:

- Website: http://hshehata.github.io/courses/zu/cse411/
- Office hours: Sunday 11:30am - 12:30pm

Outline

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1. **Logical Agents (Continued)**
   - Introduction
   - Propositional logic (Continued)
   - Weather Forecasting Example
2. **Requirements & Reading Material**

Introduction

Knowledge-based agents

- **Knowledge-based agents** are agents that can:
  - store knowledge → **knowledge base (KB)**.
  - deduce new facts → **inferencing**.

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  - deduce new facts → **inferencing**.
- KB includes set of assertions about environment:
  - **facts**.
  - **rules**.

Introduction

Knowledge-based agents

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  - store knowledge → **knowledge base (KB)**.
  - deduce new facts → **inferencing**.
- KB includes set of assertions about environment:
  - **facts**.
  - **rules**.
- These assertions are **sentences** represented in a given **logic.**

Propositional Logic

Syntax

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Propositional Logic

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- PL **syntax**:

Sentence	::=	Atomic-Sentence
	|	Complex-Sentence
Atomic-Sentence	::=	TRUE | FALSE | P | Q | R| ...
Complex-Sentence	::=	(Sentence)
	|	¬Sentence
	|	Sentence ∧ Sentence
	|	Sentence ∨ Sentence
	|	Sentence ⇒ Sentence
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Propositional Logic

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- Ex.:
  - "$(A \wedge B) \Rightarrow \neg C$" is a well-formed PL formula.
  - "$A \wedge \Rightarrow B$" is not a well-formed PL formula!

Propositional Logic

Semantics

- PL **semantics**:

P	Q	$TRUE$	$FALSE$	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
F	F	T	F	T	F	F	T	T
F	T	T	F	T	F	T	T	F
T	F	T	F	F	F	T	F	F
T	T	T	F	F	T	T	T	T

Propositional Logic

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- **Logical entailment**: α *entails* β (*i.e.*, $\alpha \models \beta$) if and only if every model that satisfies α also satisfies β.

Propositional Logic

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T	F	T	F	F	F	T	F	F
T	T	T	F	F	T	T	T	T

- **Logical entailment**: α *entails* β (*i.e.*, $\alpha \models \beta$) if and only if every model that satisfies α also satisfies β.
- Inference techniques to prove entailment in PL include:
  - Truth tables.
  - Rules.
  - Resolution.
  - Forward chaining.
  - Backward chaining (Canceled!).

Propositional Logic

Ex.: Wumpus world

- Suppose KB consists of five facts/rules:

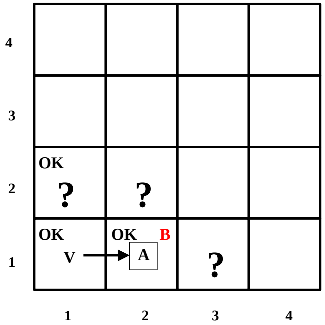
R_1: $\neg P_{11}$.

R_2: $B_{11} \Leftrightarrow (P_{12} \vee P_{21})$.

R_3: $B_{21} \Leftrightarrow (P_{11} \vee P_{22} \vee P_{31})$.

R_4: $\neg B_{11}$.

R_5: B_{21}.



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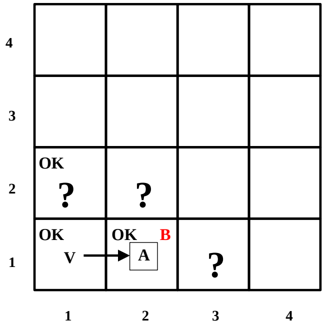
- In other words, KB := $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$



Propositional Logic

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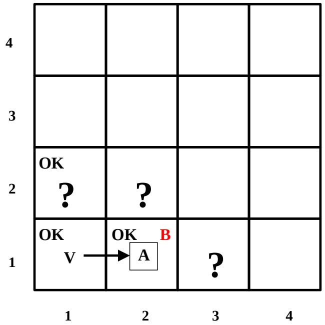
- In other words, KB := $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$
- Prove or disprove the following entailments:
  - KB $\models \neg P_{12}$.
  - KB $\models \neg P_{22}$.



Propositional Logic

Ex.: Wumpus world - inferencing with truth tables

B_{11}	B_{21}	P_{11}	P_{12}	P_{21}	P_{22}	P_{31}	R_1	R_2	R_3	R_4	R_5	KB	$\neg P_{12}$	$\neg P_{22}$
F	F	F	F	F	F	F	T	T	T	T	F	F	T	T
F	F	F	F	F	F	T	T	T	F	T	F	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
F	T	F	F	F	F	F	T	T	F	T	T	F	T	T
F	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	T	F	F	T	F	F	T	F	F	T	T	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
T	T	T	T	T	T	T	F	T	T	F	T	F	F	F

Propositional Logic

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B_{11}	B_{21}	P_{11}	P_{12}	P_{21}	P_{22}	P_{31}	R_1	R_2	R_3	R_4	R_5	KB	$\neg P_{12}$	$\neg P_{22}$
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F	F	F	F	F	F	T	T	T	F	T	F	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
F	T	F	F	F	F	F	T	T	F	T	T	F	T	T
F	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	T	F	F	T	F	F	T	F	F	T	T	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
T	T	T	T	T	T	T	F	T	T	F	T	F	F	F

- This means that:

Propositional Logic

Ex.: Wumpus world - inferencing with truth tables

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F	F	F	F	F	F	F	T	T	T	T	F	F	T	T
F	F	F	F	F	F	T	T	T	F	T	F	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
F	T	F	F	F	F	F	T	T	F	T	T	F	T	T
F	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	T	F	F	T	F	F	T	F	F	T	T	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
T	T	T	T	T	T	T	F	T	T	F	T	F	F	F

- This means that:
  - KB $\models \neg P_{12}$.

Propositional Logic

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⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
F	T	F	F	F	F	F	T	T	F	T	T	F	T	T
F	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	T	F	F	T	F	F	T	F	F	T	T	F	T	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
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- This means that:
  - KB $\models \neg P_{12}$.
  - KB $\not\models \neg P_{22}$.

Propositional logic

Definitions

- ***Validity:*** a sentence α is valid if it's TRUE in all models.
  - Ex.: $P \vee \neg P$ is a valid sentence.
  - Valid sentences are also known as **tautologies**.

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  - Ex.: KB, which equals $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$, is satisfiable because it's TRUE in three models.

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- Relationship between validity and satisfiability:

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  1. α is valid iff $\neg\alpha$ is unsatisfiable.

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- Relationship between validity and satisfiability:
  1. α is valid iff $\neg\alpha$ is unsatisfiable.
  2. α is satisfiable iff $\neg\alpha$ is not valid.
  3. $\alpha \models \beta$ iff $(\alpha \wedge \neg\beta)$ is unsatisfiable.

     → "proof by refutation (or contradiction)" technique.

Propositional logic

Definitions

Logical equivalency: Two sentences α and β are logically equivalent iff they are TRUE in the same set of models.

$$\alpha \equiv \beta \text{ iff } \alpha \models \beta \text{ and } \beta \models \alpha$$

Propositional logic

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Standard logical equivalences

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional eliminatio
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Propositional logic

Inferencing with rules

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Propositional logic

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Propositional logic

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- This method is **sound** (given that inference rules are sound) but **might not be complete** (depending on available inference rules).

Propositional logic

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- Inferencing-with-truth-tables is sound and complete.

Propositional logic

Inferencing with rules

Inference Rules

Rule Name	Premises	Derived Conclusion
Modus Ponens	$A, A \Rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Unit Resolution	$A \vee B, \neg B$	A
Resolution	$A \vee B, \neg B \vee C$	$A \vee C$

Additional rules based on logical equivalences

Biconditional Elimination	$A \Leftrightarrow B$	$(A \Rightarrow B) \wedge (B \Rightarrow A)$
Biconditional Introduction	$(A \Rightarrow B) \wedge (B \Rightarrow A)$	$A \Leftrightarrow B$
Implication Elimination	$A \Rightarrow B$	$\neg A \vee B$
Implication Introduction	$\neg A \vee B$	$A \Rightarrow B$
$\vdots$	$\vdots$	$\vdots$

Wumpus world

Ex.: Wumpus world - inferencing with rules

Show that $KB \vDash \neg P_{12}$.

R_1: $\neg P_{11}$.

R_2: $B_{11} \Leftrightarrow (P_{12} \vee P_{21})$.

R_3: $B_{21} \Leftrightarrow (P_{11} \vee P_{22} \vee P_{31})$.

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- Apply biconditional elimination to R_2.

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R_4: $\neg B_{11}$.

R_5: $\neg B_{21}$.

R_6: $(B_{11} \Rightarrow (P_{12} \vee P_{21})) \wedge ((P_{12} \vee P_{21}) \Rightarrow B_{11})$.

- Apply biconditional elimination to R_2.

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R_6: $(B_{11} \Rightarrow (P_{12} \vee P_{21})) \wedge ((P_{12} \vee P_{21}) \Rightarrow B_{11})$.

- Apply biconditional elimination to R_2.
- Apply And-elimination to R_6.

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R_6: $(B_{11} \Rightarrow (P_{12} \vee P_{21})) \wedge ((P_{12} \vee P_{21}) \Rightarrow B_{11})$.

R_7: $(P_{12} \vee P_{21}) \Rightarrow B_{11}$.

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- Apply biconditional elimination to R_2.
- Apply And-elimination to R_6.
- Apply contraposition to R_7.

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R_6: $(B_{11} \Rightarrow (P_{12} \vee P_{21})) \wedge ((P_{12} \vee P_{21}) \Rightarrow B_{11})$.

R_7: $(P_{12} \vee P_{21}) \Rightarrow B_{11}$.

R_8: $\neg B_{11} \Rightarrow \neg(P_{12} \vee P_{21})$.

- Apply biconditional elimination to R_2.
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- Apply biconditional elimination to R_2.
- Apply And-elimination to R_6.
- Apply contraposition to R_7.
- Apply Modes Ponens to R_4 and R_8.

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R_7: $(P_{12} \vee P_{21}) \Rightarrow B_{11}$.

R_8: $\neg B_{11} \Rightarrow \neg(P_{12} \vee P_{21})$.

R_9: $\neg(P_{12} \vee P_{21})$.

- Apply biconditional elimination to R_2.
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- Searching for a proof can be more efficient than enumerating models (*i.e.*, inferencing with truth tables).
  - Reason: irrelevant propositions are ignored.

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Inferencing using resolution

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  4. Distribute $\vee$ over $\wedge$ wherever possible.

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- In other words, we want to show that $KB \models \alpha$:
  - We know that $KB \models \alpha \equiv KB \Rightarrow \alpha \equiv \neg KB \vee \alpha$.
  - So we could show that $KB \wedge \neg\alpha$ is unsatisfiable.

Propositional logic

Inferencing using resolution

Resolution algorithm: returns true iff $KB \wedge \neg\alpha$ is unsatisfiable.

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
      α, the query, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  $new \leftarrow \{\,\}$
  loop do
    for each pair of clauses C_i, C_j **in** $clauses$ **do**
      $resolvents \leftarrow$ PL-RESOLVE(C_i, C_j)
      if $resolvents$ contains the empty clause **then return** *true*
      $new \leftarrow new \cup resolvents$
    if $new \subseteq clauses$ **then return** *false*
    $clauses \leftarrow clauses \cup new$

Propositional logic

Ex.: Wumpus world - inferencing using resolution

- So, consider the shown case, the KB consists of:

  R_2: $B_{11} \Leftrightarrow (P_{12} \vee P_{21})$.

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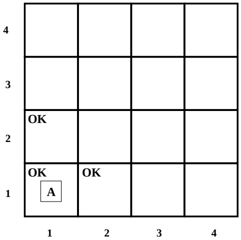

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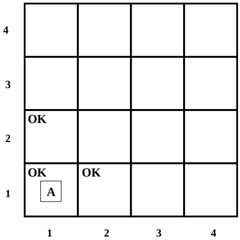

Propositional logic

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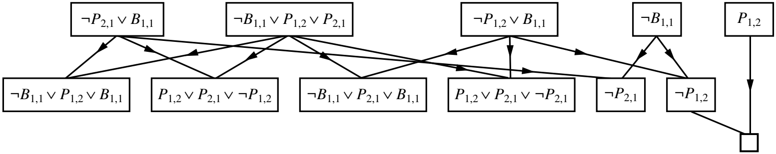

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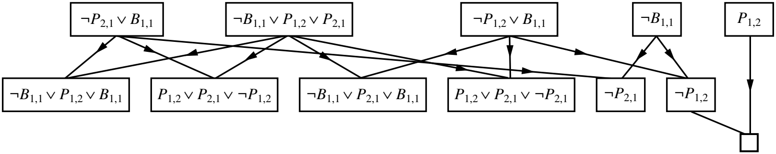

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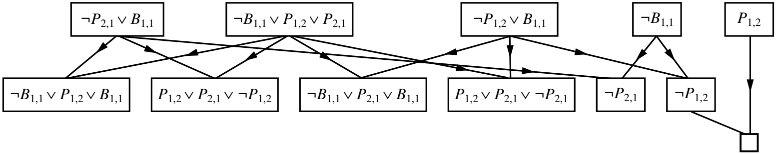
- When we resolve P_{12} and $\neg P_{12}$ we get the empty clause.



Propositional logic

Definite and Horn clauses

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- KBs with only DCs are interesting for 3 reasons:
  1. Every DC could be written as an implication:
     $$\neg l_1 \vee \neg l_2 \vee ..\neg l_k \vee l_m \equiv \neg(l_1 \wedge l_2 \wedge ..l_k) \vee l_m$$
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  2. Inference with HCs could be done through ***Forward chaining*** or ***Backward chaining*** algorithms that are easy to follow by humans.
  3. Deciding entailment could be done in linear time.

Propositional logic

Forward chaining

- It's a ***data driven*** inferencing approach, starts from what we know until it reaches the goal.

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  - Repeat until question answered, or nothing else to do.

Propositional logic

Forward chaining

Forward-chaining algorithm: returns true iff $KB \models q$

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional definite clauses
          q, the query, a proposition symbol
  count ← a table, where count[c] is the number of symbols in c's premise
  inferred ← a table, where inferred[s] is initially false for all symbols
  agenda ← a queue of symbols, initially symbols known to be true in KB

  while agenda is not empty do
     p ← POP(agenda)
     if p = q then return true
     if inferred[p] = false then
        inferred[p] ← true
        for each clause c in KB where p is in c.PREMISE do
           decrement count[c]
           if count[c] = 0 then add c.CONCLUSION to agenda
  return false
```

Propositional logic

Ex.: forward chaining

- Forward chaining is visualized using ***AND-OR graphs***.
- Ex.: show that $KB \models Q$, given KB contains: A, B, $A \wedge B \Rightarrow L$, $A \wedge P \Rightarrow L$, $B \wedge L \Rightarrow M$, $L \wedge M \Rightarrow P$, $P \Rightarrow Q$.

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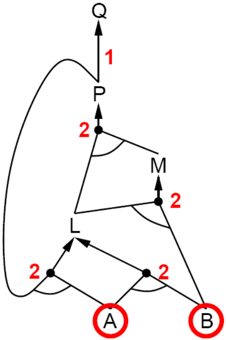

agenda = [A, B]

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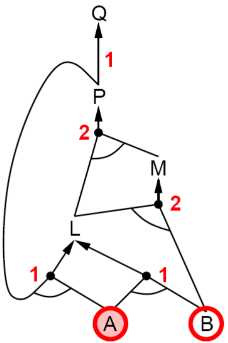

agenda = [B]

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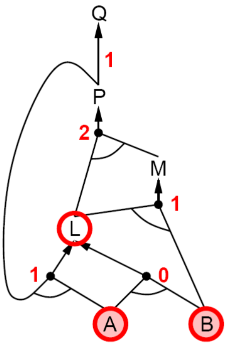

agenda = [L]

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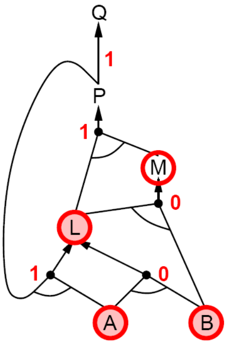

agenda = [M]

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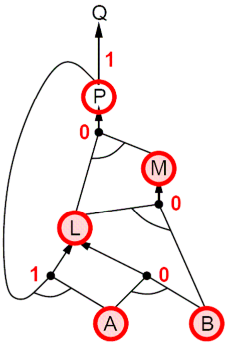

agenda = [P]

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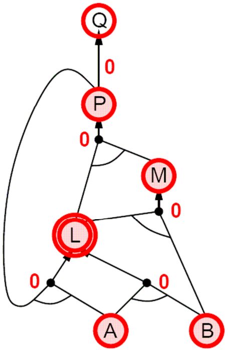

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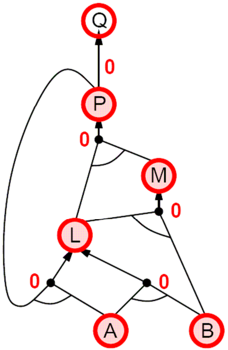

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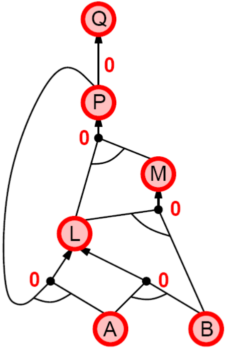

agenda = []

Weather Forecasting

Ex.: weather forecasting - problem statement

- Let's say you have three propositional symbols:
  - ***P:*** It's hot.
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Weather Forecasting

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- Is it raining?

Weather Forecasting

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- KB consists of:

Weather Forecasting

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    R_2**:** $Q \Rightarrow P.$

Weather Forecasting

Ex.: weather forecasting - the KB

- KB consists of:
  - **Rule: If** it is hot and humid, **then** it is raining:

    R_1**:** $P \wedge Q \Rightarrow R.$
  - **Rule: If** it is humid, **then** it is hot:

    R_2**:** $Q \Rightarrow P.$
  - **Fact:** It is humid:

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    R_3**:** Q.

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  - **Fact:** It is humid:

    R_3: Q.
- The KB can be represented as the conjunction of all the sentences: $R_1 \wedge R_2 \wedge R_3$.

Weather Forecasting

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    R_2**:** $Q \Rightarrow P$.
  - **Fact:** It is humid:

    R_3**:** Q.
- The KB can be represented as the conjunction of all the sentences: $R_1 \wedge R_2 \wedge R_3$.
- Goal: to show whether $KB \models R$.

Weather Forecasting

Ex.: weather forecasting - inferencing with TTs

- We check whether $KB \models R$ by checking whether R is true in every model in which KB is true.

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Weather Forecasting

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- This is the same as checking whether $KB \Rightarrow R$ is valid.

P, Q, R	$P \wedge Q \Rightarrow R$	$Q \Rightarrow P$	Q	KB	R	$KB \Rightarrow R$
T, T, T	T	T	T	T	T	T
T, T, F	F	T	T	F	F	T
T, F, T	T	T	F	F	T	T
T, F, F	T	T	F	F	F	T
F, T, T	T	F	T	F	T	T
F, T, F	T	F	T	F	F	T
F, F, T	T	T	F	F	T	T
F, F, F	T	T	F	F	F	T

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P, Q, R	$P \wedge Q \Rightarrow R$	$Q \Rightarrow P$	Q	KB	R	$KB \Rightarrow R$
T, T, T	T	T	T	T	T	T
T, T, F	F	T	T	F	F	T
T, F, T	T	T	F	F	T	T
T, F, F	T	T	F	F	F	T
F, T, T	T	F	T	F	T	T
F, T, F	T	F	T	F	F	T
F, F, T	T	T	F	F	T	T
F, F, F	T	T	F	F	F	T

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- This is the same as checking whether $KB \Rightarrow R$ is valid.

P, Q, R	$P \wedge Q \Rightarrow R$	$Q \Rightarrow P$	Q	KB	R	$KB \Rightarrow R$
T, T, T	T	T	T	T	T	T
T, T, F	F	T	T	F	F	T
T, F, T	T	T	F	F	T	T
T, F, F	T	T	F	F	F	T
F, T, T	T	F	T	F	T	T
F, T, F	T	F	T	F	F	T
F, F, T	T	T	F	F	T	T
F, F, F	T	T	F	F	F	T

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

R_1: $P \wedge Q \Rightarrow R$.

R_2: $Q \Rightarrow P$.

R_3: Q.

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

R_1: $P \wedge Q \Rightarrow R$.

R_2: $Q \Rightarrow P$.

R_3: Q.

- Apply Modes Ponens to R_2 and R_3.

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

R_1: $P \land Q \Rightarrow R$.

R_2: $Q \Rightarrow P$.

R_3: Q.

R_4: P.

- Apply Modes Ponens to R_2 and R_3.

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- Apply Modes Ponens to R_2 and R_3.
- Apply And-introduction to R_3 and R_4.

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

R_1: $P \wedge Q \Rightarrow R$.

R_2: $Q \Rightarrow P$.

R_3: Q.

R_4: P.

R_5: $P \wedge Q$.

- Apply Modes Ponens to R_2 and R_3.
- Apply And-introduction to R_3 and R_4.

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

R_1: $P \wedge Q \Rightarrow R$.

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R_3: Q.

R_4: P.

R_5: $P \wedge Q$.

- Apply Modes Ponens to R_2 and R_3.
- Apply And-introduction to R_3 and R_4.
- Apply Modes Ponens R_1 and R_5.

Weather Forecasting

Ex.: weather forecasting - inferencing with rules

Showing $KB \models R$ using the inference-rules method:

R_1: $P \wedge Q \Rightarrow R$.

R_2: $Q \Rightarrow P$.

R_3: Q.

R_4: P.

R_5: $P \wedge Q$.

R_6: R.

- Apply Modes Ponens to R_2 and R_3.
- Apply And-introduction to R_3 and R_4.
- Apply Modes Ponens R_1 and R_5.

Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \vDash R$ using the resolution method:

Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \vDash R$ using the resolution method:

- Put KB in CNF:

Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

- Put KB in CNF:

  R_1**:** $P \wedge Q \Rightarrow R$

Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

- Put KB in CNF:

$$\begin{aligned} R_1\textbf{:}\ & P \wedge Q \Rightarrow R \\ & \equiv \neg(Q \wedge P) \vee R \end{aligned}$$

Weather Forecasting

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Showing $KB \models R$ using the resolution method:

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Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

- Put KB in CNF:

$$
\begin{aligned}
R_1\textbf{:}\quad & P \wedge Q \Rightarrow R \\
& \equiv \neg(Q \wedge P) \vee R \\
& \equiv \neg Q \vee \neg P \vee R. \\
R_2\textbf{:}\quad & Q \Rightarrow P \\
& \equiv \neg Q \vee P.
\end{aligned}
$$

Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

- Put KB in CNF:

$$
\begin{aligned}
R_1\text{:}\quad & P \wedge Q \Rightarrow R \\
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& \equiv \neg Q \vee \neg P \vee R. \\
R_2\text{:}\quad & Q \Rightarrow P \\
& \equiv \neg Q \vee P. \\
R_3\text{:}\quad & Q.
\end{aligned}
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Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

- Put KB in CNF:

$$R_1\text{:}\ P \wedge Q \Rightarrow R \equiv \neg(Q \wedge P) \vee R \equiv \neg Q \vee \neg P \vee R.$$

$$R_2\text{:}\ Q \Rightarrow P \equiv \neg Q \vee P.$$

$$R_3\text{:}\ Q.$$

- Add the negation of what we want to prove:

Weather Forecasting

Ex.: weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

- Put KB in CNF:

$$
\begin{aligned}
R_1\text{:}\ & P \wedge Q \Rightarrow R \\
& \equiv \neg(Q \wedge P) \vee R \\
& \equiv \neg Q \vee \neg P \vee R. \\
R_2\text{:}\ & Q \Rightarrow P \\
& \equiv \neg Q \vee P. \\
R_3\text{:}\ & Q.
\end{aligned}
$$

- Add the negation of what we want to prove:

$$R_4\text{:}\ \neg R.$$

- Show whether $KB \wedge \neg R$ is unsatisfiable.

Weather Forecasting

Ex.: Weather forecasting - inferencing by resolution

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Weather Forecasting

Ex.: Weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

R_1: $\neg Q \vee \neg P \vee R$.

R_2: $\neg Q \vee P$.

R_3: Q.

R_4: $\neg R$.

Weather Forecasting

Ex.: Weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

R_1: $\neg Q \vee \neg P \vee R$.

R_2: $\neg Q \vee P$.

R_3: Q.

R_4: $\neg R$.

- Resolve R_2 and R_3.

Weather Forecasting

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Showing $KB \models R$ using the resolution method:

R_1: $\neg Q \vee \neg P \vee R$.

R_2: $\neg Q \vee P$.

R_3: Q.

R_4: $\neg R$.

R_5: P.

- Resolve R_2 and R_3.

Weather Forecasting

Ex.: Weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

R_1: $\neg Q \lor \neg P \lor R$.

R_2: $\neg Q \lor P$.

R_3: Q.

R_4: $\neg R$.

R_5: P.

- Resolve R_2 and R_3.
- Resolve R_1 and R_3.

Weather Forecasting

Ex.: Weather forecasting - inferencing by resolution

Showing $KB \models R$ using the resolution method:

R_1: $\neg Q \lor \neg P \lor R$.

R_2: $\neg Q \lor P$.

R_3: Q.

R_4: $\neg R$.

R_5: P.

R_6: $\neg P \lor R$.

- Resolve R_2 and R_3.
- Resolve R_1 and R_3.

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Showing $KB \models R$ using the resolution method:

R_1: $\neg Q \vee \neg P \vee R$.

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R_4: $\neg R$.

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R_3: Q.

R_4: $\neg R$.

R_5: P.

R_6: $\neg P \vee R$.

R_7: R.

- Resolve R_2 and R_3.
- Resolve R_1 and R_3.
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R_6: $\neg P \vee R$.

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R_2: $\neg Q \vee P$.

R_3: Q.

R_4: $\neg R$.

R_5: P.

R_6: $\neg P \vee R$.

R_7: R.

R_8: $false$.

resolve returns $\{\}$

- Resolve R_2 and R_3.
- Resolve R_1 and R_3.
- Resolve R_5 and R_6.
- Resolve R_4 and R_7.

Weather Forecasting

Ex.: Weather forecasting - inferencing with FC

Showing $KB \vDash R$ using the forward-chaining method:

- KB contains: $R_1 : P \wedge Q \Rightarrow R$, $R_2 : Q \Rightarrow P$, $R_3 : Q$.

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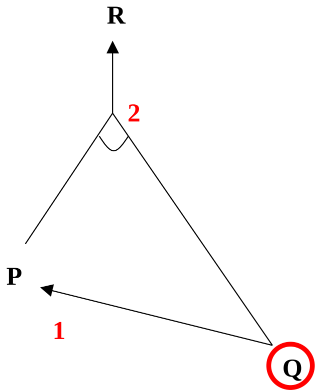

agenda = [Q]

Weather Forecasting

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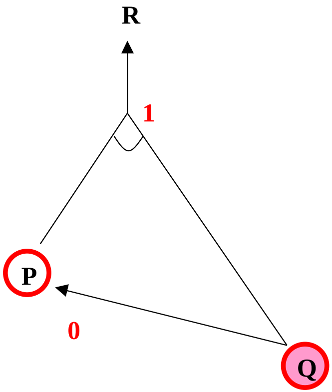

agenda = [P]

Weather Forecasting

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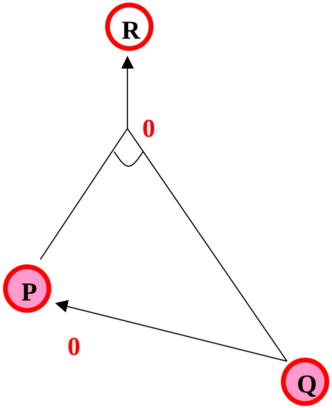

agenda = [R]

Weather Forecasting

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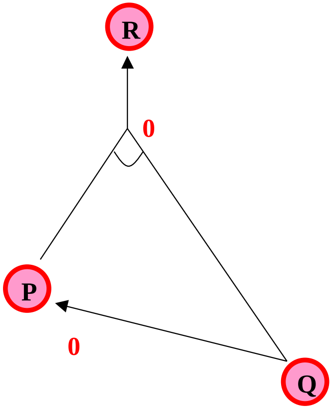

agenda = []

Outline

Requirements

What do I need from you

- When given a certain problem you should be able to:

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  - Express the problem in terms of propositional logic (i.e. write the KB in form of rules or CNF).

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  - Draw AND-OR graph of the KB.

Requirements

What do I need from you

- When given a certain problem you should be able to:
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  - Inference certain conclusions using:
    - Truth tables.
    - Rules.
    - Resolution.
    - Forward chaining.
    - Backward chaining.

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What do I need from you

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  - Inference certain conclusions using:
    - Truth tables.
    - Rules.
    - Resolution.
    - Forward chaining.
    - Backward chaining.
- Answer descriptive questions.

Reading Material

Which parts of the textbook are covered

- Russell-Norvig, Chapters 7:
  - Pages 249 - 259.