

CS 211 - Digital Logic Design 211 عال - تصميم المنطق الرقمي

First Term - 1439/1440
Lecture #5

Dr. Hazem Ibrahim Shehata

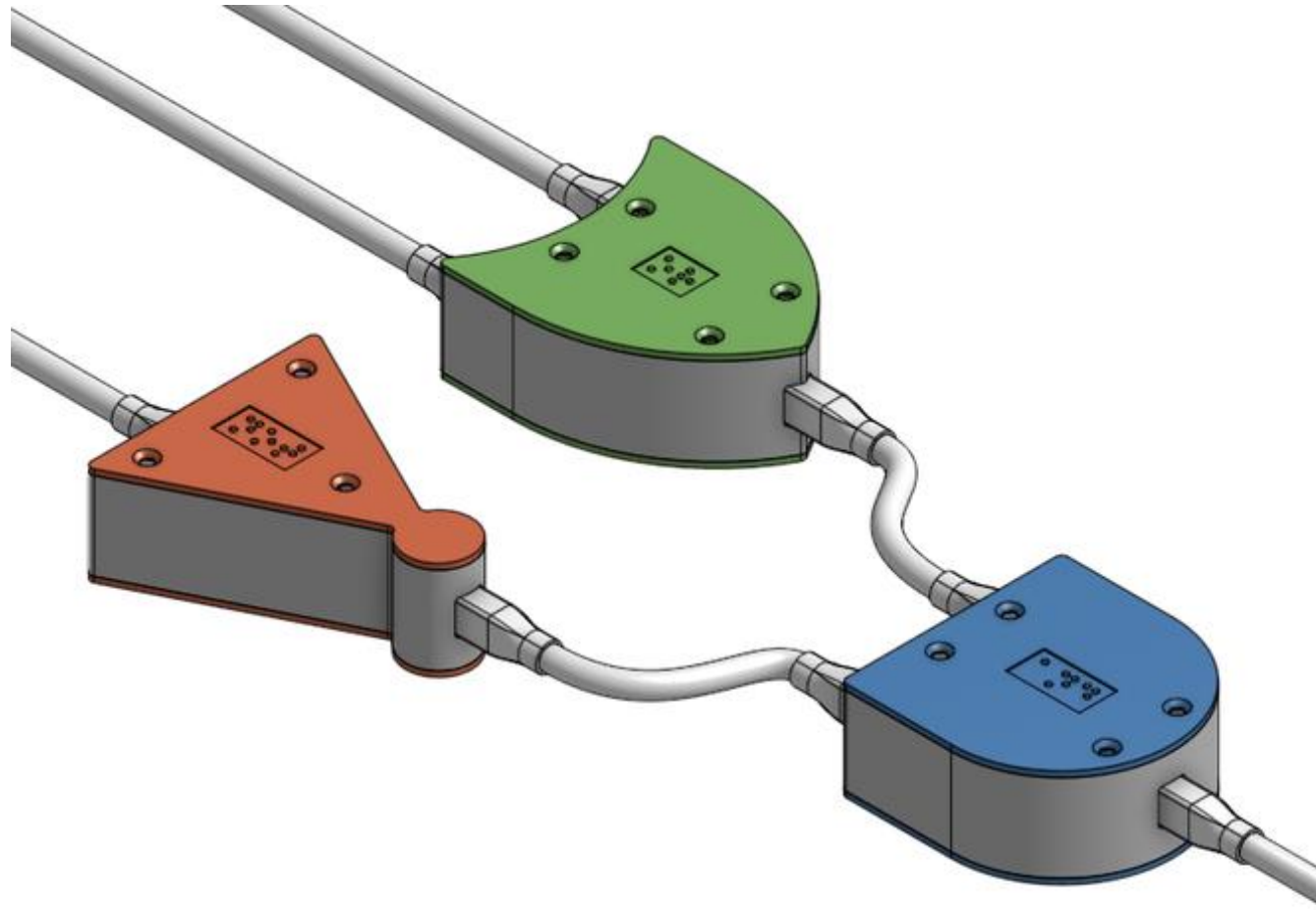
Assistant Professor

College of Computing and Information Technology

Administrivia

- Assignment #1:
 - Released on Sunday.
 - Due: **Sunday, October 7, 2018.**

Website: <http://hshehata.github.io/courses/su/cs211>

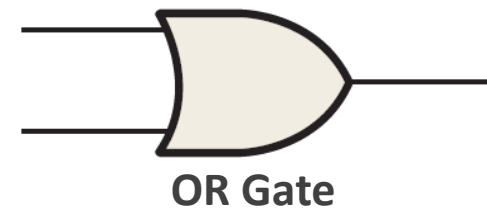
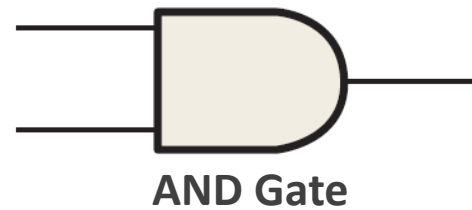
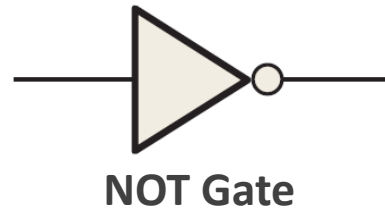


Chapter 3: Logic Gates

Logic Gates/Circuits

➤ **Logic Gate**: electronic device implementing a basic **logical operation** on **1+ binary input** and producing **1 binary output**.

◦ **Examples:**



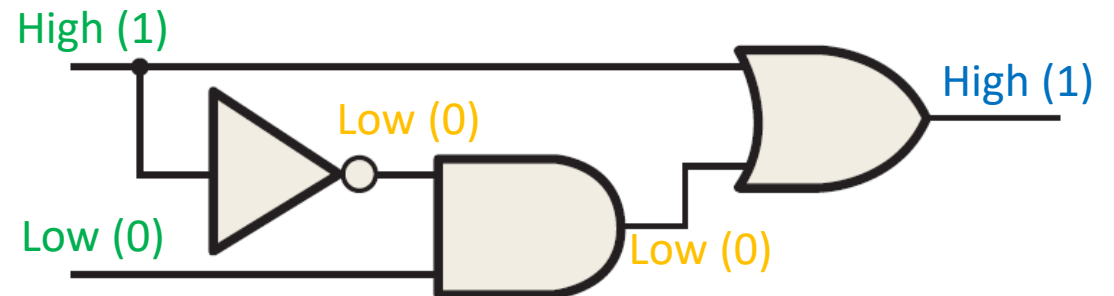
➤ **Logic Circuit**: electronic circuit built out of logic gates!

◦ Each wire carries a single bit represented in terms of voltage level.

◦ **High** voltage level (e.g., 5v) ➔ logical **1**.

◦ **Low** voltage level (e.g., 0v) ➔ logical **0**.

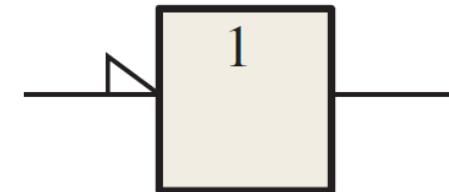
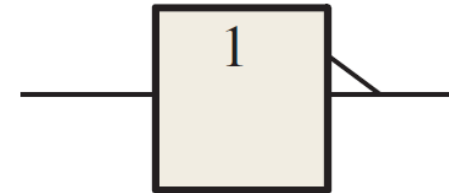
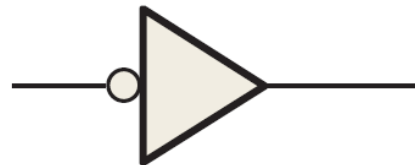
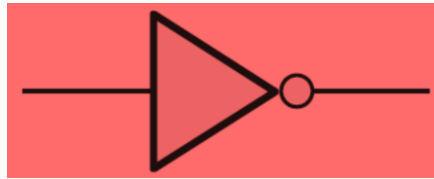
◦ **Example:**



Inverter (or NOT Gate)

- Performs operation called **inversion** or **complementation**.
 - Takes 1 (single-bit) input, and produces 1 (single-bit) output.
 - Output value equals the inverse of input value.

- Symbols:



Operation of Inverter

➤ Possible scenarios:

- Low voltage applied to input → High voltage produced at output
- High voltage applied to input → Low voltage produced at output

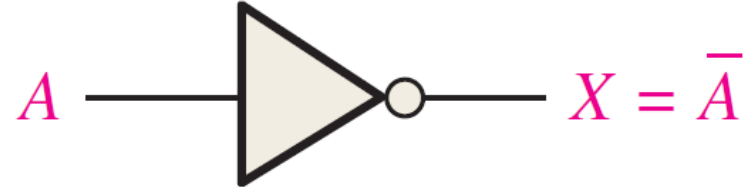
➤ Operation can be represented as a **Truth Table**, that lists all input combinations with the corresponding outputs!

Input	Output
Low (0)	High (1)
High (1)	Low (0)

Logic Expression of Inverter

➤ **Boolean Algebra**: is a type of mathematics that uses variables and operators to describe logic circuits.

➤ Boolean expression that describes inverter is: $X = \bar{A}$



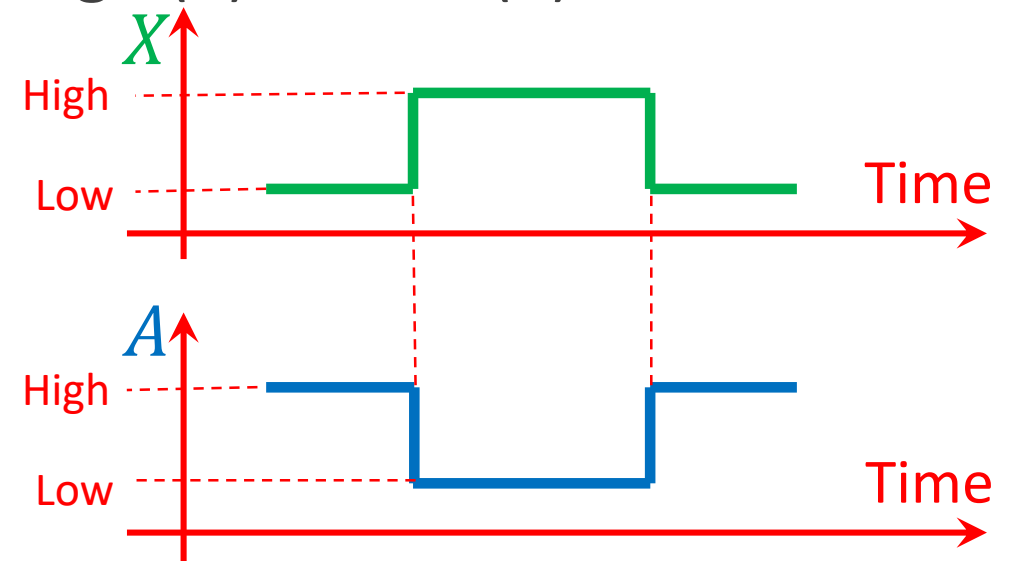
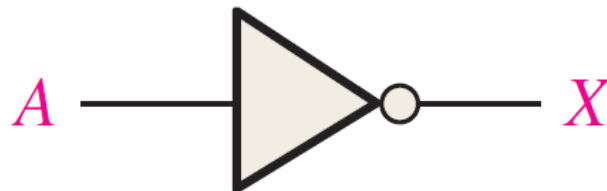
- If $A = 0$, then $X = \bar{0} = 1$
- If $A = 1$, then $X = \bar{1} = 0$

➤ Complemented variable \bar{A} is read as: “A bar” or “not A”.

Timing Diagram of Inverter

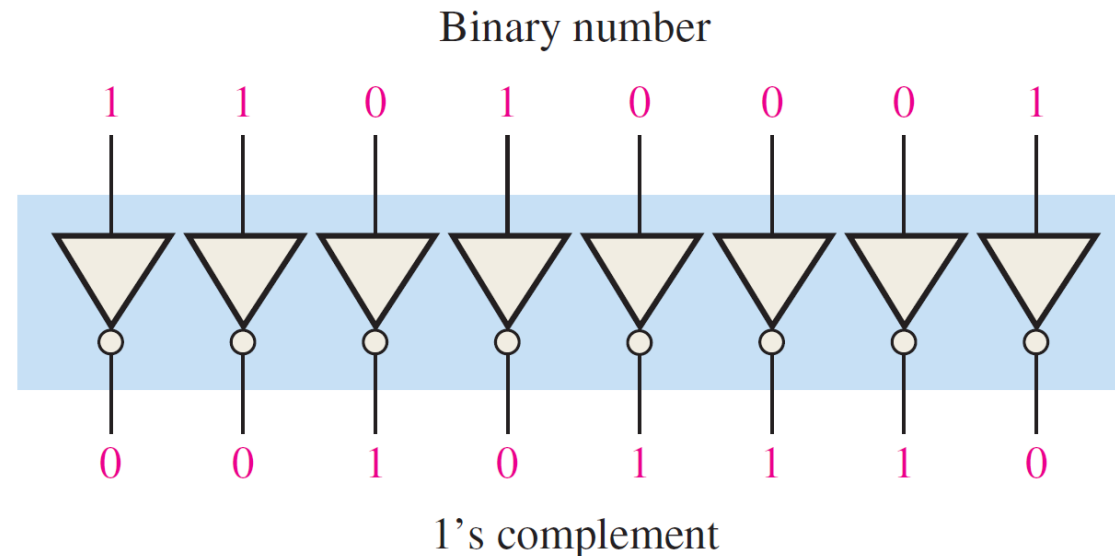
- Timing Diagram: represents how signals of a logic circuit change over time.
 - X-axis → time.
 - Y-axis → signals values. Either High (1) or Low (0).

➤ Example:



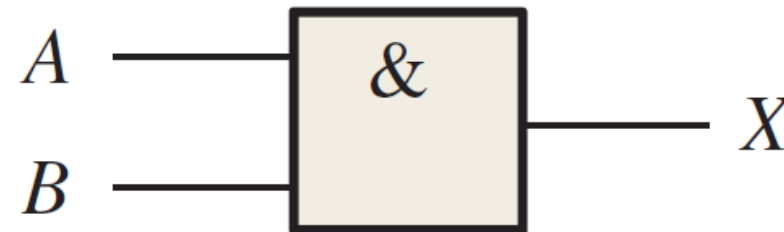
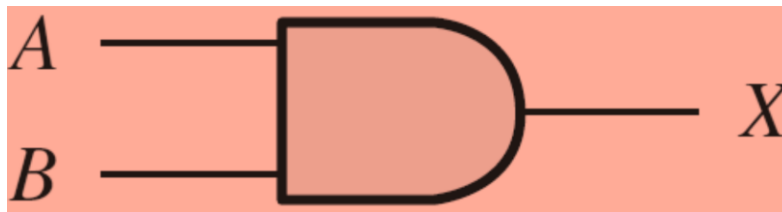
Application of Inverter

- Inverters can be used to calculate 1's complement of binary numbers.
- **Example:** Circuit to produce 1's comp. of an 8-bit number.



AND Gate

- Performs operation called **logical multiplication**.
 - Takes 2⁺ inputs, and produces 1 output.
 - Output value is **High (1)** if and only if **all inputs** are **High (1)**.
 - Output value is **Low (0)** if and only if **1⁺ inputs** are **Low (0)**.
- Symbols:



AND Gate Truth Table

➤ For a 2-input AND Gate:

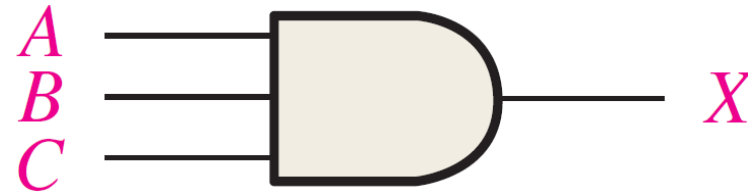


Inputs		Output
<i>A</i>	<i>B</i>	<i>X</i>
0	0	0
0	1	0
1	0	0
1	1	1

➤ **Note:** Number of possible binary input combinations to an n -input gate is: 2^n → Truth table must have 2^n rows!

Example: Truth Table for 3-input AND Gate

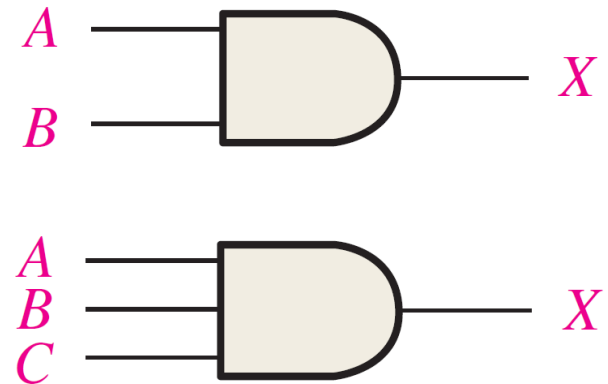
$$n = 3 \rightarrow N = 2^3 = 8$$



Inputs			Output
<i>A</i>	<i>B</i>	<i>C</i>	<i>X</i>
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

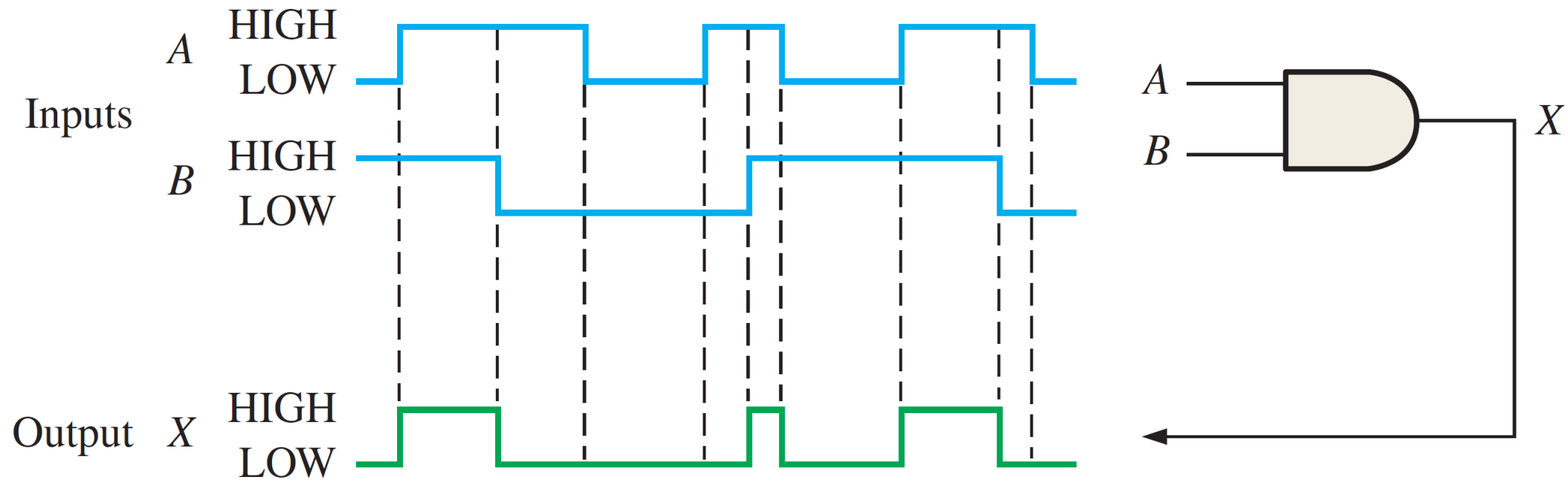
Logic Expression of AND Gate

- In Boolean algebra, AND gate is represented using **Boolean multiplication** operator \rightarrow “.”
 - Boolean multiplication is similar to binary multiplication:
 - $0 \cdot 0 = 0, 0 \cdot 1 = 0, 1 \cdot 0 = 0, 1 \cdot 1 = 1$
- Boolean expression for 2-input AND gate is:
 - $X = A \cdot B$, or $X = AB$
- Boolean expression for 3-input AND gate is:
 - $X = A \cdot B \cdot C$, or $X = ABC$



Timing Diagram of AND Gate

➤ Example: 2-input AND Gate



Application of AND Gate

➤ Example: Seat Belt Alarm System

HIGH = On
LOW = Off

HIGH = Unbuckled
LOW = Buckled

Ignition
switch

A

Seat
belt

B

C

Timer

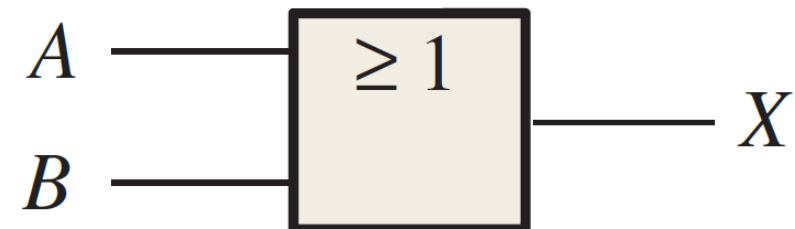
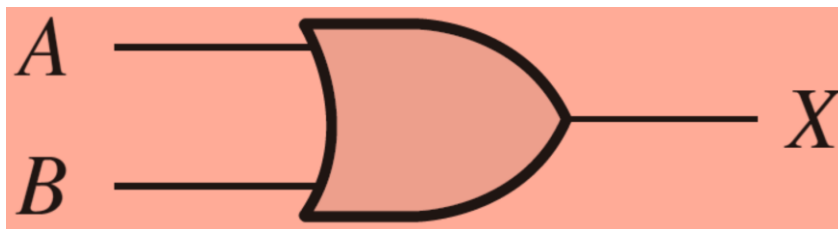
Audible
alarm
circuit

HIGH activates
alarm.

Ignition on = HIGH for 30 s

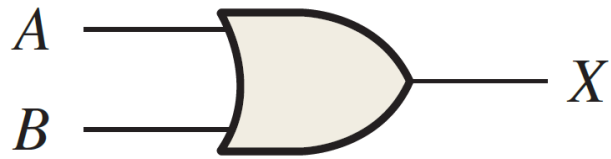
OR Gate

- Performs operation called **logical addition**.
 - Takes 2+ inputs, and produces 1 output.
 - Output value is **High (1)** if and only if **1+ inputs** are **High (1)**.
 - Output value is **Low (0)** if and only if **all inputs** are **Low (0)**.
- Symbols:



OR Gate Truth Table

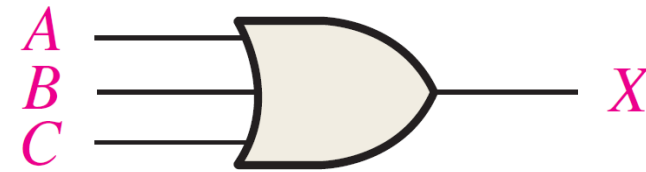
➤ For a 2-input OR Gate:



Inputs		Output
<i>A</i>	<i>B</i>	<i>X</i>
0	0	0
0	1	1
1	0	1
1	1	1

Example: Truth Table for 3-input OR Gate

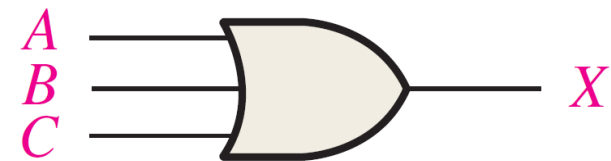
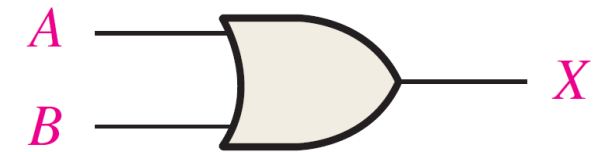
$$n = 3 \rightarrow N = 2^3 = 8$$



Inputs			Output
<i>A</i>	<i>B</i>	<i>C</i>	<i>X</i>
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

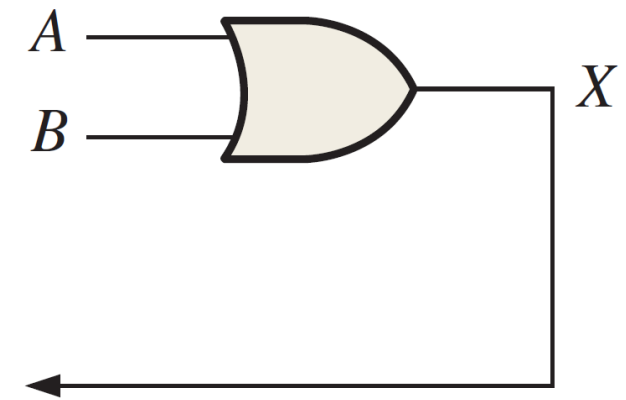
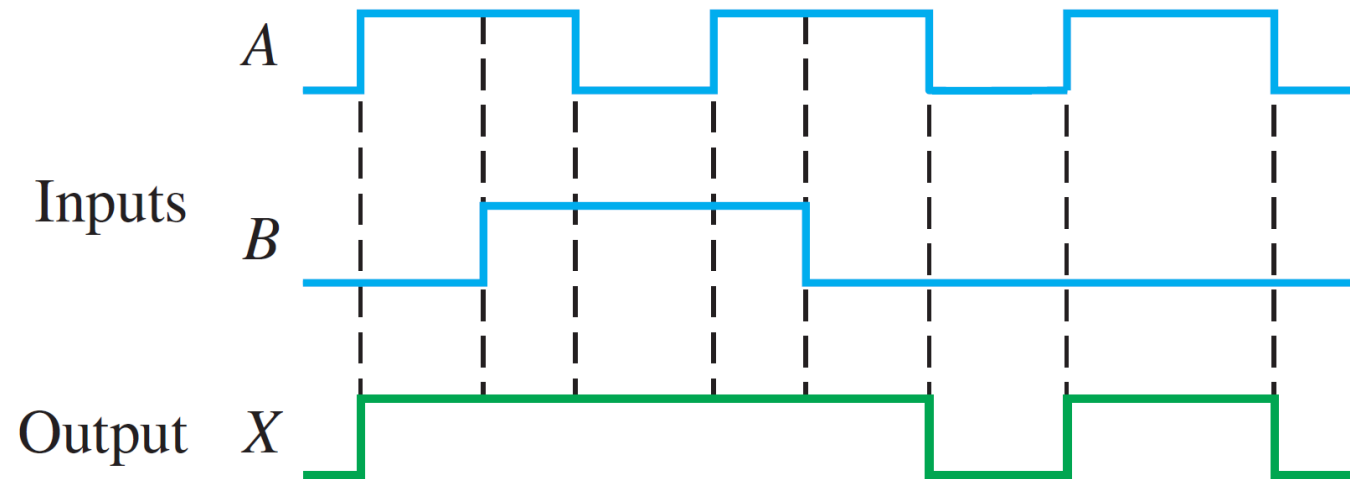
Logic Expression of OR Gate

- In Boolean algebra, OR gate is represented using **Boolean addition** operator \rightarrow “ + ”
 - Boolean addition differs from binary addition in one case ($1+1=?$):
 - $0 + 0 = 0$, $0 + 1 = 1$, $1 + 0 = 1$, **$1 + 1 = 1$**
- Boolean expression for 2-input AND gate is:
 - **$X = A + B$**
- Boolean expression for 3-input AND gate is:
 - **$X = A + B + C$**



Timing Diagram of OR Gate

➤ **Example:** 2-input OR Gate

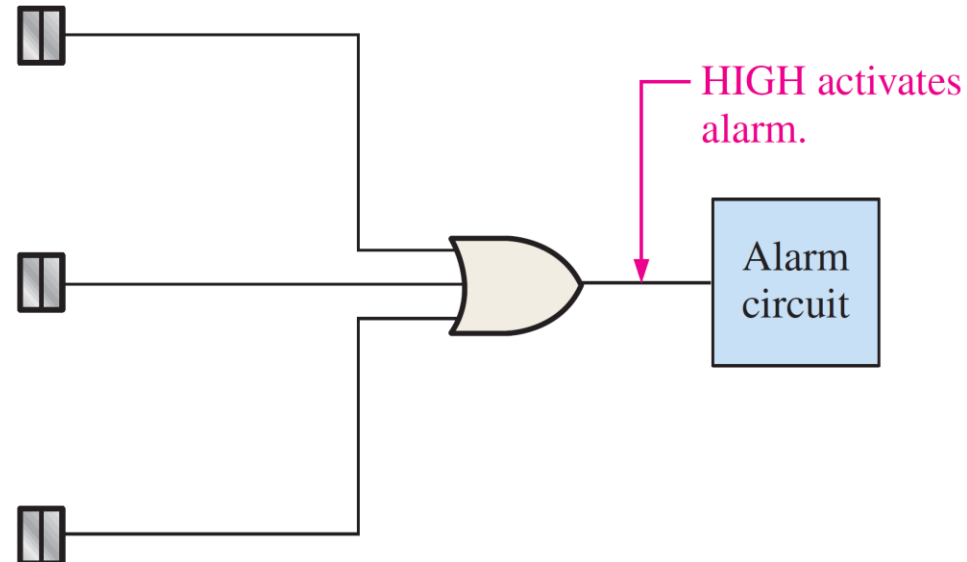


Application of OR Gate

➤ Example: Simplified Intrusion Detection System

Open door/window
sensors

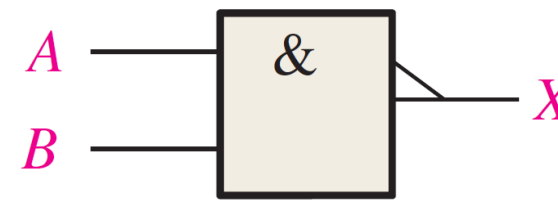
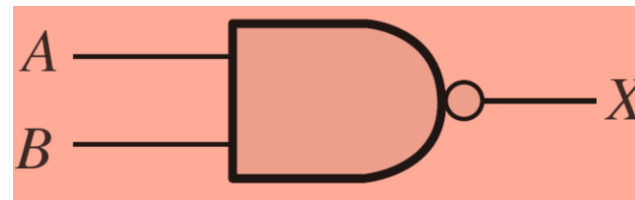
HIGH = Open
LOW = Closed



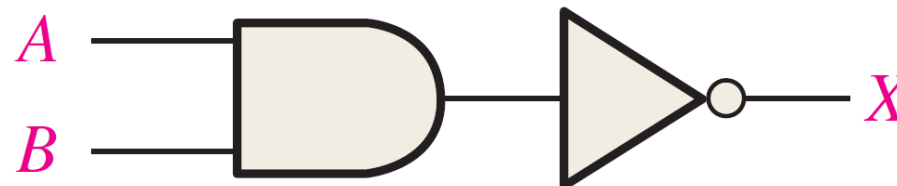
NAND Gate

- Contraction of **NOT-AND** → AND with an inverted output.
 - Takes 2+ inputs, and produces 1 output.
 - Output value is **Low (0)** if and only if **all inputs** are **High (1)**.
 - Output value is **High (1)** if and only if **1+ inputs** are **Low (0)**.

➤ Symbols:



➤ Equivalent to:



NAND Gate Truth Table

➤ For a 2-input NAND Gate:



Inputs		Output
<i>A</i>	<i>B</i>	<i>X</i>
0	0	1
0	1	1
1	0	1
1	1	0

➤ **Note:** NAND Gate is equivalent to Negative-OR Gate.



Logic Expression of NAND Gate

➤ In Boolean algebra, NAND gate is represented by multiplication combined with complementation → “ \cdot ”

➤ Boolean expression for 2-input NAND gate is:

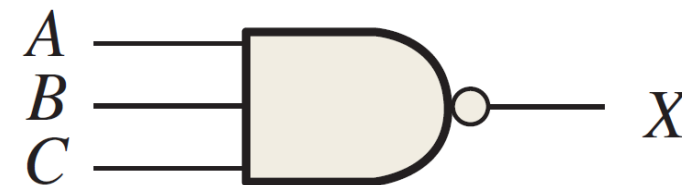
◦ $X = \overline{A \cdot B}$, or $X = \overline{AB}$

A	B	$\overline{AB} = X$
0	0	$\overline{0 \cdot 0} = \overline{0} = 1$
0	1	$\overline{0 \cdot 1} = \overline{0} = 1$
1	0	$\overline{1 \cdot 0} = \overline{0} = 1$
1	1	$\overline{1 \cdot 1} = \overline{1} = 0$



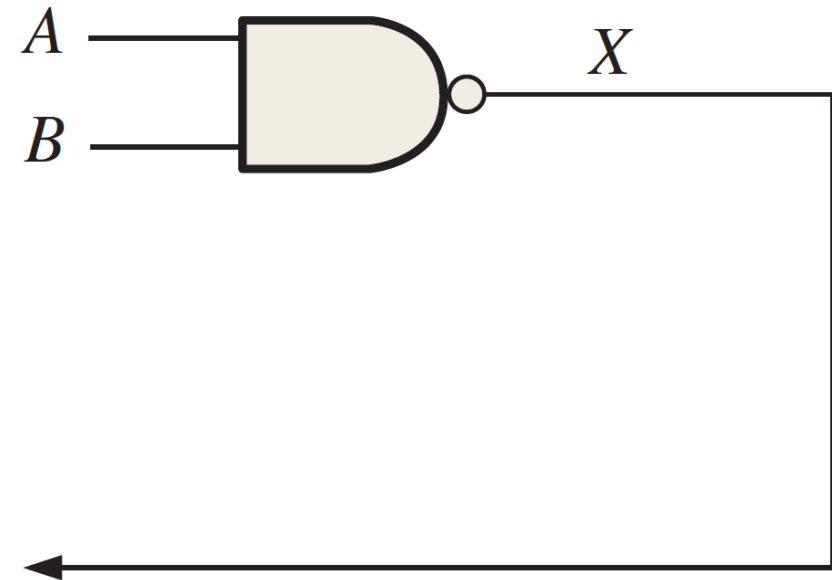
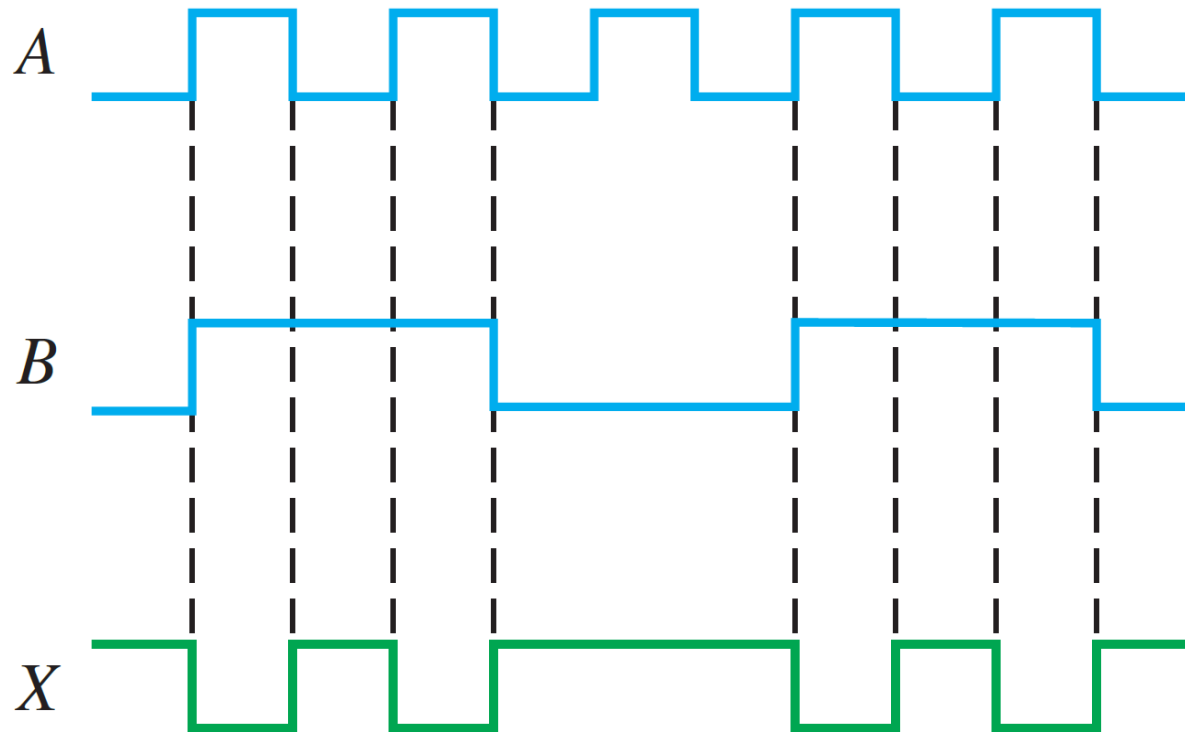
➤ Boolean expression for 3-input NAND gate is:

◦ $X = \overline{A \cdot B \cdot C}$, or $X = \overline{ABC}$



Timing Diagram of NAND Gate

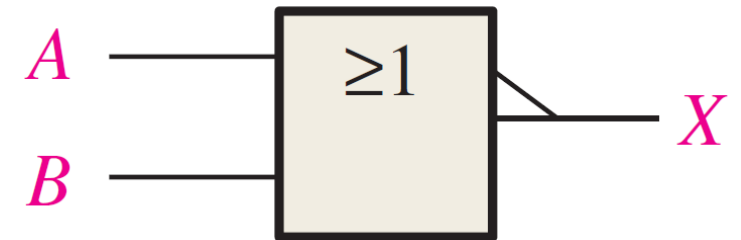
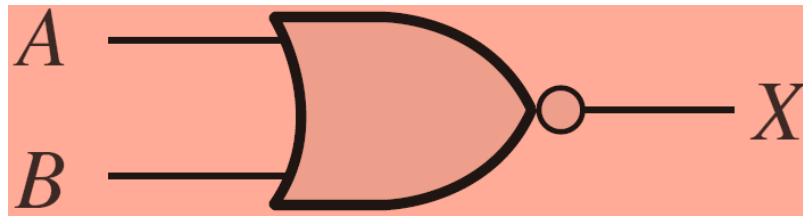
➤ **Example:** 2-input NAND Gate



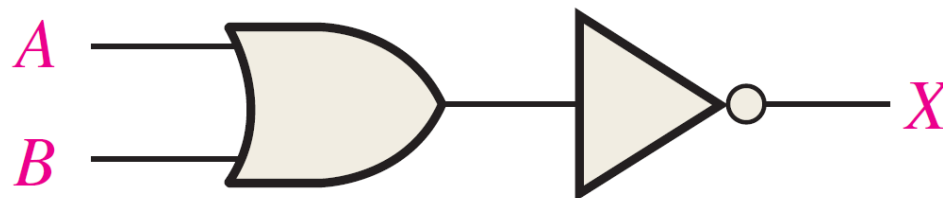
NOR Gate

- Contraction of **NOT-OR** → OR with an inverted output.
 - Takes 2+ inputs, and produces 1 output.
 - Output value is **Low (0)** if and only if 1+ inputs are **High (1)**.
 - Output value is **High (1)** if and only if **all inputs** are **Low (0)**.

➤ Symbols:

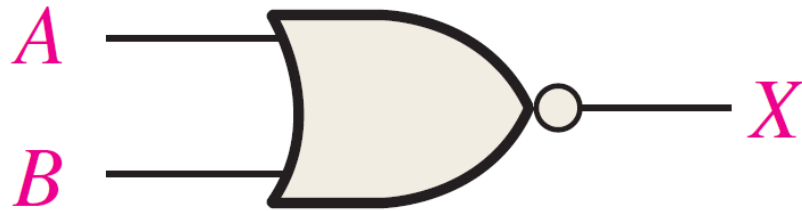


➤ Equivalent to:



NOR Gate Truth Table

➤ For a 2-input NOR Gate:



Inputs		Output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

➤ Note: NOR Gate is equivalent to Negative-AND Gate.



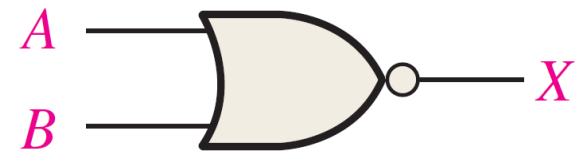
Logic Expression of NOR Gate

➤ In Boolean algebra, NOR gate is represented by addition combined with complementation → “ $\overline{+}$ ”

➤ Boolean expression for 2-input NOR gate is:

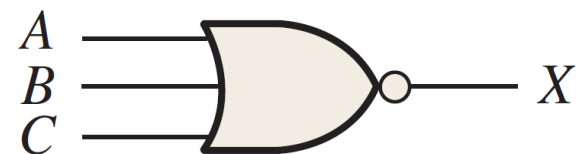
◦ $X = \overline{A + B}$

A	B	$\overline{A + B} = X$
0	0	$\overline{0 + 0} = \overline{0} = 1$
0	1	$\overline{0 + 1} = \overline{1} = 0$
1	0	$\overline{1 + 0} = \overline{1} = 0$
1	1	$\overline{1 + 1} = \overline{1} = 0$



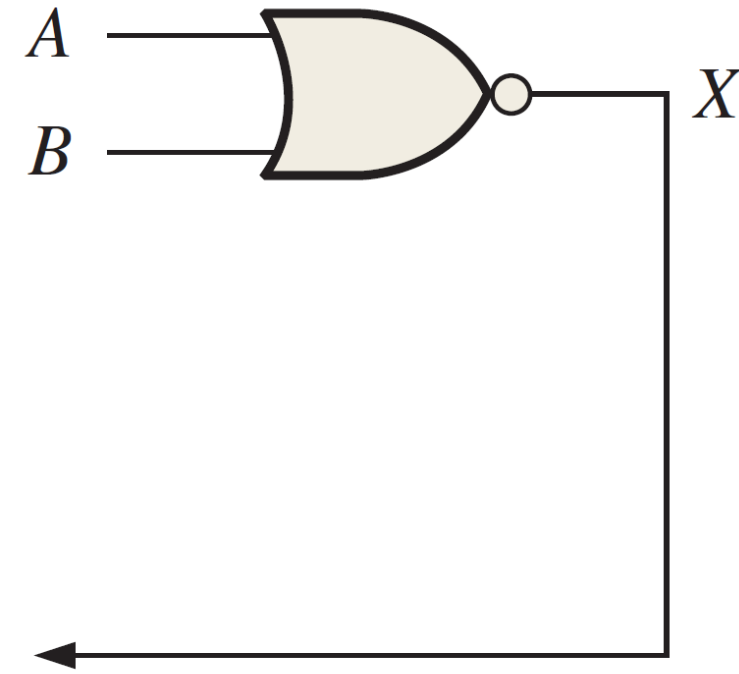
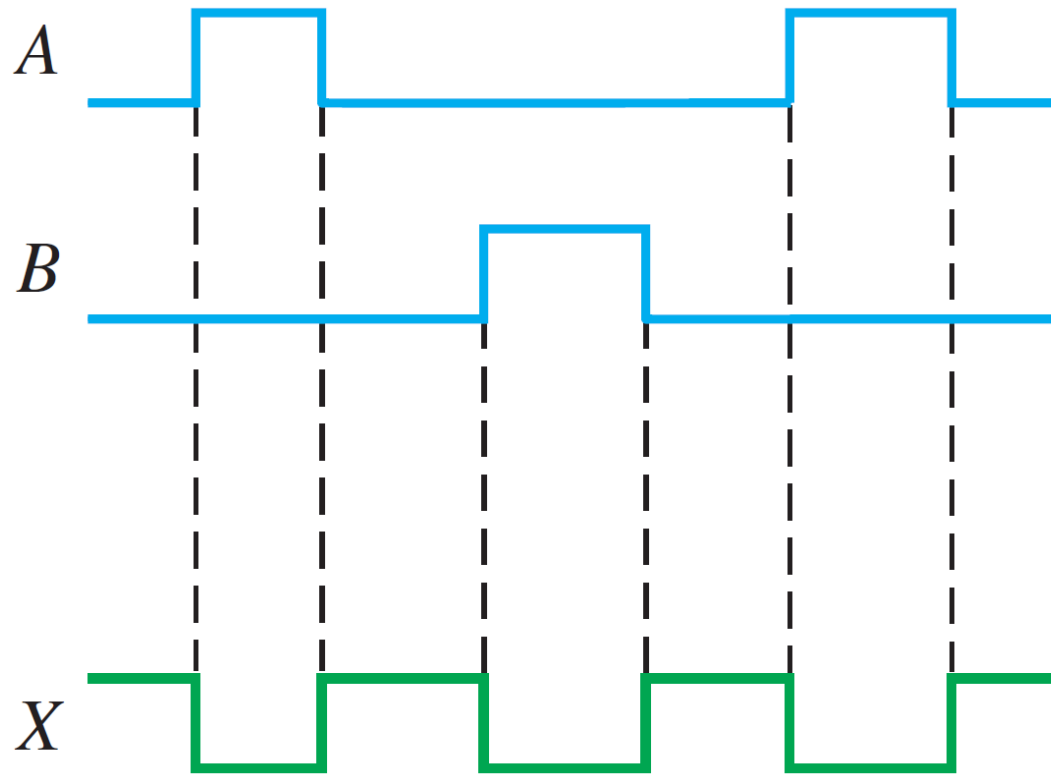
➤ Boolean expression for 3-input NOR gate is:

◦ $X = \overline{A + B + C}$



Timing Diagram of NOR Gate

➤ **Example:** 2-input NOR Gate



Reading Material

- Floyd, Chapter 3:
 - Pages 107 - 130