CSE 411: Artificial Intelligence (Elective Course #6)

400 Level, Mechatronics Engineering
2^{nd} Term 2016/2017, Lecture #6

Hazem Shehata

Dept. of Computer & Systems Engineering
Zagazig University

April 3^{rd}, 2017

Credits to Dr. Mohamed El Abd for the slides

Adminstrivia

Notes

- Assignment #2:
  - Due on Thursday.

Course Info:

- Website: http://hshehata.github.io/courses/zu/cse411/
- Office hours: Sunday 11:30am - 12:30pm

Adminstrivia

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- Assignment #2:
  - Due on Thursday.
- Assignment #3:
  - Released Today.
  - Due on Sunday, April 16, 2017.

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Outline

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1. Adversarial Search

2. Requirements & Reading Material

Adversarial search

Introduction

- Multi-agent environments:
  - Cooperative.
  - Competitive.

Adversarial search

Introduction

- Multi-agent environments:
  - Cooperative.
  - Competitive.
- In competitive environments, different agents have conflicting goals.

Adversarial search

Introduction

- Multi-agent environments:
  - Cooperative.
  - Competitive.
- In competitive environments, different agents have conflicting goals.
- This gives rise to ***adversarial search***, also known as ***games***.

Adversarial search

Introduction

- In games, other agents:
  - Want our agent to lose.
  - Introduce uncertainty, don't know what will they do.

Adversarial search

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- The unpredictability of other agents can introduce many possible ***contingencies***.

Adversarial search

Introduction

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  - Want our agent to lose.
  - Introduce uncertainty, don't know what will they do.
- The unpredictability of other agents can introduce many possible ***contingencies***.
- We will consider a specific type:
  - Deterministic.
  - Fully observable.
  - Two agents with alternating actions.

Adversarial search

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

Adversarial search

Introduction

- The main problem with games is that the search space is too huge:
  - A search tree for chess has an average branching factor of 35.
  - An average chess game lasts for 50 moves per player.
  - The average search tree has 35^{100} nodes.

Adversarial search

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- There is no enough time to search the whole tree and calculate the exact consequences of every move.
- If the optimal decision is not feasible (due to time or memory constraints), some decision has to be taken.

Adversarial search

Introduction

We will investigate:

- What is an optimal move and how to find it.

Adversarial search

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Adversarial search

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We will investigate:

- What is an optimal move and how to find it.
- Pruning the search tree.
- How to choose a good move when time is limited.
- How to define a heuristic evaluation function to approximate the utility of a state.

Adversarial search

Minimax Algorithm

- First thing to consider is how to make an optimal move.

Adversarial search

Minimax Algorithm

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- Certain assumptions exist:
  - A two player game.
  - The play is sequential (agents taking turns).
  - The opponent is playing perfectly.

Adversarial search

Minimax Algorithm

- First thing to consider is how to make an optimal move.
- Certain assumptions exist:
  - A two player game.
  - The play is sequential (agents taking turns).
  - The opponent is playing perfectly.
- The two players are referred to as **MAX** and **MIN**:
  - **MAX** wants to maximize the utility.
  - **MIN** wants to minimize the utility.

Adversarial search

Minimax Algorithm

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Adversarial search

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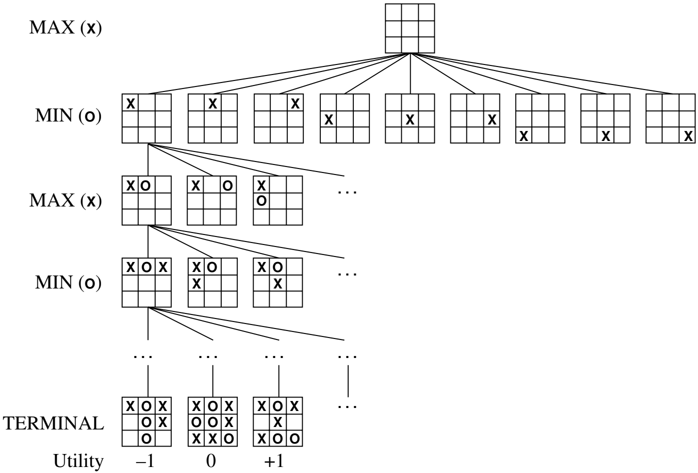
- S_0, ACTIONS, and RESULT define the ***game tree***.

Adversarial search

Example: game tree for tic-tac-toe

A partial game tree for the game of tic-tac-toe:



Adversarial search

Minimax Algorithm

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Adversarial search

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Adversarial search

Minimax Algorithm

- **MAX** is the first player to move, starting at the root node.
- The utility values at the leaf nodes are from **MAX's** point of view.
- The optimal strategy is found by examining the ***minimax value*** of each node.

Adversarial search

Minimax Algorithm

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Adversarial search

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- However, since **MIN** affects the search, a contingent strategy should be found.
- The strategy defines:
  - **MAX's** move at the initial state.
  - **MAX's** move at all states generated by all possible moves by **MIN**.
  - ... and so on.

Adversarial search

Minimax Algorithm

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- However, since **MIN** affects the search, a contingent strategy should be found.
- The strategy defines:
  - **MAX's** move at the initial state.
  - **MAX's** move at all states generated by all possible moves by **MIN**.
  - ... and so on.
- This means that **MAX** needs a winning strategy independent of what **MIN** does.

Adversarial search

Minimax Algorithm

- **MAX** will choose a move that maximizes its utility value.

Adversarial search

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- **MAX** will choose a move that maximizes its utility value.
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- This results in **MAX** (at the root) choosing its best move given available information.

Adversarial search

Minimax Algorithm

- **MAX** will choose a move that maximizes its utility value.
- **MIN** will choose a move that minimizes the utility value of **MAX**.
- This results in **MAX** (at the root) choosing its best move given available information.
- Available information is via look ahead (the search down the tree).

Adversarial search

Minimax Algorithm

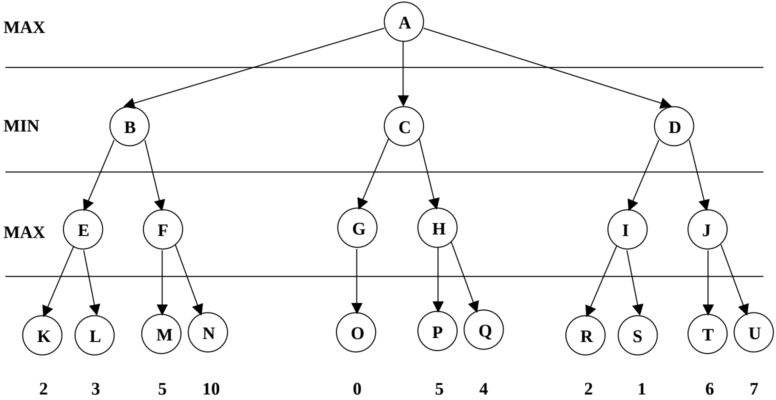
- The minimax value of a node is defined as follows:

$$\text{MINIMAX}(s) = \begin{cases} \text{UTILITY}(s) & \textbf{if } \text{TERMINAL-TEST}(s) \\ max_{a \in \text{ACTIONS}(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \textbf{if } \text{PLAYER}(s) = \text{MAX} \\ min_{a \in \text{ACTIONS}(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \textbf{if } \text{PLAYER}(s) = \text{MIN} \end{cases}$$

Adversarial search

Example: applying minimax

Calculate the minimax value for the node A.



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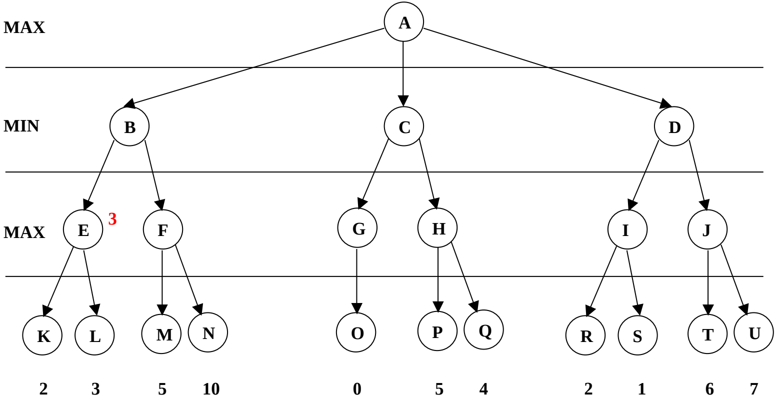

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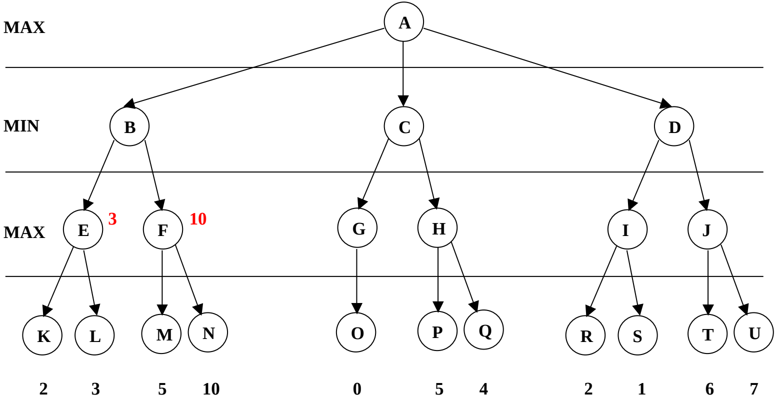

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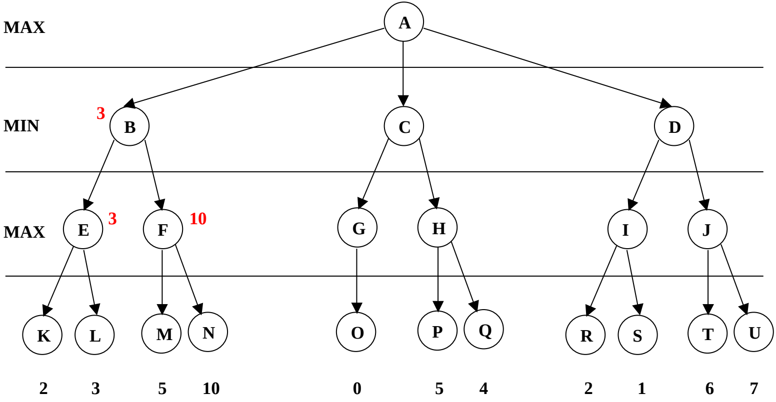

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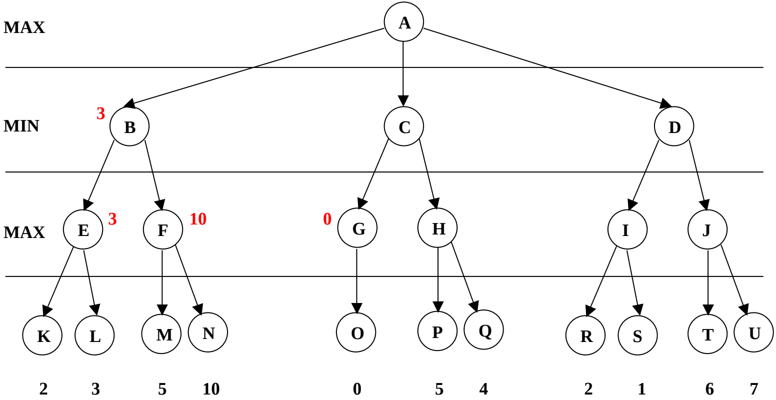

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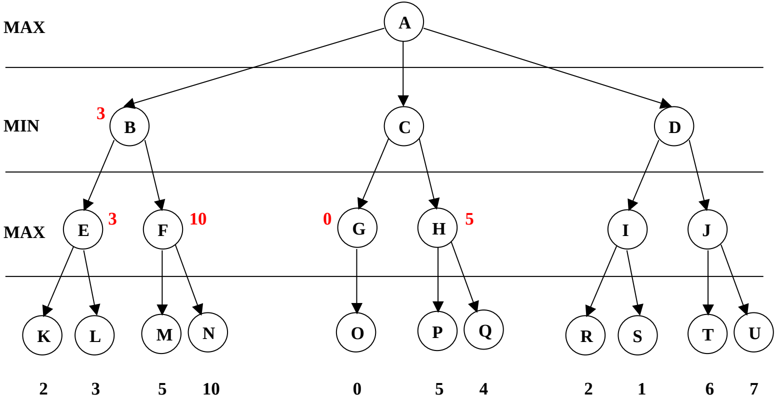

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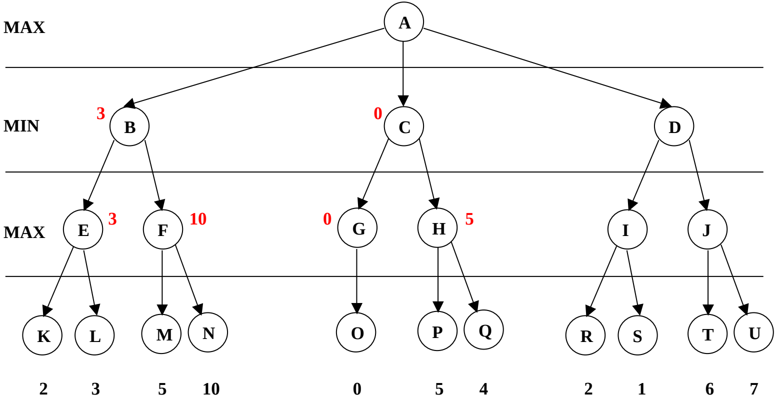

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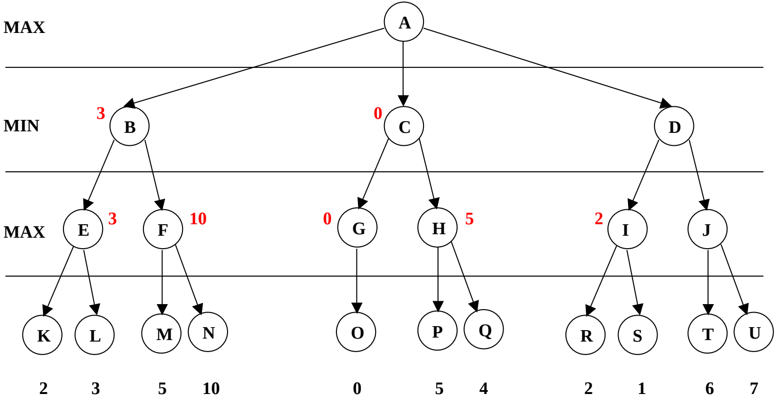

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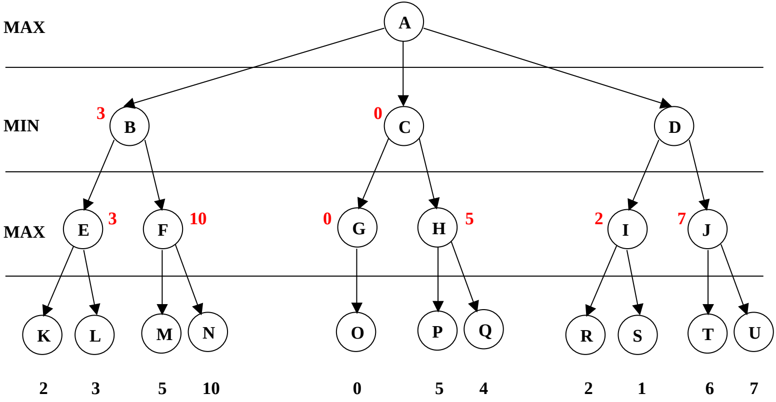

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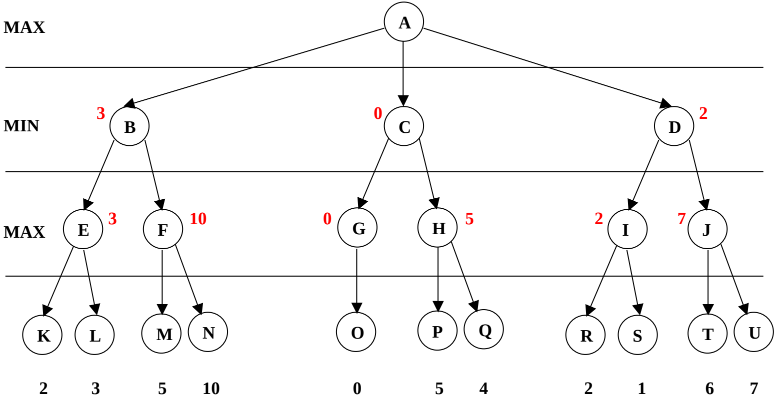

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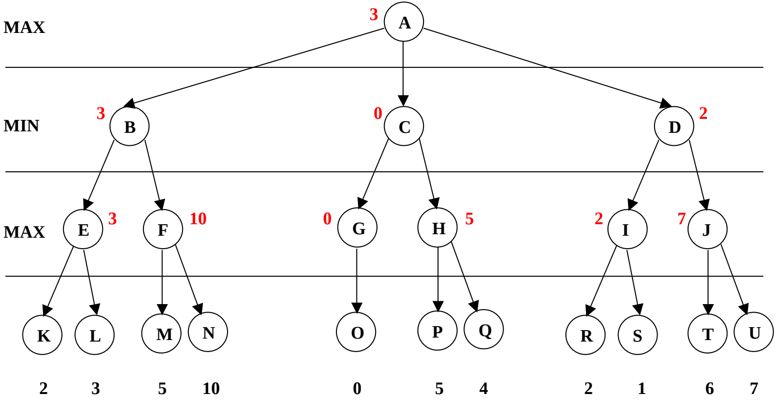

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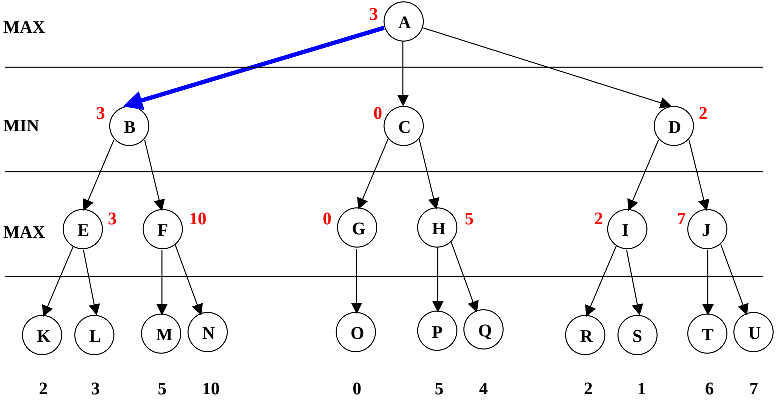

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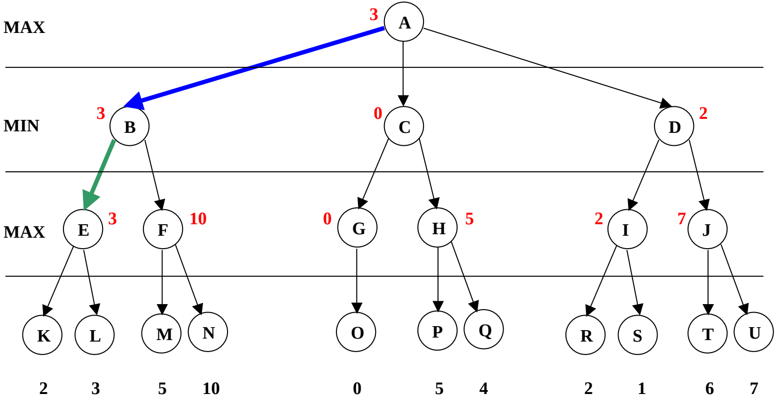

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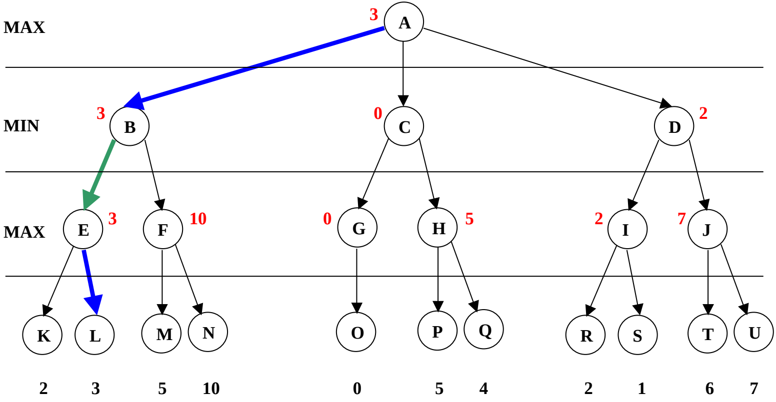

Adversarial search

Example: applying minimax to NIM

- Consider the game of NIM:
  - 7 matches placed in a pile.
  - Each player divides a pile of matches into 2 non-empty piles with a different number of matches.
  - The player who cannot make a move is the loser.

Adversarial search

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- The value at each node represents the best value of the best terminal state the current player can hope to achieve.

Adversarial search

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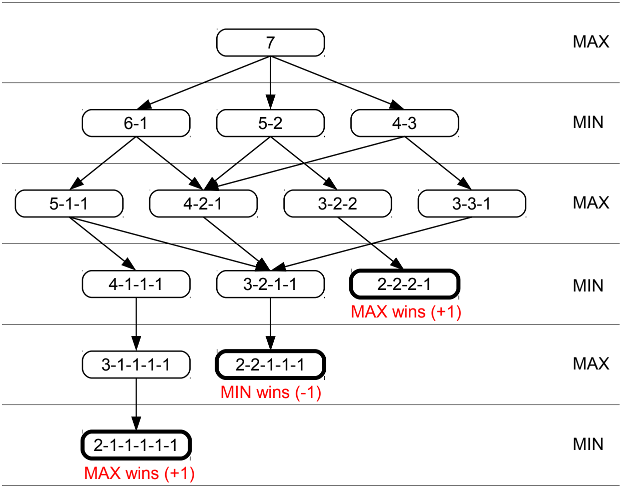

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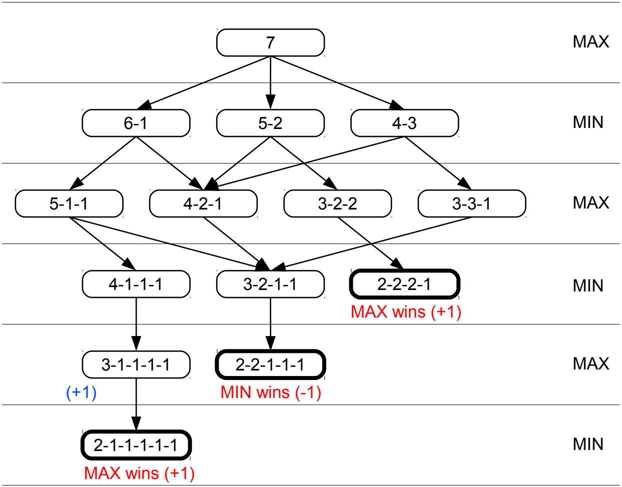

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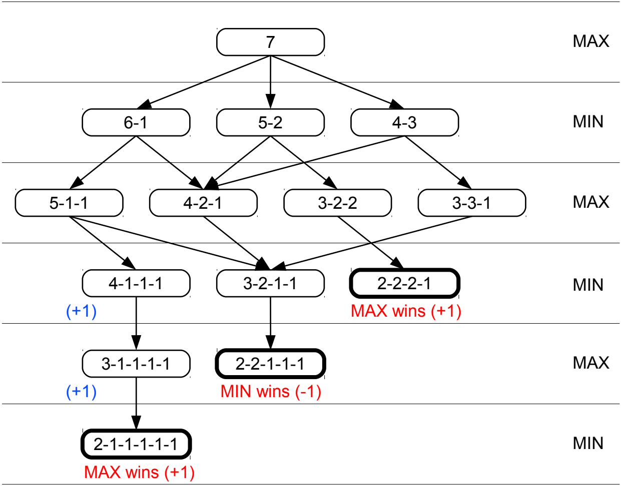

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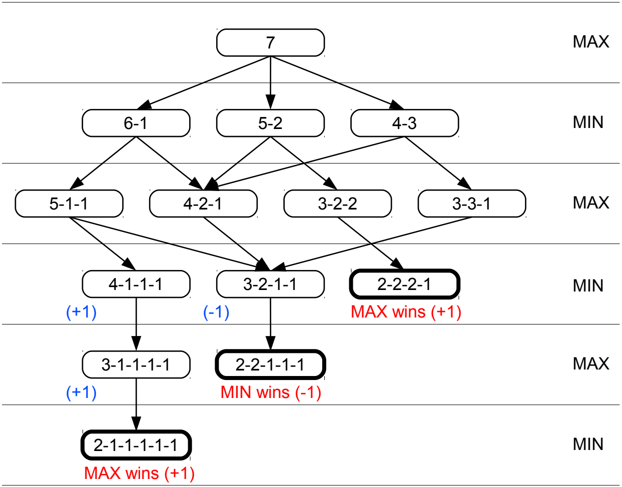

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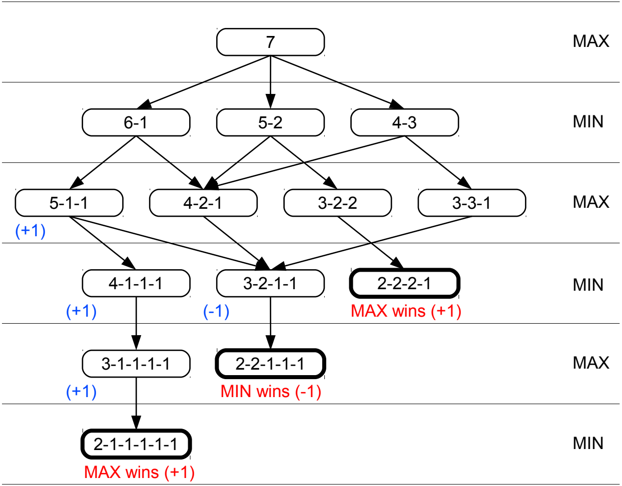

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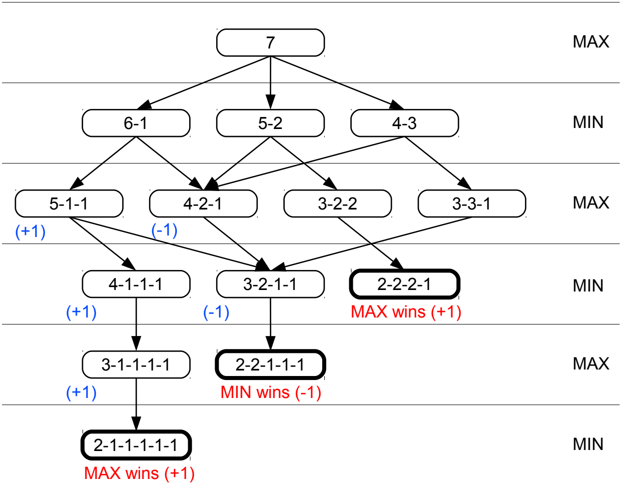

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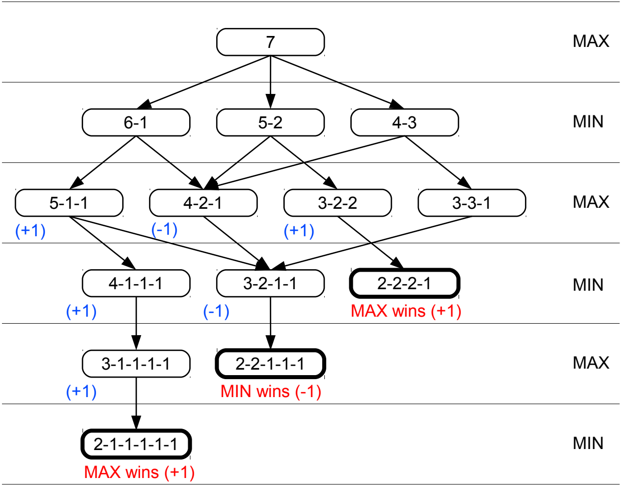

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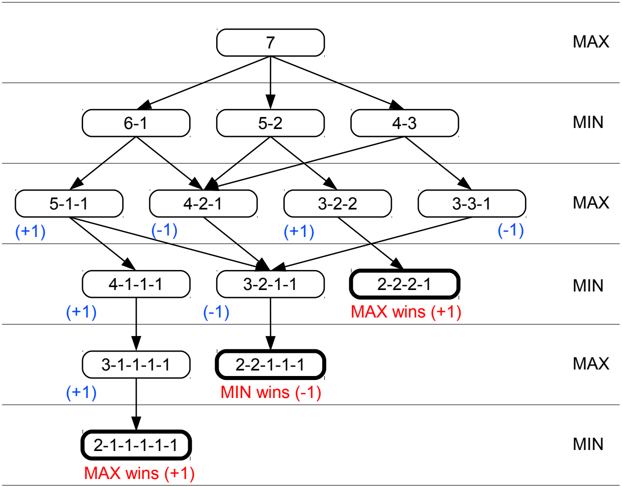

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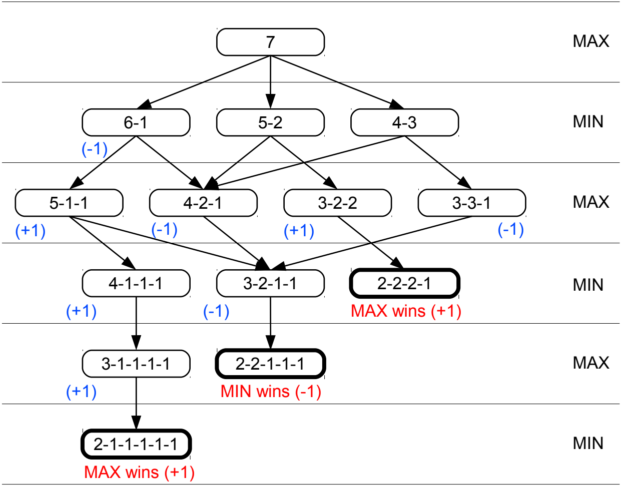

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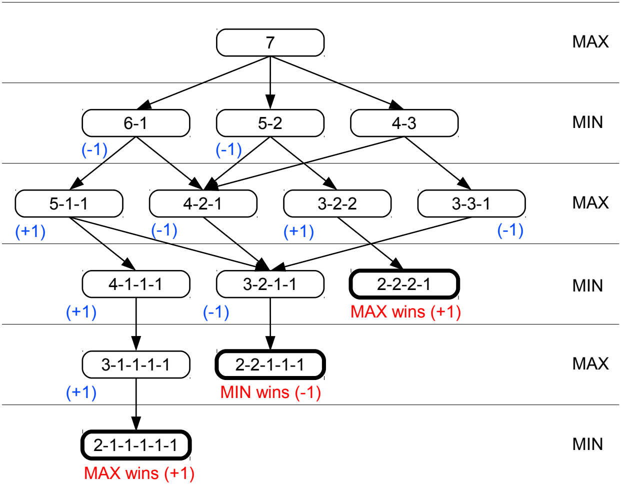

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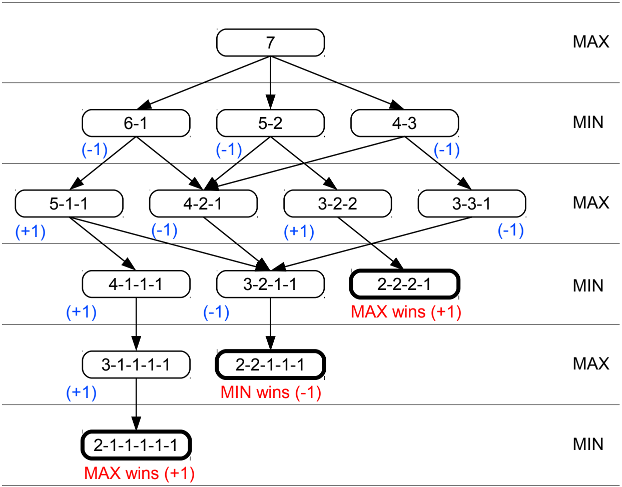

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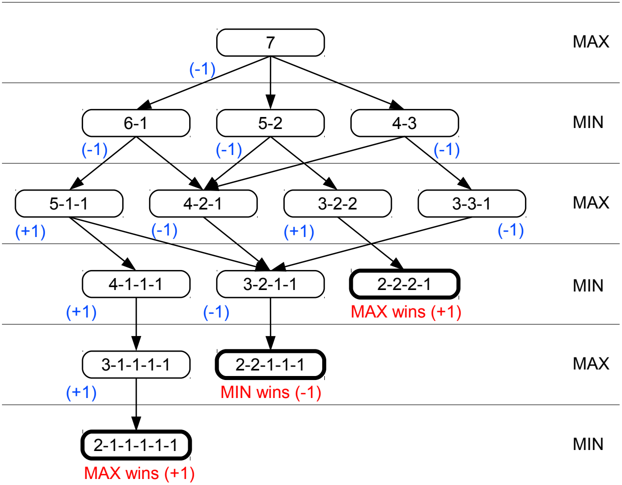

Adversarial search

The minimax search algorithm

function MINIMAX-SEARCH(s) **returns** an action
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function MAX-VALUE(s) **returns** a utility value
    if TERMINAL-TEST(s) **then return** UTILITY(s)
    $v \leftarrow -\infty$
    for each a **in** ACTIONS(s) **do**
        $v \leftarrow$ MAX(v,MIN-VALUE(RESULT(s,a)))
    return v

Adversarial search

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Adversarial search

Properties of the minimax search algorithm

- Complete if the tree is finite.

Adversarial search

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- Space complexity = $O(bm)$.

Adversarial search

α-β Pruning

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Adversarial search

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Adversarial search

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- Minimax has to generate the entire tree, compute the utility values at the leaf nodes, and then propagate these values up the tree.
- We want a way to do less work by removing unnecessary branches thus exploring less nodes.
- However, we want to guarantee the same decision as Minimax.

Adversarial search

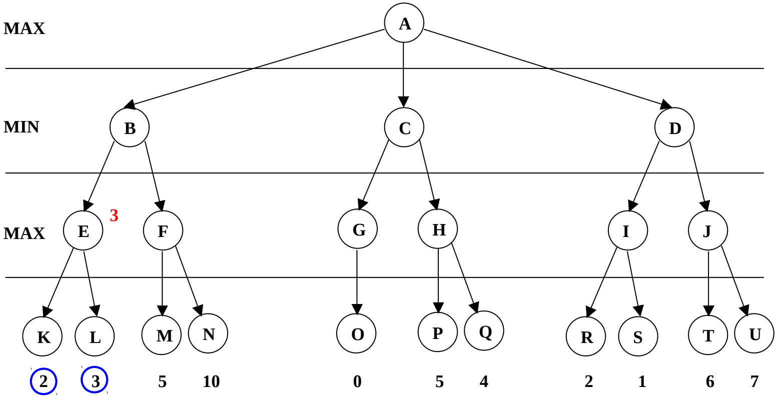
α-β Pruning

- Game trees are too huge to be solved to optimality.
- Minimax has to generate the entire tree, compute the utility values at the leaf nodes, and then propagate these values up the tree.
- We want a way to do less work by removing unnecessary branches thus exploring less nodes.
- However, we want to guarantee the same decision as Minimax.
- We use the **alpha-beta (α-β) pruning** technique.

Adversarial search

Example: illustrating α-β pruning

Show a scenario where α-β pruning becomes applicable.



Adversarial search

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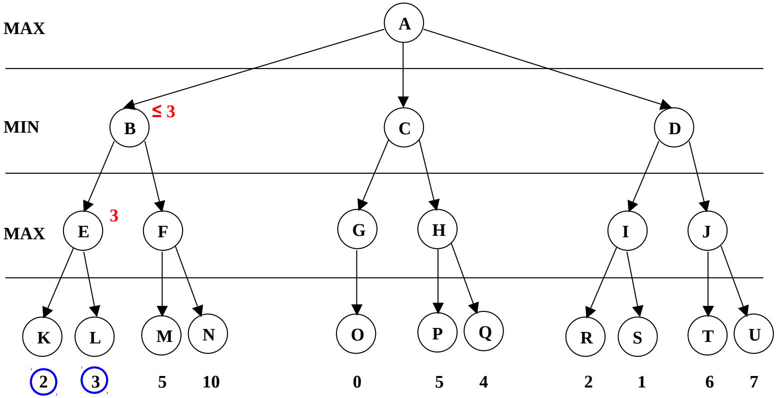

Adversarial search

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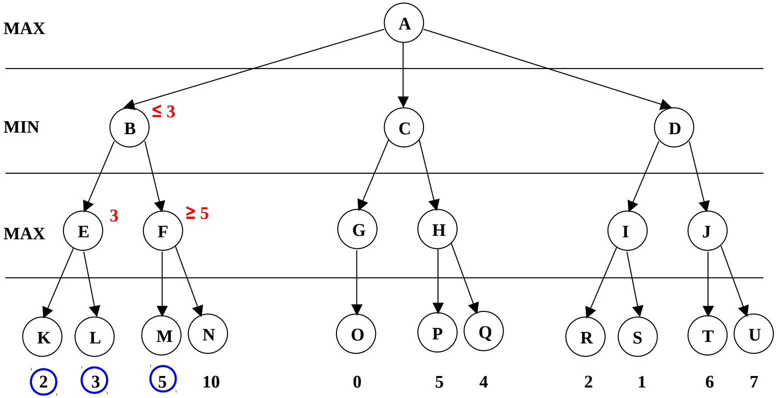

Adversarial search

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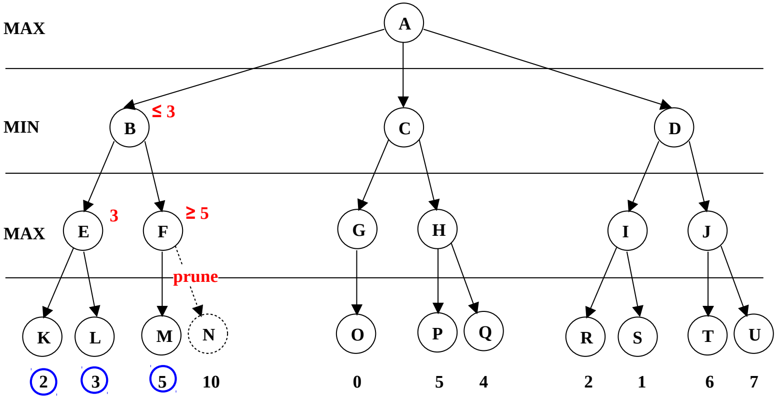
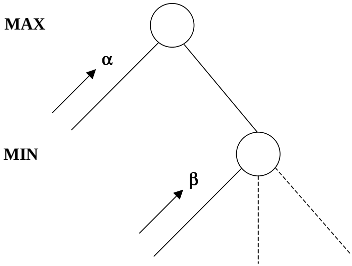


Adversarial search

α-cutoff

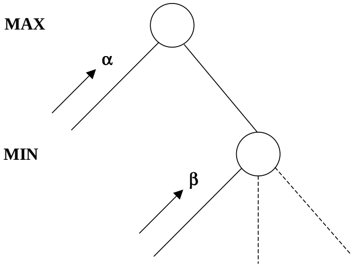
- α is the best (maximum) value **MAX** is assured so far off the current path.



Adversarial search

α-cutoff

- α is the best (maximum) value **MAX** is assured so far off the current path.
- If β is worse than α (from **MAX's** point of view), **MAX** will avoid it, dashed branches will be pruned.



Adversarial search

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- α is the best (maximum) value **MAX** is assured so far off the current path.
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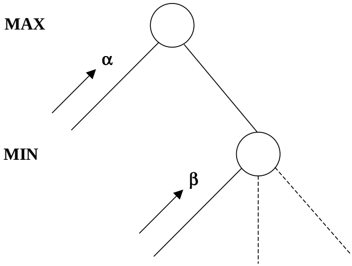
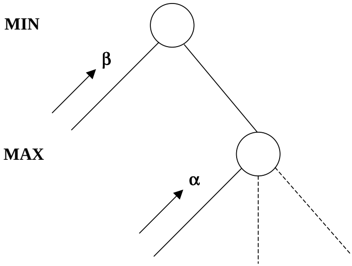
- Happens when $\alpha \geq \beta$, referred to as α-cutoff.



Adversarial search

β-cutoff

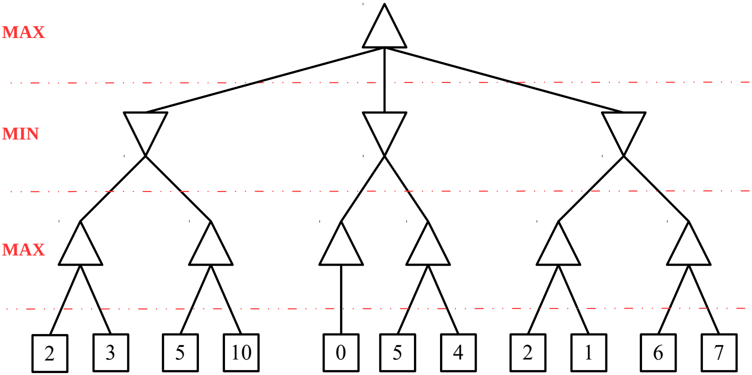
- β is the best (minimum) value **MIN** is assured so far off the current path.
- If α is worse than β (from **MIN's** point of view), **MIN** will avoid it, dashed branches will be pruned.
- Happens when $\alpha \geq \beta$, referred to as β-cutoff.



Adversarial search

Example: applying α-β pruning

Apply α-β pruning to calculate minimax value for node A.



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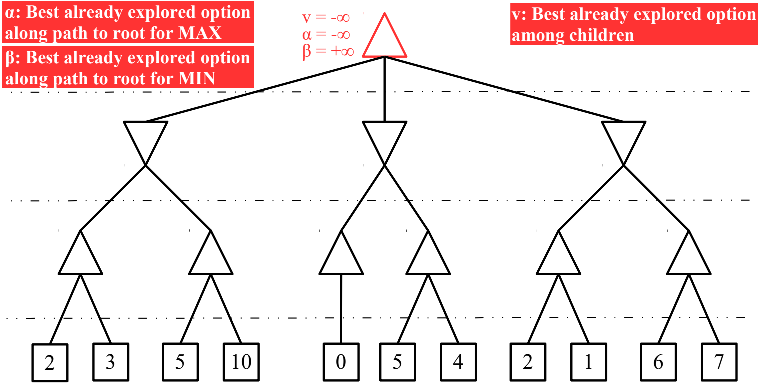

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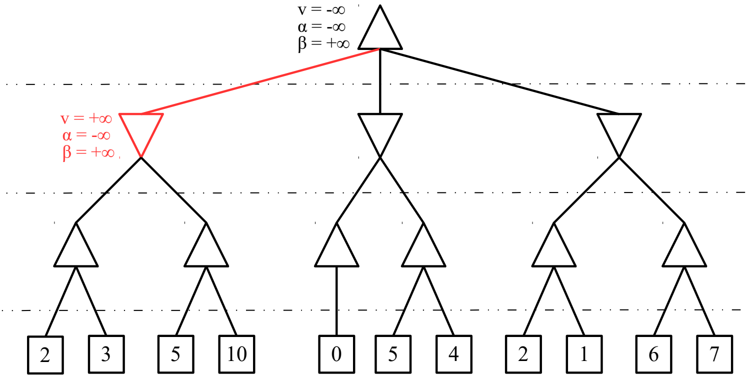

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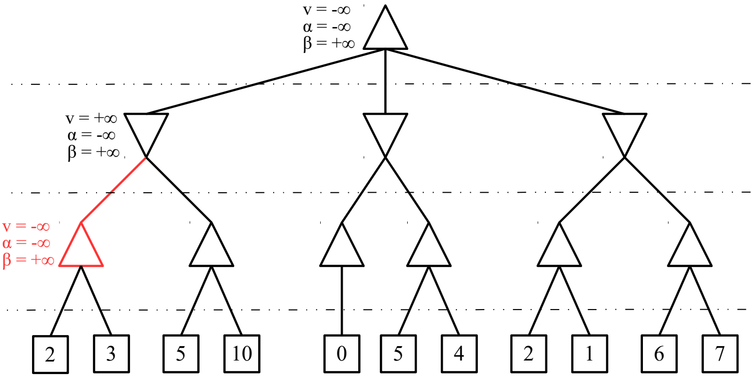

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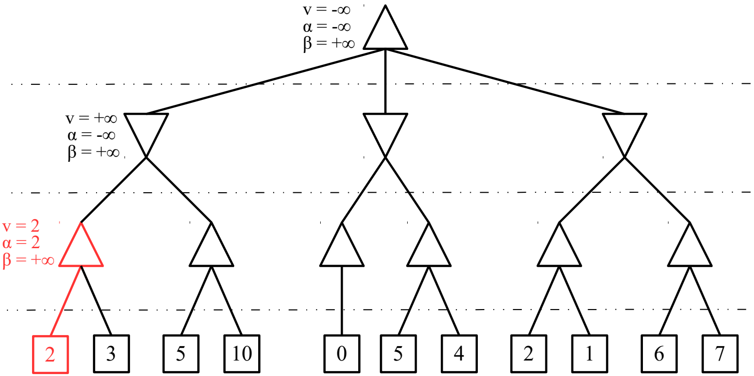

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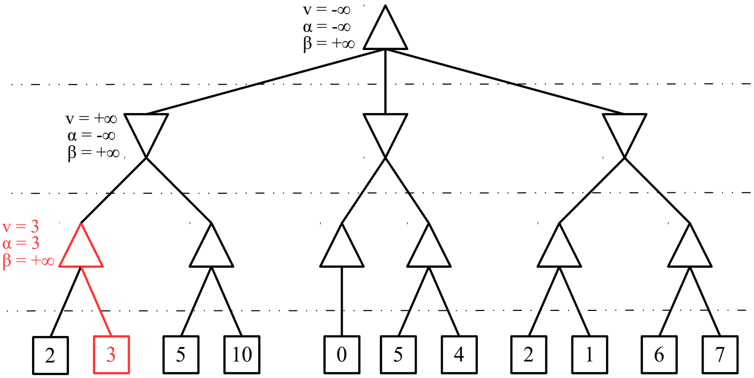

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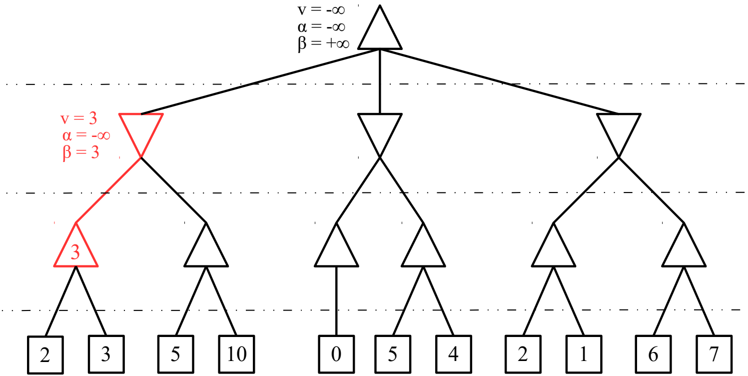

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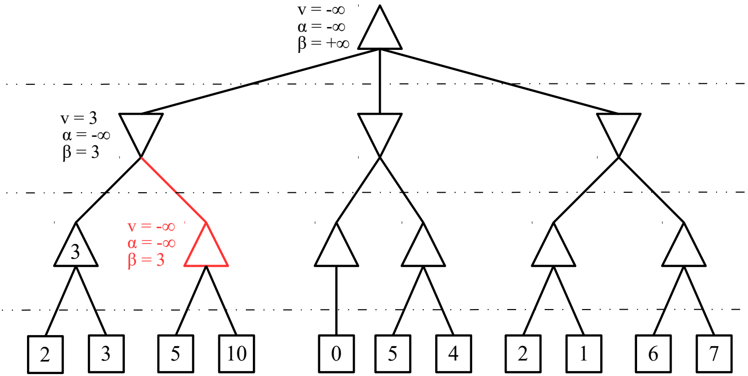

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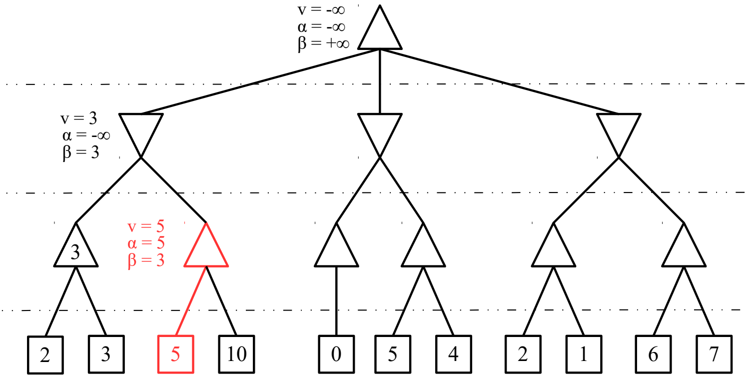

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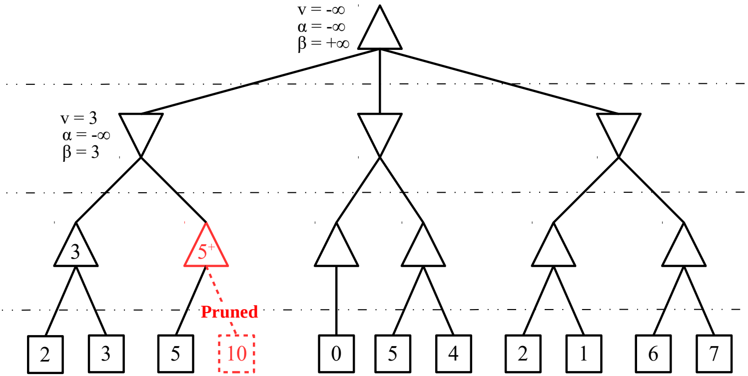

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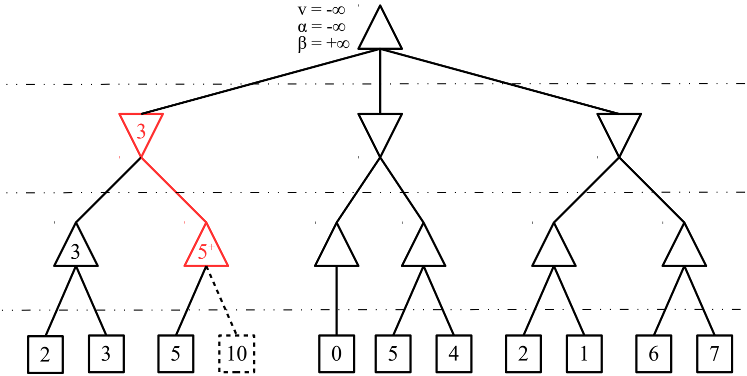

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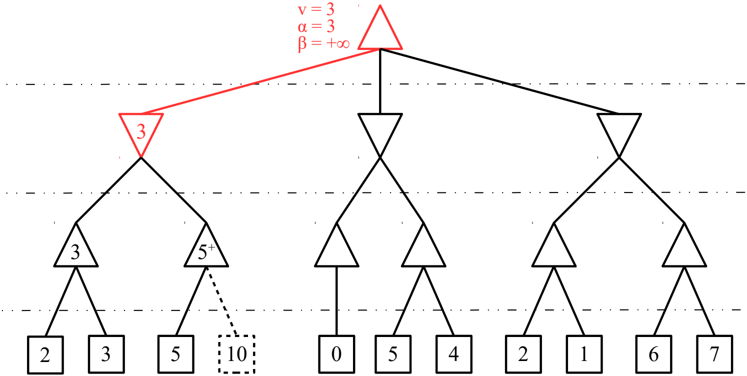

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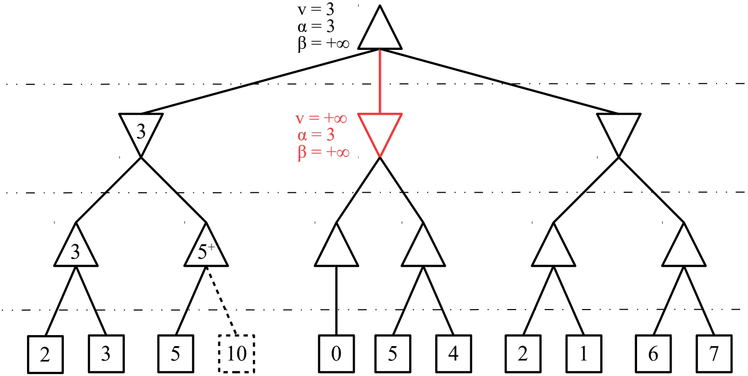

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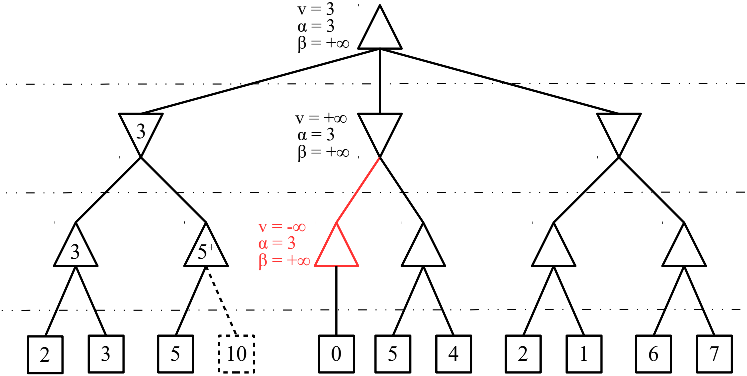

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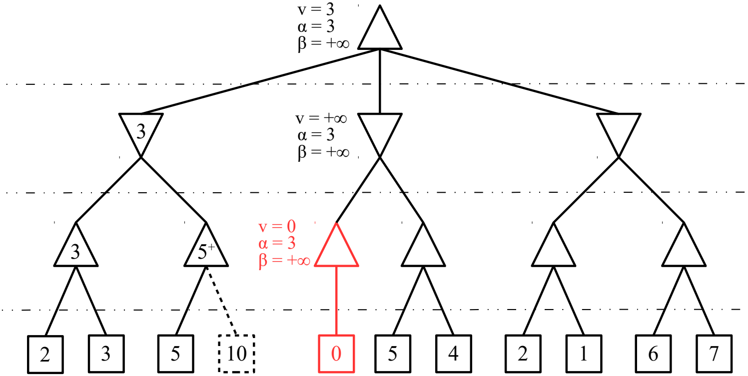

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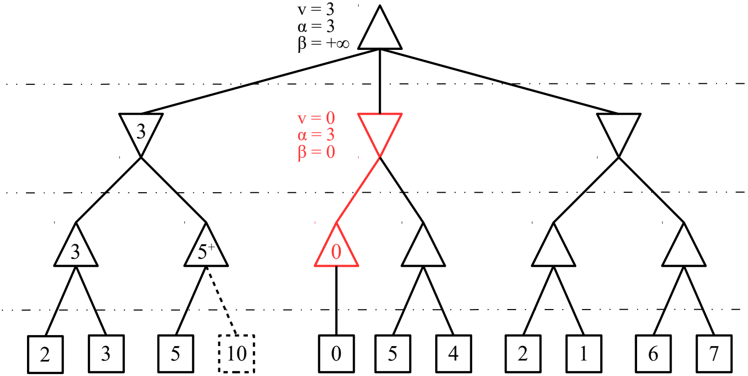

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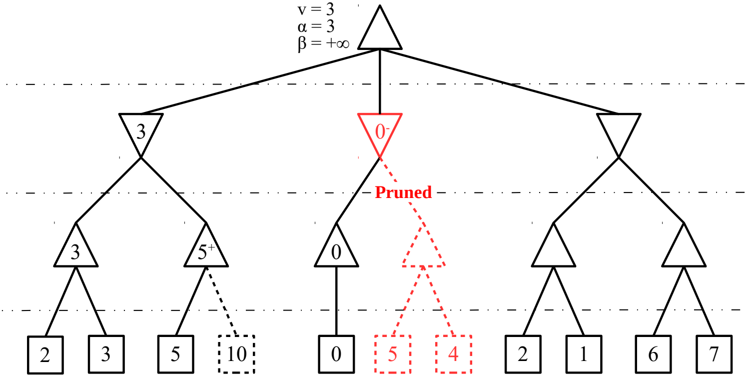

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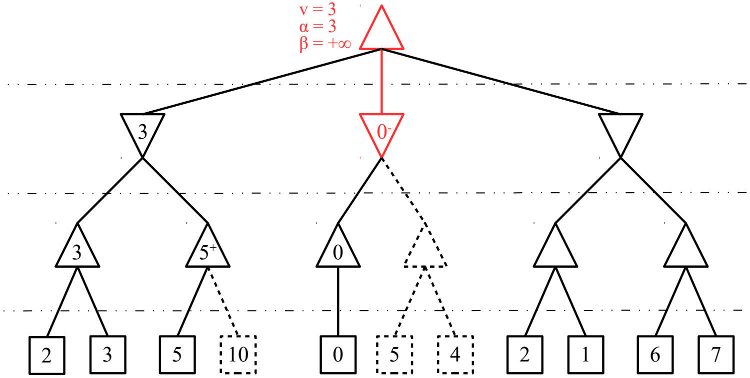

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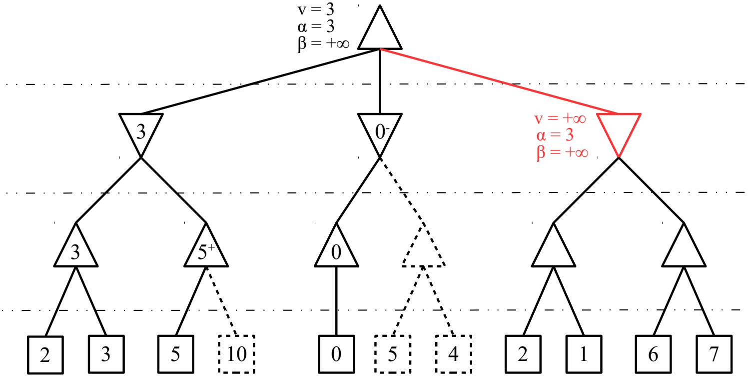

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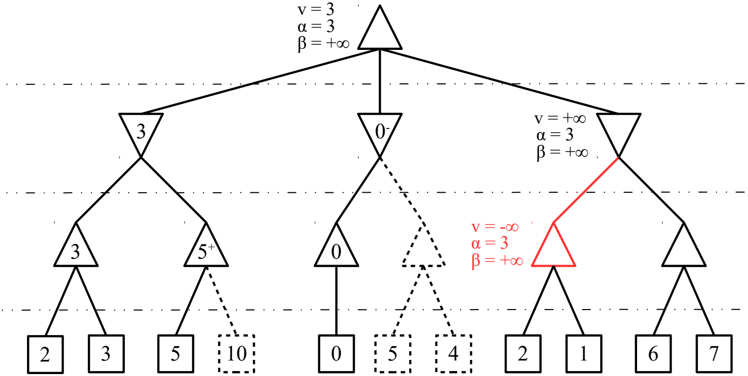

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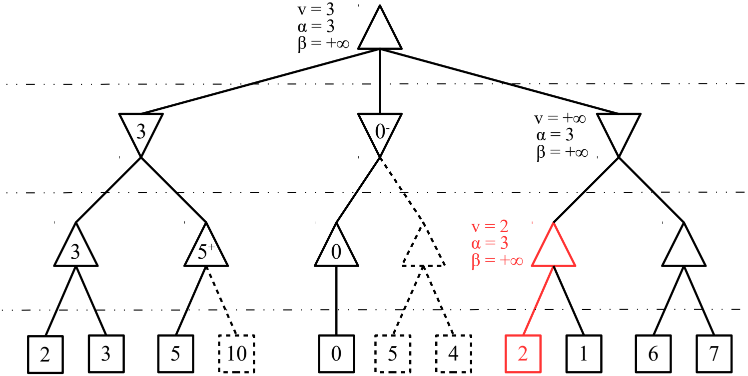

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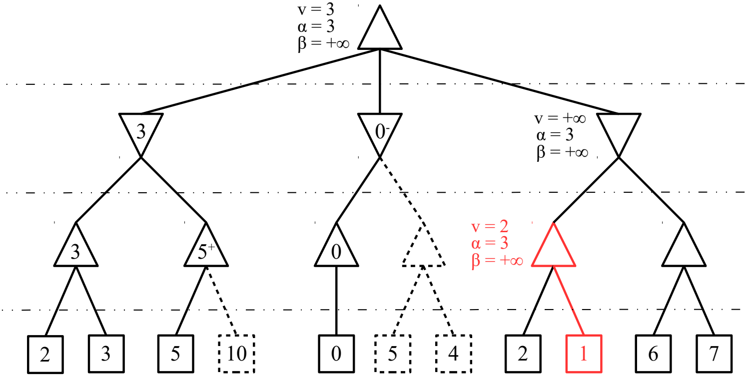

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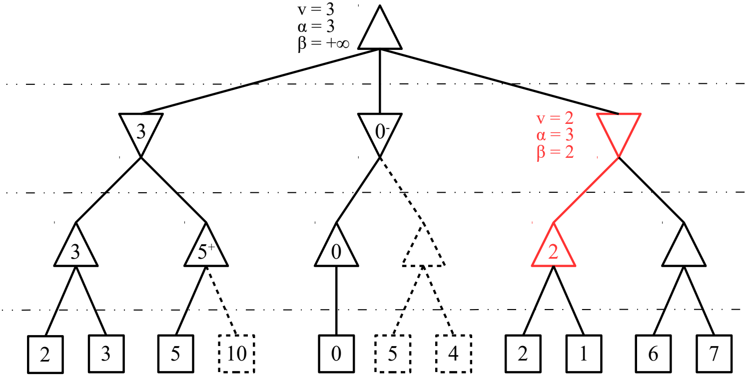

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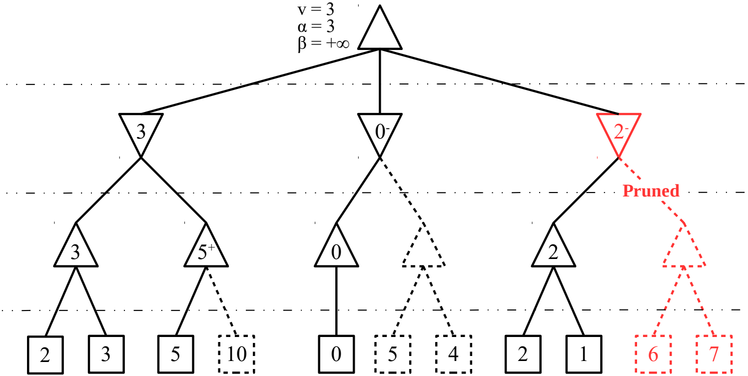

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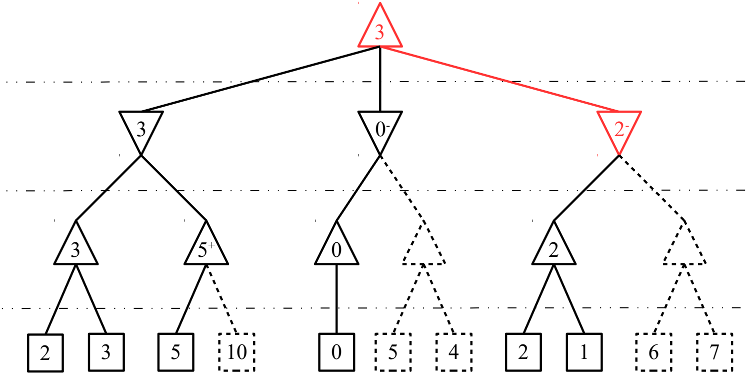

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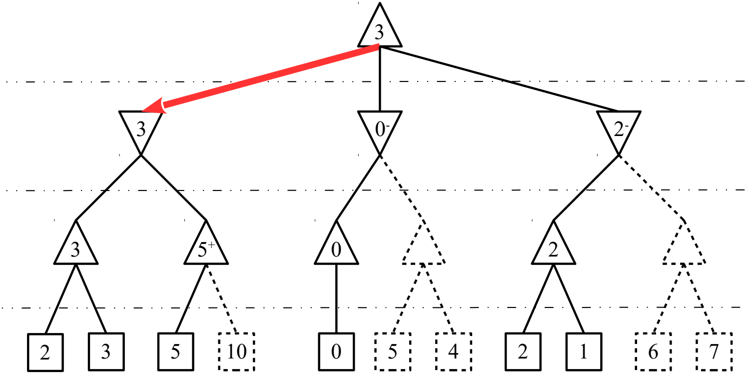

Adversarial search

α-β Pruning

- In the previous example, α-β pruning explored 14 nodes instead of 21 nodes explored by Minimax.

Adversarial search

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- α-β pruning also reached the same decision reached by Minimax.

Adversarial search

The α-β search Algorithm

function ALPHA-BETA-SEARCH(*s*) **returns** an action
  v ← MAX-VALUE(*s*, -∞, +∞)
  return the **action** in ACTIONS(*s*) with value *v*

Adversarial search

The α-β search Algorithm

function ALPHA-BETA-SEARCH(s) **returns** an action
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---

function MAX-VALUE(s, α, β) **returns** a utility value
  if TERMINAL-TEST(s) **then return** UTILITY(s)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(s) **do**
    $v \leftarrow$ MAX(v,MIN-VALUE(RESULT(s,a), α, β))
    $\alpha \leftarrow$ MAX(α,v)
    if $\alpha \geq \beta$ **then return** α /* β-cutoff */
  return v

Adversarial search

The α-β search Algorithm

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    β ← MIN(β,*v*)
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  return *v*

Outline

Requirements

What do I need from you

- When given a certain problem you should be able to:
  - Formulate the game problem.

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  - Formulate the game problem.
  - Build the game tree up to a given depth.
  - Apply the minimax search algorithm.
  - Apply the $\alpha - \beta$ pruning algorithm.
- Answer descriptive questions.

Reading Material

Which parts of the textbook are covered

- Russell-Norvig, Chapters 5:
  - Pages 161 - 170.