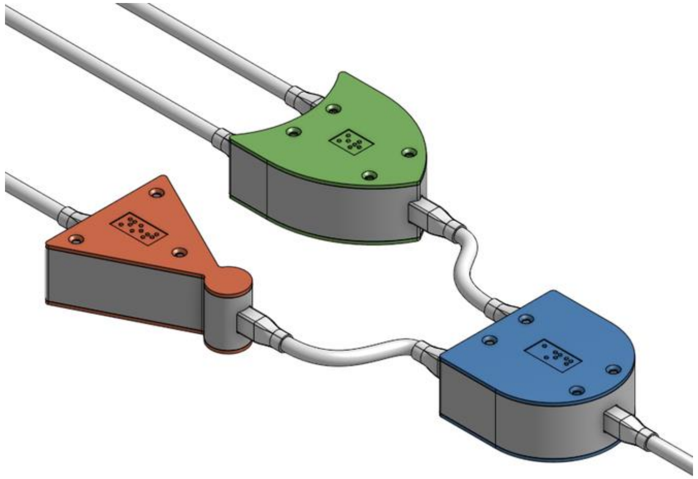


CS 211 - Digital Logic Design
211 عال - تصميم المنطق الرقمي

First Term - 1439/1440
Lecture #6

Dr. Hazem Ibrahim Shehata

Assistant Professor

College of Computing and Information Technology

Administrivia

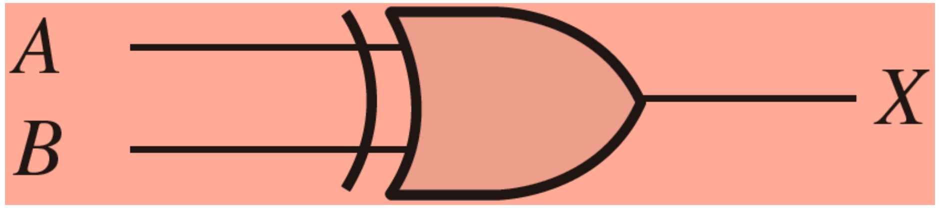
- Assignment #1:
  - Due: **Today**.
  - Solution to be posted tomorrow.
- Midterm #1:
  - Date: **Wednesday, October 24, 2018**.
  - Time: 8:30am - 9:30am
  - Scope: Chapters 2 and 3 (Lectures 1,2,3,4,5, and half-of-6).

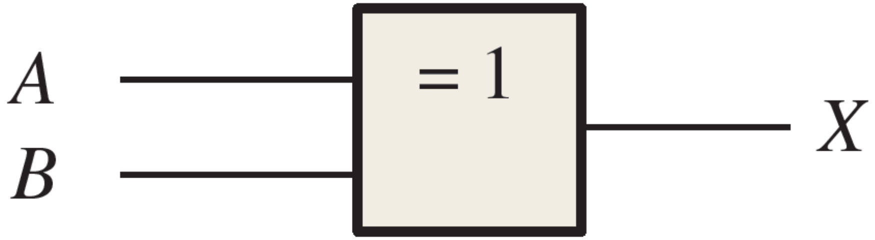
Website: http://hshehata.github.io/courses/su/cs211

Chapter 3: Logic Gates (… Continuing …)

Exclusive-OR (XOR) Gate

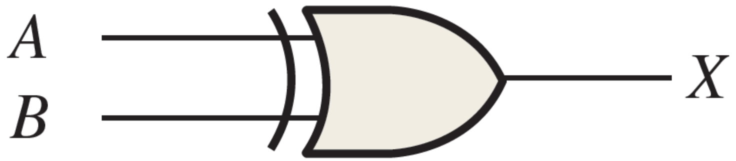
- Performs operation called modulo-2 addition, which is the same as binary addition with no carry.
  - Takes 2^+ inputs, and produces 1 output.
  - Output is High (1) if and only if number of High (1) inputs is odd.
  - Output is Low (0) if and only if number of High (1) inputs is even.
- Symbols:





XOR Gate Truth Table

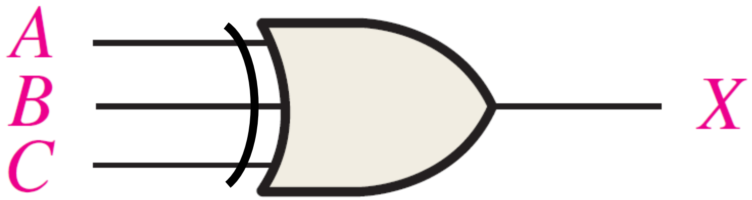
- For a 2-input XOR Gate:



Inputs		Output
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Example: Truth Table for 3-input XOR Gate

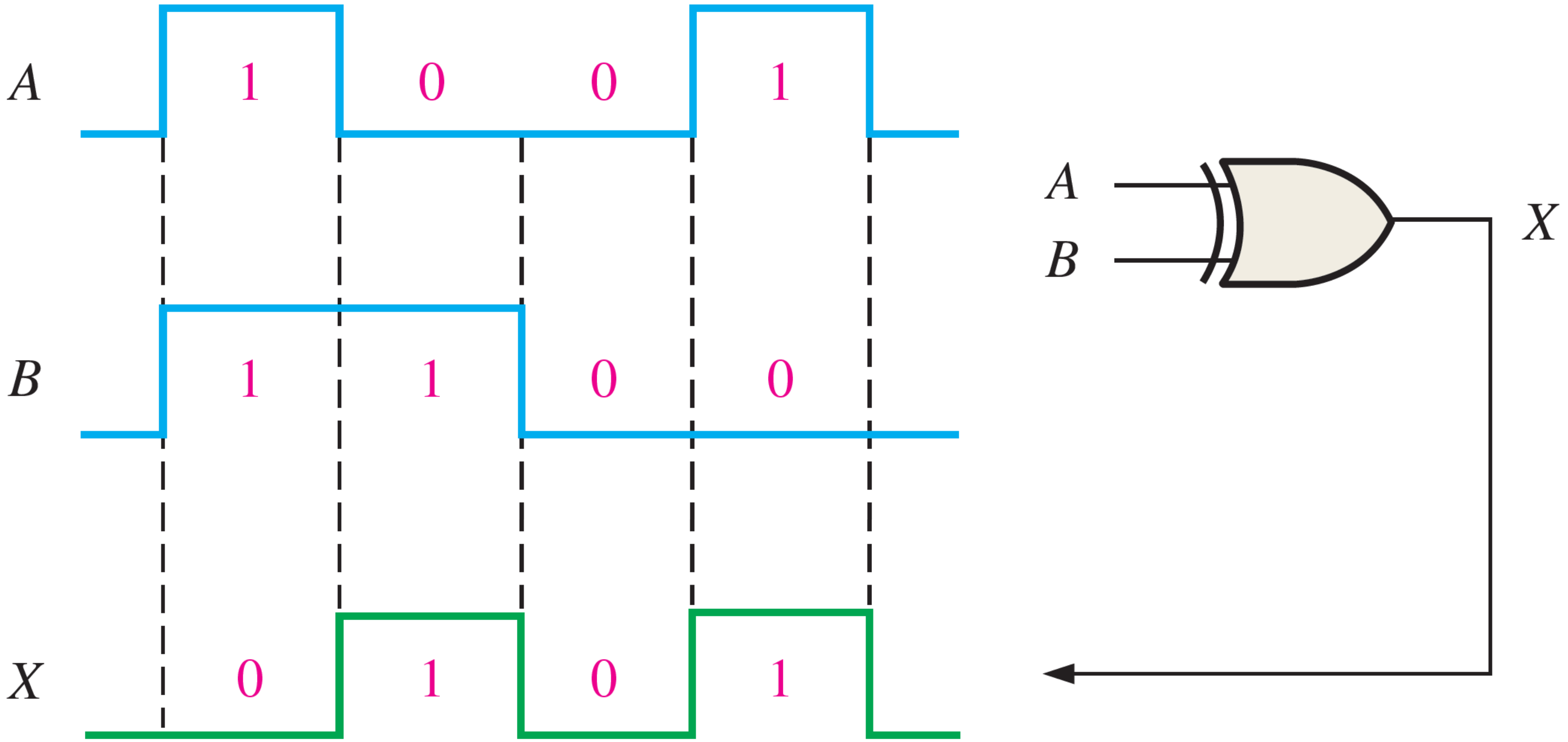
$n = 3 \rightarrow N = 2^3 = 8$



Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

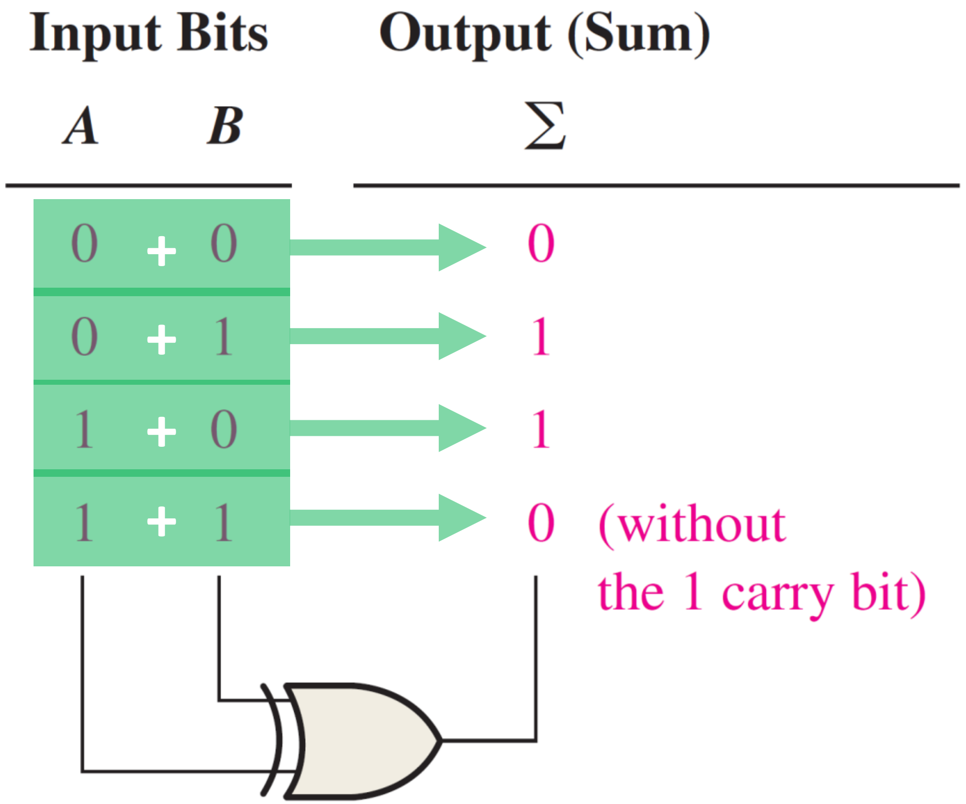
Timing Diagram of XOR Gate

➢ Example: 2-input XOR Gate



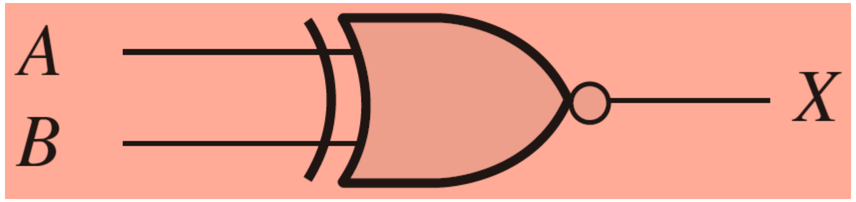
Application of XOR Gate

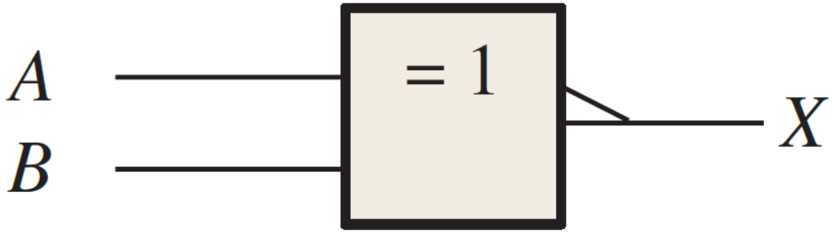
- Example: 1-bit Modulo-2 adder



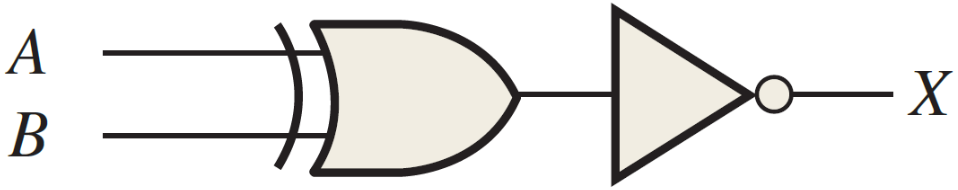
Exclusive-NOR (XNOR) Gate

- XOR with an inverted output.
  - Takes 2^+ inputs, and produces 1 output.
  - Output is Low (0) if and only if number of High (1) inputs is odd.
  - Output is High (1) if and only if number of High (1) inputs is even.
- Symbols:



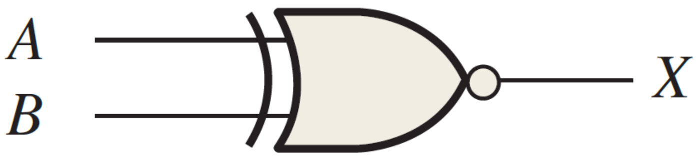


- Equivalent to:



XNOR Gate Truth Table

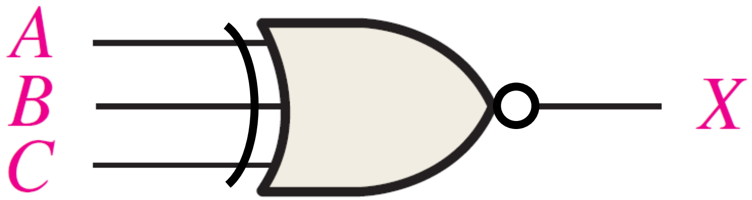
- For a 2-input XNOR Gate:



Inputs		Output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

Example: Truth Table for 3-input XNOR Gate

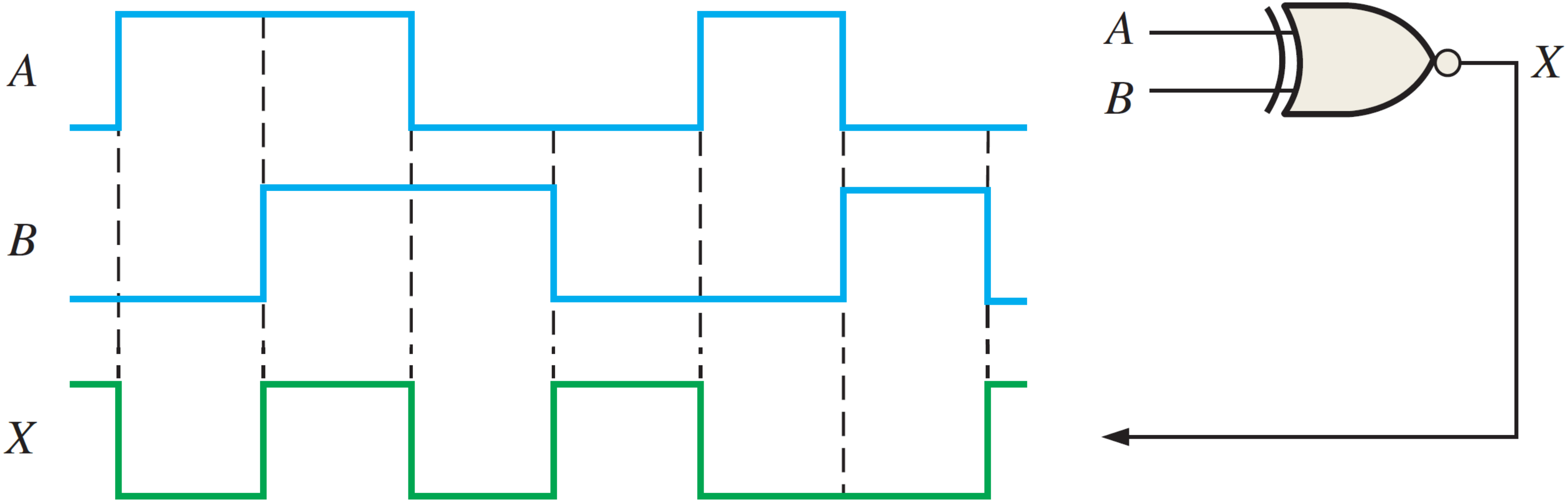
$n = 3 \rightarrow N = 2^3 = 8$



Inputs			Output
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

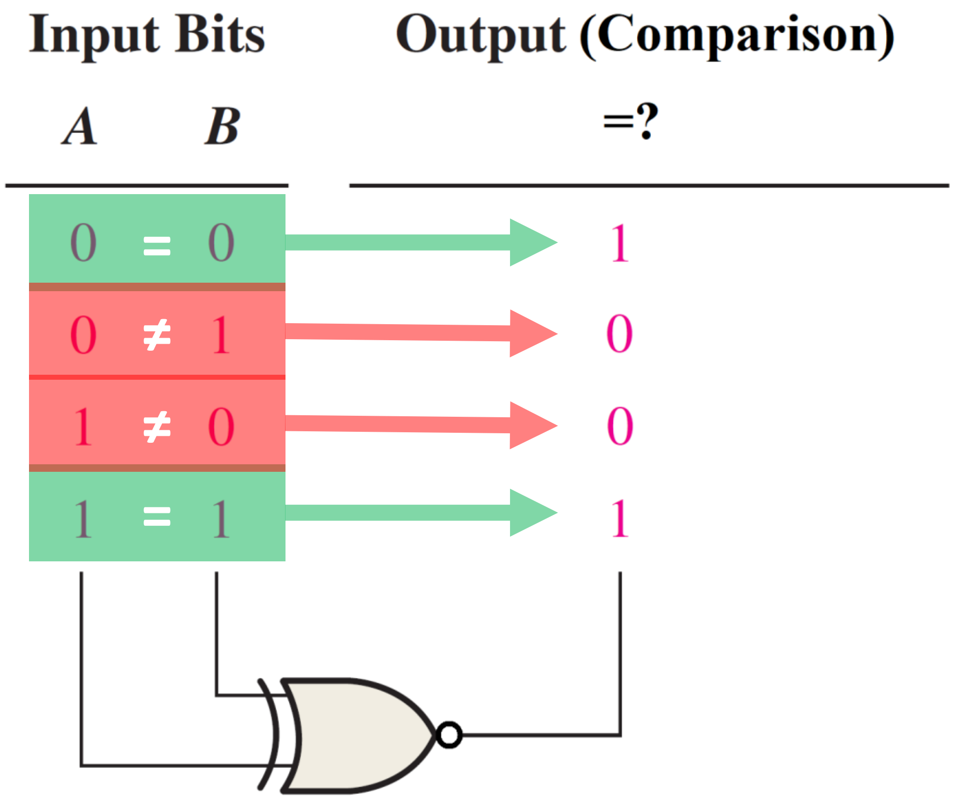
Timing Diagram of XNOR Gate

- **Example**: 2-input XNOR Gate



Application of XNOR Gate

- Example: 1-bit Binary Comparator



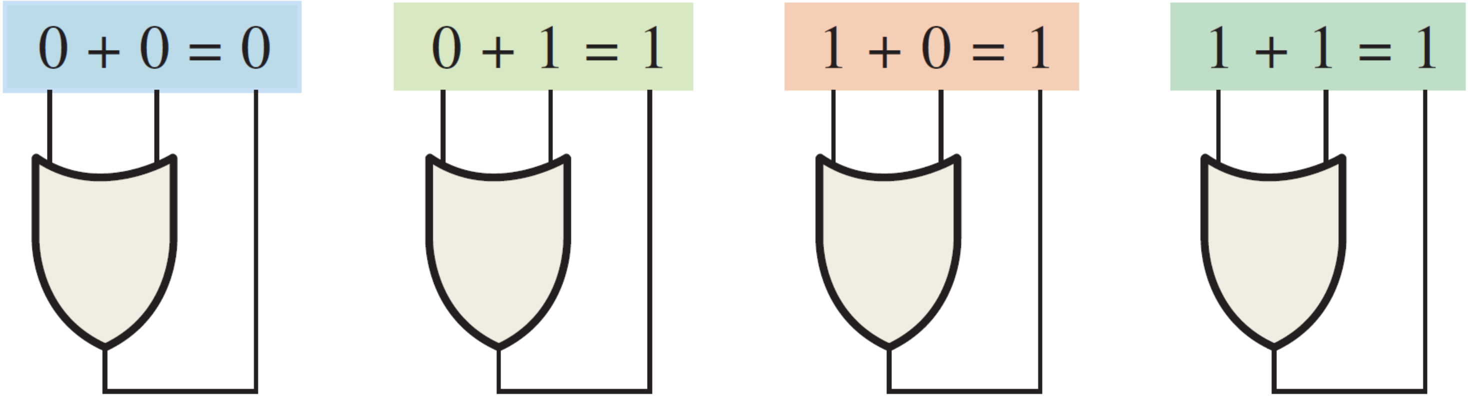
Chapter 4: Boolean Algebra and Logic Simplification

Basic Boolean Terms

- **Boolean Algebra (BA)**: the mathematics of digital logic.
- **Basic terms** in BA: Variable, Complement, Literal.
  1. **Variable**: symbol (usually an italic uppercase letter or word) to represent an action, a condition, or data. Any single variable can have only a 1 or a 0 value. Example: A or X.
  2. **Complement**: the inverse of a variable and is indicated by a bar over the variable (overbar). Example: $\bar{A}$ or $\bar{X}$.
  3. **Literal**: variable or complement of a variable. Example: A or $\bar{X}$.

Boolean Addition

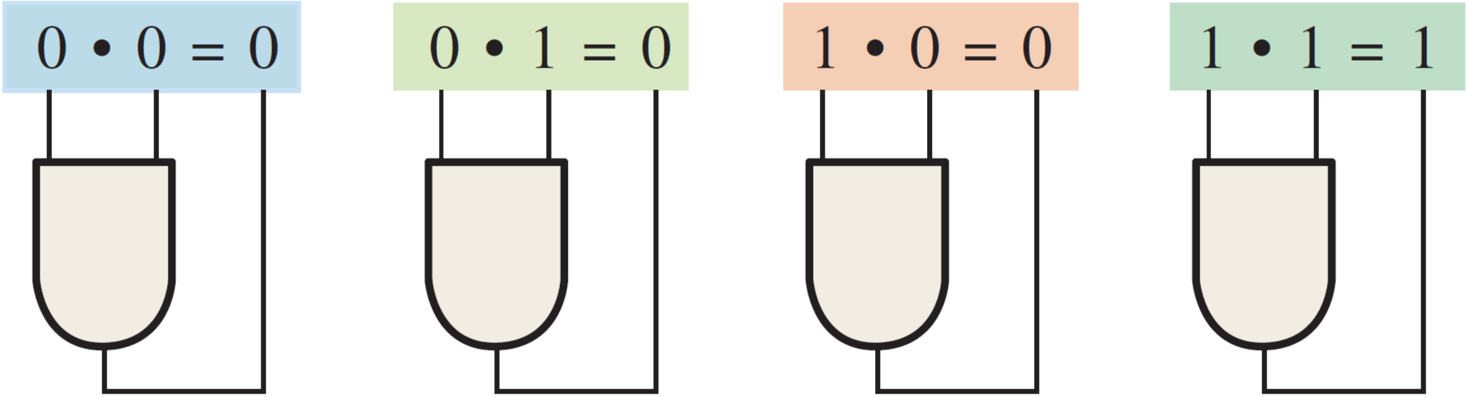
- **Boolean addition** (+) is equivalent to **OR operation**.



- **Sum term**: sum of literals.
  - Produced in logic circuits by OR operation with no AND's involved.
  - Equals 1 if 1^+ literals equals 1. Equals 0 if all literals equal 0.
  - Examples: $A + \bar{B}$, $\bar{A} + B + C + \bar{D}$

Boolean Multiplication

- Boolean multiplication (·) is equivalent to AND operation.



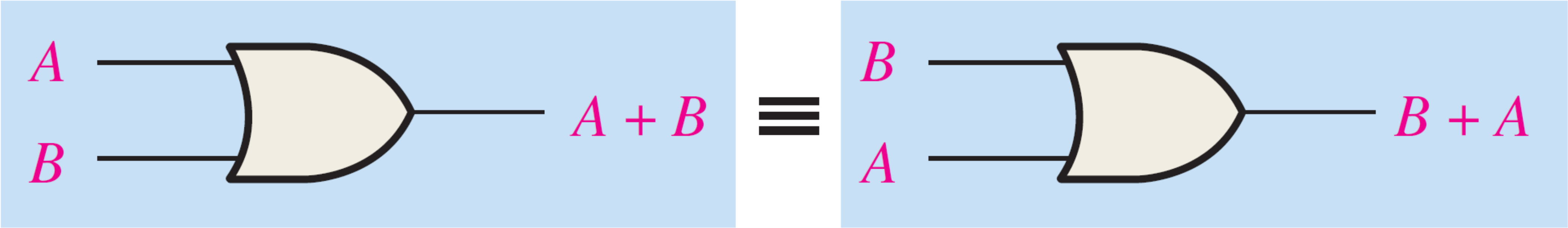
- Product term: product of literals.
  - Produced in logic circuits by AND operation with no OR's involved.
  - Equals 1 if all literals equal 1. Equals 0 if 1^+ literals equals 0.
  - Examples: $\bar{A} \cdot \bar{B}$, $A\bar{B}\bar{C}D$

Laws and Rules of Boolean Algebra

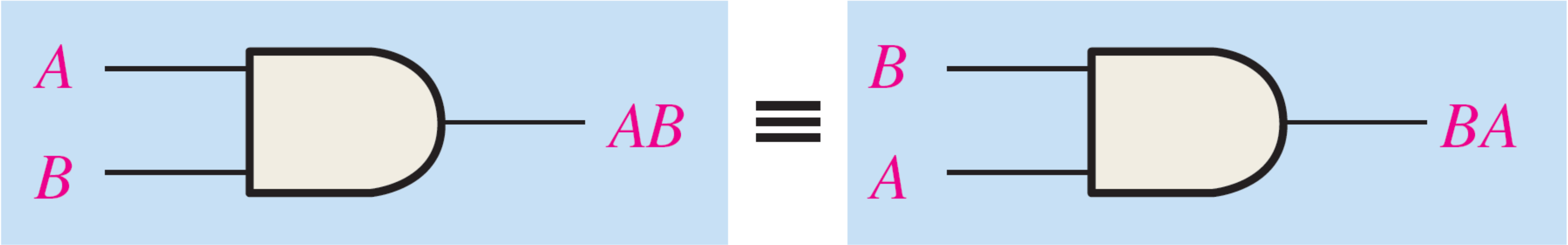
- Commutative Laws (+, ·)
- Associative Laws (+, ·)
- Distributive Law
- Rules of Boolean Algebra (12 rules)

Commutative Laws

- Commutative law of addition: $A + B = B + A$

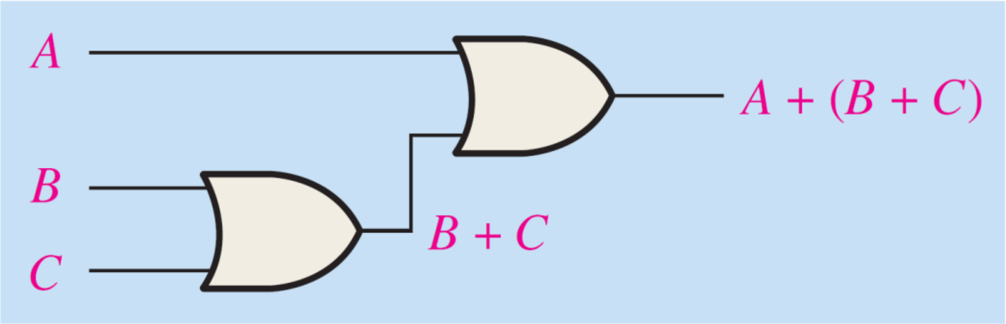


- Commutative law of multiplication: $AB = BA$



Associative Laws

- Associative law of addition: $A + (B + C) = (A + B) + C$



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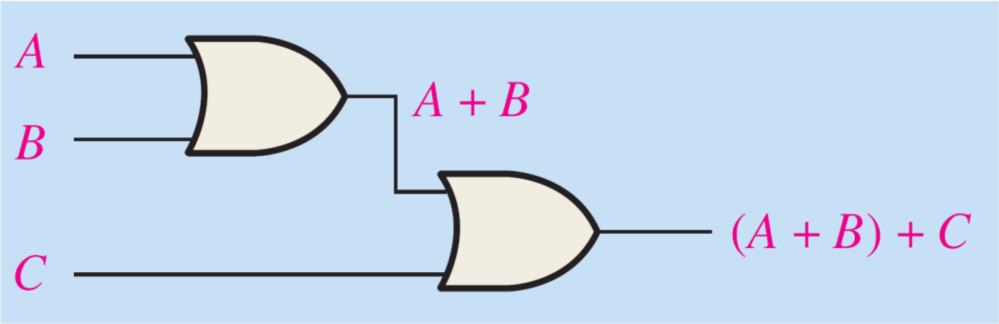
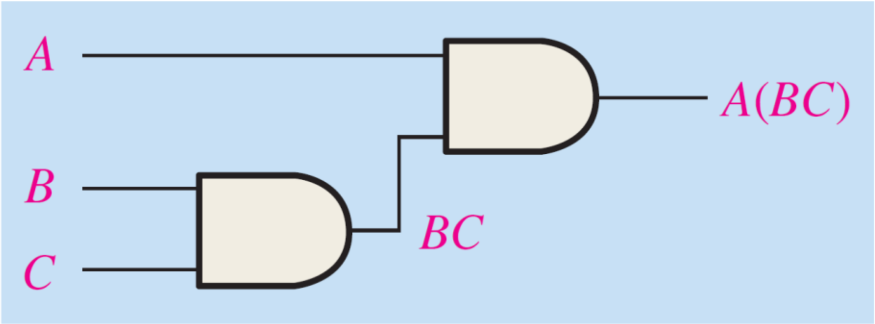


- Associative law of multiplication: $A(BC) = (AB)C$



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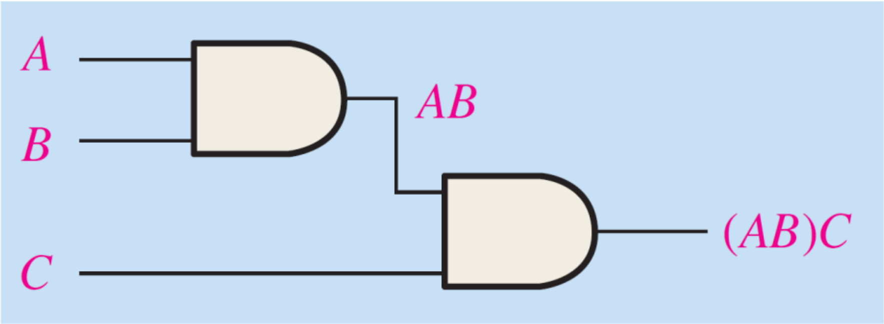
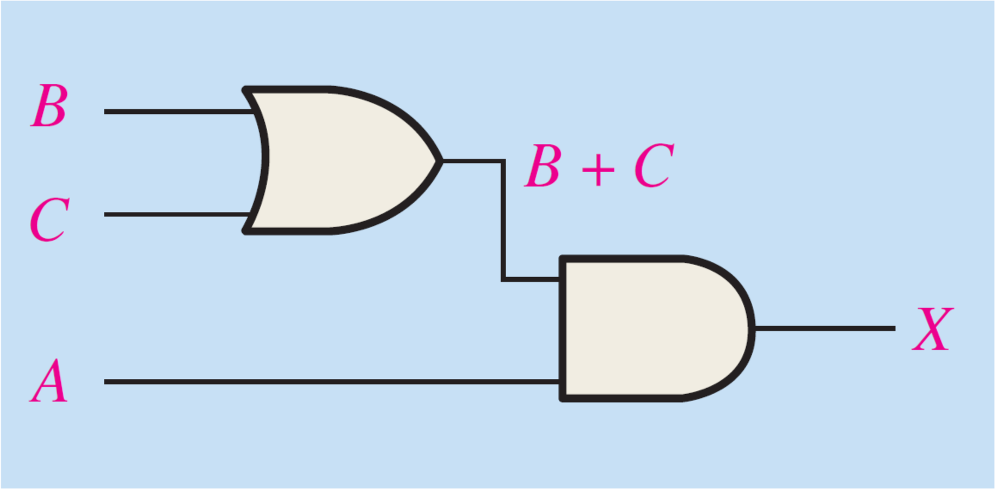


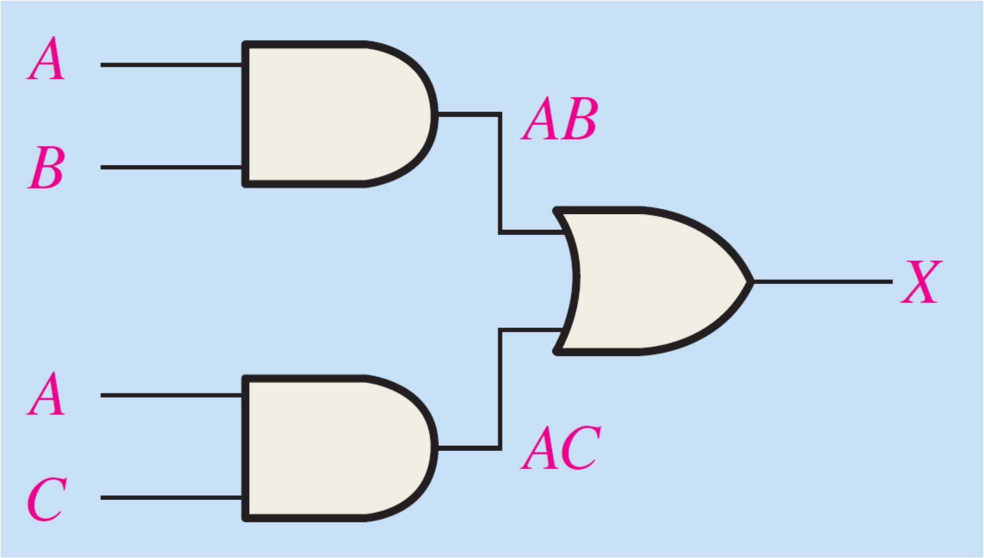
Distributive Law

- Distributive Law: $A(B + C) = AB + AC$



$X = A(B + C)$

$\equiv$

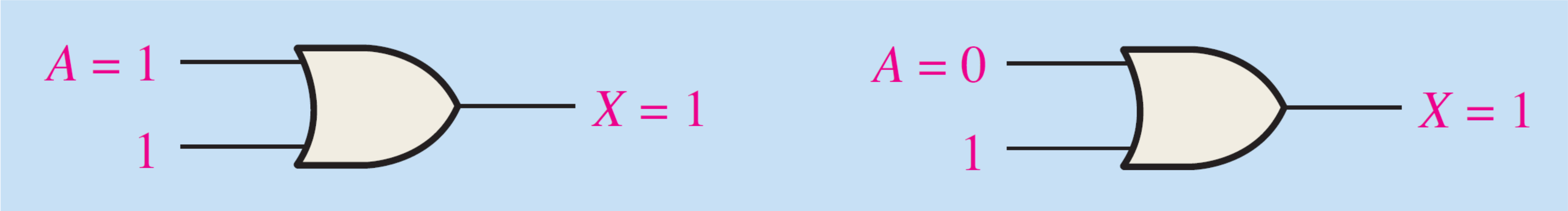


$X = AB + AC$

Rules of Boolean Algebra

- Rule 1: $A + 0 = A$

- Rule 2: $A + 1 = 1$



Rules of Boolean Algebra

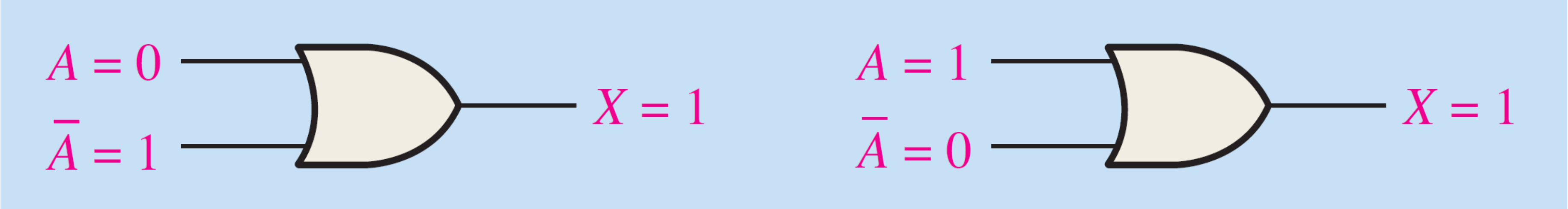
- **Rule 3:** $A \cdot 0 = 0$

- **Rule 4:** $A \cdot 1 = A$

Rules of Boolean Algebra

- Rule 5: $A + A = A$

- Rule 6: $A + \bar{A} = 1$



Rules of Boolean Algebra

- **Rule 7:** $A \cdot A = A$

- **Rule 8:** $A \cdot \bar{A} = 0$

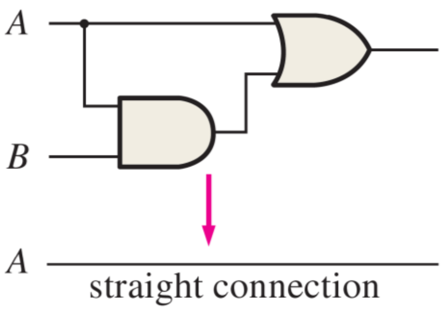
Rules of Boolean Algebra

- **Rule 9:** $\bar{\bar{A}} = A$

- **Rule 10:** $A + AB = A$
  - Proof:
    - L.H.S. = $A \cdot 1 + AB$ [R4]
    - = $A(1 + B)$ [Dist. Law]
    - = $A \cdot 1$ [R2]
    - = A [R4]
    - = R.H.S.

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal

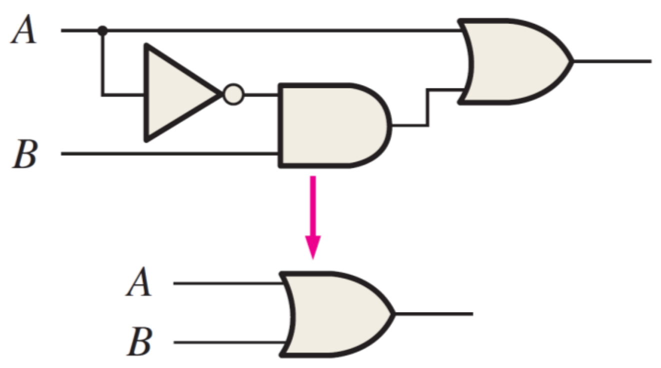


Rules of Boolean Algebra

- Rule 11: $A + \bar{A}B = A + B$
  - Proof:
    - L.H.S. = $A \ + \ AB + \bar{A}B$ [R10] $= A + (A + \bar{A})B$ [Dist. Law]
    - $= A + 1 \cdot B$ [R6] $= A + B$ [R4] = R.H.S.

A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Rules of Boolean Algebra

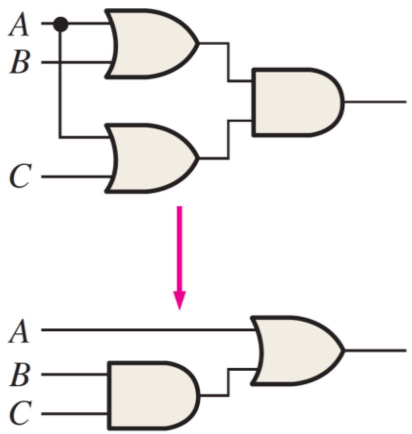
- **Rule 12:** $(A + B)(A + C) = A + BC$
  - Proof:
    - L.H.S. = $AA + AC + AB + BC$ [Dist. Law]
    - = $A + AC + AB + BC$ [R7]
    - = $A(1 + C) + AB + BC$ [Dist. Law]
    - = $A \cdot 1 + AB + BC$ [R2]
    - = $A + AB + BC$ [R4]
    - = $A(1 + B) + BC$ [Dist. Law]
    - = $A \cdot 1 + BC$ [R2]
    - = $A + BC$ [R4]
    - = R.H.S.

Rules of Boolean Algebra

- Rule 12: $(A + B)(A + C) = A + BC$ (...Continuing...)
  - Proof:

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



Rules of Boolean Algebra - Summary

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, *B*, or *C* can represent a single variable or a combination of variables.

Reading Material

- Floyd, Chapter 3:
  - Pages 130 - 134
- Floyd, Chapter 4:
  - Pages 172 - 179