CSE 411: Artificial Intelligence (Elective Course #6)

400 Level, Mechatronics Engineering
2nd Term 2016/2017, Lecture #5

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March 27th, 2017

Credits to Dr. Mohamed El Abd for the slides

Adminstrivia

Notes

- Assignment #2:
  - To be released on Thursday (Due 1 week after).
  - Constructing search trees on paper.

Course Info:

- Website: http://hshehata.github.io/courses/zu/cse411/
- Office hours: Sunday 11:30am - 12:30pm

Adminstrivia

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- Assignment #2:
  - To be released on Thursday (Due 1 week after).
  - Constructing search trees on paper.
- Assignment #3:
  - To be released next week.
  - Implementing search algorithms in Python.

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Types of search algorithms

- **Uninformed Search:**
  Only has the information provided by the problem formulation (initial state, available actions, transition model, goal test, and step/path cost).

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  - Algorithms: **GBFS**, **A***.

Outline

1. Informed Search

2. Requirements & Reading Material

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Informed Search

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- They just repeat a cycle of:
  - Choosing the next node to expand.
  - Check whether it is the goal.
  - If it is not, expand the node.
- Nodes are chosen in a specific order (imposed by a search strategy), which is not necessarily the ”best” order in terms of getting closer/faster to the goal.

Informed Search

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- How should we compute $f(n)$?

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  - Path cost is distance from root to state n.
- So, uniform-cost search uses an evaluation function $f(n) = g(n)$.
- The path cost $g(n)$ only accounts for cost to reach n. Hence, uniform-cost search is not goal directed/oriented.

Informed Search

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- More generally:
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- This estimate is called a **heuristic function**.
- Cannot be done based on problem formulation:
  Need to add additional information.

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  - $h(goal) = 0$.

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- Evaluation function $f(n)$:
  - $f(n) = g(n)$: Uniform-cost search.
  - $f(n) = h(n)$: Greedy best-first search.
  - $f(n) = g(n) + h(n)$: A* search.

Informed Search

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- It uses $f(n) = h(n)$ and ignores the path cost $g(n)$ entirely.
- Consider it to be the complement of uniform-cost search.
- GBFS algorithm is identical to UNIFORM-COST-SEARCH except that h is used instead of g.

Search Algorithms

Greedy best-first search algorithm (tree version)

```
function GREEDY-BEST-FIRST-SEARCH(problem) returns a solution, or failure
  node ← node with STATE=problem.INITIAL-STATE, PATH-COST=0
  frontier ← priority queue ordered by [heuristic h], with node as only element
  loop do
     if frontier.EMPTY?() then return failure
     node ← frontier.POP()          /* choose lowest-cost node in frontier */
     if problem.GOAL-TEST(node.STATE) then return node.SOLUTION()
     for each action in problem.ACTIONS(node.STATE) do
        child ← node.CHILD-NODE(problem, action)
        frontier.INSERT(child, [h(child)])
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  node ← node with STATE=*problem*.INITIAL-STATE, PATH-COST=0
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    add *node*.STATE to *explored*
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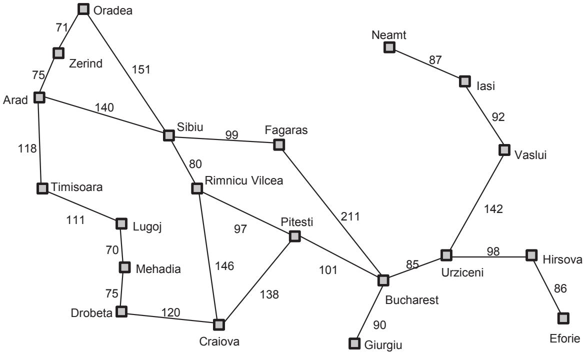
      if *child*.STATE is not in *explored* and not in *frontier* **then**
        frontier.INSERT(*child*, *h*(*child*))
      else if *child*.STATE is in *frontier* with higher *h*-value **then**
        replace that frontier node with *child*

Informed Search

GBFS example - Romania map

Find a route from Arad to Bucharest.



straight-line distances to Bucharest

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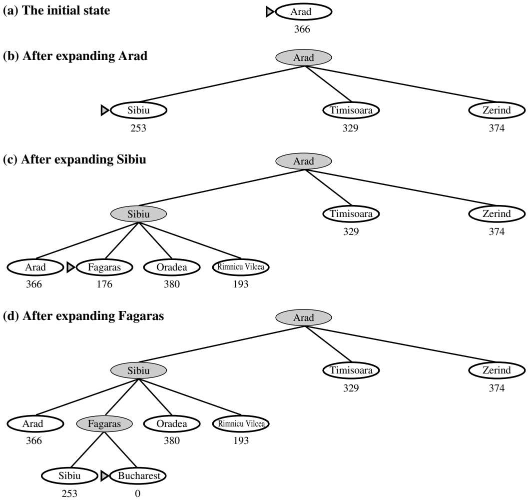

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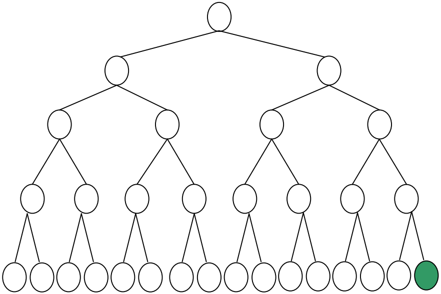
GBFS properties:

- Upper-bound case: goal is last node of the tree:
  - Number of nodes generated: b nodes for each node of m levels (entire tree).
  - Time and space complexity: all generated nodes $O(b^m)$.



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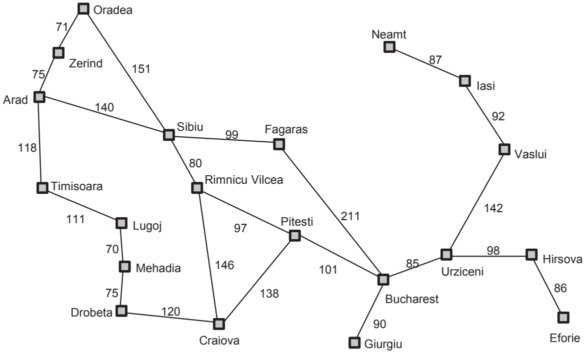

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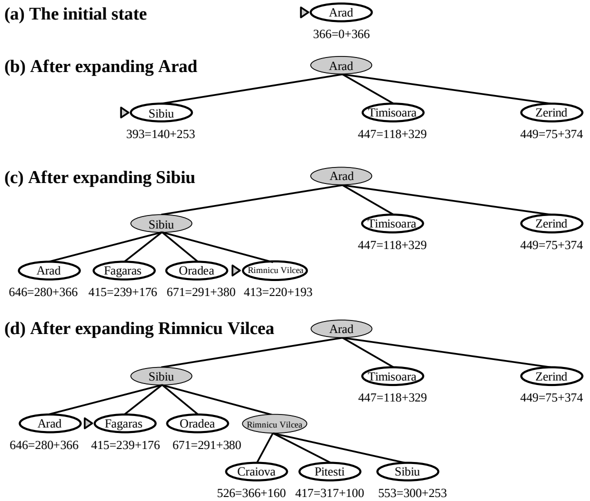

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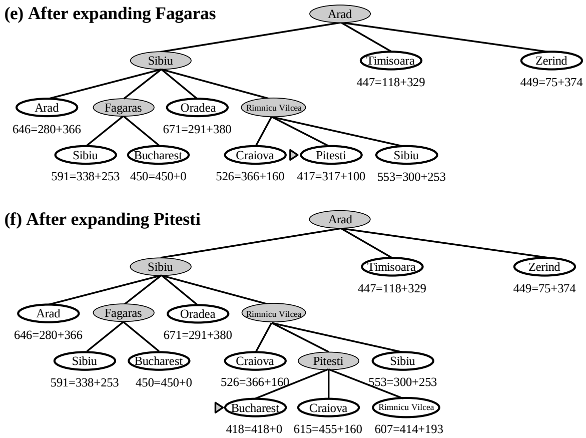

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- Complete (if b and ε are finite).
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  - Reason: number of nodes with cost $\leq C^*$ is finite.
- Optimal, given an admissible heuristic.
- Time and space complexity: not straightforward!
  - Number of nodes explored depends on the difference between h and h^* (true cost).
  - If $h = h^*$, A* expands only the nodes on the optimal solution path(s).
  - If $h = 0$, A* consumes as much (time/space) resources as UCS.

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- If $h(n)$ never overestimates, then $f(n)$ never overestimates the true cost to the goal through node n.

Informed Search

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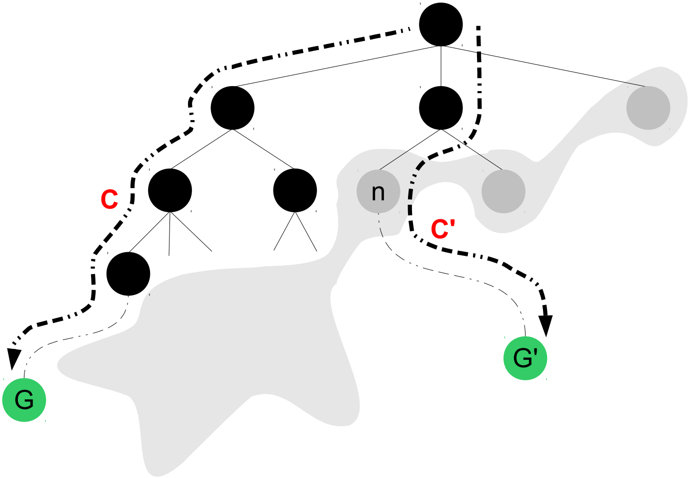
- Proving that ”admissibility guarantees optimality”:



Informed Search

A* search

- Proving that "admissibility guarantees optimality":
  - Suppose an A* search (that uses an admissible heuristic h) finds a goal G whose path cost of is C.
    - $h(G) = 0$, and hence: $f(G) = g(G) = C$
  - Let G' be another goal node whose path cost is C'.
  - Frontier contains a node n on the optimal path to G'.
    - h is admissible, and hence: $f(n) = g(n) + h(n) \leq C'$
  - Node G has been popped from frontier before node n.
    - $f(G) \leq f(n)$, and hence: $C \leq C'$.
  - Using an admissible heuristic, A* would always discover the lowest-cost (*i.e.*, optimal) solution.

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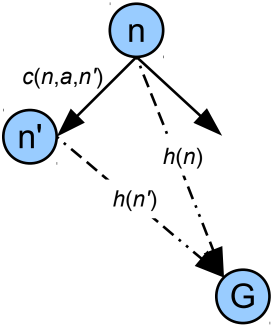
- This means that the graph version of A* search is not optimal any more!
- To ensure optimality, A* must use a **consistent** heuristic function h.
  - Consistency is a stronger condition than admissibility.

Informed Search

A* search

- **Consistency** (monotonicity) means that for every node n and child node n' reachable from n by action a, the estimated cost $h(n)$ is never greater than the estimated cost $h(n')$ plus the step cost of getting to n':

$$h(n) \leq h(n') + c(n, a, n')$$



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- With some mathematical arrangements:

$$\begin{aligned} f(n) &= h(n) + g(n) \\ &\leq h(n') + c(n, a, n') + g(n) \\ &\leq h(n') + g(n') \\ &\leq f(n') \end{aligned}$$

So, $f(n)$ never decreases as we approach the goal.

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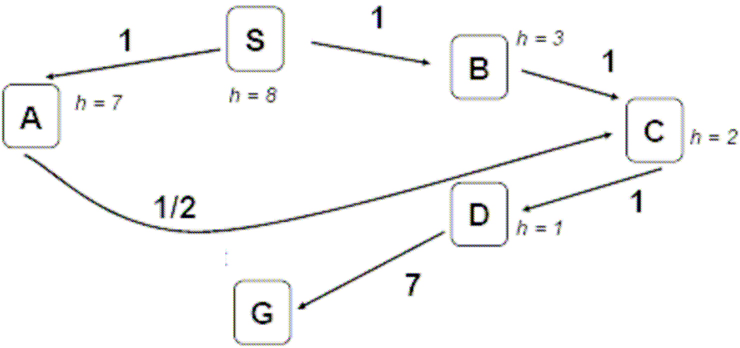
  So, $f(n)$ never decreases as we approach the goal.

- Consistency guarantees that states are always visited by the cheapest path first; no need to check if subsequent paths are better than the first.

Informed Search

A* example - consistency

Check whether h is consistent, and perform A* search.



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- Frontier nodes are ordered using an evaluation function f defined in terms of h.
  - GBFS: $f(n) = h(n)$.
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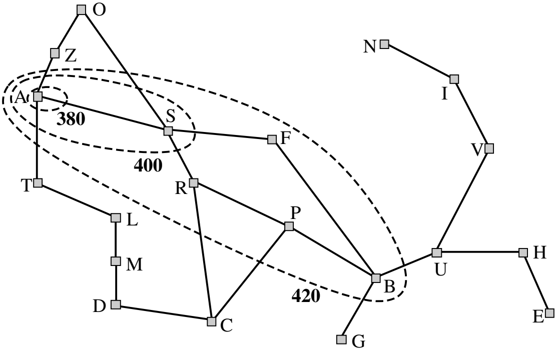
  - GBFS: $f(n) = h(n)$.
  - A*: $f(n) = h(n) + g(n)$.
- A* has the advantage of being optimal given that:
  - h is admissible (tree version).
  - h is consistent (graph version).

Informed Search

Effect of heuristic functions

- Uniform-cost search expands in circular cost contours ($h(n) = 0$).



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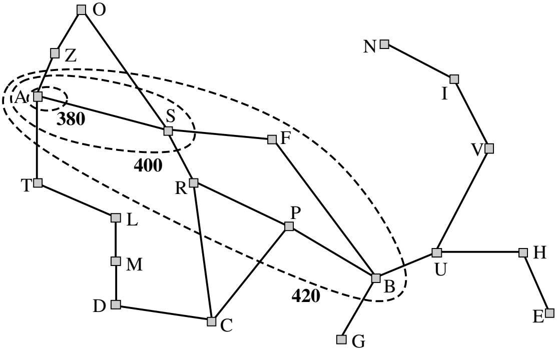
- A* search elongates & rotates contours towards goal:
  - More narrow/elongated, the better $h(n)$ is. More directed.



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- **Informedness:**
  For two admissible heuristic functions, $h1$ and $h2$:

  if $h2(n) \geq h1(n)$
  $h2(n)$ is more informed than $h1(n)$

  Alternatively we say that $h2(n)$ dominates $h1(n)$.

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- More informedness implies fewer expanded states.

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- The heuristic function is usually chosen by means of ***problem relaxation***.
- Problem relaxation means making the problem easier by dropping some constraints.

Informed Search

Creating a heuristic function

- How to choose the heuristic function for a given problem?
- The heuristic function is usually chosen by means of ***problem relaxation***.
- Problem relaxation means making the problem easier by dropping some constraints.
- Can also have different heuristics and always choose the best one:

$$h(n) = \max\{h_1(n), h_2(n), ..., h_m(n)\}$$

Informed Search

Creating a heuristic function

- For the Romania map problem, the heuristic function was selected as the straight line distance between the current city and the goal.

Informed Search

Creating a heuristic function

- For the Romania map problem, the heuristic function was selected as the straight line distance between the current city and the goal.
- How is this a relaxed problem?

Informed Search

Creating a heuristic function

- For the Romania map problem, the heuristic function was selected as the straight line distance between the current city and the goal.
- How is this a relaxed problem?
  - By dropping the traveling on roads constraint.

Informed Search

Example: creating heuristic for 8-puzzle

- Consider the 8-puzzle problem.

7	2	4
5		6
8	3	1

Start State

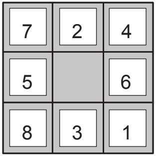
	1	2
3	4	5
6	7	8

Goal State

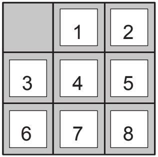
Informed Search

Example: creating heuristic for 8-puzzle

- Consider the 8-puzzle problem.
  - It would take at least *26 moves* to solve the problem instance shown below.



Start State

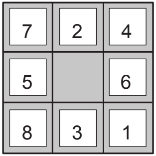


Goal State

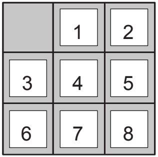
Informed Search

Example: creating heuristic for 8-puzzle

- Consider the 8-puzzle problem.
  - It would take at least *26 moves* to solve the problem instance shown below.
  - An admissible heuristic shouldn't overestimate that cost of 26 moves.
    - $h(Start) \leq 26$



Start State

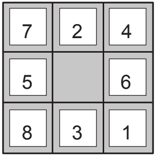


Goal State

Informed Search

Example: creating heuristic for 8-puzzle

- Consider the 8-puzzle problem.
  - It would take at least *26 moves* to solve the problem instance shown below.
  - An admissible heuristic shouldn't overestimate that cost of 26 moves.
    - $h(Start) \leq 26$
  - Generally speaking, an admissible heuristic shouldn't overestimate the cost of solving the puzzle starting from any node.
    - $h(n) \leq$ minimum number of moves to get from n to $Goal$.



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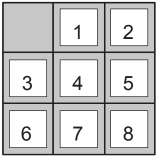
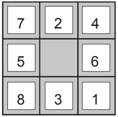


Goal State

Informed Search

Example: creating heuristic for 8-puzzle

- Description of legal moves in 8-puzzle:
  A tile can move from location A to location B if A,B are adjacent and B is blank.



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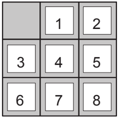
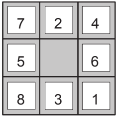


Goal State

Informed Search

Example: creating heuristic for 8-puzzle

- Description of legal moves in 8-puzzle:
  A tile can move from location A to location B if A,B are adjacent and B is blank.
- Problem relaxation:



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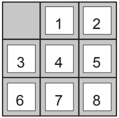

Goal State

Informed Search

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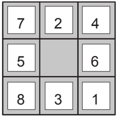

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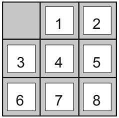

Goal State

Informed Search

Example: creating heuristic for 8-puzzle

- Description of legal moves in 8-puzzle:
  A tile can move from location A to location B if A,B are adjacent and B is blank.
- Problem relaxation:
  - A tile can move from location A to location B:
    - $h_1(n) =$ number of tiles out of place.

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    - $h_1(Start) = 1+1+1+1+1+1+1+1 = 8 \leq 26.$

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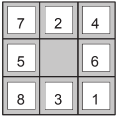
  - A tile can move from loc. A to loc. B if A,B are adjacent:



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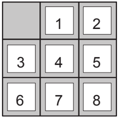

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    - $h_1(Start) = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8 \leq 26.$
  - A tile can move from loc. A to loc. B if A,B are adjacent:
    - $h_2(n) =$ sum of distances of tiles from goal locations.

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5		6
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Goal State

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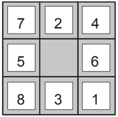
Example: creating heuristic for 8-puzzle

- Description of legal moves in 8-puzzle:
  A tile can move from location A to location B if A,B are adjacent and B is blank.
- Problem relaxation:
  - A tile can move from location A to location B:
    - $h_1(n) =$ number of tiles out of place.
    - $h_1(Start) = 1+1+1+1+1+1+1+1 = 8 \leq 26.$
  - A tile can move from loc. A to loc. B if A,B are adjacent:
    - $h_2(n) =$ sum of distances of tiles from goal locations.
    - $h_2(Start) = 3+1+2+2+2+3+3+2 = 18 \leq 26.$



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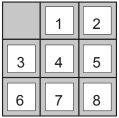

Goal State

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  - A tile can move from location A to location B:
    - $h_1(n) =$ number of tiles out of place.
    - $h_1(Start) = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8 \leq 26$.
  - A tile can move from loc. A to loc. B if A,B are adjacent:
    - $h_2(n) =$ sum of distances of tiles from goal locations.
    - $h_2(Start) = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18 \leq 26$.
- Notice that h_1 and h_2 are admissible, and $h_1(n) \leq h_2(n)$.

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Goal State

Informed Search

A* search - 8-puzzle search costs

Data are averaged over 100 instances for the 8-puzzle problem across various solution lengths.

Optimal Solution Length (d)	Search Cost (nodes generated)		
	IDS	A* using h_1	A* using h_2
4 steps	112	13	12
8 steps	6384	39	25
12 steps	3644035	227	73
16 steps	-	1301	211
20 steps	-	7276	676
24 steps	-	39135	1641

Outline

Requirements

What do I need from you

- When given a certain problem you should be able to:

Requirements

What do I need from you

- When given a certain problem you should be able to:
  - Build the search tree up to a given depth.

Requirements

What do I need from you

- When given a certain problem you should be able to:
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  - Traverse the search tree according to a given strategy.

Requirements

What do I need from you

- When given a certain problem you should be able to:
  - Build the search tree up to a given depth.
  - Traverse the search tree according to a given strategy.
  - Propose a good heuristic function for the problem.
- Answer descriptive questions.

Requirements

What do I need from you

- When given a certain problem you should be able to:
  - Build the search tree up to a given depth.
  - Traverse the search tree according to a given strategy.
  - Propose a good heuristic function for the problem.
  - Indicate whether a given heuristic is admissible/consistent or not.
- Answer descriptive questions.

Reading Material

Which parts of the textbook are covered

- Russell-Norvig, Chapters 3:
  - Pages 92 - 98.
  - Pages 102 - 106.