Kingdom of Saudi Arabia
Ministry of Education
Shaqra University
CCIT

CS 211 – Digital Logic Design
First Term – 1439/1440
Solution to Assignment #1

1. Convert the following numbers from the given base to the other three bases …

- 69.3125_{10} to binary/octal/hexadecimal:

  - 69.3125_{10} to binary:

69 / 2 = 34, **1** 34 / 2 = 17, **0** 17 / 2 = 8, **1** 8 / 2 = 4, **0** 4 / 2 = 2, **0** 2 / 2 = 1, **0** 1 / 2 = 0, **1** ➔ 1000101_2	0.3125 * 2 = **0**.625 0.625 * 2 = **1**.25 0.25 * 2 = **0**.5 0.5 * 2 = **1**.0 ➔ 0.0101_2
➔ 1000101.0101_2	

  - 69.3125_{10} to octal:

69 / 8 = 8, **5** 8 / 8 = 1, **0** 1 / 8 = 0, **1** ➔ 105_8	0.3125 * 8 = **2**.5 0.5 * 8 = **4**.0 ➔ 0.24_8
➔ 105.24_8	

  - 69.3125_{10} to hexadecimal:

69 / 16 = 4, **5** 4 / 16 = 1, **4** ➔ 45_{16}	0.3125 * 16 = **5**.0 ➔ 0.5_{16}
➔ 45.5_{16}	

- 10111101.101_2 to decimal/octal/hexadecimal

  - 10111101.101_2 to decimal:

    ➔ $2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 + 2^{-1} + 2^{-3} = 189.625_{10}$

  - 10111101.101_2 to octal:
    ➔ $010\ 111\ 101\ .\ 101_2$ ➔ 275.5_8

  - 10111101.101_2 to hexadecimal:

    ➔ $1011\ 1101\ .\ 1010_2$ ➔ $BD.A_{16}$

- 326.5_8 to decimal/binary/hexadecimal
  - 326.5_8 to decimal:

    ➔ $3 * 8^2 + 2 * 8^1 + 6 * 8^0 + 5 * 8^{-1} = 214.625_{10}$

  - 326.5_8 to binary:

    ➔ 326.5_8 ➔ $011\ 010\ 110\ .\ 101_2$

  - 326.5_8 to hexadecimal:

    ➔ 326.5_8 ➔ $011\ 010\ 110\ .\ 101_2$ ➔ $1101\ 0110\ .\ 1010_2$ ➔ $D6.A_{16}$

- $C7.A_{16}$ to decimal/binary/octal
  - $C7.A_{16}$ to decimal:

    ➔ $12 * 16^1 + 7 * 16^0 + 10 * 16^{-1} = 199.625_{10}$

  - $C7.A_{16}$ to binary:

    ➔ $C7.A_{16}$ ➔ $1100\ 0111\ .\ 1010_2$

  - $C7.A_{16}$ to octal:

    ➔ $C7.A_{16}$ ➔ $1100\ 0111\ .\ 1010_2$ ➔ $011\ 000\ 111\ .\ 101_2$ ➔ 307.5_8

- Summary:

Decimal	**Binary**	**Octal**	**Hexadecimal**
69.3125_{10}	1000101.0101_2	105.24_8	45.5_{16}
189.625_{10}	10111101.101_2	275.5_8	$BD.A_{16}$
214.625_{10}	011010110.101_2	326.5_8	$D6.A_{16}$
199.625_{10}	11000111.1010_2	307.5_8	$C7.A_{16}$

2. Perform the following arithmetic operations using 2's complement …

a. $17_{10} - 69_{10}$

$$\begin{array}{rl}
0\,0\,0\,1\,0\,0\,0\,1 & \leftarrow 17_{10} \\
-\ 0\,1\,0\,0\,0\,1\,0\,1 & \leftarrow 69_{10} \\
\hline
0\,0\,0\,1\,0\,0\,0\,1 & \\
+\ 1\,0\,1\,1\,1\,0\,1\,1 & \leftarrow \text{2's comp. of } 01000101 \\
\hline
1\,1\,0\,0\,1\,1\,0\,0 & \rightarrow \text{-2's comp. of } 11001100 \rightarrow -00110100_2 \rightarrow -52_{10}
\end{array}$$

b. $-12_{10} \times 11_{10}$

Note: Here 8-bits will not be sufficient to represent the result since the multiplicand (-12_{10}) and the multiplier ($+11_{10}$) require 5 bits each to be represented in the two's complement format, and hence the multiplication product needs to be represented by 2*5=10 bits. So we choose to represent all numbers using 10 bits (instead of 8 bits).

$$\begin{array}{rl}
0\,0\,0\,0\,0\,0\,1\,1\,0\,0 & \leftarrow |\,\text{Multiplicand}\,| = 12_{10} \\
\times\ 0\,0\,0\,0\,0\,0\,1\,0\,1\,1 & \leftarrow |\,\text{Multiplier}\,| = 11_{10} \\
\hline
0\,0\,0\,0\,0\,0\,1\,1\,0\,0 & \\
+\ 0\,0\,0\,0\,0\,1\,1\,0\,0 & \\
+\ 0\,0\,0\,0\,0\,0\,0\,0 & \\
+\ 0\,0\,0\,1\,1\,0\,0 & \\
+\ 0\,0\,0\,0\,0\,0 & \\
+\ \ldots & \\
\hline
0\,0\,1\,0\,0\,0\,0\,1\,0\,0 & \rightarrow |\,\text{Product}\,| = 132_{10}
\end{array}$$

Multiplicand is negative & multiplier is positive → Product is negative!
Product = 2's comp. of 0010000100 = 1101111100

c. $-116_{10} \div -21_{10}$

$$\begin{array}{rr|l l}
 & & 0\,0\,0\,0\,0\,1\,0\,1 & \rightarrow |\,\text{Quotient}\,| = 5_{10} \\
|\,\text{Divisor}\,| = 21_{10} \rightarrow & 0\,0\,0\,1\,0\,1\,0\,1 & 0\,1\,1\,1\,0\,1\,0\,0 & \leftarrow |\,\text{Dividend}\,| = 116_{10} \\
 & & \underline{0\,1\,0\,1\,0\,1} & \\
 & & 0\,0\,1\,0\,0\,0\,0\,0 & \\
 & & \underline{0\,0\,0\,1\,0\,1\,0\,1} & \\
 & & 0\,0\,0\,0\,1\,0\,1\,1 & \rightarrow |\,\text{Remainder}\,| = 11_{10}
\end{array}$$

Dividend and divisor are negative → Quotient is positive & remainder is negative
Quotient = 00000101
Remainder = 2's complement of 00001011 = 11110101

3. Calculate the decimal value which is equivalent to the binary value: 100010010110 …

a. If it represents a BCD number.

$1000\ 1001\ 0110_{BCD} \rightarrow 896_{10}$

b. If it represents a Gray Code.

$100010010110_{Gray} \rightarrow 111100011011_2 \rightarrow 2^{11} + 2^{10} + 2^9 + 2^8 + 2^4 + 2^3 + 2^1 + 2^0 = 3867_{10}$

c. If it represents a signed number in the sign-magnitude form.

$100010010110_{SM} \rightarrow -00010010110 \rightarrow -150_{10}$

d. If it represents a signed number in the 1's complement form.

$100010010110_{1's} \rightarrow$ -1's comp. of $100010010110 \rightarrow -011101101001 \rightarrow -(2^{10} + 2^9 + 2^8 + 2^6 + 2^5 + 2^3 + 2^0) = -1897_{10}$

4. Represent 69.3125_{10} as a single-precision floating-point binary number.

$69.3125_{10} = 1000101.0101_2 = 1.0001010101 * 2^6$
$S = 0$, $E = 6_{10}+127_{10} = 133_{10} = 10000101$, $F= 00010101010000000000000$
Number $\rightarrow$ 0 10000101 00010101010000000000000