

Outline

**Adversarial
Search
(Continued)**

Ordering in α - β
Multiplayer Games
Limited Time
Evaluation
functions
Stochastic games

**Requirements
& Reading
Material**

CSE 411: Artificial Intelligence (Elective Course #6)

**400 Level, Mechatronics Engineering
2nd Term 2016/2017, Lecture #7**

Hazem Shehata

Dept. of Computer & Systems Engineering
Zagazig University

April 19th, 2017

Credits to Dr. Mohamed El Abd for the slides

Adminstrivia

Notes

- Assignment #3:
 - Due: **today**.

Course Info:

- Website: <http://hshehata.github.io/courses/zu/cse411/>
- Office hours: Sunday 11:30am - 12:30pm

Adminstrivia

Notes

- Assignment #3:
 - Due: **today**.
- Midterm:
 - Date: **Wednesday, Apr. 26, 2017**.
 - Time: **1:00pm - 2:30pm**.
 - Scope: lectures 1 \rightarrow 6.

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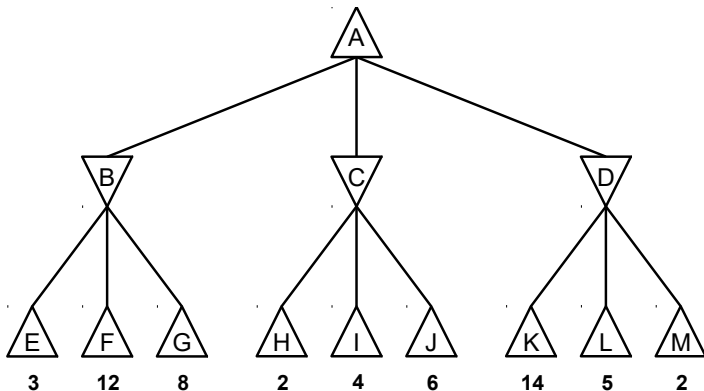
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Apply α - β pruning to calculate minimax value for node A.



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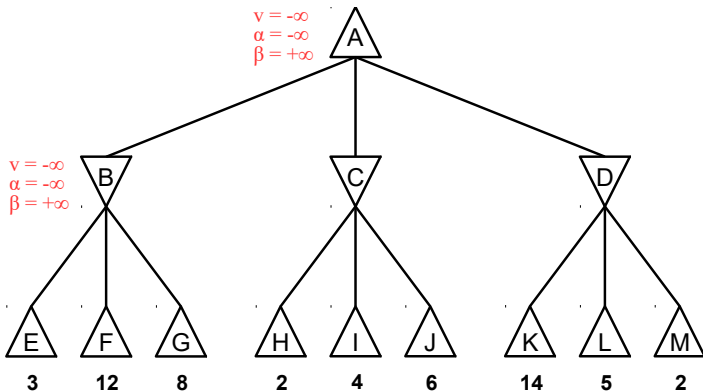
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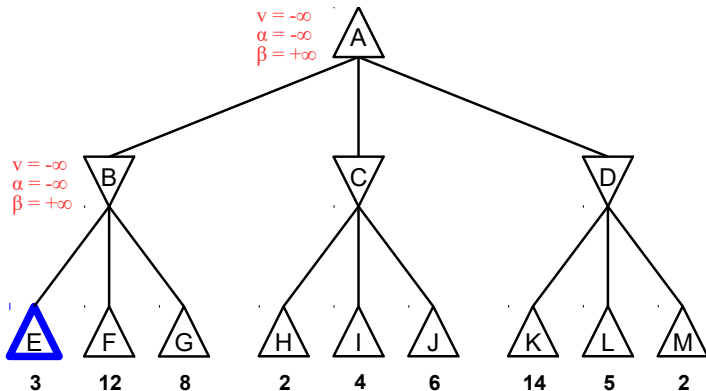
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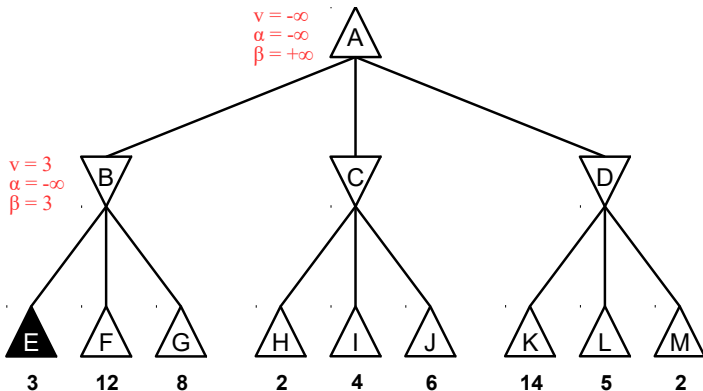
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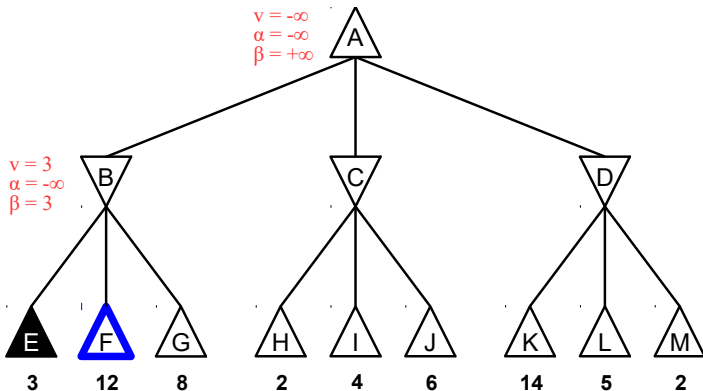
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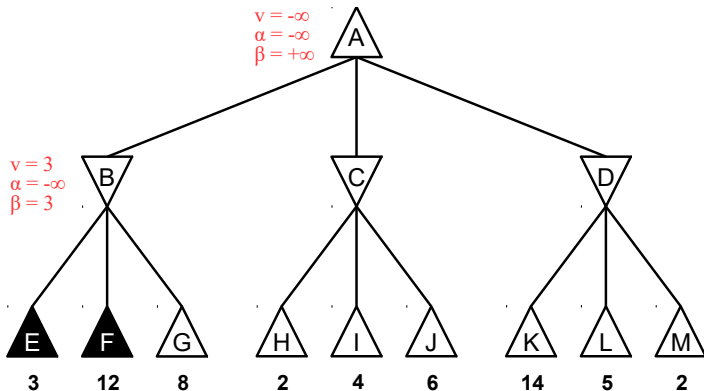
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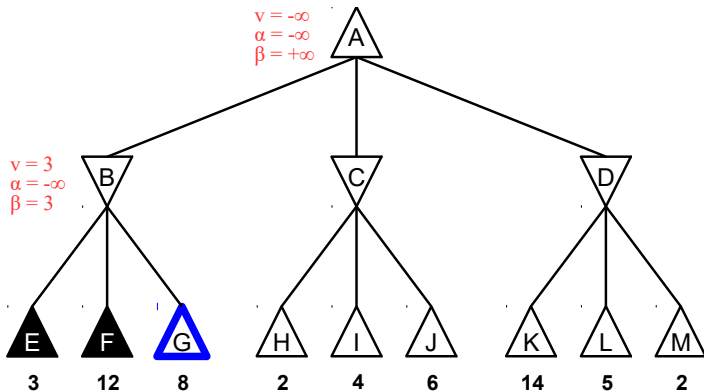
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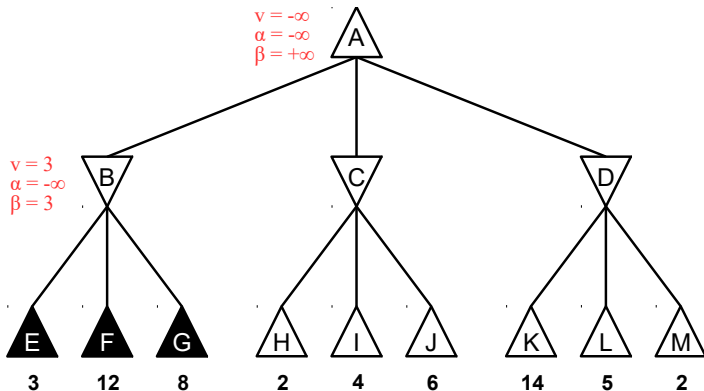
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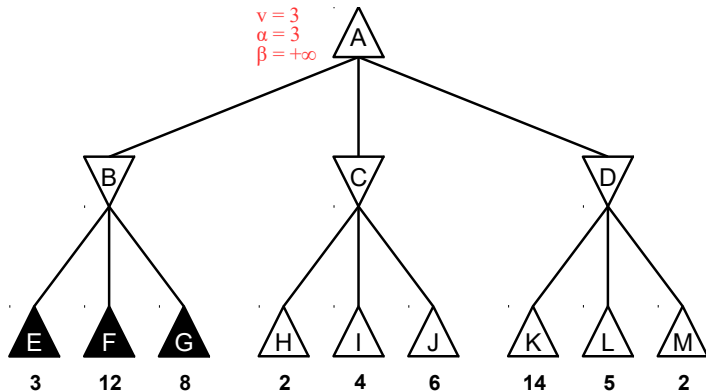
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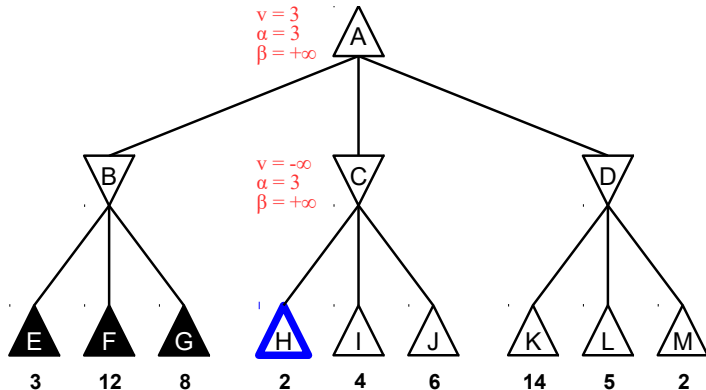
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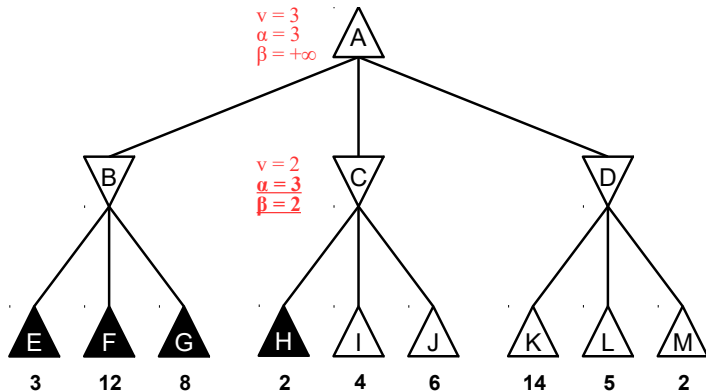
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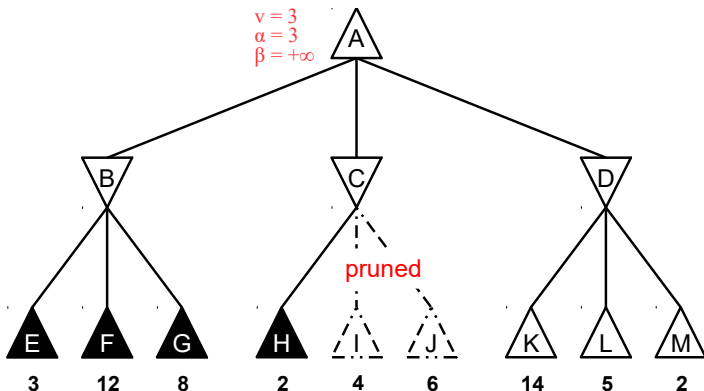
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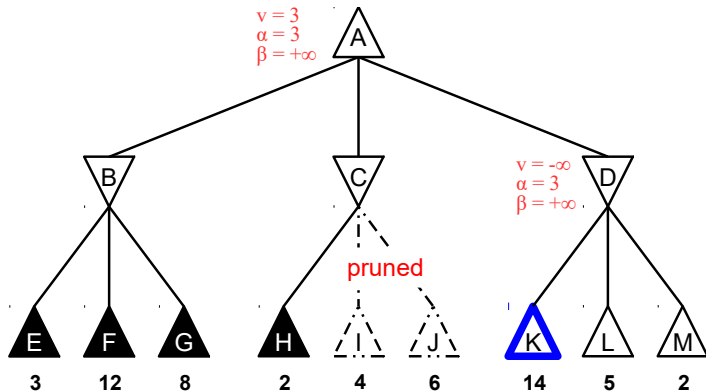
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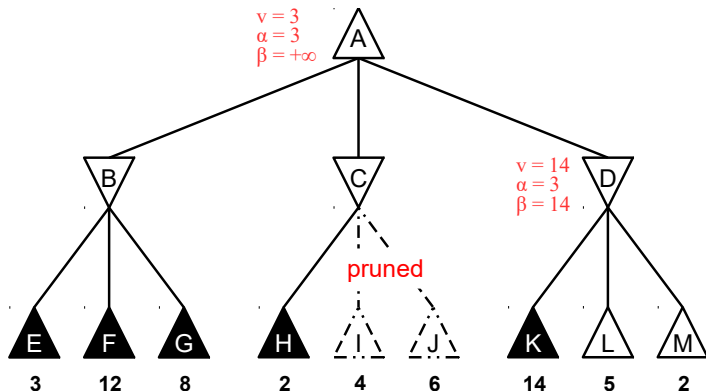
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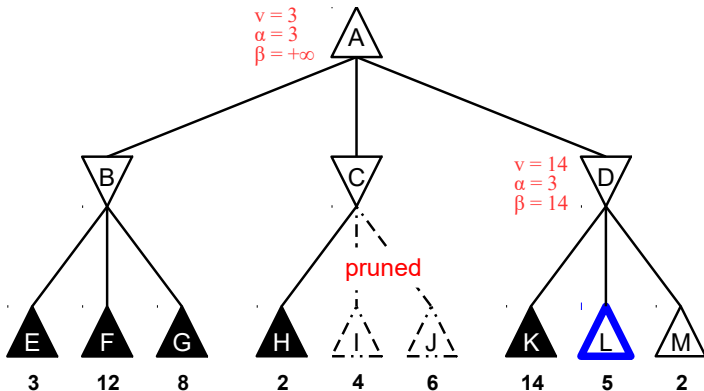
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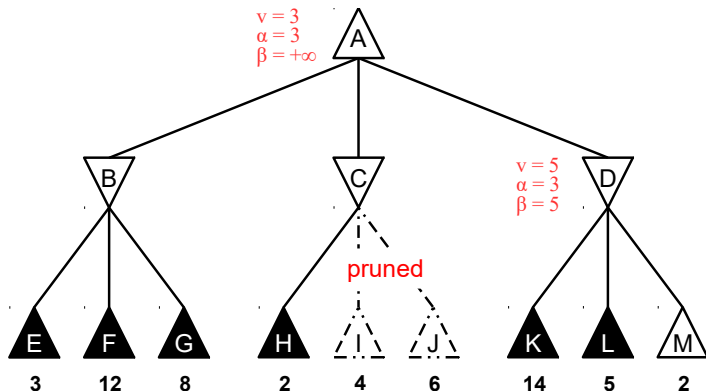
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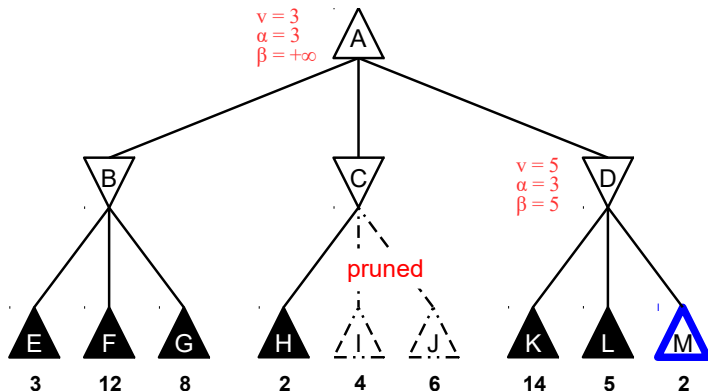
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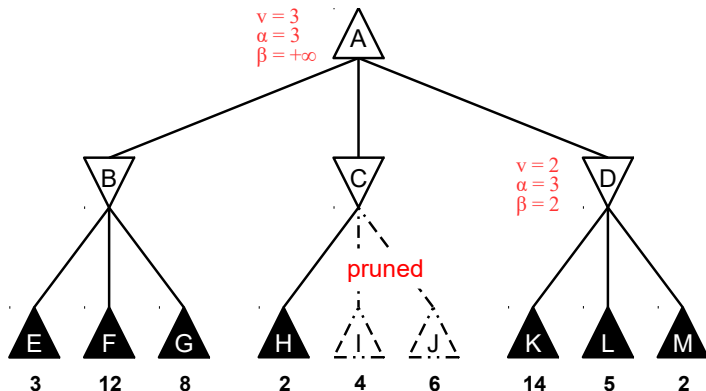
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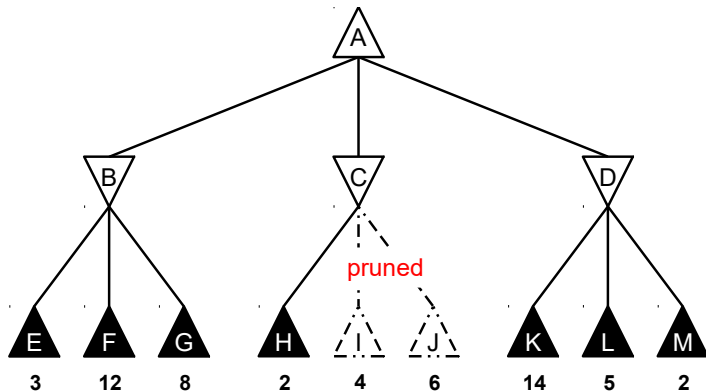
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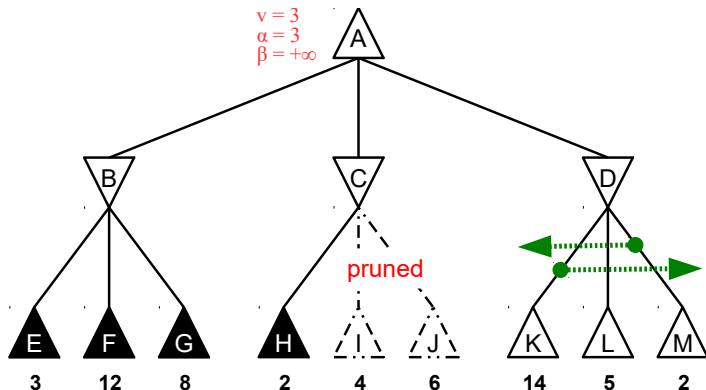
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Example: Effect of move ordering on α - β pruning

Reverse the order of the utility values for K, L, and M.



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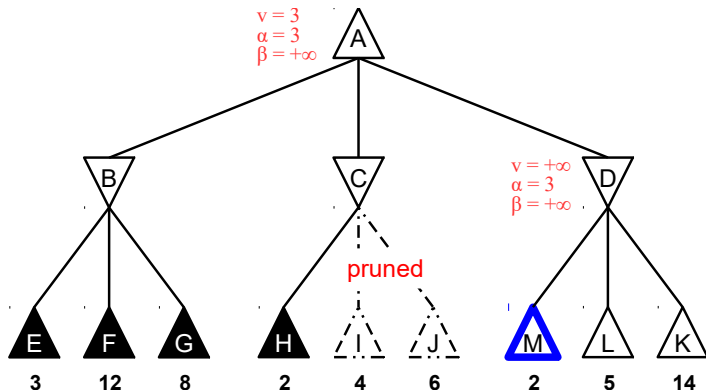
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Example: Effect of move ordering on α - β pruning

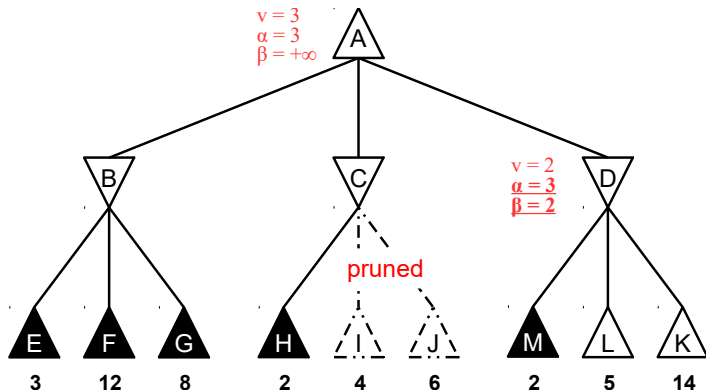
Reverse the order of the utility values for K, L, and M.



Adversarial search

Example: Effect of move ordering on α - β pruning

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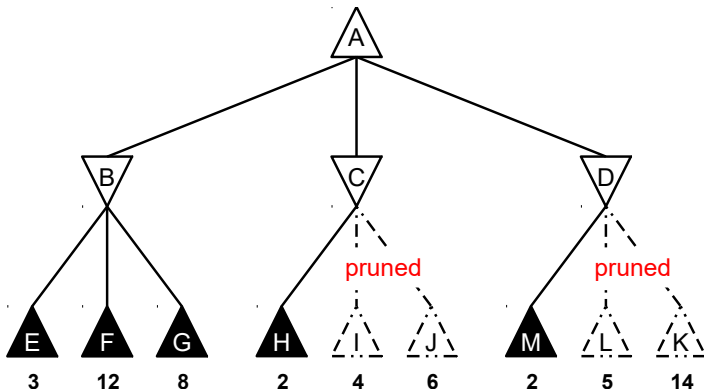
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Effect of move ordering on α - β pruning

- In the previous example, changing the order of nodes increases the efficiency of α - β pruning.

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Effect of move ordering on α - β pruning

- In the previous example, changing the order of nodes increases the efficiency of α - β pruning.
- Efficiency dependant on the ordering of children:
 - Checks each of **MAX's** children until finding one with a value higher than β .
 - Checks each of **MIN's** children until finding one with a value lower than α .

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Effect of move ordering on α - β pruning

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- Efficiency dependant on the ordering of children:
 - Checks each of **MAX's** children until finding one with a value higher than β .
 - Checks each of **MIN's** children until finding one with a value lower than α .
- Can use heuristics to order the nodes to check:
 - Check the highest-value children first for **MAX**.
 - Check the lowest-value children first for **MIN**.

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- Efficiency dependant on the ordering of children:
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- Can use heuristics to order the nodes to check:
 - Check the highest-value children first for **MAX**.
 - Check the lowest-value children first for **MIN**.
- Good ordering can reduce time complexity to $O(b^{m/2})$, random ordering gives roughly $O(b^{3m/4})$, remember that Minimax is $O(b^m)$.

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Optimal decisions in multiplayer games

- Minimax can be extended to multiplayer games:

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Optimal decisions in multiplayer games

- Minimax can be extended to multiplayer games:
 - The single value for each node is replaced with a vector.

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Optimal decisions in multiplayer games

- Minimax can be extended to multiplayer games:
 - The single value for each node is replaced with a vector.
 - For instance, a vector $\langle v_a, v_b, v_c \rangle$ is associated with each node in a 3-player game with players A, B, and C.

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 - For instance, a vector $\langle v_a, v_b, v_c \rangle$ is associated with each node in a 3-player game with players A, B, and C.
 - For a terminal state, UTILITY function returns a vector representing utility of that state from each player's viewpoint.

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 - For a terminal state, UTILITY function returns a vector representing utility of that state from each player's viewpoint.
- The multiplayer version of minimax (M-MAX) can be defined as follows:

$$\text{M-MAX}(s) = \begin{cases} \text{UTILITY}(s) & \text{if } \text{TERMINAL-TEST}(s) \\ \max_{a \in \text{ACTIONS}(s)}^p \text{M-MAX}(\text{RESULT}(s, a)) & \text{if } \text{PLAYER}(s) = p \end{cases}$$

where \max^p is a function that chooses from among a group of vectors the one whose p^{th} element is the maximum.

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Example: multiplayer game tree

A partial game tree with 3 players (A, B, C).

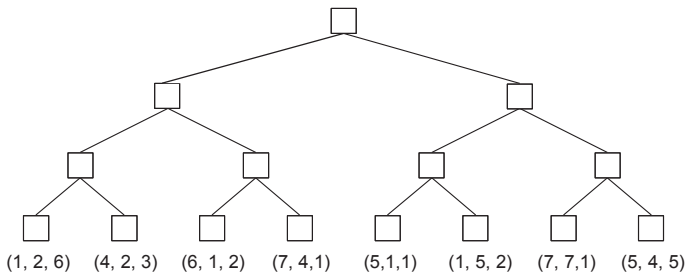
to move

A

B

C

A



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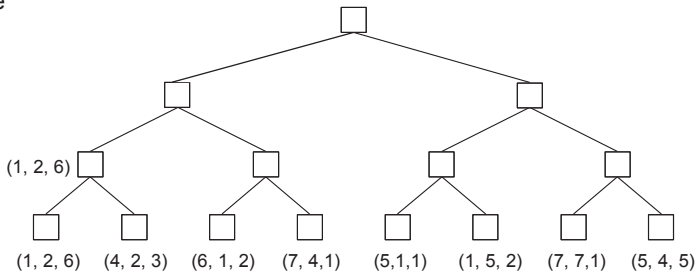
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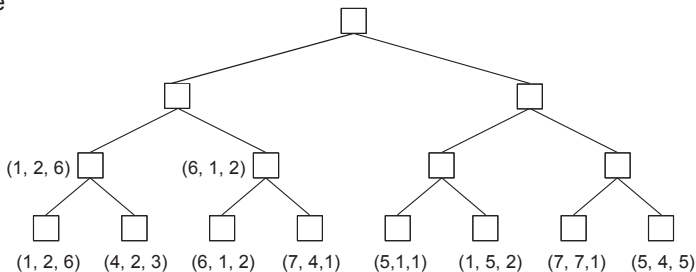
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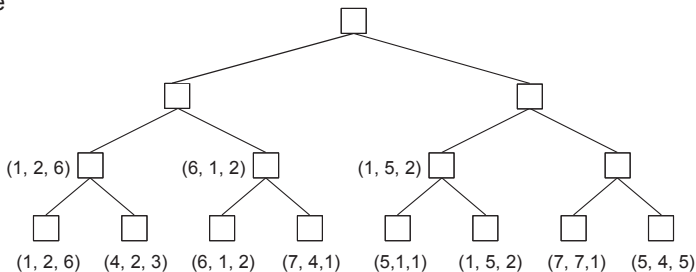
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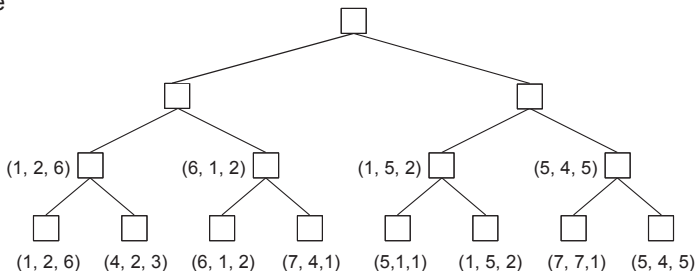
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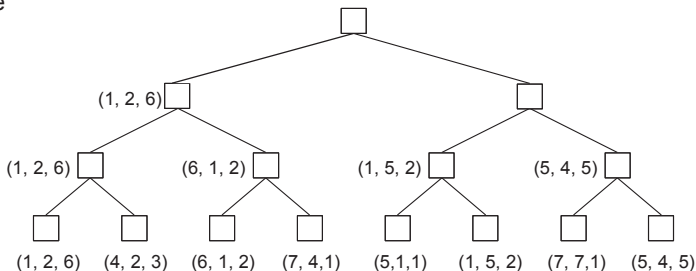
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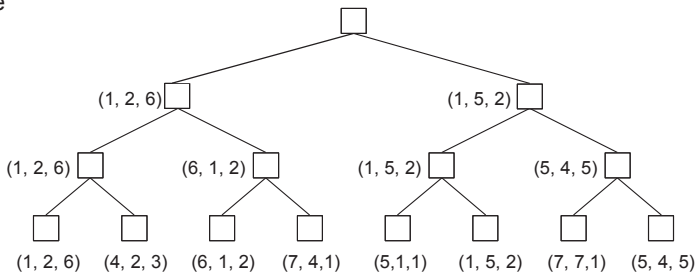
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Example: multiplayer game tree

A partial game tree with 3 players (A, B, C).

to move

A

(1, 2, 6)

B

(1, 2, 6)

(1, 5, 2)

C

(1, 2, 6)

(6, 1, 2)

(1, 5, 2)

(5, 4, 5)

A

(1, 2, 6)

(4, 2, 3)

(6, 1, 2)

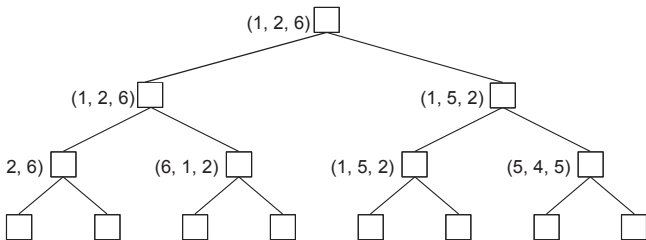
(7, 4, 1)

(5, 1, 1)

(1, 5, 2)

(7, 7, 1)

(5, 4, 5)



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to move

A

(1, 2, 6)

B

(1, 2, 6)

(1, 5, 2)

C

(1, 2, 6)

(6, 1, 2)

(1, 5, 2)

(5, 4, 5)

A

(1, 2, 6)

(4, 2, 3)

(6, 1, 2)

(7, 4, 1)

(5, 1, 1)

(1, 5, 2)

(7, 7, 1)

(5, 4, 5)

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Imperfect real-time decisions

- How to reach a good decision when time is limited?

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Imperfect real-time decisions

- How to reach a good decision when time is limited?
 - **Cut off** search earlier (before reaching terminal states).

Adversarial search

Imperfect real-time decisions

- How to reach a good decision when time is limited?
 - **Cut off** search earlier (before reaching terminal states).
 - Apply a **heuristic evaluation function** to evaluate the desirability of states (instead of their utility values).

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Imperfect real-time decisions

- How to reach a good decision when time is limited?
 - **Cut off** search earlier (before reaching terminal states).
 - Apply a **heuristic evaluation function** to evaluate the desirability of states (instead of their utility values).
- For inst., suppose we have 100 sec's to make a move in a chess game, and we explore 10^4 nodes/sec's:

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Imperfect real-time decisions

- How to reach a good decision when time is limited?
 - **Cut off** search earlier (before reaching terminal states).
 - Apply a **heuristic evaluation function** to evaluate the desirability of states (instead of their utility values).
- For inst., suppose we have 100 sec's to make a move in a chess game, and we explore 10^4 nodes/sec's:
 - We can explore 10^6 nodes per move $\approx 35^{8/2}$.

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Imperfect real-time decisions

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- For inst., suppose we have 100 sec's to make a move in a chess game, and we explore 10^4 nodes/sec's:
 - We can explore 10^6 nodes per move $\approx 35^{8/2}$.
 - α - β can reach depth of 8 \Rightarrow pretty good chess program.

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Heuristic minimax

- Need to alter minimax (or α - β) in two ways:

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- Need to alter minimax (or α - β) in two ways:
 - 1 Replace utility function (UTILITY) by heuristic evaluation function (EVAL) \Rightarrow estimates position's utility.

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- Need to alter minimax (or α - β) in two ways:
 - 1 Replace utility function (UTILITY) by heuristic evaluation function (EVAL) \Rightarrow estimates position's utility.
 - 2 Replace terminal test (TERMINAL-TEST) by cutoff test (CUTOFF-TEST) \Rightarrow decides when to apply EVAL.

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Heuristic minimax

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 - 1 Replace utility function (UTILITY) by heuristic evaluation function (EVAL) \Rightarrow estimates position's utility.
 - 2 Replace terminal test (TERMINAL-TEST) by cutoff test (CUTOFF-TEST) \Rightarrow decides when to apply EVAL.

- Modified version of minimax is called **heuristic minimax**:

$$\text{H-MINIMAX}(s, d) = \begin{cases} \text{EVAL}(s) & \text{if CUTOFF-TEST}(s, d) \\ \max_{a \in \text{ACTIONS}(s)} \text{H-MINIMAX}(\text{RESULT}(s, a), d + 1) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in \text{ACTIONS}(s)} \text{H-MINIMAX}(\text{RESULT}(s, a), d + 1) & \text{if PLAYER}(s) = \text{MIN} \end{cases}$$

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- To implement this modification in the minimax and α - β pruning algorithms, all that has to be done to is to replace the line:

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if TERMINAL-TEST(s) **then return** UTILITY(s)

Adversarial search

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Adversarial search

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if CUTOFF-TEST(s, d) **then return** EVAL(s)

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if TERMINAL-TEST(s) **then return** UTILITY(s)
with the line:
if CUTOFF-TEST(s, d) **then return** EVAL(s)
- Also, some extra bookkeeping is needed to increment the depth on each recursive call of the algorithm.

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Evaluation functions

- Heuristic evaluation functions are functions to estimate the expected utility of a game from a non-leaf node.

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Evaluation functions

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 - Should not take too much time to compute.
 - For non-leaf nodes, it should be strongly correlated with the actual chances of winning.
- Mostly work by calculating various **features** of state.
 - Chess example: features could be number of white queens, black queens, white pawns, black pawns, *etc.*

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- A simple version is to set it up by calculating a weighted average of the numerical values of all the features.

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Evaluation functions

- A simple version is to set it up by calculating a weighted average of the numerical values of all the features.
- In this case, the evaluation function takes the form of a **weighted linear function** expressed as follows:

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$$\text{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

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- Chess example: f_i could be number of each kind of piece on board, w_i could represent strength of that kind.

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Evaluation functions

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$$\text{EVAL}(s) = w_1f_1(s) + w_2f_2(s) + \dots + w_nf_n(s)$$
 - Chess example: f_i could be number of each kind of piece on board, w_i could represent strength of that kind.
- Evaluation function need not return actual expected values as long as the ordering of states is the same.

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- In some games, there are many unpredictable external events that could affect the course of the game.

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- In some games, there are many unpredictable external events that could affect the course of the game.
- These games have stochastic environments, and hence, called **stochastic games**.

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Stochastic games

- In some games, there are many unpredictable external events that could affect the course of the game.
- These games have stochastic environments, and hence, called **stochastic games**.
- Typical sources of unpredictability in stochastic games include events such as: roll of a dice, toss of a coin, withdrawal of a card, *etc.*

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Stochastic games

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- Due to such uncertainty, we cannot construct a **complete/exact** game tree for a stochastic game.

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Stochastic games

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- Typical sources of unpredictability in stochastic games include events such as: roll of a dice, toss of a coin, withdrawal of a card, *etc.*
- Due to such uncertainty, we cannot construct a **complete/exact** game tree for a stochastic game.
- Minimax strategy becomes inapplicable to such games!

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- To model such uncertainty in stochastic games, a game tree would include **chance nodes** in addition to MAX and MIN nodes.

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Stochastic games

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 - Branches leading from each chance node denote the possible outcomes of an unpredictable event (*e.g.*, dice roll).

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Stochastic games

- To model such uncertainty in stochastic games, a game tree would include **chance nodes** in addition to MAX and MIN nodes.
 - Branches leading from each chance node denote the possible outcomes of an unpredictable event (*e.g.*, dice roll).
 - Each branch is labeled with an **outcome** r and its **probability** $P(r)$.

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- To model such uncertainty in stochastic games, a game tree would include **chance nodes** in addition to MAX and MIN nodes.
 - Branches leading from each chance node denote the possible outcomes of an unpredictable event (*e.g.*, dice roll).
 - Each branch is labeled with an **outcome** r and its **probability** $P(r)$.
 - **Average expected utility** of a chance node
 $= \sum_r P(r) * \text{NEXT}(r)$, where $\text{NEXT}(r)$ represents the average expected utility of the node reached as a consequence of the outcome r .

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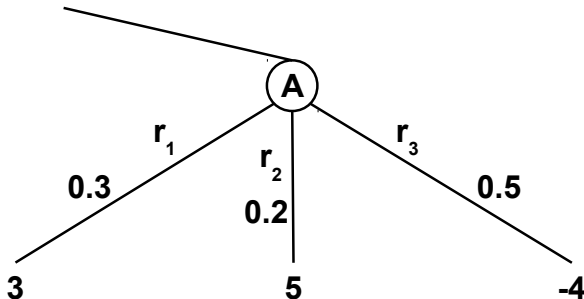
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Example: chance nodes

The expected utility of the chance node given below
 $= 0.3 * 3 + 0.2 * 5 + 0.5 * (-4) = -0.1$.



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- In **single-player stochastic games** (*e.g.*, solitaire and minesweeper), the game tree has only two types of nodes:

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- In **single-player stochastic games** (*e.g.*, solitaire and minesweeper), the game tree has only two types of nodes:
 - 1 MAX nodes to model the player's behavior.

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Stochastic games

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- In **single-player stochastic games** (*e.g.*, solitaire and minesweeper), the game tree has only two types of nodes:
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- Average expected utility of a node in such tree is called **expectimax**.

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- The expectimax value of a node is defined as follows:

$$\text{EXPECTIMAX}(s) = \begin{cases} \text{UTILITY}(s) & \text{if } \text{TERMINAL-TEST}(s) \\ \max_a \text{EXPECTIMAX}(\text{RESULT}(s, a)) & \text{if } \text{PLAYER}(s) = \text{MAX} \\ \sum_r P(r) * \text{EXPECTIMAX}(\text{RESULT}(s, r)) & \text{if } \text{PLAYER}(s) = \text{CHANCE} \end{cases}$$

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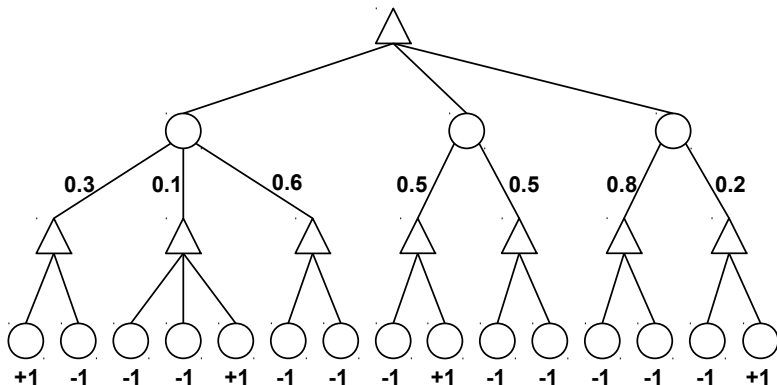
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Example: expectimax calculation

Calculate the expectimax value of the root node for the given game tree.



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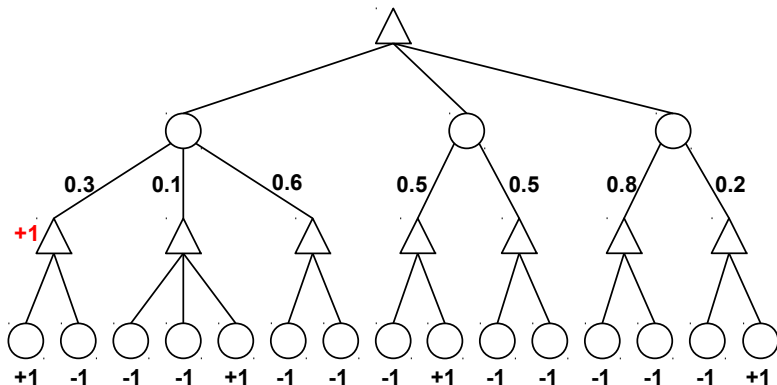
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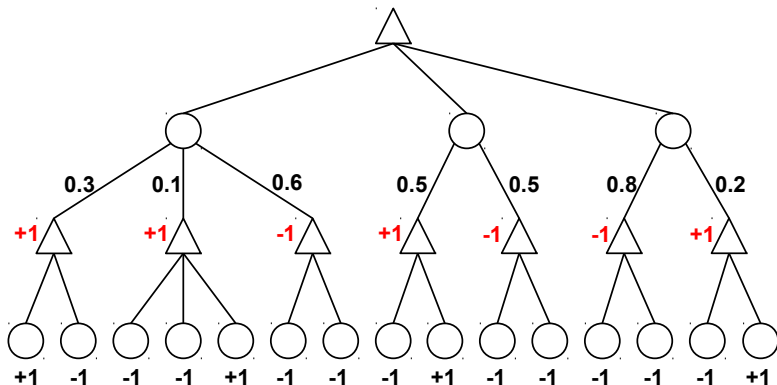
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Example: expectimax calculation

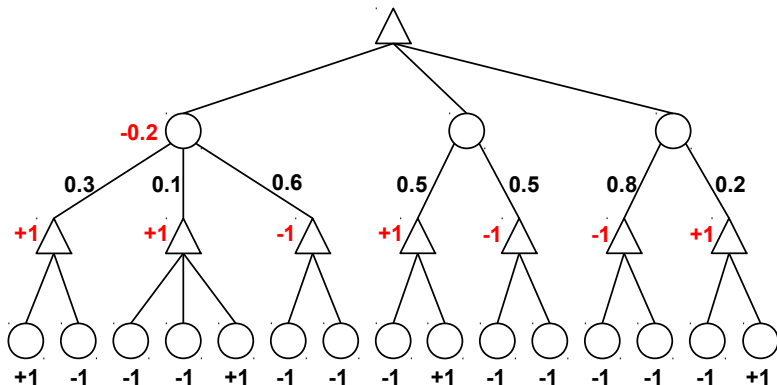
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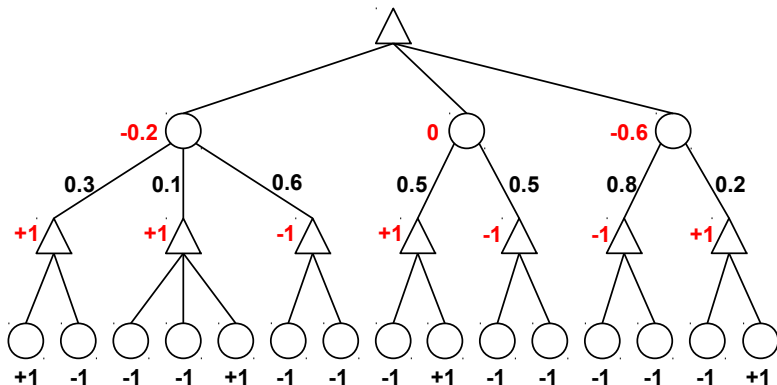
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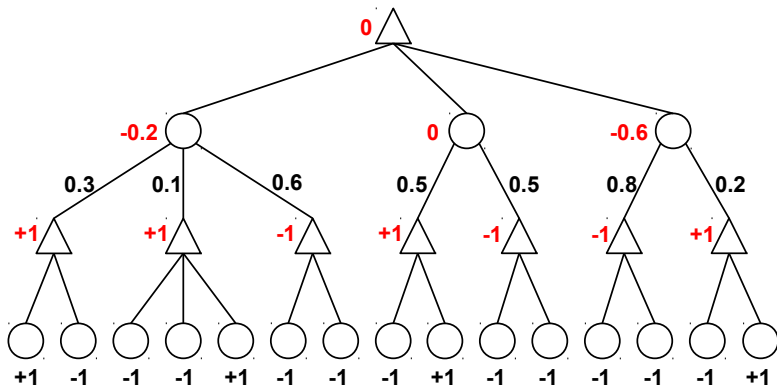
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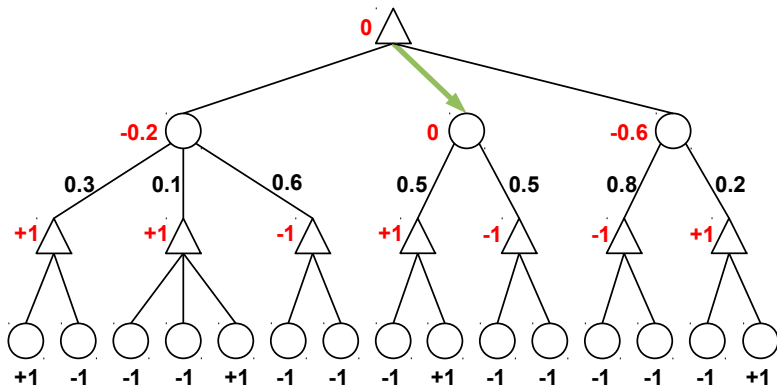
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- Notice that in expectimax, we choose the move that maximizes the **average** outcome of the game.

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- Notice that in expectimax, we choose the move that maximizes the **average** outcome of the game.
- The actual outcome (*i.e.*, utility value when game ends) may be **more or less** than the expectimax value.

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Stochastic games

- Notice that in expectimax, we choose the move that maximizes the **average** outcome of the game.
- The actual outcome (*i.e.*, utility value when game ends) may be **more or less** than the expectimax value.
- Unlike minimax where the actual outcome is guaranteed to be **no less** than the minimax value.

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- Notice that in expectimax, we choose the move that maximizes the **average** outcome of the game.
- The actual outcome (*i.e.*, utility value when game ends) may be **more or less** than the expectimax value.
- Unlike minimax where the actual outcome is guaranteed to be **no less** than the minimax value.
- However, expectimax may still be applicable to 2-player deterministic games (*e.g.*, tic-tac-toe and chess) if a probability distribution for opponent's actions is known.

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- In **two-player stochastic games** (e.g., backgammon), aim is to minimize opponent's chances of winning in presence of a stochastic environment.

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Stochastic games

- In **two-player stochastic games** (e.g., backgammon), aim is to minimize opponent's chances of winning in presence of a stochastic environment.
- The game tree would have three types of nodes:
 - 1 MAX nodes to model the player's behavior.
 - 2 MIN nodes to model the opponent's behavior.
 - 3 Chance nodes to model the stochastic environment.

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- The average expected value in this case is called **expectiminimax**, and defined as follows:

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Example: Backgammon

White has rolled 6-5 and has 4 possible moves.

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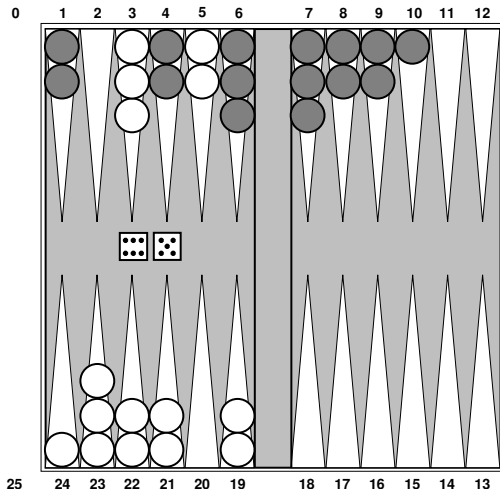
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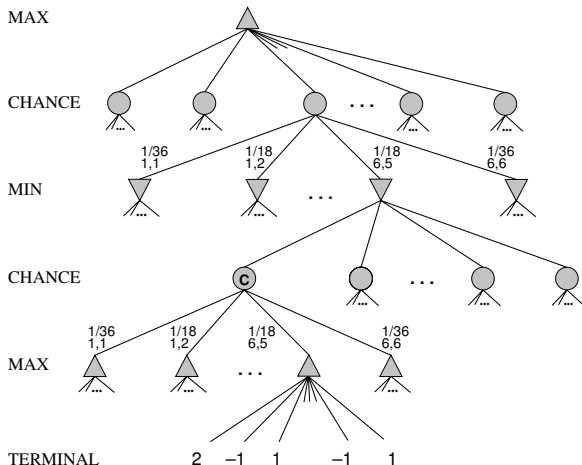
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Example: Backgammon

Schematic game tree for a backgammon position.



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What do I need from you

- When given a certain problem you should be able to:

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What do I need from you

- When given a certain problem you should be able to:
 - Build the game tree up to a given depth.

Requirements

What do I need from you

- When given a certain problem you should be able to:
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 - Apply the expectimax and expectiminimax algorithms.

Requirements

What do I need from you

- When given a certain problem you should be able to:
 - Build the game tree up to a given depth.
 - Apply the expectimax and expectiminimax algorithms.
 - Propose a good heuristic evaluation function

Requirements

What do I need from you

- When given a certain problem you should be able to:
 - Build the game tree up to a given depth.
 - Apply the expectimax and expectiminimax algorithms.
 - Propose a good heuristic evaluation function
- Answer descriptive questions.

Reading Material

Which parts of the textbook are covered

- Russell-Norvig, Chapters 5:
 - Pages 171 - 173.
 - Pages 177 - 178