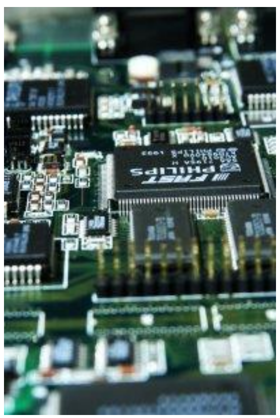
CSE 401

Computer Engineering (2)

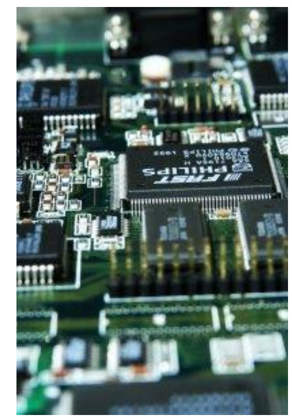
هندسة الحاسبات (2)



4th year, Comm. Engineering

Winter 2016

Lecture #11



Dr. Hazem Ibrahim Shehata

Dept. of Computer & Systems Engineering

Credits to Dr. Ahmed Abdul-Monem Ahmed for the slides

Adminstrivia

- Midterm:
  - Solution has been posted
  - Marks will be available by the end of this week
- Assignment #3 (optional):
  - To be released today
  - Due: Thursday, May 19, 2016
  - Assignments mark = max(A1+A2, A2+A3, A1+A3)
- Lab exam:
  - Date: TBA
  - Location: CSE labs (3^{rd} floor, Industrial Eng. Building)

Website: http://hshehata.github.io/courses/zu/cse401/
Office hours: Monday 11:30am – 12:30pm

Chapter 9. Computer Arithmetic (*Cont.*)

Outline

- Integer Representation
  - Sign-Magnitude, Two’s Complement, Biased
- Integer Arithmetic
  - Negation, Addition, Subtraction
  - Multiplication, Division
- Floating-Point Representation
  - IEEE 754
- Floating-Point Arithmetic
  - Addition, Subtraction
  - Multiplication, Division
  - Rounding

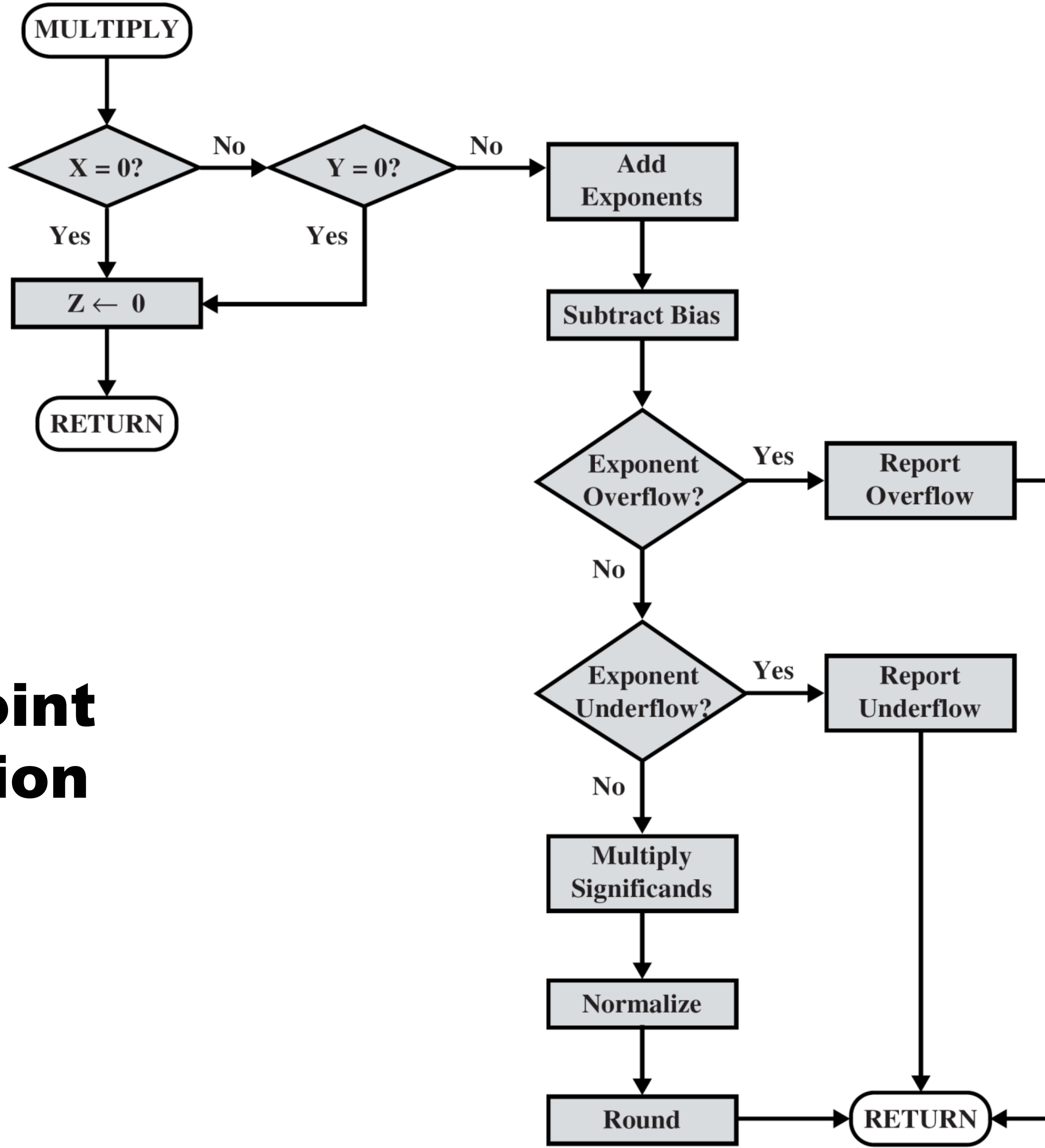
FP Arithmetic +/-

- Algorithm:
  1. Check for zeros (and other special cases, e.g., NaN).
  2. Align significands (adjusting exponents).
  3. Add or subtract significands.
  4. Normalize result.
  5. Round result.

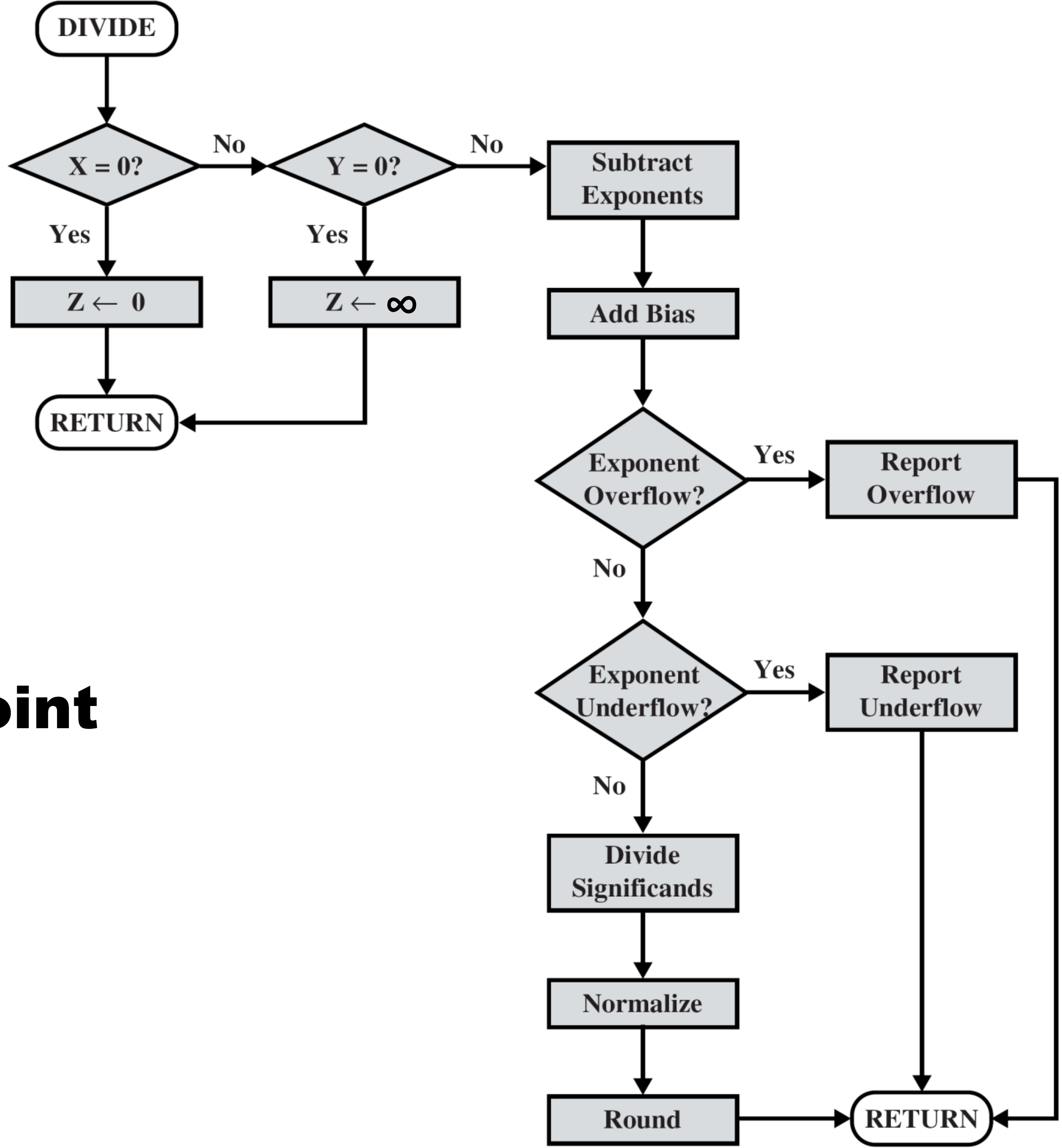
FP Arithmetic x/÷

- Algorithm:
  1. Check for zeros (and other special cases, e.g., NaN).
  2. Add/subtract exponents.
  3. Multiply/divide significands (watch sign).
  4. Normalize result.
  5. Round result.
- All intermediate results should be in double length storage.

Floating Point Multiplication



Floating Point Division



Guard Bits

- Extra bits added to the right of the mantissa during intermediate calculations.
- Maintains good precision.

$$\begin{array}{r} 1\,.\,0\,0\,0\,\ldots\,0\,0 \times 2^{1} \\ -\,1\,.\,1\,1\,1\,\ldots\,1\,1 \times 2^{0} \end{array}$$

$$\begin{array}{r} 1\,.\,0\,0\,0\,\ldots\,0\,0 \times 2^{1} \\ -\,0\,.\,1\,1\,1\,\ldots\,1\,1 \times 2^{1} \\ \hline 0\,.\,0\,0\,0\,\ldots\,0\,1 \times 2^{1} \end{array}$$

$$= 2^{-23} \times 2^{1} = \underline{\mathbf{2^{-22}}}$$

$$\begin{array}{r} 1\,.\,0\,0\,0\,\ldots\,0\,0\;\;0\,0\,0\,0 \times 2^{1} \\ -\,1\,.\,1\,1\,1\,\ldots\,1\,1\;\;0\,0\,0\,0 \times 2^{0} \end{array}$$

$$\begin{array}{r} 1\,.\,0\,0\,0\,\ldots\,0\,0\;\;0\,0\,0\,0 \times 2^{1} \\ -\,0\,.\,1\,1\,1\,\ldots\,1\,1\;\;1\,0\,0\,0 \times 2^{1} \\ \hline 0\,.\,0\,0\,0\,\ldots\,0\,0\;\;1\,0\,0\,0 \times 2^{1} \end{array}$$

$$= 2^{-24} \times 2^{1} = \underline{\mathbf{2^{-23}}}$$

Rounding

- The result of any operation on significands is stored in a longer register.
- When the result is to be stored as an FP number, extra bits have to be dropped off ➔ rounding.
- Round to nearest representable number.
- Round toward $+\infty$: **round up** to the next number.
  - Ex.: +1.1…001 001 ➔ +1.1…010
  - Ex.: – 1.1…001 001 ➔ – 1.1…001
- Round toward $-\infty$: **round down** to the next number.
  - Ex.: +1.1…001 001 ➔ +1.1…001
  - Ex.: – 1.1…001 001 ➔ – 1.1…010
- Round toward zero: **truncate** the extra bits.
  - Ex.: +1.1…001 001 ➔ +1.1…001
  - Ex.: – 1.1…001 001 ➔ – 1.1…001

Round to Nearest

- Default technique listed in the IEEE standard.
- Deliver the representable value nearest to the infinitely precise result. If the two nearest representable values are equally near, the one with LSB 0 will be delivered.
- Examples:
  - If the guard bits are 10010 ➔ they amount to more than one half of the last representable bit position ➔ **Round away from zero**.
  - If the guard bits are 01111 ➔ they amount to less than one half of the last representable bit position ➔ **Truncate**.
  - If the guard bits are 10000 ➔ midway
    - If we always truncate ➔ biased toward zero.
    - If we choose randomly ➔ not predictable/deterministic results.
    - IEEE standard:
      - Force the result to be even.
      - If last bit is 1, round away from zero, else, truncate.

Round to $\pm\infty$

- Useful in implementing interval arithmetic.
- Interval arithmetic: produce two values for every result. These two values correspond to the lower and upper endpoints of an interval that contains the true result.
- Used in monitoring and controlling errors.

Reading Material

- Stallings, Chapter 10:
  - Pages 352-356