Econ 8210: Computational Economics
Homework II

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Model

Consider the stochastic neoclassical growth model, where there is a representative household with preferences over consumption c_t and labor l_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(\log c_t - \frac{l_t^2}{2} \Big)$$

where $\beta = 0.97$.

The household consumes, saves, and works with a budget constraint:

$$c_t + i_t = w_t l_t + r_t k_t$$

The production function is Cobb-Douglas with capital share $\alpha = 0.33$:

$$c_t + i_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha}$$

with the law of motion of capital:

$$k_{t+1} = (1-\delta) k_t + i_t$$

where depreciation $\delta = 0.1$, and technology level z_t that follows an AR(1) process:

$$z_t = \rho z_{t-1} + \sigma \varepsilon_t$$

where $\rho = 0.95$, $\sigma = 0.007$, and $\varepsilon_t \sim N(0, 1)$.

Social Planner's Problem

Putting together the resource constraints, the social planner's problem in recursive formulation is

$$
\begin{aligned}
V(k,z) = \max_{c,l,k'} & \left\{ \log c - \frac{l^2}{2} + \beta \mathbb{E}\big[V(k',z') \mid z\big] \right\} \\
\text{s.t.} \quad & c + k' = e^z k^\alpha l^{1-\alpha} + (1-\delta)k \\
& z' = \rho z + \sigma \varepsilon', \quad \varepsilon \sim \mathcal{N}(0,1)
\end{aligned}
$$

Equilibrium Characterization

Taking the FOCs and invoking the envelope theorem, we obtain the optimality conditions that characterize the equilibrium. Namely, we have the resource constraint, intertemporal Euler equation, and the intratemporal condition:

$$k_{t+1} = (1-\delta)k_t + e^{z_t} k_t^\alpha l_t^{1-\alpha} - c_t \tag{1}$$

$$\frac{1}{c_t} = \beta \, \mathbb{E}_t \left[\frac{1}{c_{t+1}} \Big(\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + (1-\delta) \Big) \right] \tag{2}$$

$$c_t = \frac{e^{z_t} k_t^\alpha (1-\alpha)}{l_t^{\alpha+1}} \tag{3}$$

Assume that at the steady state, productivity shock $z_{ss} = 0$. The steady state equilibrium is characterized by:

$$c_{ss} = k_{ss}^\alpha l_{ss}^{1-\alpha} - \delta k_{ss} \tag{4}$$

$$1/\beta = \alpha k_{ss}^{\alpha-1} l_{ss}^{1-\alpha} + (1-\delta) \tag{5}$$

$$c_{ss} = \frac{k_{ss}^\alpha (1-\alpha)}{l_{ss}^{\alpha+1}} \tag{6}$$

1 Chebyshev

Compute the solution to the model when you use six Chebyshev polynomials on capital and a three-point finite approximation z_t using Tauchen's method.

2 Finite Elements

Compute the solution to the model when you use eight finite elements on capital and a three-point finite approximation to z_t using Tauchen's method. Use a Galerkin weighting scheme.

3 Perturbation

Find a third-order perturbation solution to the model.

4 Deep Learning

Compute the solution to the model when you use a neural network with three layers and 25 nodes per layer on capital and a three-point finite approximation to z_t using Tauchen's method.

5 Comparison