Econ 8210 Replication Project: Pilossoph and Wee (*AEJ Macro*, 2021)

Haosi Shen

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The paper I am replicating is Pilossoph and Wee (2021) [2]. This study examines the sources of the marital wage premium (MWP) through a novel framework grounded in household search behavior within frictional labor markets. The authors argue that, beyond selection into marriage, the MWP is predominantly driven by joint household decision-making during the job search process. Married individuals benefit from two primary mechanisms: (i) income pooling, which facilitates intra-household risk-sharing, increases tolerance for labor market risk, and raises reservation wages; and (ii) enhanced job ladder progression (i.e., greater search effort), as spouses internalize the positive impact of transitioning to better-paying jobs on their partner's search and employment outcomes. Additionally, the model aligns with empirical evidence by demonstrating the positive correlation between MWP and spousal education. When specialization is based on comparative advantage, individuals with higher returns to labor market search—proxied by skills and education levels—invest greater effort in job search, thereby enhancing household income and their spouse's selectivity in wage offers.

To empirically evaluate these mechanisms, the study utilizes data from the March Current Population Survey (CPS) for the years 2000–2007. The sample comprises married and single individuals aged 25–60, with controls for age, race, and the presence of children. Using maximum likelihood methods, the authors calibrate the model parameters for each gender-skill type, alongside the shares of unobservable skill types within each education and gender category for singles and joint households. For identification, the paper assumes that individuals in joint households face the same labor market parameters as their single counterparts. To account for selection into marriage and marital sorting, the model allows the shares of unobservable types to vary between singles and joint households.

In this replication project, I reproduce the tables and figures corresponding to the main results of the paper. Specifically, I replicated Tables 2 to 4 using MATLAB. In the following sections, I describe the algorithms and numerical methods used for each component, along with the results.

Singles

The following analysis reproduces Table 2 in Pilossoph and Wee (2021).

For each gender $i \in \{m, f\}$, skill $\xi \in \{H, L\}$, and education $\xi \in \{H, L\}$, I estimate the parameters governing the labor market as well as the shares $p_{x,i}(\xi)$ of each unobservable type x within each education group ξ. Estimation of the model parameters uses data on the singles and applies the Expectations Maximization (EM) algorithm in Dempster, Laird, and Rubin 1977 [1], under the identifying assumption that an individual's marital status has no effect on their labor market parameters. The parameters include

- $\mu(x_i), \sigma(x_i)$: parameters of the log-normal wage offer distribution
- $q(x_i)$: job arrival rate while in non-employment
- $\delta(x_i)$: exogenous separation rate while in employment
- γ_i: search cost parameter
- $w^R(x_i)$: reservation wages
- b_i: home production value while in non-employment

The EM algorithm estimates the shares and labor market parameters for each gender-skill type according to the following steps:

1. For each gender-skill type x_i, guess the share of x_i in each education category $\xi \in \{HS, Col\}$, $p_{x,i}(\xi)$. Conditional on this guess, estimate $\delta(x_i)$ using the empirical employment- to-nonemployment rate for singles of gender i with education ξ.

2. Given the guesses $p_{x,i}(\xi)$ and $\delta(x_i)$, choose $\{\mu(x_i), \sigma(x_i), q(x_i)\}$ that yields the maximum likelihood. Then, given these guesses, pin down jointly the job search parameters γ_i and $w^R(x_i)$ by targeting an average monthly employment-to-employment transition rate of 2 percent in accordance to the CPS.

3. Then, imposing the actual minimum wage (across education groups) as a consistent estimator for the reservation wage of the low type, and back out b_i using the indifference condition for low-type workers.

4. Given the estimated parameters, update the guess for shares $p_{x,i}(\xi)$ based on the conditional probabilities of employment status for each x_i with ξ.

5. Repeat the above iteratively until the estimated parameters and shares best explain (in terms of maximum likelihood) the data for singles. [1]

Panel A: Fixed Parameters

Parameter	Description	Value
ρ	Time Discount rate	0.004
ι	CRRA parameter	3

Panel B: Estimated Parameters

x_i	γ_i	b_i	$q(x_i)$	$\mu(x_i)$	$\sigma(x_i)$	$\delta(x_i)$	$w^R(x_i)$
(M, L)	6.57	1.22	0.20	2.15	0.46	0.05	1.58
	(3e-03)	(1e-03)	(1e-04)	(5e-04)	(2e-04)	(8e-18)	(3e-03)
(M, H)	6.57	1.22	0.15	2.51	0.56	0.02	1.62
	(3e-03)	(1e-03)	(2e-04)	(6e-04)	(3e-04)	(3e-18)	(2e-03)
(F, L)	7.19	0.62	0.19	1.95	0.50	0.05	1.28
	(5e-03)	(1e-03)	(9e-05)	(7e-04)	(4e-04)	(1e-18)	(2e-03)
(F, H)	7.19	0.62	0.12	2.38	0.53	0.02	1.28
	(5e-03)	(1e-03)	(4e-05)	(5e-04)	(3e-04)	(3e-18)	(2e-03)

Panel C: Model-Generated Probabilities and Non-employment Rates

	(M, HS)	(M, Col)	(F, HS)	(F, Col)
$p^{sin}_{i,H}(\mathcal{E})$	0.105	0.854	0.040	0.935
Model $u^{sin}_i(\mathcal{E})$	0.208	0.153	0.215	0.169
Data $u^{sin}_i(\mathcal{E})$	0.205	0.155	0.217	0.168
Model $EE^{sin}_i(\mathcal{E})$	0.023	0.016	0.024	0.013

[1] Section 3 - Part B of the paper [2] contains derivations of the log-likelihood function.

Panel D: Average EE Rates by Gender		
	M	F
Model	0.019	0.017
Data	0.020	0.020

Joint Households

Solution Methods

The paper employs the spectral method to solve the joint household problem. In particular, the analysis represents the households' value functions as Chebyshev polynomials and uses the Levenberg-Marquardt algorithm for guessing optimal coefficients on the Chebyshev polynomials. The state space is the wage grid. To update the coefficients on the Chebyshev polynomials, evaluate the cost function-defined as the sum of squared distance between the initial guesses and the updated value functions-at each point on the wage grid, and then use the gradient descent method to lower the cost function.

Given guesses for the reservation wage policy of the nonemployed and the estimated value functions, one can determine the optimal search effort for each joint household type as well as the wage distribution received by individual spouses in equilibrium. Note that using Chebyshev polynomials in approximation of the value functions enables us to efficiently approximate the numerical integral in our search policy functions through using quadrature weights. Then, iterate this process until the value functions converge.

Specific functional forms and equations can be found in Section 2 and Appendix E of the paper.

Estimating Selection and Sorting in Marriage

To account for selection into marriage based on unobservable characteristics as well as sorting in marriage, the paper estimates the proportion of high-skilled individuals in households with a specific educational pairing (ξ_m, ξ_f). This proportion is selected to best align with the observed joint distribution of marital wage premia and nonemployment rates among couples. Let $\Pr(x_m \cap x_f \mid \xi_m, \xi_f)$ represent the joint probability that m and f possess skill levels x_m and x_f, respectively, given their educational pair (ξ_m, ξ_f). Then, the paper solves for joint probabilities $\Pr(x_m \cap x_f \mid E_m, E_f)$ that minimize the gap between the model-predicted and observed MWP and nonemployment rates.

Define $h_i(w \mid x_m, x_f)$ as the probability density indicating that individual $i \in \{m, f\}$ earns a wage w, conditional on being part of a joint household with skill types (x_m, x_f). The average wage of individual i in a joint household of the education pair (ξ_m, ξ_f) is then expressed as:

$$\sum_{x_m} \sum_{x_f} \Pr(x_m \cap x_f \mid \xi_m, \xi_f) \int_{w}^{\bar{w}} y \cdot h_i(y \mid x_m, x_f)\, dy.$$

The MWP is defined as 1 plus the logarithm of the ratio of an individual's steady-state average wage in a joint household of type (ξ_m, ξ_f) to the average wage of their single counterpart with education level ξ_i. Therefore, an MWP greater than 1 indicates that being in a joint household increases one's wage relative to being in a single household. Analogously, the nonemployment rates for males and females in a joint household of education (x_m, x_f) can be calculated using the steady state mass of each type of joint household, as defined by the Laws of Motion in Appendix D.

Table 3 presents the implied joint probabilities along with the corresponding MWP and nonemployment rates. Panel B calculates the empirical MWP using the regression coefficients reported in Table 1. [2]

Panel A: Estimated Joint Probabilities				
(ξ_m, ξ_f)	$P(L \cap L)$	$P(L \cap H)$	$P(H \cap L)$	$P(H \cap H)$
(HS, HS)	0.64	0.00	0.33	0.03
(HS, Col)	0.18	0.38	0.00	0.44
(Col, HS)	0.00	0.00	0.88	0.12
(Col, Col)	0.00	0.00	0.00	1.00

[2] In the replication exercise, I omitted Table 1 as it only involves data cleaning and standard OLS regression.

Panel B: Implied MWP and Non-employment Rates				
(ξ_m, ξ_f)	Implied MWP		Non-employed	
	Data	Model	Data	Model
(HS, HS)	1.20, 1.04	1.20, 1.05	0.17, 0.18	0.23, 0.20
(HS, Col)	1.33, 0.92	1.35, 0.91	0.15, 0.15	0.24, 0.16
(Col, HS)	1.07, 1.16	1.00, 1.19	0.14, 0.17	0.16, 0.23
(Col, Col)	1.24, 1.09	1.12, 1.05	0.13, 0.15	0.15, 0.17

Decomposing the MWP

Finally, I replicate the paper's approach to analyze the fully estimated model and decompose the MWP into its underlying components: household search versus selection and sorting. Specifically, I reproduce the analysis that estimates the MWP under the scenario when only joint household search is present, excluding the portion of the MWP attributed to selection or sorting. For this decomposition, we assume that (i) there is no selection in marriage, meaning the marginal distribution of skill types for each gender in marriage matches that of singles, and (ii) there is no sorting, such that the distribution of marriage types aligns with what would result from random matching.

Accordingly, the share of high-skilled males with education level ξ_m married to high-skilled females with education level ξ_f is given by $p_{H,m}(E_m) \times p_{H,f}(E_f)$. Following the same approach as before, the average wage of individual i in a joint household with education pair (ξ_m, ξ_f) is defined as:

$$\sum_{x_m} \sum_{x_f} p_{x_m}(\xi_m) p_{x_f}(\xi_f) \int_{w}^{\bar{w}} y \cdot h_i(y \mid x_m, x_f)\, dy.$$

The MWP is defined the same way as before.

Table 4 reports the model-implied moments assuming no selection or sorting into marriage.

Estimated Statistics for Skill Combinations					
(ξ_m, ξ_f)	MWP	Nonemployment Rate	$w_i^R(\cdot)$	Mean $\widehat{w}(\cdot)$	w^c
(HS, HS)	(1.02, 1.02)	(0.22, 0.22)	(1.42, 0.84)	(1.91, 1.71)	(1.61, 1.04)
(HS, Col)	(1.11, 0.91)	(0.25, 0.17)	(1.44, 0.81)	(2.91, 1.01)	(1.53, 0.89)
(Col, HS)	(0.92, 1.11)	(0.16, 0.28)	(1.40, 0.85)	(1.44, 2.59)	(1.65, 1.16)
(Col, Col)	(1.04, 1.01)	(0.16, 0.19)	(1.44, 0.83)	(2.31, 1.72)	(1.65, 1.04)

where $\hat{w}(w, \cdot)$ denotes the reservation wage for the nonemployed in a joint household where the spouse is employed at wage w, and w^c is the cutoff wage for which the breadwinner cycle ends and the individual prefers dual employment over being in a worker-searcher household.

References

[1] Dempster, A. P., Laird, N. M., and Rubin, D. B. Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society: Series B (Methodological) 39*, 1 (1977), 1–22.

[2] Pilossoph, L., and Wee, S. L. Household search and the marital wage premium. *American Economic Journal: Macroeconomics 13*, 4 (October 2021), 55–109.