

```

% Load the dataset
load('dataset3.mat');

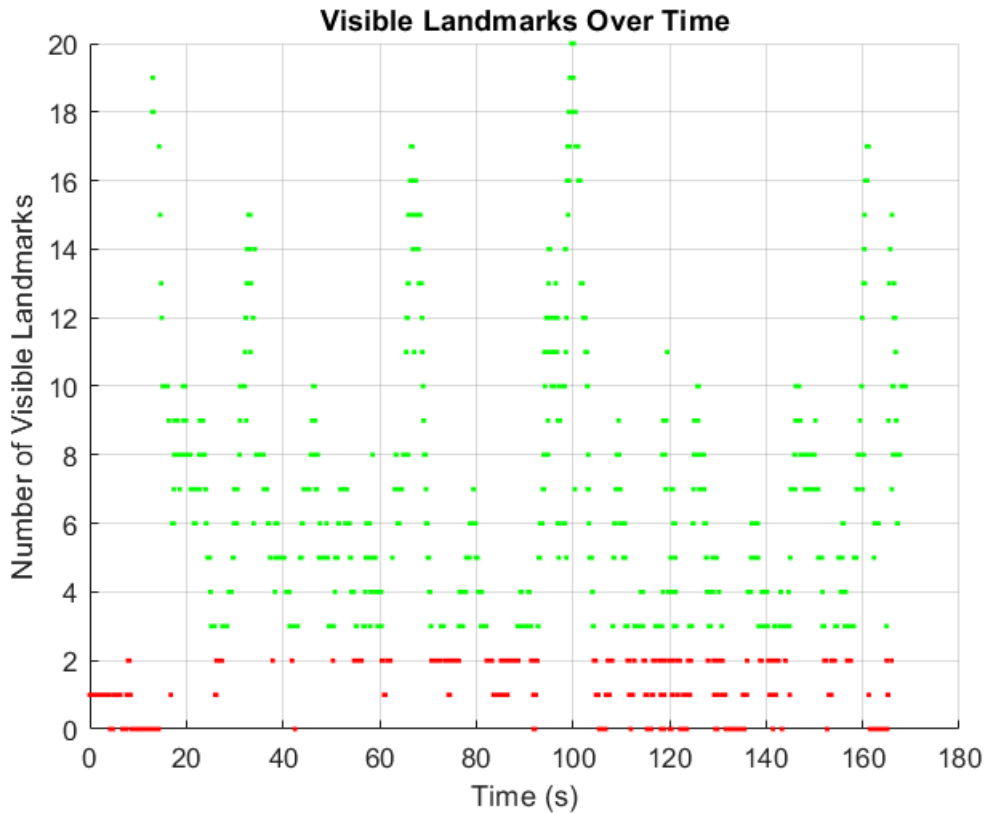
% Number of timesteps and landmarks
K = size(t, 2); % Number of time steps
M = size(rho_i_pj_i, 2); % Number of landmarks

% Preallocate visibility array
visible_landmarks = zeros(1, K);

% Compute visible landmarks at each timestep
for k = 1:K
    for j = 1:M
        % Check if any component of y_k_j(:,k,j) is valid (not equal to -1)
        if any(y_k_j(:, k, j) ~= -1)
            visible_landmarks(k) = visible_landmarks(k) + 1;
        end
    end
end

% Plot visibility
figure;
hold on;
for k = 1:K
    if visible_landmarks(k) >= 3
        plot(t(k), visible_landmarks(k), 'g.', 'MarkerSize', 5); % Green for 3 or more landmarks
    else
        plot(t(k), visible_landmarks(k), 'r.', 'MarkerSize', 5); % Red otherwise
    end
end
xlabel('Time (s)');
ylabel('Number of Visible Landmarks');
title('Visible Landmarks Over Time');
grid on;
hold off;

```



```

% calculate the errors
% use  $\hat{e}_{v,k}(x_{op}) = \ln(\Xi_k \cdot \text{Top}_{k-1} \cdot \text{inv}(\text{Top}_k))$ 
%  $\hat{x} = \ln(\text{var})$  just means  $\exp(\hat{x}) = \text{var}$ , ie its the inverse operation
%  $\Xi_k = \exp(w_{k-1} \cdot T_k)$ 
%  $F_{k-1} = \text{inv}(J(-\hat{e}_{v,k}(x_{op}))) \cdot \text{Ad}(\text{Top}_k, \text{inv}(\text{Top}_{k-1}))$ 
%  $E_k = \text{inv}(J(-\hat{e}_{v,k}(x_{op})))$ 
% then find  $\hat{e}_{y,jk}(x) = \hat{e}_{y,jk}(x_{op}) - G_{jk} \cdot Z_{jk} \cdot \epsilon_{k-1}$ 
% where  $G_{jk} = \text{delg/delp}$  at  $p_{jk}(x_{op})$   $[f_u/2, 0, -f_u \cdot x/z^2; 0, f_v/z, -f_v \cdot y/z^2; f_u/2, 0, -f_u \cdot (x-b)]$ 
%  $Z_{jk} = [-C_{cv} \cdot C_{k,op} \quad C_{cv} \cdot C_{k,op} \cdot (p_j - r_{k,op})_{circ}] \cdot \epsilon_{k-1}$ 
% Then  $\hat{e}_{y,k}(x) \approx \hat{e}_{y,k}(x_{op}) - G_k \cdot \epsilon_{k-1}$ 
% where  $G_k = [G_{1k} \cdot Z_{1k}, \dots, G_{Mk} \cdot Z_{Mk}]$ 
%  $\hat{e}_{y,k}(x) = [\hat{e}_{y,1k}(x); \dots, \hat{e}_{y,Mk}(x)]$ 
%  $\hat{e}_{y,k}(x_{op}) = [\hat{e}_{y,1k}(x_{op}); \dots, \hat{e}_{y,Mk}(x_{op})]$ 
% and circ is defined as swap of  $\wedge$ . ie  $a_{circ} \cdot b = b \wedge a$ 
% the gradient  $G_{jk}$  is evaluated at  $p_{jk}(x_{op}) = C_{cv} \cdot C_{k,op} \cdot (p_j - r_{k,op}) - p_c$ 
% note matlab variables on right side:  $C_{cv} = C_{c_v}$ ,  $p_j = \text{rho}_{i_{pj_i}}(:, j)$ ,  $p_c = \text{rho}_{v_{c_v}}$ 

% Then set up Gauss-Newton solver
% H = two big matrices stacked on top of each other =  $[H_1; H_2]$ 
% H1 has  $E_{k1}, \dots, E_{k2}$  along main diag and  $-F_{k1}, \dots, -F_{k2}$  in diagonal
% above main one
% H2 is block diag with diag elements being  $G_{k1} \dots G_{k2}$ 
%  $e(x_{op}) =$  two vectors stacked on top of each other =  $[e_v(x_{op}); e_y(x_{op})]$ 
%  $e_v(x_{op}) = [e_{v,k1}(x_{op}); \dots; e_{v,k2}(x_{op})]$ 
%  $e_y(x_{op}) = [e_{y,k1}(x_{op}); \dots; e_{y,k2}(x_{op})]$ 
%  $\text{delx} = [\epsilon_{k1}; \dots; \epsilon_{k2}]$  is what we are to find. each  $\epsilon_{k?}$  is 6x1
% with first 3 for translation and last 3 for rotation

```

```
% W = diag(Q_k1, ..., Q_k2, R_k1, ... R_k2)
% all the Qks are 6x6 diagonal. first 3 diag elements given by elements of Tk*v_var and
% last 3 by Tk*w_var where v_var, w_var are matlab knowns.
% all the Rkjs are just 3x3 diag with elements Tk*y_var. If time k has M
% landmarks visisble I guess Rk would be M*3 by M*3 diagonal.

% finally gauss newton solution is to solve the equation below
% A*delx = b
% where A = H'*inv(W)*H, b = H'*inv(W)*e(x_op)
% A is block tridiagonal
% this gives up the optimal perturbation which allows us to update Top
% Top,k = exp(eps^_k)*Top,k
% then repeat process
```

```
% Parameters for the problem
```

```
k1 = 1215;
k2 = 1714;
```

```
% Load necessary data (e.g., t, r_i_vk_i, etc.)
```

```
% Assume the data is loaded here as variables t, r_i_vk_i, theta_vk_i, ...
```

```
% v_vk_vk_i, w_vk_vk_i, y_k_j, rho_i_pj_i, C_c_v, rho_v_c_v, fu, fv, b,
```

```
% v_var, w_var, y_var.
```

```
% Call Gauss-Newton algorithm
```

```
[Top_final, converged] = gauss_newton_se3(k1, k2, t, r_i_vk_i, theta_vk_i, ...
    v_vk_vk_i, w_vk_vk_i, y_k_j, rho_i_pj_i, C_c_v, rho_v_c_v, fu, fv, b, cu, cv, ...
    v_var, w_var, y_var);
```

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0
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.490854e-17.
```

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1
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.161991e-16.
```

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2
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Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 4.891261e-21.
```

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3
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 3.554426e-19.
```

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4
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 3.695894e-20.
```

```
5
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 7.189868e-23.
```

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6
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 4.309087e-31.
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7
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 7.393888e-19.
```

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Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 8.467418e-19.
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Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 8.467418e-19.
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Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.757605e-19.
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.179761e-18.
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.033768e-19.
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Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.179761e-18.
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```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.610725e-19.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 3.057109e-18.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.068233e-19.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 3.057109e-18.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 3.893926e-20.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 5.465731e-20.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 9.345450e-20.
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 5.465731e-20.
```





























[illegible]























[illegible]





Warning: Matrix is singular to working precision.  
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 4.703296e-59.  
Warning: Principal matrix logarithm is not defined for A with nonpositive real eigenvalues. A non-principal matrix logarithm is returned.  
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.494046e-160.  
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.491095e-204.  
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.494046e-160.  
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.707642e-95.  
Warning: Principal matrix logarithm is not defined for A with nonpositive real eigenvalues. A non-principal matrix logarithm is returned.

```
function [Top_final, converged] = gauss_newton_se3(k1, k2, t, r_i_vk_i, theta_vk_i, v_vk_vk_i,
    % Function to perform Gauss-Newton optimization for SE(3) transformations.
    % Inputs:
    %   k1, k2: Start and end indices for the time window
    %   t: Time vector
    %   r_i_vk_i: Translation vectors
    %   theta_vk_i: Rotation vectors
    %   v_vk_vk_i: Linear velocities
    %   w_vk_vk_i: Angular velocities
    %   y_k_j: Landmark observations
    %   rho_i_pj_i: Landmark positions
    %   C_c_v: Camera intrinsic matrix
    %   rho_v_c_v: Camera offset vector
    %   fu, fv, b: Camera parameters
    %   v_var, w_var, y_var: Covariances for motion and observation models
    % Outputs:
    %   Top_final: Final state estimate (4x4xK)
    %   converged: Boolean indicating whether convergence was achieved

    % Initialize Top using the ground truth at k1
    K_window = k2 - k1 + 1; % Define the size of the window
    Top = zeros(4, 4, K_window); % Preallocate space for Top within the window
    r_k1 = r_i_vk_i(:, k1);
    theta_k1 = theta_vk_i(:, k1);
    R_k1 = expm(skew(theta_k1));
    Top(:, :, 1) = [R_k1, r_k1; 0, 0, 0, 1];

    % Dead-reckon to initialize Top for subsequent timesteps
    for k = 2:K_window
        k_curr = k1 + k - 1;
        Tk = t(k_curr) - t(k_curr - 1);
        v_vk_vk = v_vk_vk_i(:, k_curr - 1);
        w_vk_vk = w_vk_vk_i(:, k_curr - 1);
        r_prev = Top(1:3, 4, k - 1);
        R_prev = Top(1:3, 1:3, k - 1);

        % Update rotation
        theta_delta = w_vk_vk * Tk;
        R_curr = expm(skew(theta_delta)) * R_prev;

        % Update translation
        r_delta = R_prev' * (v_vk_vk * Tk);
        r_curr = r_prev + r_delta;
    end
end
```

```

    % Update Top
    Top(:, :, k) = [R_curr, r_curr; 0, 0, 0, 1];
end

% Gauss-Newton loop
converged = false;
iteration = 0;
max_iterations = 200;
epsilon_threshold = 1e-6;

while ~converged && iteration < max_iterations
    disp(iteration)

    % Initialize variables for this iteration
    H1 = zeros(6 * K_window, 6 * K_window);
    H2_blocks = {};
    Q_blocks = {};
    R_blocks = {};
    e_v = [];
    e_y = [];

    % Compute e_{v,0}(x_{op})
    Top_k = Top(:, :, 1);
    % Use ground truth from r_i_vk_i and theta_vk_i for T_gt_0
    r_gt_0 = r_i_vk_i(:, k1); % Ground truth translation
    theta_gt_0 = theta_vk_i(:, k1); % Ground truth rotation
    R_gt_0 = expm(skew(theta_gt_0)); % Compute rotation matrix from theta
    T_gt_0 = [R_gt_0, r_gt_0; 0, 0, 0, 1]; % Ground truth transformation matrix
    ev_0_matrix = logm(T_gt_0 / T_op_k);
    ev_0 = [vec(ev_0_matrix(1:3, 1:3)); ev_0_matrix(1:3, 4)];
    E_k1 = inv(SE3_jacobian(-ev_0));
    e_v = [e_v; ev_0];
    H1(1:6, 1:6) = E_k1;
    dummy = t(1,2) - t(1,1);
    Q_k1 = diag([dummy^2 * v_var; dummy^2 * w_var]);
    Q_blocks{end + 1} = Q_k1;

    % Include ey,0 and the first R covariance matrix
    visible_landmarks_k1 = find(any(y_k_j(:, k1, :) ~= -1, 1));
    Gk1 = [];
    ey_k1 = [];

    if isempty(visible_landmarks_k1)
        % If no measurements exist, append zeros
        Gk1 = zeros(0, 6); % No observation Jacobians
        ey_k1 = zeros(0, 1); % No observation errors
    else
        for idx = 1:numel(visible_landmarks_k1)
            j = visible_landmarks_k1(idx); % Extract the current element
            C_k = Top(1:3, 1:3, 1);
            r_k = Top(1:3, 4, 1);
            p_j = rho_i_pj_i(:, j);
            p_jk_xop = C_c_v * C_k * (p_j - r_k) - rho_v_c_v;

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% Extract x, y, z from p_jk_xop
x = p_jk_xop(1);
y = p_jk_xop(2);
z = p_jk_xop(3);

% Apply g function to compute predicted observation
g_p_jk_xop = g_function(x, y, z, fu, fv, b, cu, cv);

% Observation error
ey_jk = y_k_j(:, k1, j) - g_p_jk_xop;

% Compute Gjk
x = p_jk_xop(1); y = p_jk_xop(2); z = p_jk_xop(3);
G_jk = [fu / 2, 0, -fu * x / z^2;
        0, fv / z, -fv * y / z^2;
        fu / 2, 0, -fu * (x - b) / z^2;
        0, fv / 2, -fv * y / z^2];
Z_jk = [-C_c_v * C_k, -C_c_v * C_k * skew(p_j - r_k)];
Gk1 = [Gk1; G_jk * Z_jk];
ey_k1 = [ey_k1; ey_jk];
end
end
H2_blocks{end + 1} = Gk1;
e_y = [e_y; ey_k1];
% Create R_k1 with M blocks, each block being diag(y_var)
M = numel(visible_landmarks_k1); % Number of visible landmarks
diag_blocks = repmat({diag(y_var)}, 1, M); % Cell array of diag(y_var)
R_k1 = blkdiag(diag_blocks{:}); % Construct block diagonal matrix
R_blocks{end + 1} = R_k1;

% Loop through timesteps for k = 2 to K_window
for k = 2:K_window
    k_curr = k1 + k - 1;

    % Compute  $\Xi_k = \exp(w_{(k-1)} * T_k)$ 
    Tk = t(k_curr) - t(k_curr - 1);
    psi_k = w_vk_vk_i(:, k_curr - 1) * Tk;
    rho_k = v_vk_vk_i(:, k_curr - 1) * Tk; % Linear velocity * time

    % Construct the SE(3) matrix  $\Xi_k$ 
    Xi_k = expm([skew(psi_k), rho_k; 0, 0, 0, 0]); % SE(3) exponential map

    % Compute motion model error ev,k(xop)
    Top_k = Top(:, :, k);
    Top_km1 = Top(:, :, k - 1);
    ev_k = logm(Xi_k * Top_km1 / Top_k); % Lie logarithm
    ev_hat = vee(ev_k(1:3, 1:3)); % Rotation vector
    ev_trans = ev_k(1:3, 4); % Translation error
    e_v = [e_v; ev_hat; ev_trans]; % Append motion model errors

    % Compute Jacobians Fk and Ek for the motion model
    F_k = inv(SE3_jacobian(-[ev_trans; ev_hat])) * SE3_adjoint(Top_k * inv(Top_km1));
    E_k = inv(SE3_jacobian(-[ev_trans; ev_hat]));
end
end

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```

% Place E_k and -F_k into H1
H1((k - 1) * 6 + 1:k * 6, (k - 1) * 6 + 1:k * 6) = E_k; % Main diagonal
H1((k - 1) * 6 + 1:k * 6, (k - 2) * 6 + 1:(k - 1) * 6) = -F_k; % Subdiagonal

% Observation model Jacobians
visible_landmarks_k = find(any(y_k_j(:, k_curr, :) ~= -1, 1)); % Get visible landmarks
Gk = [];
ey_k = [];

if isempty(visible_landmarks_k)
    % If no measurements exist, append zeros
    Gk = zeros(0, 6); % No observation Jacobians
    ey_k = zeros(0, 1); % No observation errors
    R_k = zeros(0, 0); % No observation covariance
else
    Gk = []; % Reset Gk for this timestep
    ey_k = []; % Reset ey_k for this timestep
    for idx = 1:numel(visible_landmarks_k)
        j = visible_landmarks_k(idx); % Extract the current element
        % Compute observation model error ey,jk(xop)
        C_k = Top(1:3, 1:3, k);
        r_k = Top(1:3, 4, k);
        p_j = rho_i_pj_i(:, j);
        p_jk_xop = C_c_v * C_k * (p_j - r_k) - rho_v_c_v;

        % Extract x, y, z from p_jk_xop
        x = p_jk_xop(1);
        y = p_jk_xop(2);
        z = p_jk_xop(3);

        % Apply g function to compute predicted observation
        g_p_jk_xop = g_function(x, y, z, fu, fv, b, rho_v_c_v(1), rho_v_c_v(2));

        % Observation error
        ey_jk = y_k_j(:, k_curr, j) - g_p_jk_xop;

        % Compute Gjk
        G_jk = [fu / 2, 0, -fu * x / z^2;
                0, fv / z, -fv * y / z^2;
                fu / 2, 0, -fu * (x - b) / z^2;
                0, fv / 2, -fv * y / z^2];

        % Compute Zjk
        Z_jk = [-C_c_v * C_k, -C_c_v * C_k * skew(p_j - r_k)];
        Gk = [Gk; G_jk * Z_jk];
        ey_k = [ey_k; ey_jk];
    end

    % Create R_k with M blocks, each block being diag(y_var)
    M = numel(visible_landmarks_k); % Number of visible landmarks
    diag_blocks = repmat({diag(y_var)}, 1, M); % Cell array of diag(y_var)
    % disp('size of diagblock')
    % disp(size(diag_blocks))
    R_k = blkdiag(diag_blocks{:}); % Construct block diagonal matrix

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%           disp('size of R_k')
%           disp(size(R_k))
%           disp('size of Gk')
%           disp(size(Gk))
end

% Add Gk and ey_k to H2_blocks and e_y
H2_blocks{end + 1} = Gk;
e_y = [e_y; ey_k]; % Append observation model errors

% Add R_k to R_blocks
R_blocks{end + 1} = R_k;

% Add Q_k to Q_blocks
Q_k = diag([Tk^2 * v_var; Tk^2 * w_var]); % Motion model covariance
Q_blocks{end + 1} = Q_k;

end

% Combine motion and observation model errors
e_x_op = [e_v; e_y];

% Combine H2_blocks into a block diagonal matrix
H2 = blkdiag(H2_blocks{:});

% Combine Q and R blocks
% disp(size(Q_blocks));
% disp(size(R_blocks));
% disp(size(blkdiag(Q_blocks{:})));
% disp(size(blkdiag(R_blocks{:})));
W = blkdiag(blkdiag(Q_blocks{:}), blkdiag(R_blocks{:}));

% Combine H1 and H2
% disp(size(H1))
% disp(size(H2))
H = [H1; H2];
A = H' * (W \ H);
bans = H' * (W \ e_x_op);

% Solve for delta x
delx = A \ bans;

% Update Top,k
for k = 1:K_window
    epsilon_k = delx((k - 1) * 6 + 1:k * 6);
    rho = epsilon_k(1:3);
    phi = epsilon_k(4:6);
    perturbation = expm([skew(phi), rho; zeros(1, 4)]);
    Top(:, :, k) = perturbation * Top(:, :, k);
end

% Check for convergence
if norm(delx) < epsilon_threshold
    converged = true;

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        else
            iteration = iteration + 1;
        end
    end

    Top_final = Top;
end

function S = skew(v)
    % Converts vector v into a skew-symmetric matrix
    S = [0, -v(3), v(2); v(3), 0, -v(1); -v(2), v(1), 0];
end

% Supporting function for skew-symmetric matrix
function v = vee(S)
    % Converts skew-symmetric matrix to vector
    v = [S(3, 2); S(1, 3); S(2, 1)];
end

function J = S03_jacobian(phi)
    % Compute Jacobian for SE(3) exponential map
    norm_phi = norm(phi);
    if norm_phi < 1e-6
        J = eye(3); % Small-angle approximation
        return;
    end
    a = phi / norm_phi; % Unit vector
    phi_hat = skew(a); % Skew-symmetric form of a
    J = sin(norm_phi) / norm_phi * eye(3) + ...
        (1 - sin(norm_phi) / norm_phi) * (a * a') + ...
        (1 - cos(norm_phi)) / norm_phi * phi_hat;
end

function J_inv = S03_jacobian_inverse(phi)
    % Compute the inverse Jacobian for S0(3)
    % Inputs:
    %   phi: 3x1 vector (axis-angle representation)
    % Outputs:
    %   J_inv: 3x3 inverse Jacobian matrix

    norm_phi = norm(phi); % Magnitude of the rotation vector

    if norm_phi < 1e-6
        % Small-angle approximation
        J_inv = eye(3) - 0.5 * skew(phi);
        return;
    end

    % Unit vector along the rotation axis
    a = phi / norm_phi;

    % Precompute terms for efficiency
    cot_term = (norm_phi / 2) * cot(norm_phi / 2);
    a_hat = skew(a); % Skew-symmetric matrix of 'a'

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% Compute the inverse Jacobian
J_inv = (cot_term) * eye(3) + ...
        (1 - cot_term) * (a * a') - ...
        (norm_phi / 2) * a_hat;
end

function AdT = SE3_adjoint(T)
% Compute the adjoint matrix for a transformation T in SE(3)
C = T(1:3, 1:3); % Rotation matrix
r = T(1:3, 4); % Translation vector
AdT = [C, skew(r) * C; zeros(3), C];
end

function J = SE3_jacobian(rho_phi)
% Compute the Jacobian for SE(3) exponential map
% Inputs:
%   rho_phi: 6x1 vector [rho; phi], where
%           rho is the translation vector (3x1)
%           phi is the rotation vector (3x1)
% Outputs:
%   J: 6x6 Jacobian matrix for SE(3)

% Split input into rho and phi
rho = rho_phi(1:3); % Translation part
phi = rho_phi(4:6); % Rotation part
norm_phi = norm(phi); % Norm of the rotation vector

% Compute the SO(3) Jacobian (J_S03) for the rotation part
J_S03 = S03_jacobian(phi);

% Compute the Q matrix for the translation part
if norm_phi < 1e-6
    % Small-angle approximation
    Q = 0.5 * skew(rho);
else
    phi_hat = skew(phi); % Skew-symmetric matrix of phi
    rho_hat = skew(rho); % Skew-symmetric matrix of rho
    Q = 0.5 * rho_hat + ...
        (norm_phi - sin(norm_phi)) / norm_phi^3 * (phi_hat*rho_hat + rho_hat*phi_hat + phi_hat*rho_hat) +
        (1 - norm_phi^2 / 2 - cos(norm_phi)) / norm_phi^4 * (phi_hat*phi_hat*rho_hat + rho_hat*phi_hat*rho_hat + rho_hat*phi_hat*phi_hat) +
        0.5 * ((1 - norm_phi^2/2 - cos(norm_phi))/norm_phi^4 - 3*(norm_phi - sin(norm_phi))/norm_phi^3) * rho_hat;
end

% Assemble the SE(3) Jacobian matrix
J = [J_S03, Q; zeros(3), J_S03];
end

function J_inv = SE3_jacobian_inverse(rho_phi)
% Compute the inverse Jacobian for SE(3)
% Inputs:
%   rho_phi: 6x1 vector [rho; phi], where
%           rho is the translation vector (3x1)
%           phi is the rotation vector (3x1)

```

```

% Outputs:
%   J_inv: 6x6 inverse Jacobian matrix for SE(3)

% Split input into rho and phi
rho = rho_phi(1:3); % Translation part
phi = rho_phi(4:6); % Rotation part
norm_phi = norm(phi); % Norm of the rotation vector

% Compute the inverse SO(3) Jacobian (J_SO3_inv) for the rotation part
J_SO3_inv = SO3_jacobian_inverse(phi);

% Compute the Q matrix for the translation part
if norm_phi < 1e-6
    % Small-angle approximation
    Q = 0.5 * skew(rho);
else
    phi_hat = skew(phi); % Skew-symmetric matrix of phi
    rho_hat = skew(rho); % Skew-symmetric matrix of rho
    Q = 0.5 * rho_hat + ...
        (norm_phi - sin(norm_phi)) / norm_phi^3 * (phi_hat*rho_hat + rho_hat*phi_hat + phi_
        (1 - norm_phi^2 / 2 - cos(norm_phi)) / norm_phi^4 * (phi_hat*phi_hat*rho_hat + rho_
        0.5 * ((1 - norm_phi^2/2 - cos(norm_phi))/norm_phi^4 - 3*(norm_phi - sin(norm_phi)
end

% Assemble the SE(3) inverse Jacobian matrix
J_inv = [J_SO3_inv, -J_SO3_inv * Q * J_SO3_inv; ...
        zeros(3), J_SO3_inv];
end

% Compute the g function as per the formula
function g_val = g_function(x, y, z, fu, fv, b, cu, cv)
    % Compute the intrinsic camera parameters
    g_val = (1 / z) * [fu * x + cu;
                      fv * y + cv;
                      fu * (x - b) + cu;
                      fv * y + cv];
end

```