ECE356 Lab 3

Juho Kim (1005955085)
Hshmat Sahak (1005903710)

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Section 3: Preparation Report

Question: Consider the block diagram in Figure 2 and assume that the road is flat, i.e., θ = 0 and hence D(s) = 0. Suppose that a step voltage $v_m(t) = V_0 \cdot 1(t)$ ($V_0 > 0$) is applied to the DC motor and that at time t = 0 the cart is still (i.e., v(0) = 0). Using the final value theorem, determine $v(+\infty) = \lim(t\to\infty)\, v(t)$ in terms of V_0, a, and b.

Answer: The block diagram is shown in Figure 2 below. In the scenario described above, the road is flat, so disturbance is 0: D(s) = 0. So, the output is just the product of the input and plant (in the Laplace domain).

We thus have:
$V(s) \ = \ V_m(s) \ \cdot \ G(s) \ (1)$

Now:
$v_m(t) \ = \ V_0 \ \cdot \ 1(t)$

Taking the Laplace Transform of $v_m(t)$ to obtain $V_m(s)$ gives:
$V_m(s) \ = \ V_0/s$

Substitutiing into (1) gives:

$$V(s) = \frac{V_0 \cdot a}{s(s+b)}$$

Now, we can use Final Value Theorem (FVT) because we meet the conditions that all poles are in the OLHP, except possibly one pole at 0. Specifically, we have one pole at 0 and one at -b < 0 (since $b = B/M$ and both the friction coefficient and mass are greater than 0).

Applying FVT gives:

$$\lim_{t\to\infty} v(t) \ = \ \lim_{s\to 0} sV(s) \ = \ \lim_{s\to 0} \frac{V_0 \cdot a}{s+b} = \frac{V_0 \cdot a}{b}$$

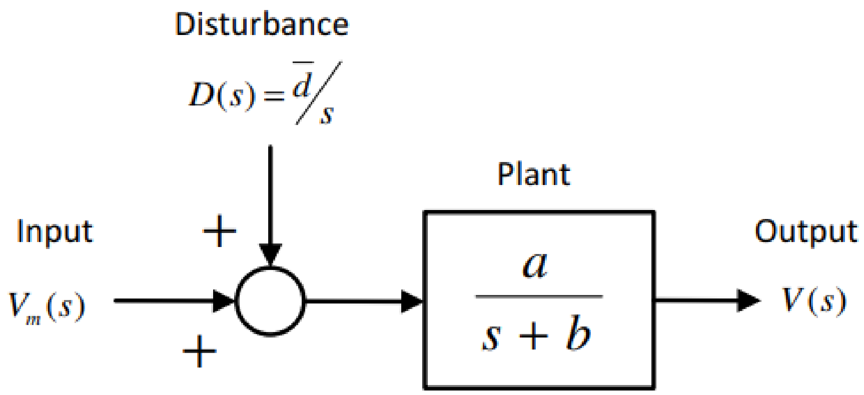


Figure 1: Block diagram of the plant. In our calculations, D(s) = 0.

Section 4.1

Question: Output of the Scope in lab3_4_1.slx with labels + description

Answer:

The Simulink model used to simulate the nonlinear cart's response to a sinusoidal input is shown below. The sinusoidal input had an amplitude of 1000 volts and a frequency of 1Hz.

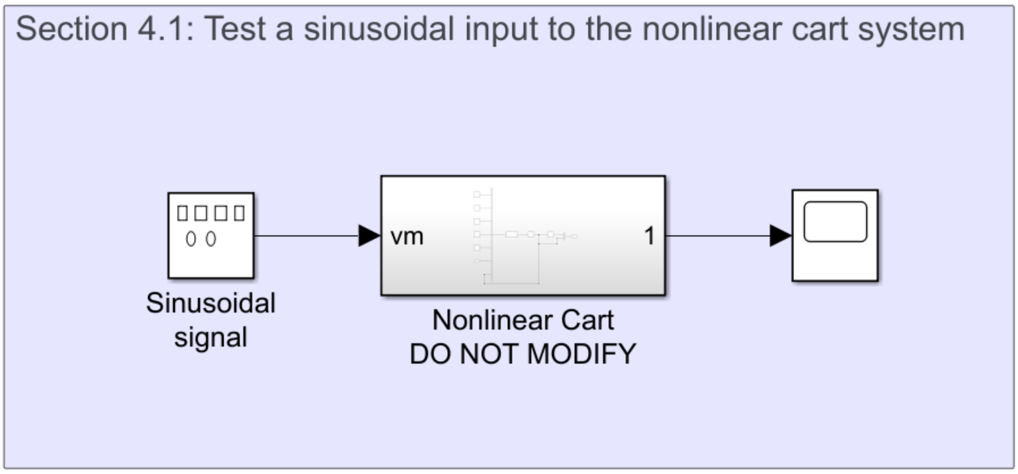
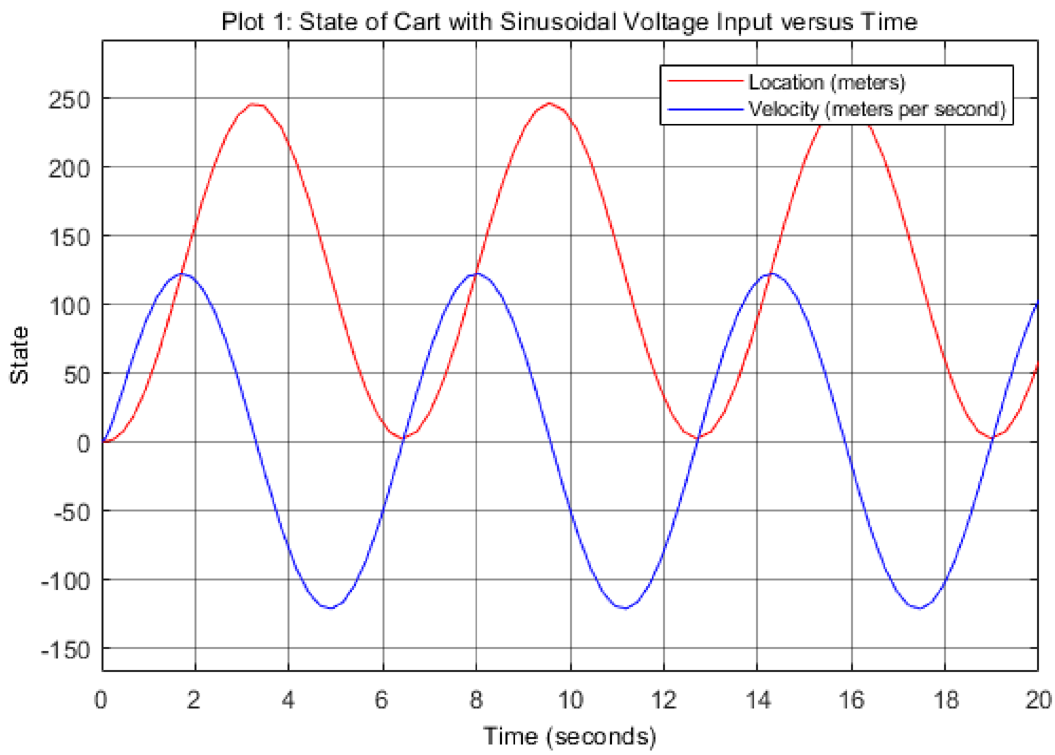


Figure 2: SIMULINK model of nonlinear cart with a sinusoidal signal

The graph below shows the result of running the provided lab3_4_1.slx file. We observe that the response to a sinusoidal input is also a sinusoid. Intuitively, this makes sense because the force u is proportional to the input voltage, so by changing voltage in a certain way (e.g, sinusoid), we also change the force imparted on the cart in that way, which in turn affects its velocity as dictated by Newton's laws.



Section 4.2

Question: Provide the output of "experimental values (from Arduino, but simulated in .slx)" and the transfer function output in lab3_4_2.slx:

- One figure for the difference between the initial guess of a,b and the "experimental" + description
- One figure for the final estimated a,b and the "experimental" + description
- Steps 6-8

Answer:

The output of transfer function in lab3_4_2.slx:

The Simulink model for the transfer function $V(s)/V_m(s)$ derived in Section 3 is shown below:

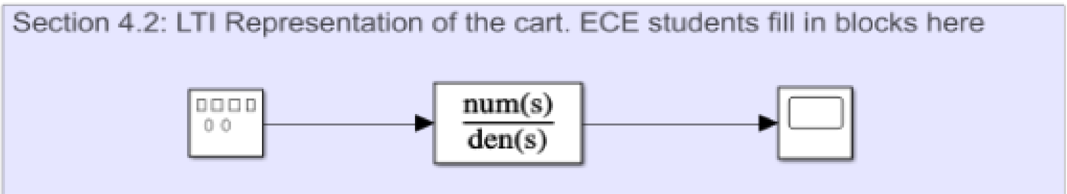
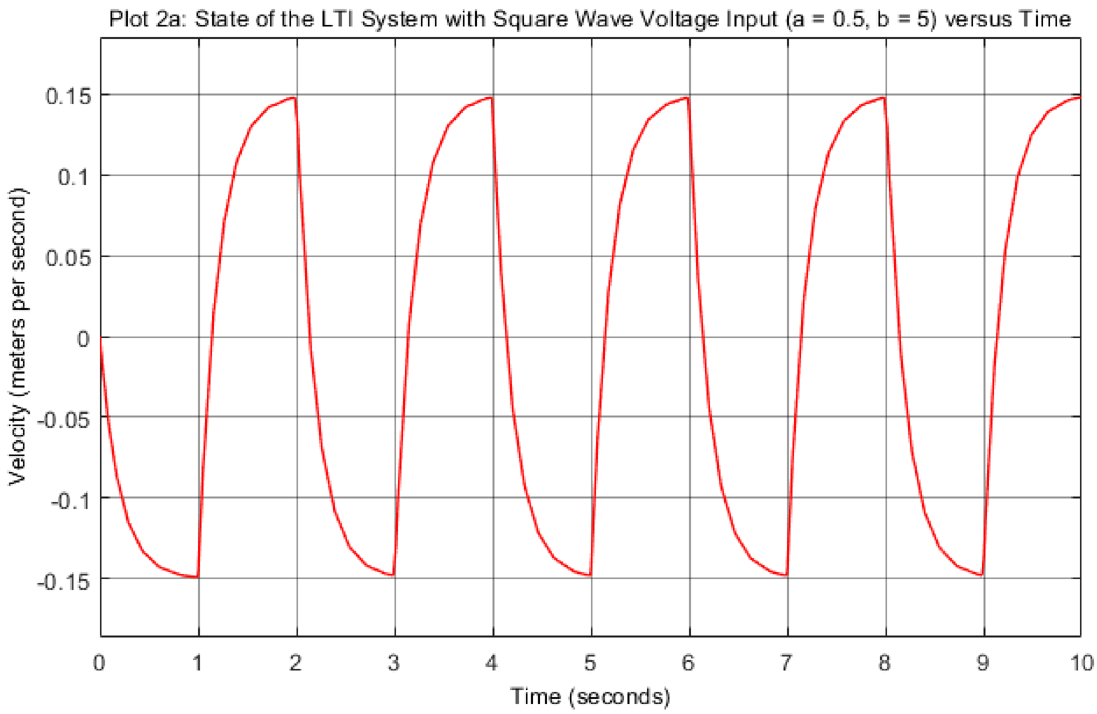


Figure 3: SIMULINK model for LTI representation of the cart

The result of applying a square wave voltage input to the transfer function with parameters a=0.5 and b=5 (corresponding to our initial guesses) is shown below. We observe that the output is periodic and reaches a maximum of 0.15m/s in magnitude. Further, the output signal appears to be symmetric in shape. This makes sense intuitively as our input is a square wave, and each portion of the square wave can be approximated as a step that pushes the system to a steady-state. Since the square wave amplitude is the same in magnitude in both halves of the period, it is reasonable that the steady-state values should be the same in magnitude for the two half periods (i.e., giving the symmetric property).



The output of "experimental values (from Arduino, but simulated in .slx)":

The Simulink model for the experimental cart nonlinear system is shown below.

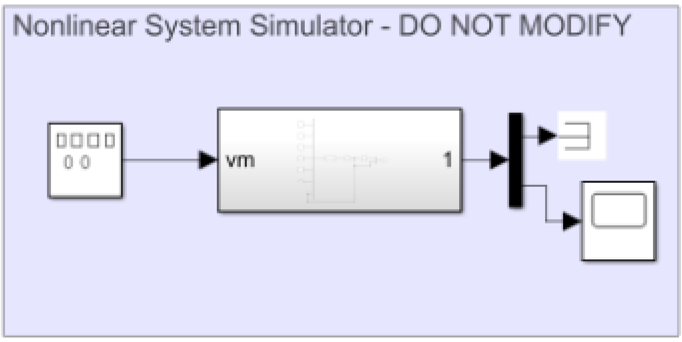
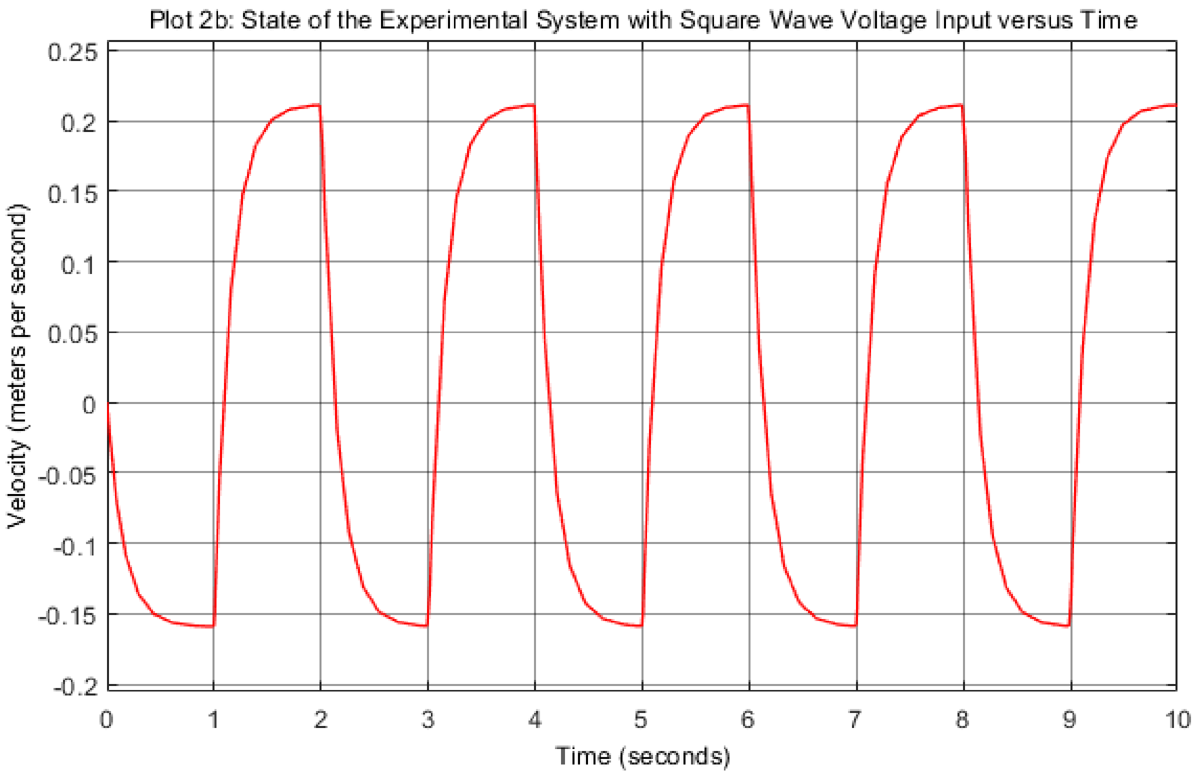


Figure 4: SIMULINK model for Nonlinear System Simulator

The next plot shows the result of running the experimental system with the same input voltage $v_m(t)$:



We observe a similar shape as the scope output using our initial guesses for a and b. However, we note that the output is not perfectly symmetric. This is because we are comparing an LTI system with a nonlinear cart. The nonlinear cart has a tilt of 1 degree (θ = 1). So, there is a nonzero disturbance D(s), which we assumed to be 0 in the transfer function we derived (Section 3). This is the cause for the discrepancy between Plot 2a and Plot 2b, shown below in Plot 2c.

The detailed model of the nonlinear cart is also shown below. We can see that it is nonlinear as θ = pi/180 rad.

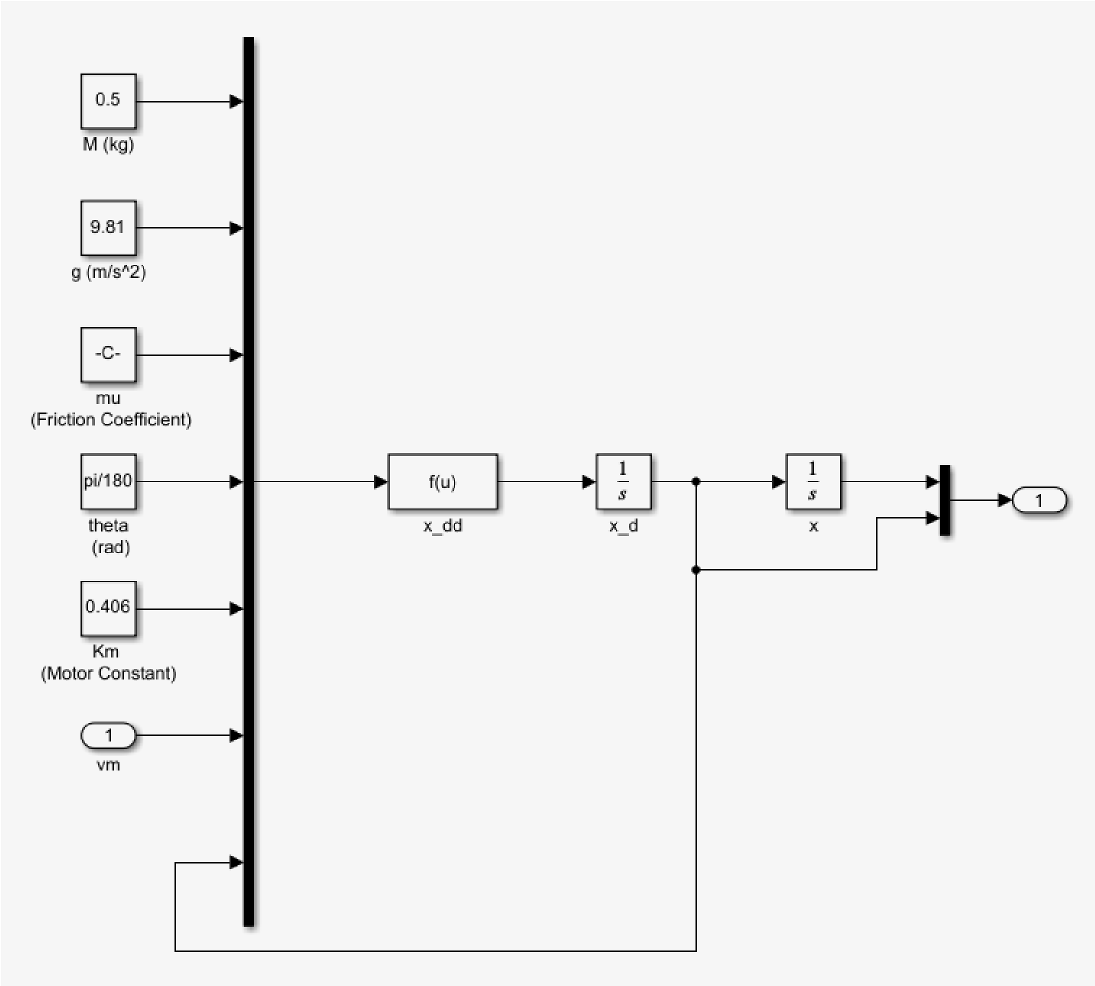


Figure 5: SIMULINK diagram of nonlinear cart.

Difference between initial guess of a,b and the "experimental" + description:

The SIMULINK model required to compute the difference between our initial guess (a,b) = (0.5,5) and the experimental results is shown below:

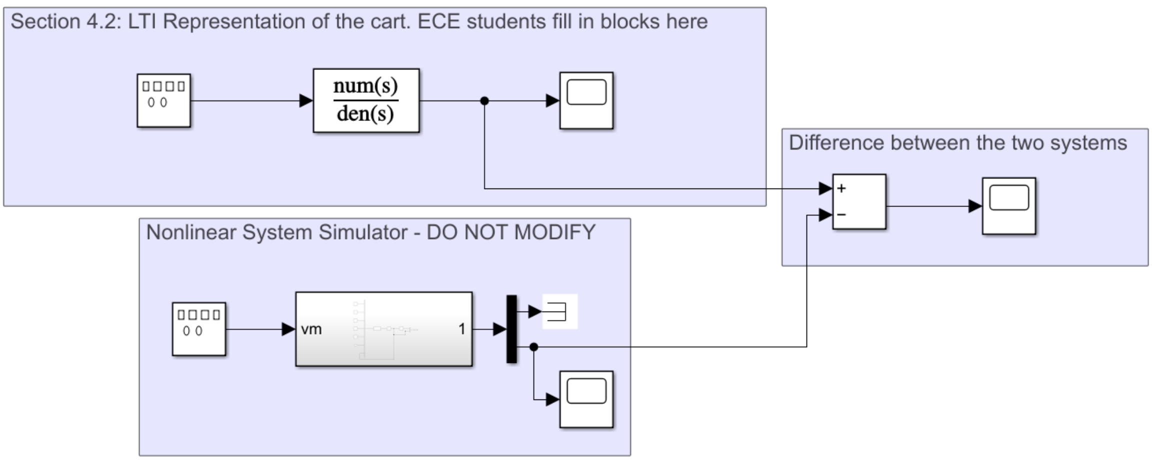
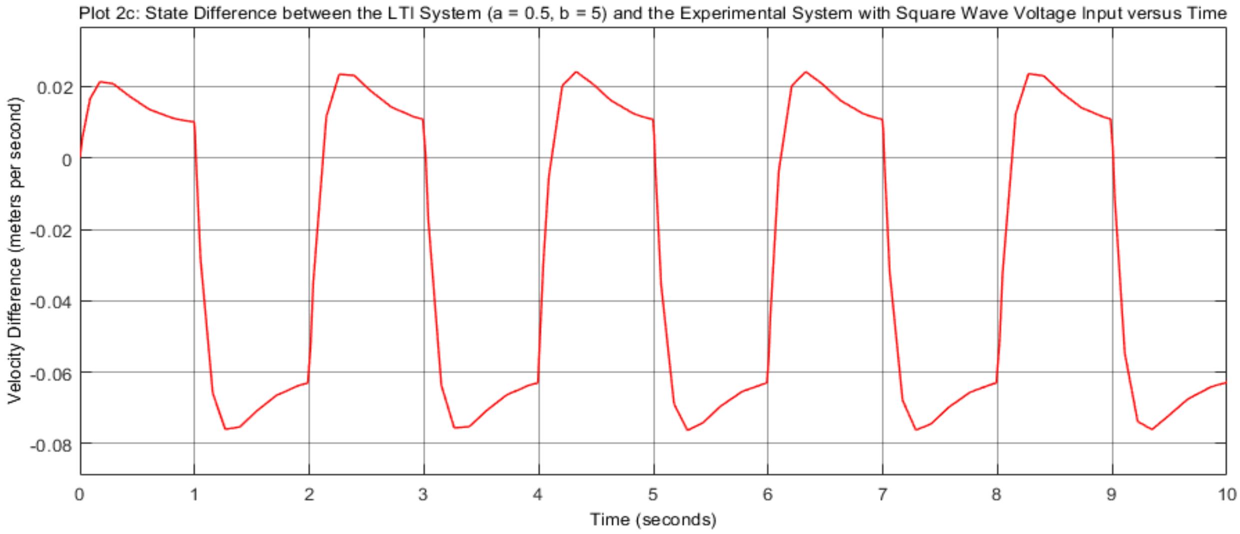


Figure 6: SIMULINK model to display the difference between LTI and nonlinear system outputs to the square wave voltage input of amplitude 1.5 V and frequency 0.5 Hz.

The result obtained in the scope when applying the square wave input is shown in Plot 2c:



As we can read off, our transfer function approximates the experimental results with a maximum error of about 0.075m/s. In the following section, we will try to improve on this error by following steps 6 to 8 of the lab.

Steps 6-8:

Step 6:

Over the half period under consideration, the signal $v_m(t)$ performs a step of amplitude V_0 = 3V . We'll make the approximation to consider the signal v(t) to be in steady state at the end of the half period. So we deduce that, in response to an input step of amplitude 3V, the plant output has a total variation of Δv. Using the formula we found in our lab preparation and the value Δv, we will find the relationship between a and b; specifically, an expression of the type a = f(b).

First, we determine the value of Δv. We do this by finding the peak-to-peak difference in the output velocity as we step from -1.5V to +1.5V in our square wave input. Simply reading values from our graph, we see that we go from a steady state of -0.16m/s to 0.21m/s. This allows us to compute Δv:

$$\begin{aligned}\Delta v &= 0.21 - (-0.16)\\ &= 0.37\end{aligned}$$

Treating this as steady state value of the step response with amplitude $\mathrm{V}_0 = 3V$, we can obtain the relationship a = f(b) as follows:

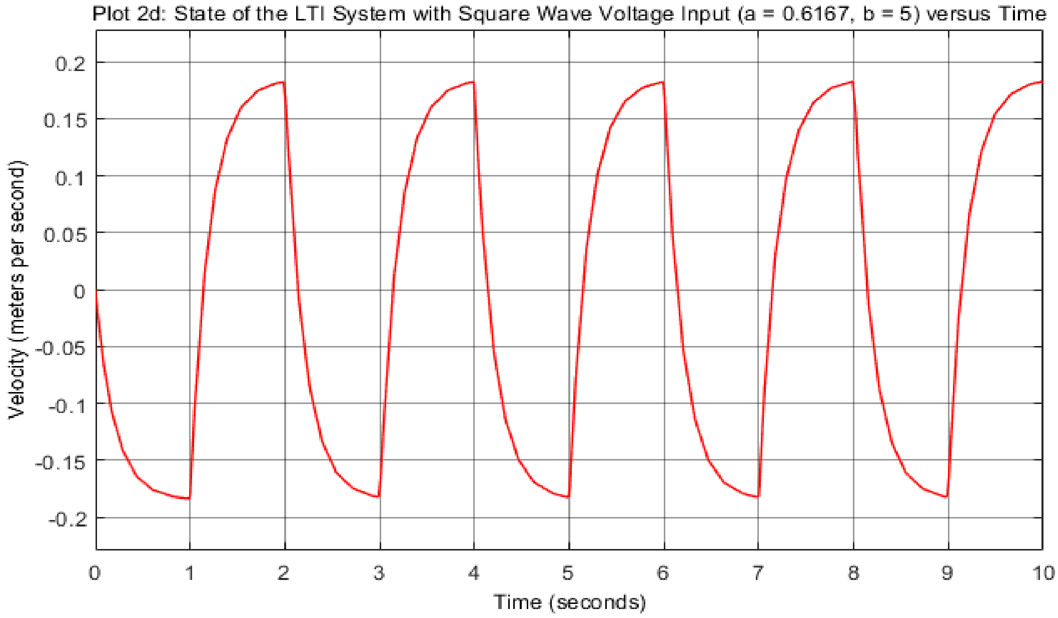
$$\frac{V_0 \cdot a}{b} = \Delta v$$

$$\frac{3 \cdot a}{b} = 0.37$$

$$=> a = \frac{37 \cdot b}{300}$$

Step 7:

We keep b = 5 and set $a = f(b) \approx 0.6167$. Running the simulation gives Plot 2d below:

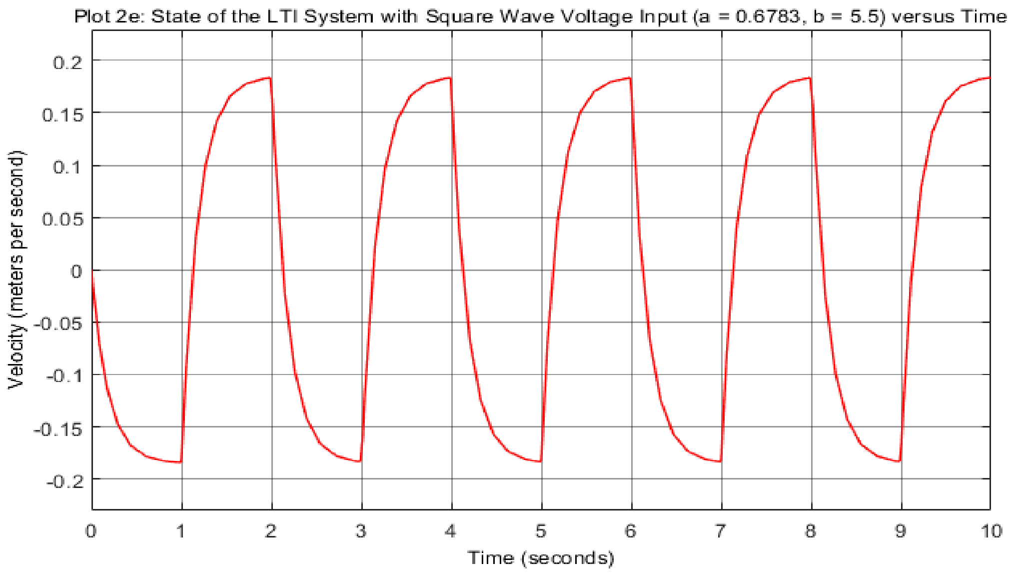


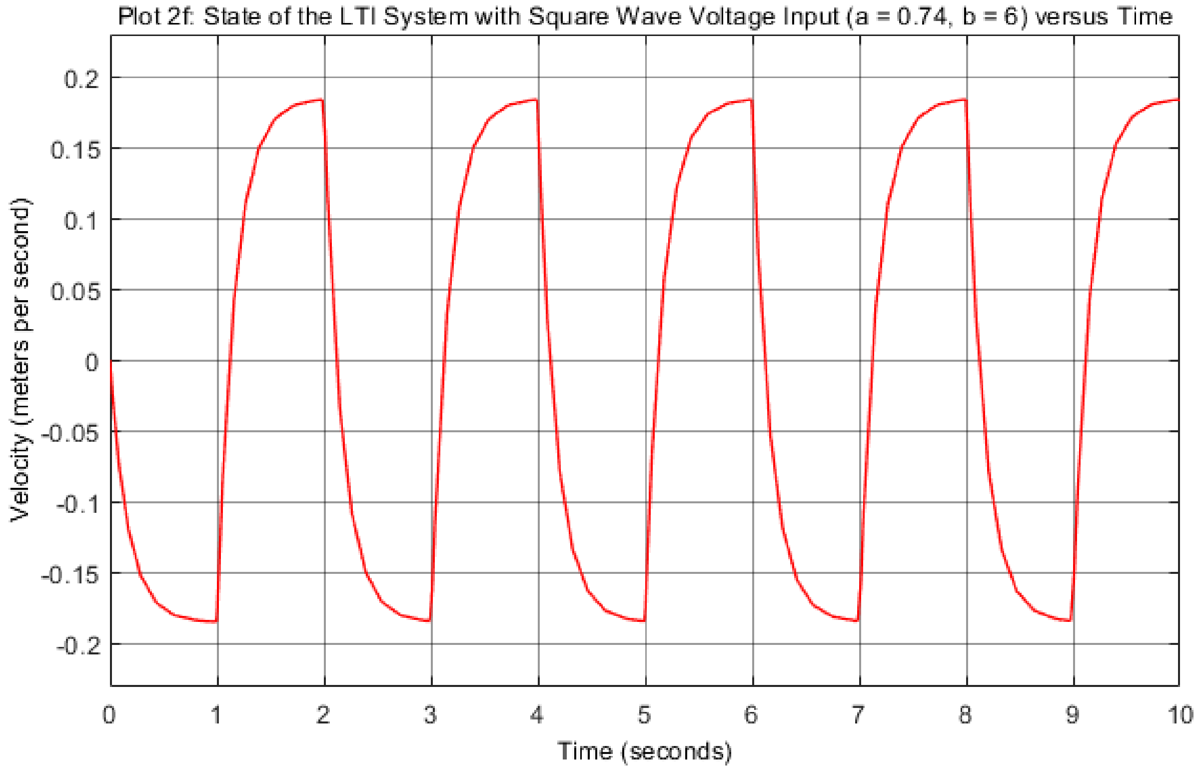
We can verify that the steady-state values of the model and actual plant outputs coincide. Specifically, we know that to coincide, we want the peak-to-peak difference in velocity to be 0.37. Since our linear transfer function outputs symmetric results, we want a peak of 0.37/2 = 0.185m/s in magnitude. Indeed, looking at the graph, it seems like 0.185 is a plausible value for what the steady state value is.

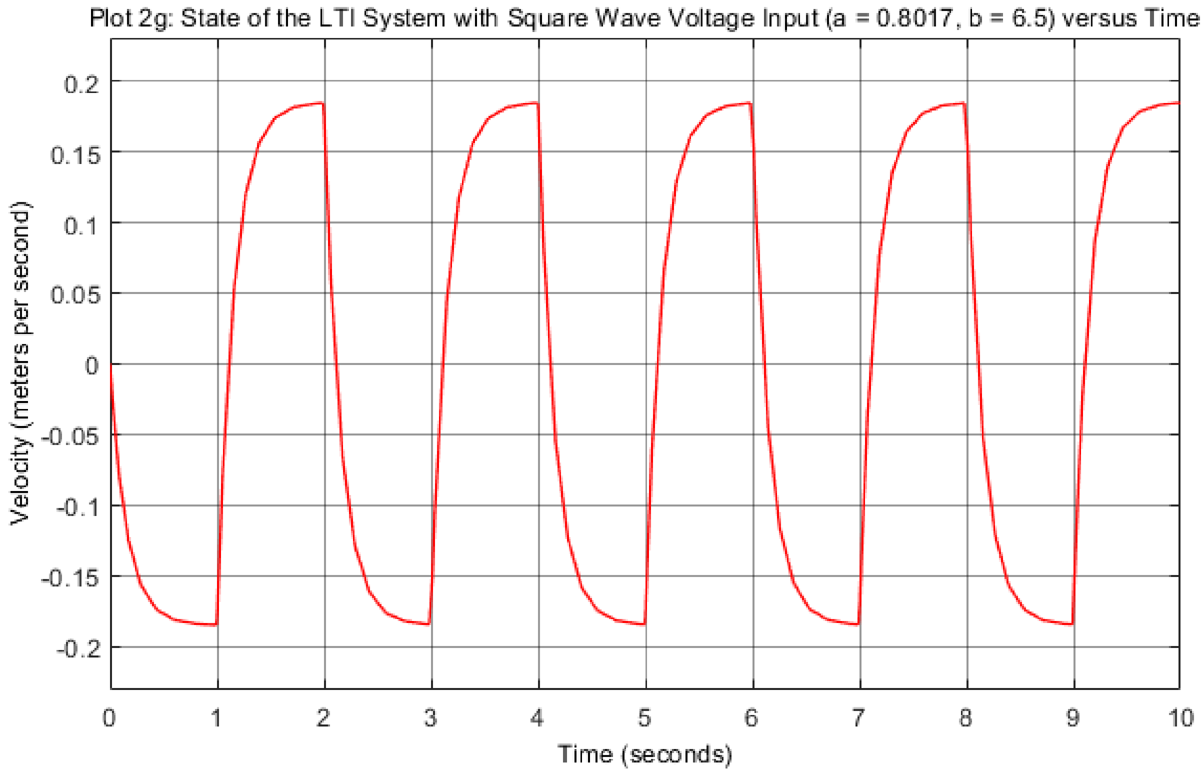
Step 8:

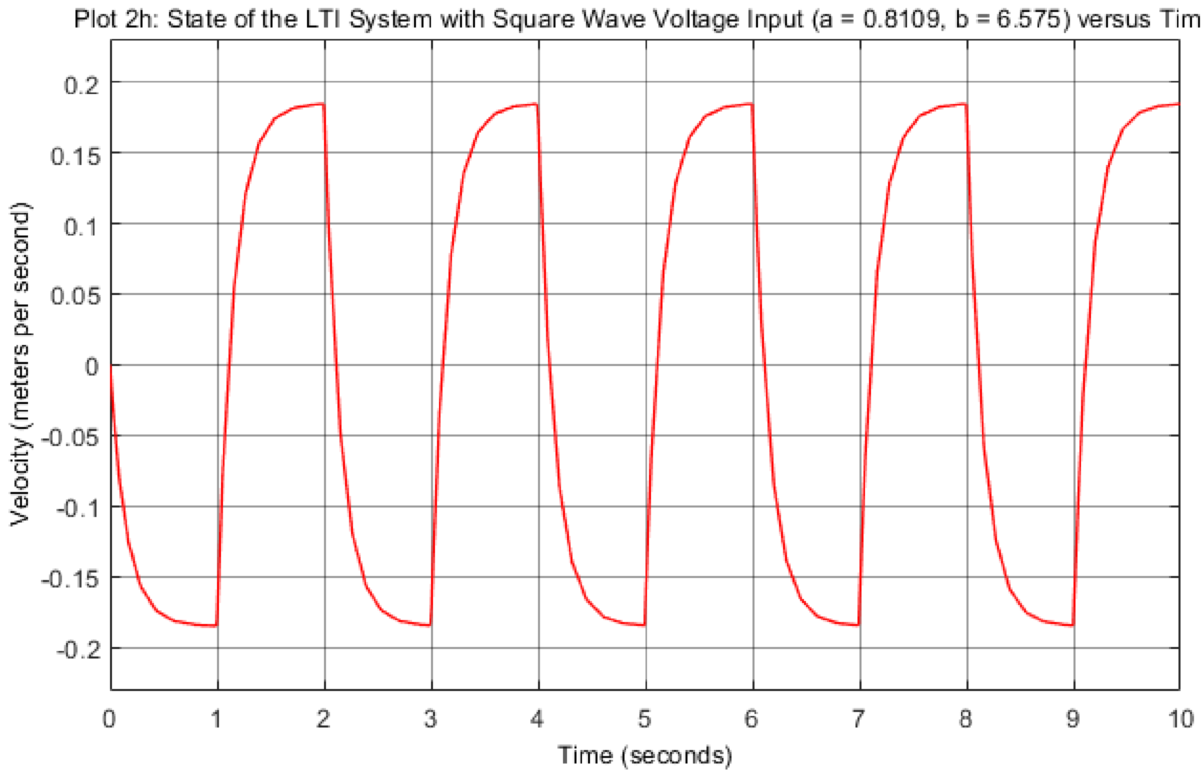
Now, we will try to increase b. Every time we modify b, we also update a = f(b) in Simulink to maintain the steady state value. We will run the simulations to see if the new values of b yields better results.

The plots below, Plot 2e to Plot 2h display our tuning process. We keep tuning b until we minimise the discrepancy between the actual plant and model outputs. The final values tuple is (a,b) = (0.8109, 6.575).

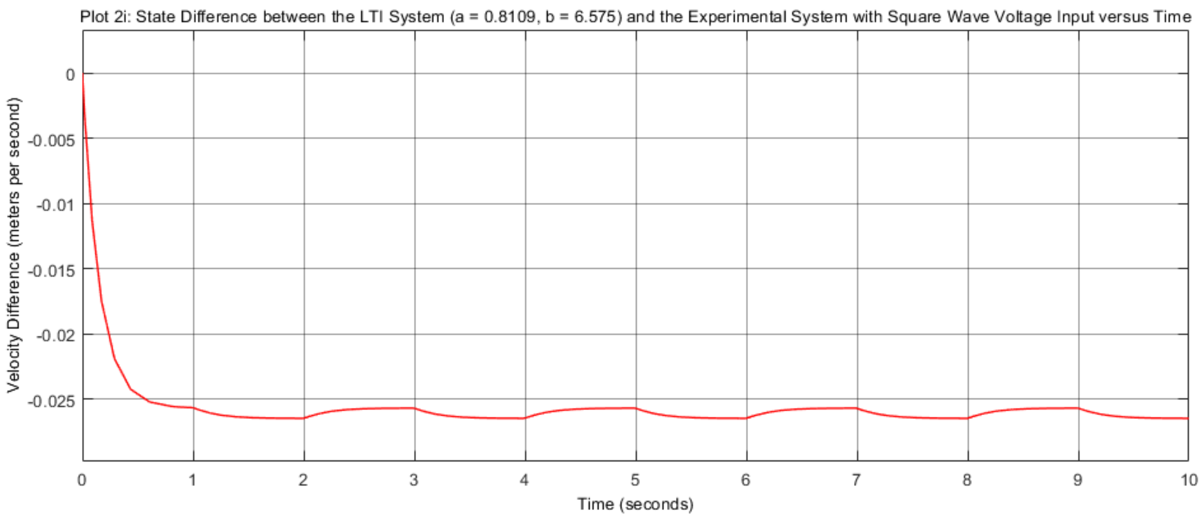



Plot 2f: State of the LTI System with Square Wave Voltage Input (a = 0.74, b = 6) versus Time
Velocity (meters per second)
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
1
2
3
4
5
6
7
8
9
10
Time (seconds)


Plot 2g: State of the LTI System with Square Wave Voltage Input (a = 0.8017, b = 6.5) versus Time
Velocity (meters per second)
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
1
2
3
4
5
6
7
8
9
10
Time (seconds)



The difference between our transfer function scope output and the output of the experimental results for our final values (a,b) = (0.8109, 6.575) is shown below. We can verify that we have met the steady-state condition, as well as improved on transient response as the velocity difference only reaches a maximum of about 0.025m/s in magnitude.



Section 4.3: Proportional Controller

Question: Plot and describe outputs as required with the appropriate Simulink files

Answer:

The diagram below shows the Simulink model for the proportional controller.

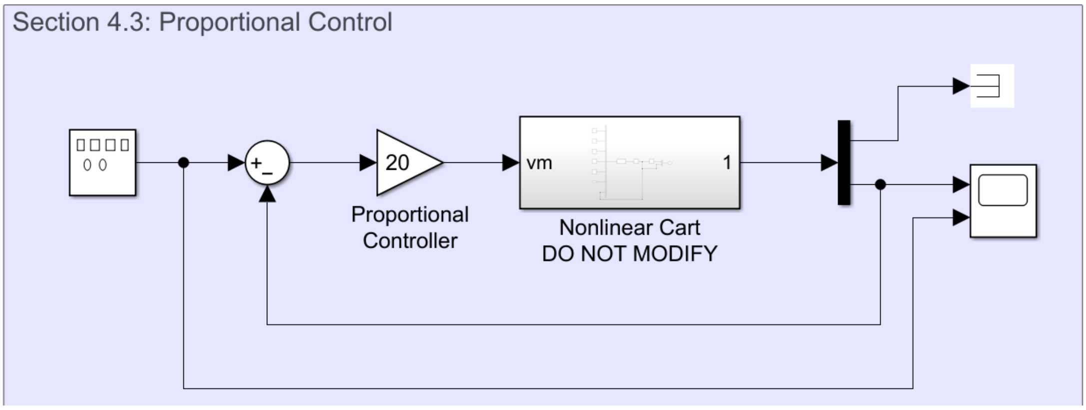
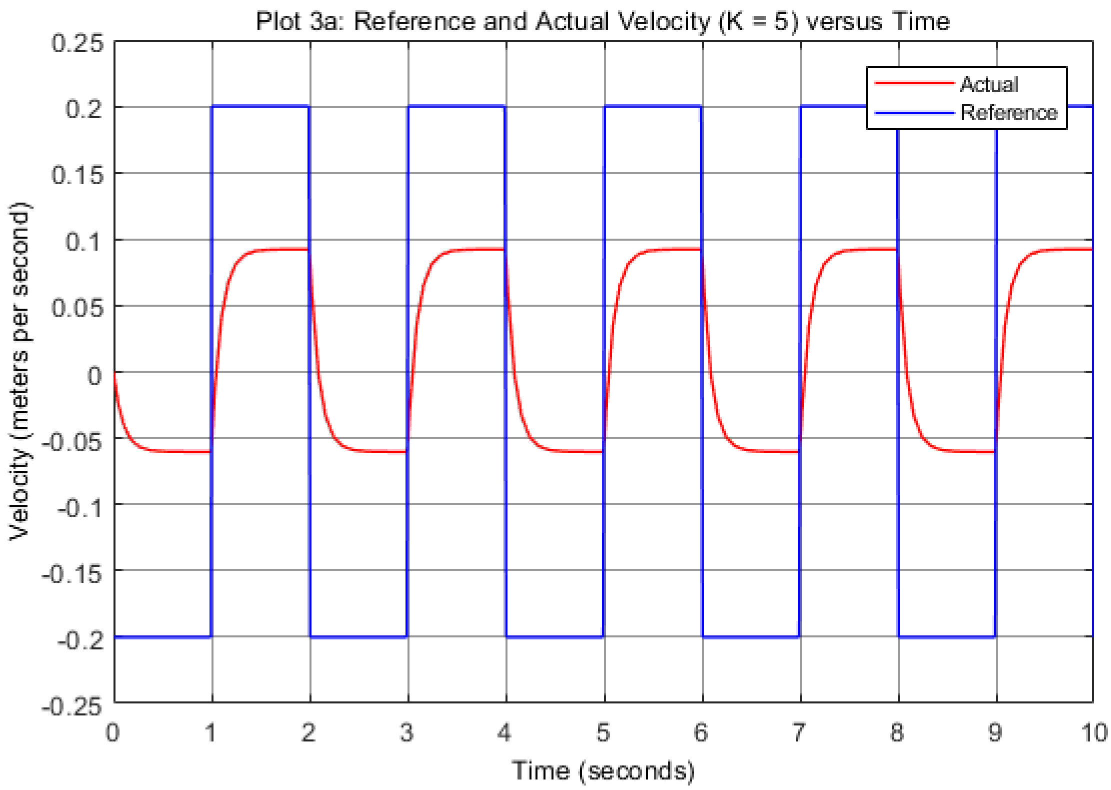


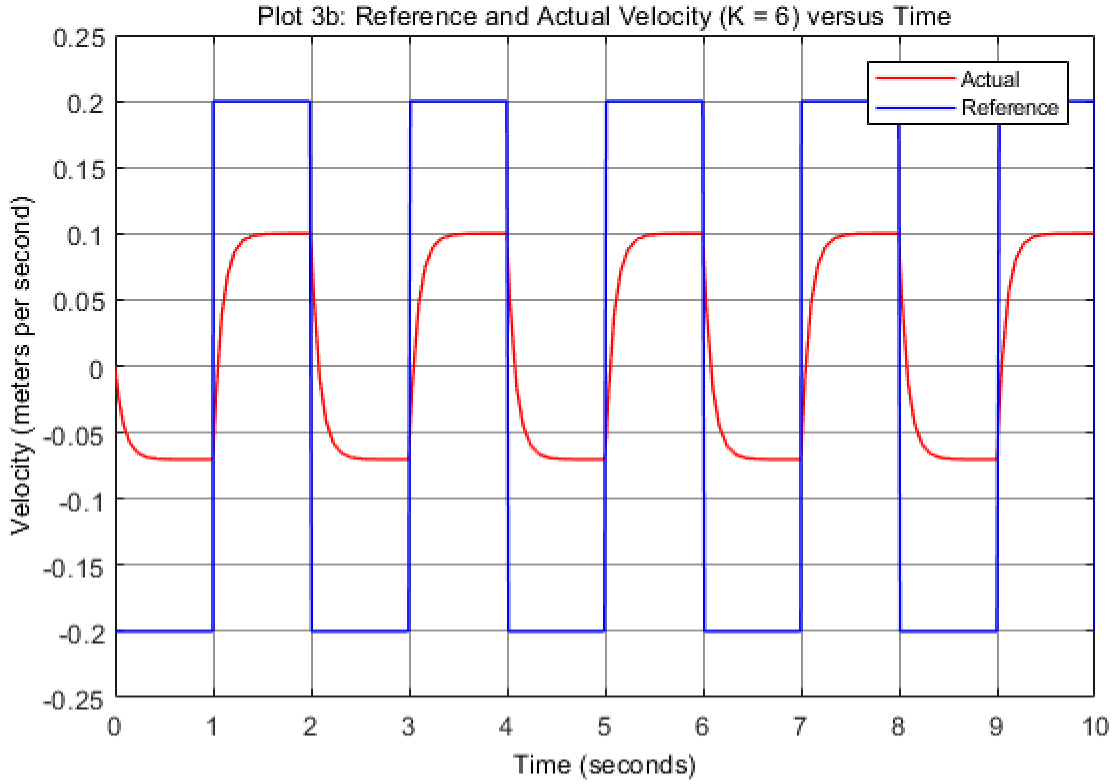
Figure 7: SIMULINK model for proportional controller

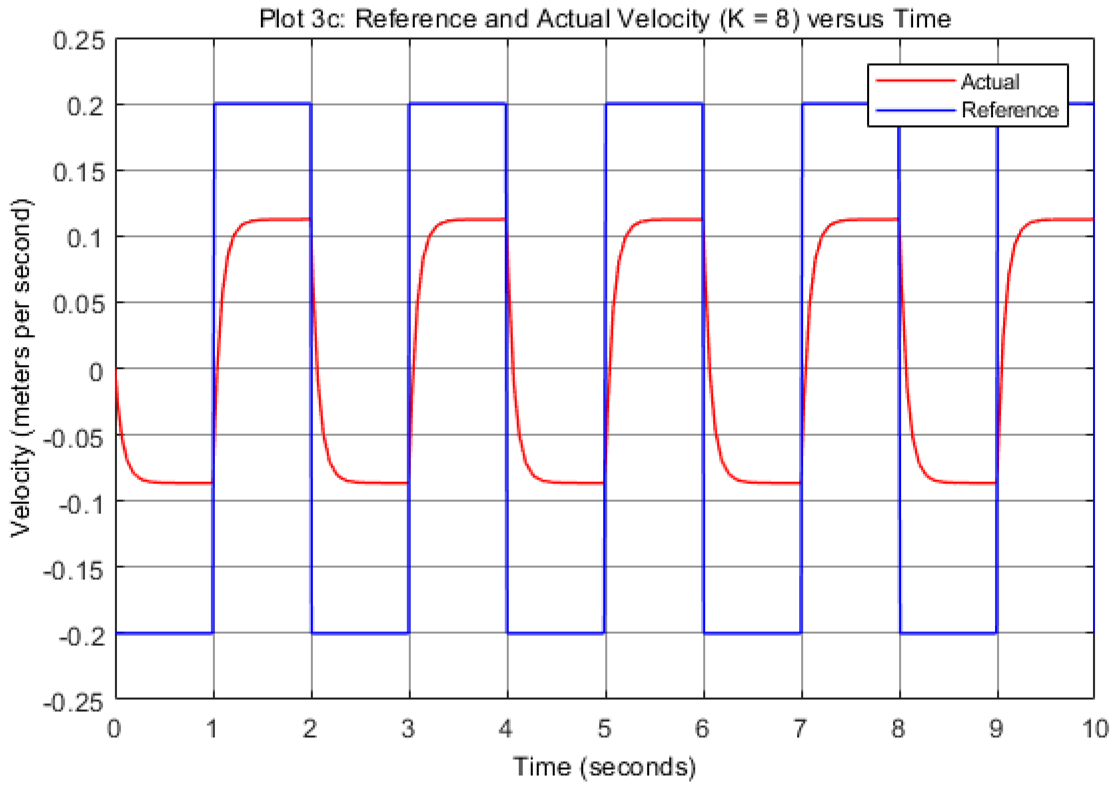
We use the lab_3_4_3.slx Simulink file. The velocity reference is set to a square waveform with an amplitude of 0.2 m/s and a frequency of 0.5 Hz.

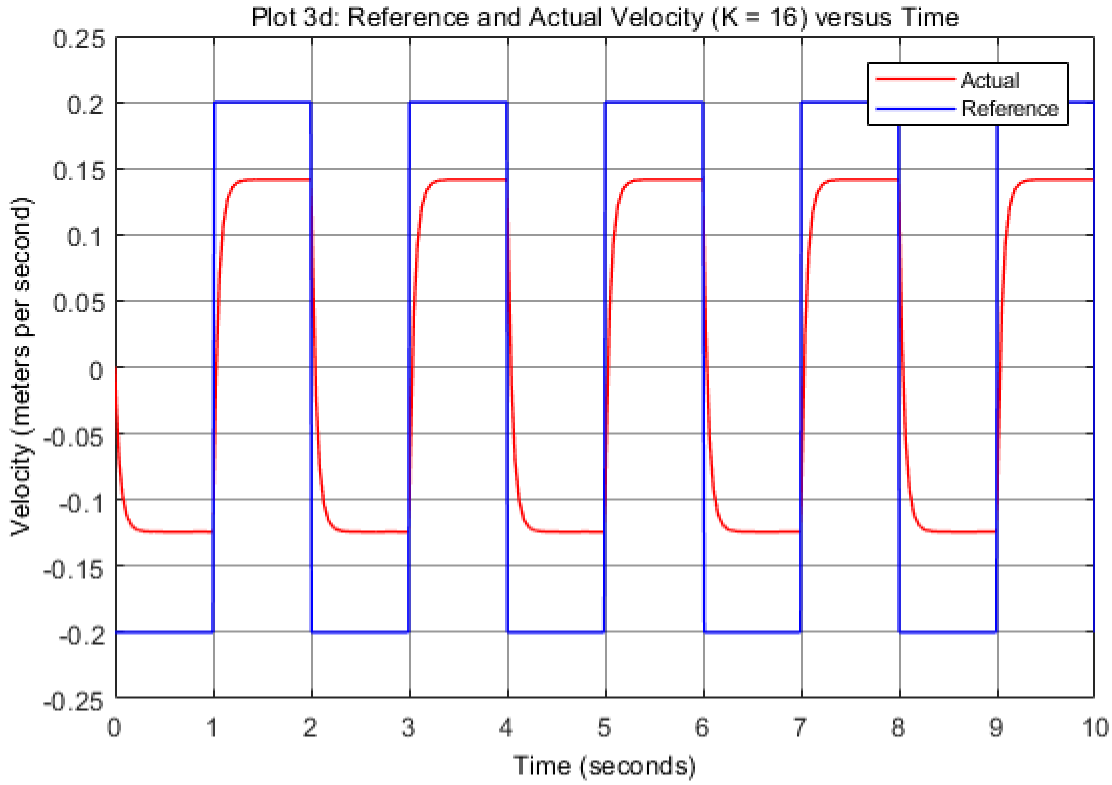
We set K = 5 as per lab instructions. We then observe the obtained experimental results. Both displacement and velocity of the cart are recorded, and are represented in Plot 3a, shown below:

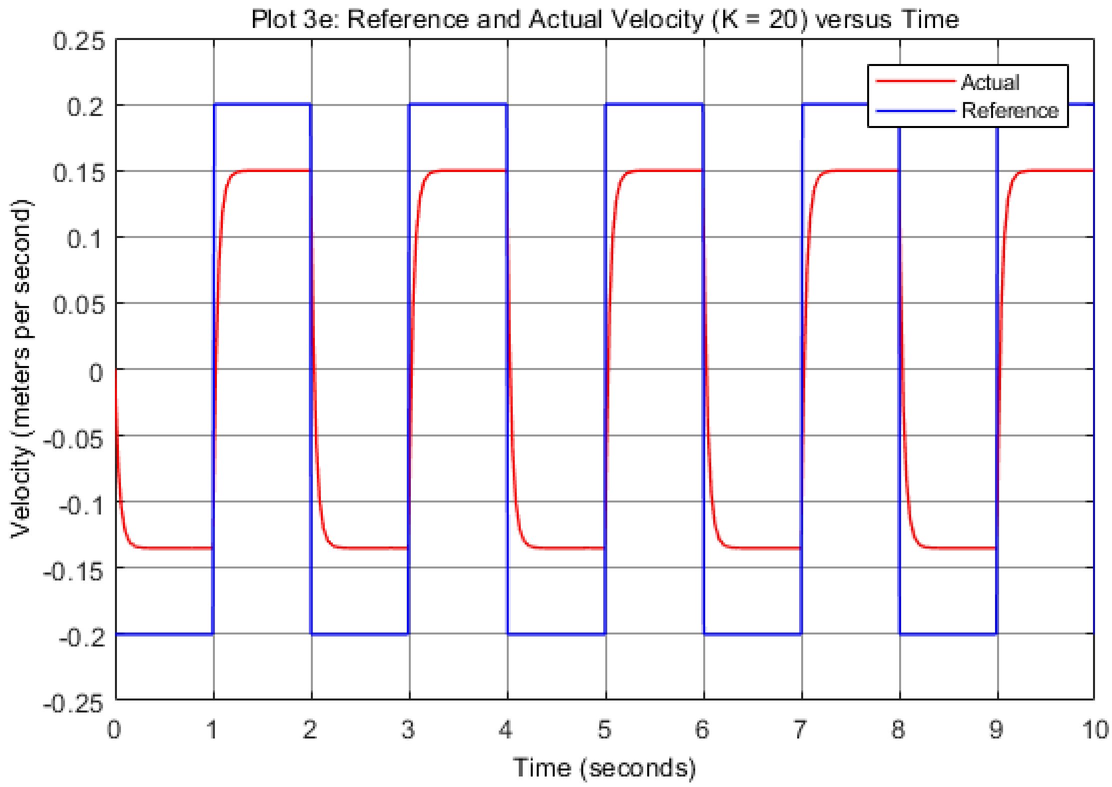


As stated in the lab document, we notice that the controller does not succeed in regulating the speed to the desired value. As suggested, we increase the controller gain and run the system again. We repeat this operation a few times; the results are shown in Plots 3b to 3e. Note that we are careful not to increase K beyond 20.









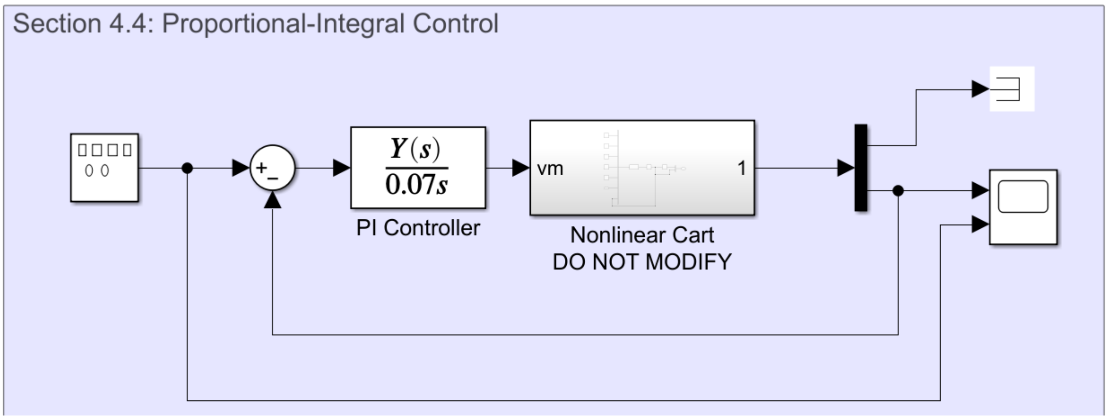
Plot 3e shows the output response obtained when K = 20. From our iterations, we note that increasing the gain K increases the steady-state velocity. However, it never reaches the reference velocity. At K=20, the actual velocity reaches a steady state of magnitude 0.15m/s, but the reference velocity is 0.2m/s.

Section 4.4: Proportional-Integral Control

Question: Plot and describe outputs as required with the appropriate Simulink files

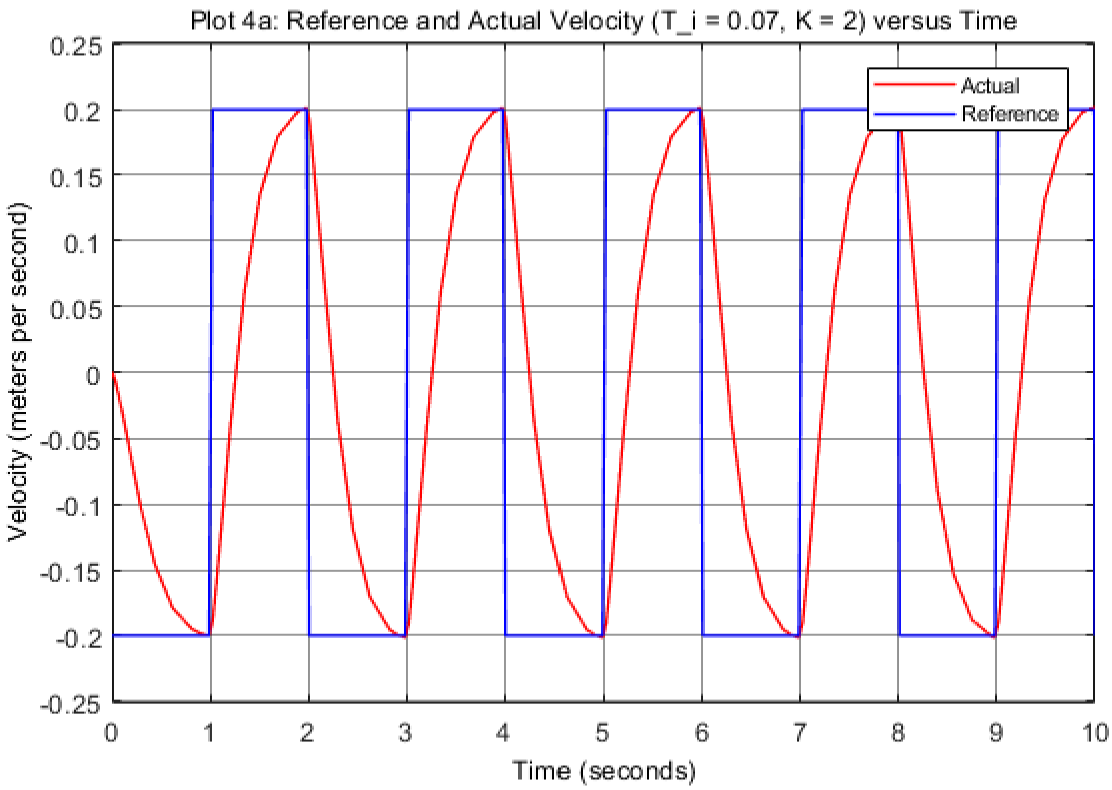
Answer:

The diagram below shows the Simulink model for the proportional-integral controller.

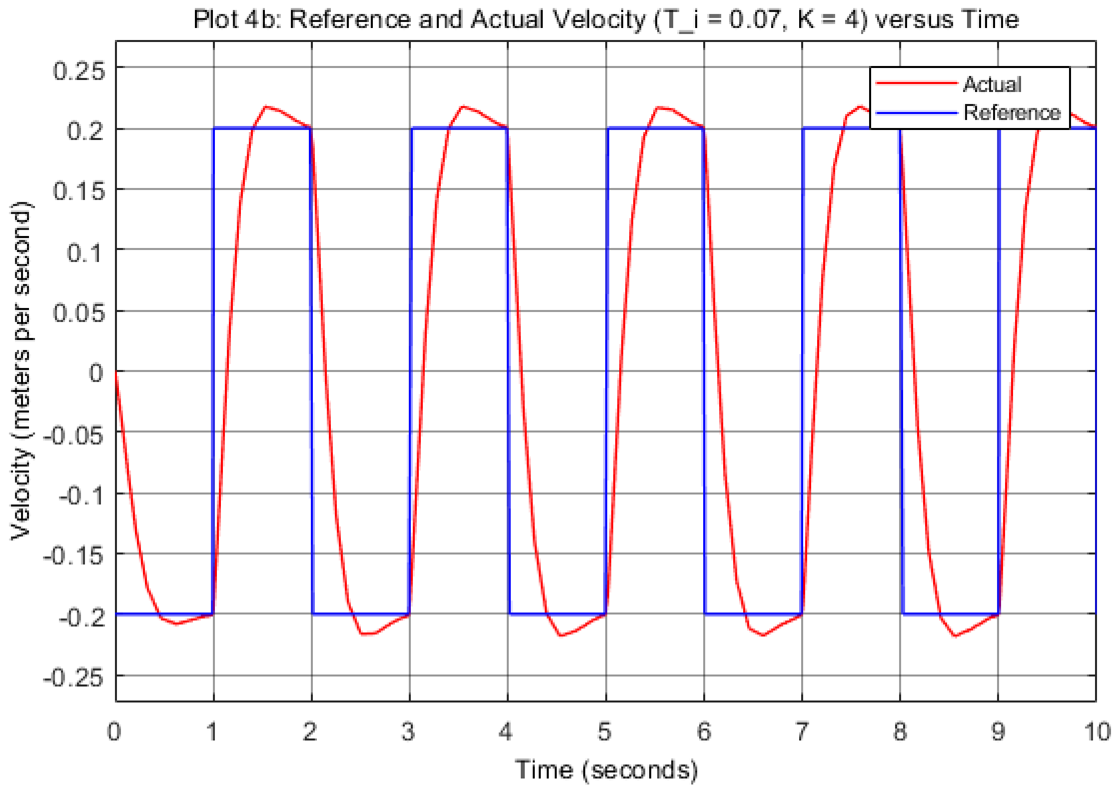


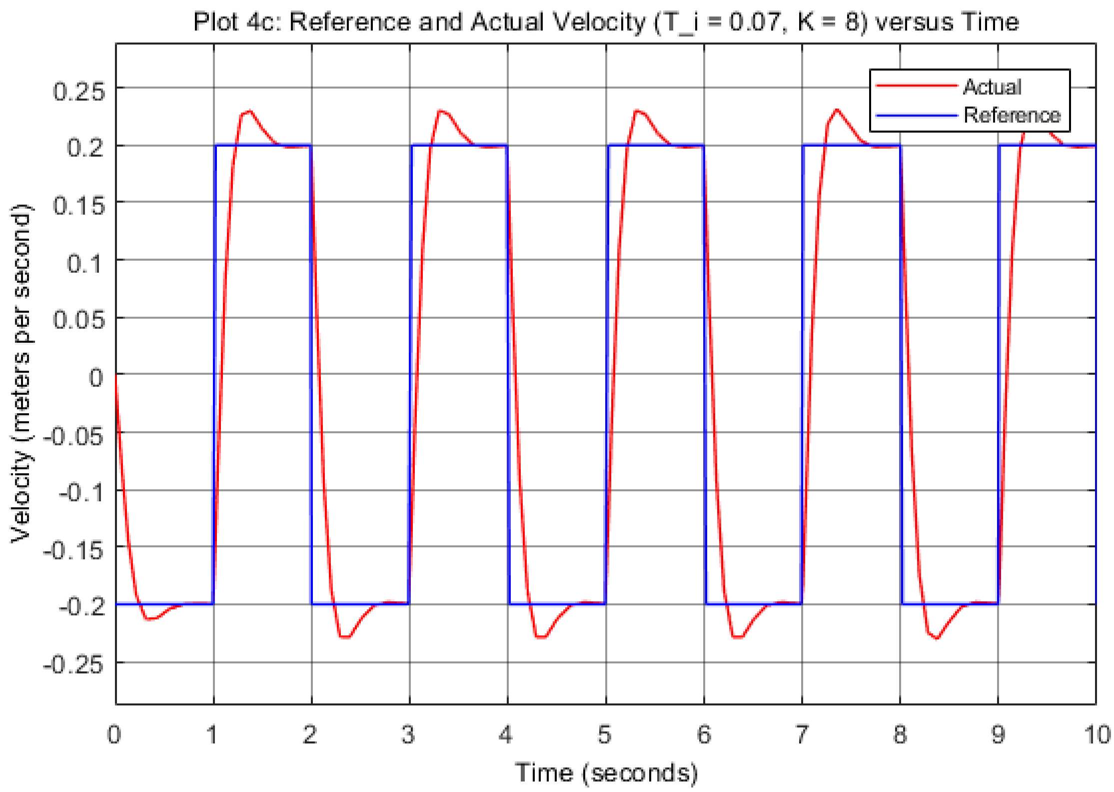
We use the lab_3_4_4.slx Simulink file. The velocity reference is set to a square waveform with an amplitude of 0.2 m/s and a frequency of 0.5 Hz.

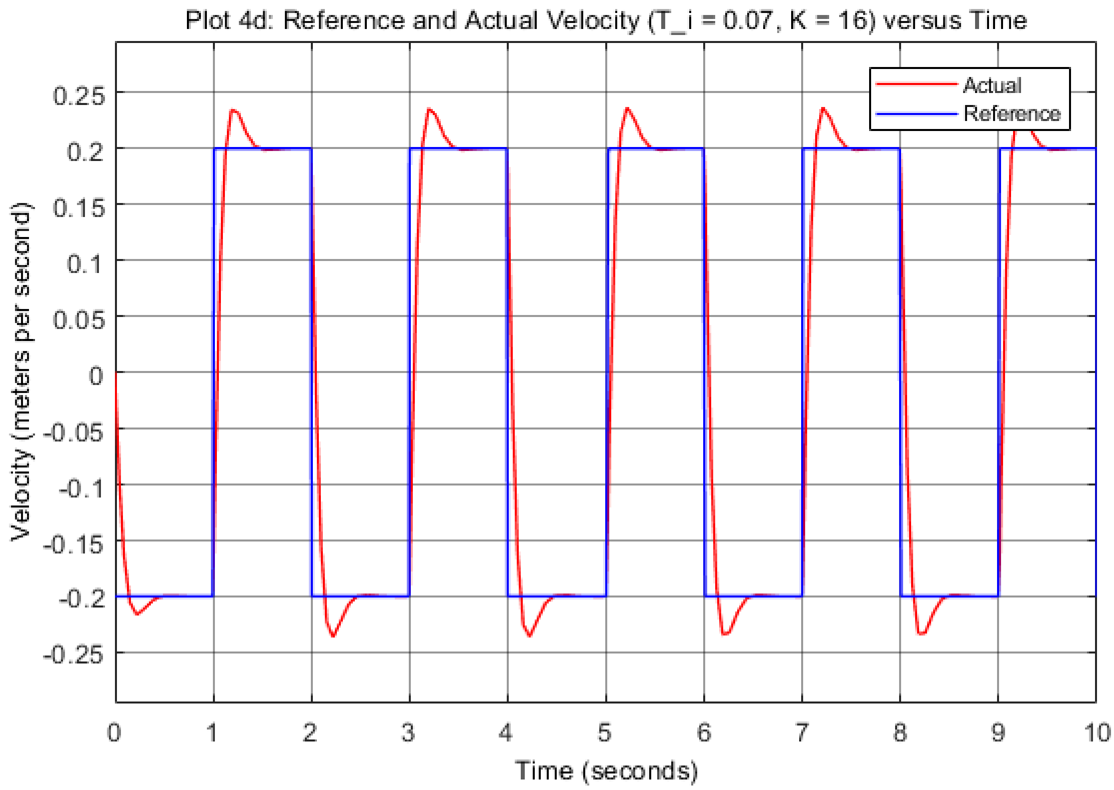
We set T_i = 0.07 and K = 2 as per lab instructions. We then observe the obtained experimental results. Both displacement and velocity of the cart are recorded, and are represented in the plot below:

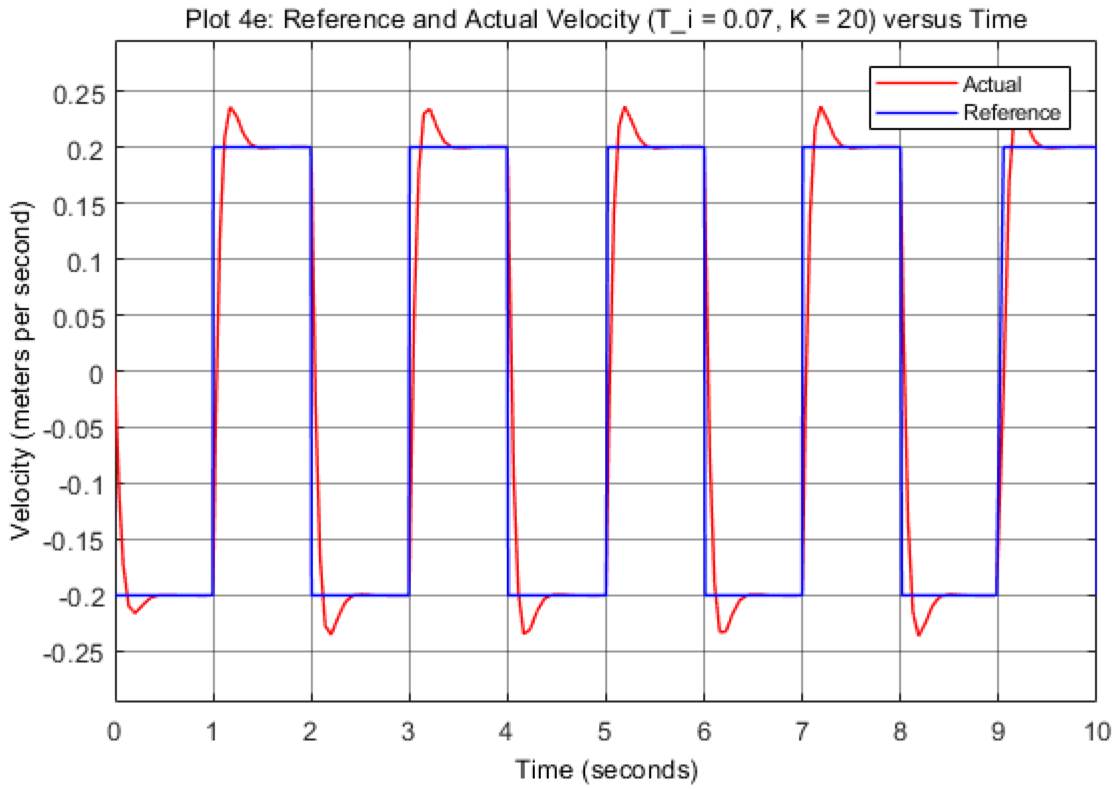


Next, we keep T_i constant and start increasing K. We are careful not to increase K beyond 20. The results of our iterations are shown below.









Plot 4e shows the output response obtained when T_i = 0.07 and K = 20. In general, increasing K helps us reach the steady-state value earlier (visually). The sudden rise of actual velocity at time = 1 second for K=20 compared to the slower rise for K=2 clearly shows that increasing K leads to faster progression towards the steady-state.

This makes sense intuitively. If we have a reference velocity we would like the cart to move at, and use $v_m(t) = K*(r(t) - v(t))$, then increasing K would mean applying a higher voltage when there is a difference between reference and current speed. This higher voltage means more force imparted to the cart, as given by the formula $u = K_m v_m$. This causes the cart to respond more quickly because there is a more sudden change in the dynamics of the system to move towards steady-state.

We also witness that increasing K causes overshoot in the velocity response. Visually, for K=2, velocity over the period never jumps over the steady-state velocity. However, for K=20, we first overshoot the steady-state before returning to steady-state. This is not desirable if there is a constraint on the maximum velocity. For example, in the real world, we might set a constraint on the velocity to ensure the safety of passengers in a car. Increasing the parameter K causes higher overshoot, which could push us to an unsafe velocity region.

Remarks about Section 4.3 and 4.4:

Through our visual observations in Section 4.3 and 4.4, we verify that assuming the CLS is BIBO stable, asymptotic tracking of polynomial references of order k-1 occurs if and only if C(s)G(s) has at least k poles at 0.

The CLS for both proportional and proportional-integral controllers in this lab is BIBO stable. In both cases, BIBO stability is met because G(s) has only one pole at s = -b, which is negative as established in Section 3.

The reference here is approximated as a step. A step is just a constant function, so has order 0. Thus, we have asymptotic tracking if and only if C(s)G(s) has type 1; i.e., at least 1 pole at 0. Now, compute C(s)G(s) for both controllers:

For proportional control:

$$C(s)G(s) \ = \ K \cdot \frac{a}{s+b}$$

For proportional-integral control:

$$C(s)G(s) \ = \ K\left(\frac{T_i \cdot s + 1}{T_i \cdot s}\right) \cdot \frac{a}{s+b}$$

It is obvious by looking at the denominator of C(s)G(s) for both controllers that the proportional-integral achieves this, but the proportional controller does not. This is the reason that our actual values did not reach the real values for the proportional controller in steady state, but they did for the PI controller.