ECE557 Lab 2: Position Tracking for the Cart System

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Introduction

The purpose of this lab is to develop an output feedback controller to force a cart system to track a square wave reference signal. A state feedback system will first be simulated in Simulink to analyze the effect of pole placement on tracking, specifically, the settling time of the system. The output feedback controller will be formed by combining a state estimator (observer) with the state feedback system. It will similarly be simulated, but also experimentally tested with a physical cart system. The poles of both the observer and state feedback will be tuned to achieve the best tracking in the physical experimentation section.

The initial simulations use a simple state feedback controller for the case when the state can be measured directly. For the case when the state cannot be measured, an observer can be used to estimate the state based on the output and reference signal. The following block diagram describes the closed-loop system representing the output feedback controller.

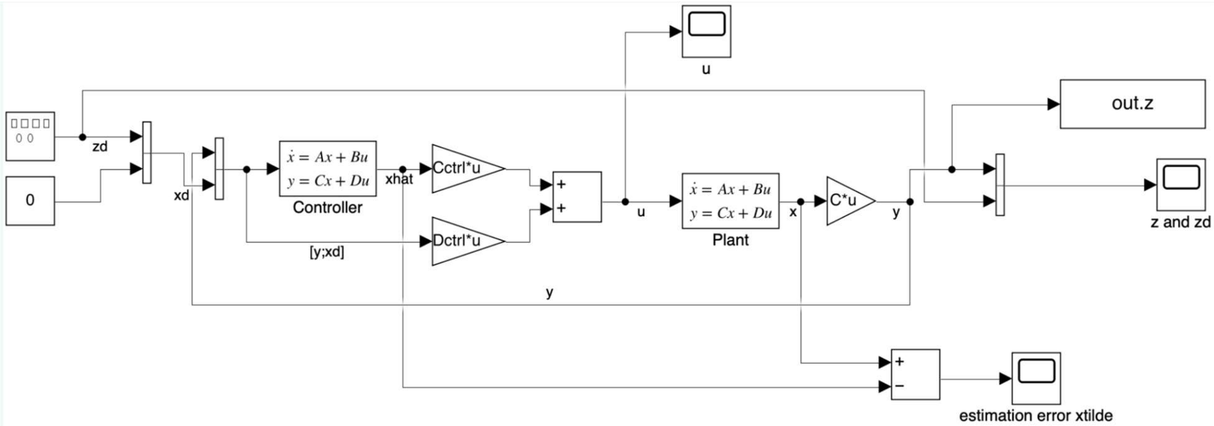

Figure 1. Closed Loop System Block Diagram

The plant block, labelled ‘plant’ in Fig 1, is defined for the cart system with the following state equations:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$A = \begin{bmatrix} 0 & 1 \\ 0 & -12.79 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 2.10 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ D = 0$$

The controller block, labelled ‘controller’, is determined by the plant state and associated pole placement through the following state equations:

$$\dot{\hat{x}} = A_{ctrl}\hat{x} + B_{ctrl}\begin{bmatrix} y \\ x_d \end{bmatrix}$$
$$u = C_{ctrl}\hat{x} + D_{ctrl}\begin{bmatrix} y \\ x_d \end{bmatrix}$$

$$A_{ctrl} = A + BK - LC$$

$$B_{ctrl} = [L \quad -BK]$$
$$C_{ctrl} = K$$
$$D_{ctrl} = [0 \quad -K]$$

The input to the controller block is the plant output and the reference signal, labelled 'y' and 'x_d' respectively. The output of the controller block is the observer state estimation, labelled 'xhat'. The plant is fed with a proportional control signal determined by the difference between the state estimate and the reference square wave. It is expected that the observer will estimate the unknown plant state and force it to track the reference signal.

Output 1 - State Feedback Controller Simulation

The gain, K, of the proportional controller can be found such that the eigenvalues of the closed loop state feedback system are specific values. To obtain eigenvalues {-5, -5}, the gain was found to be $\boxed{K = [-11.89\ 1.33]}$. The resulting state output, z(t), can be compared to the reference square wave $z_d(t)$ in Fig 2. The derivative of the state output, $\dot{z}(t)$, is shown in Fig 3.

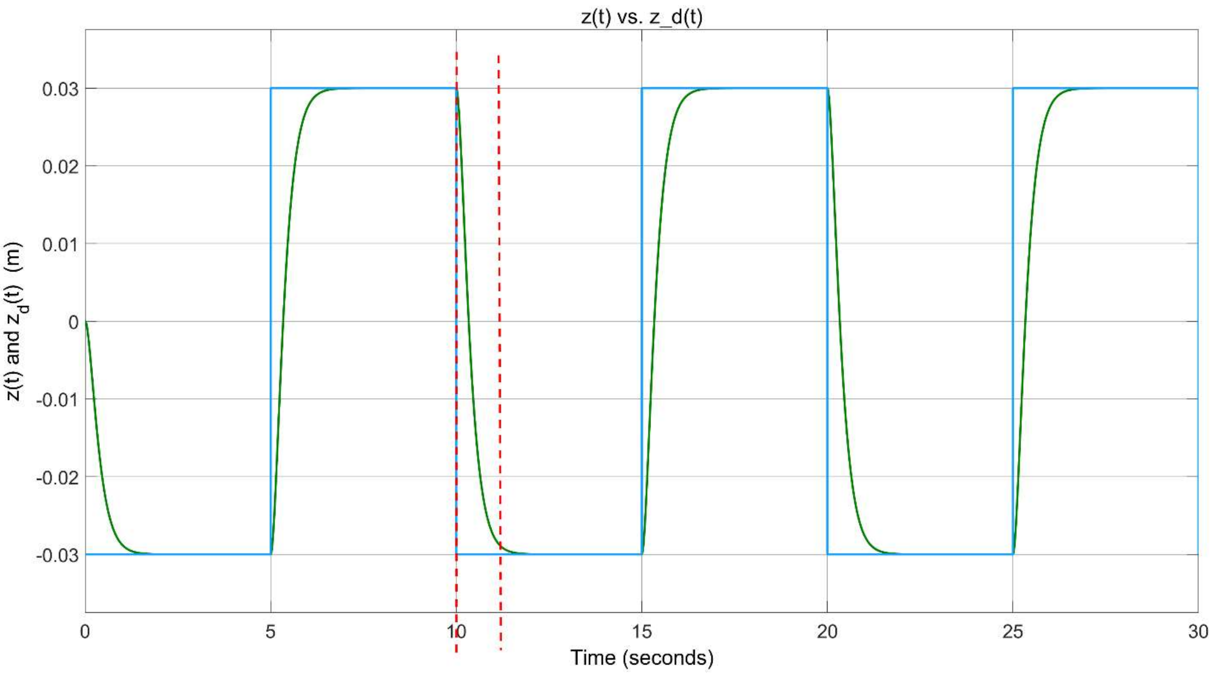

Figure 2. z(t) vs $z_d(t)$ for eigenvalues = {-5, -5}

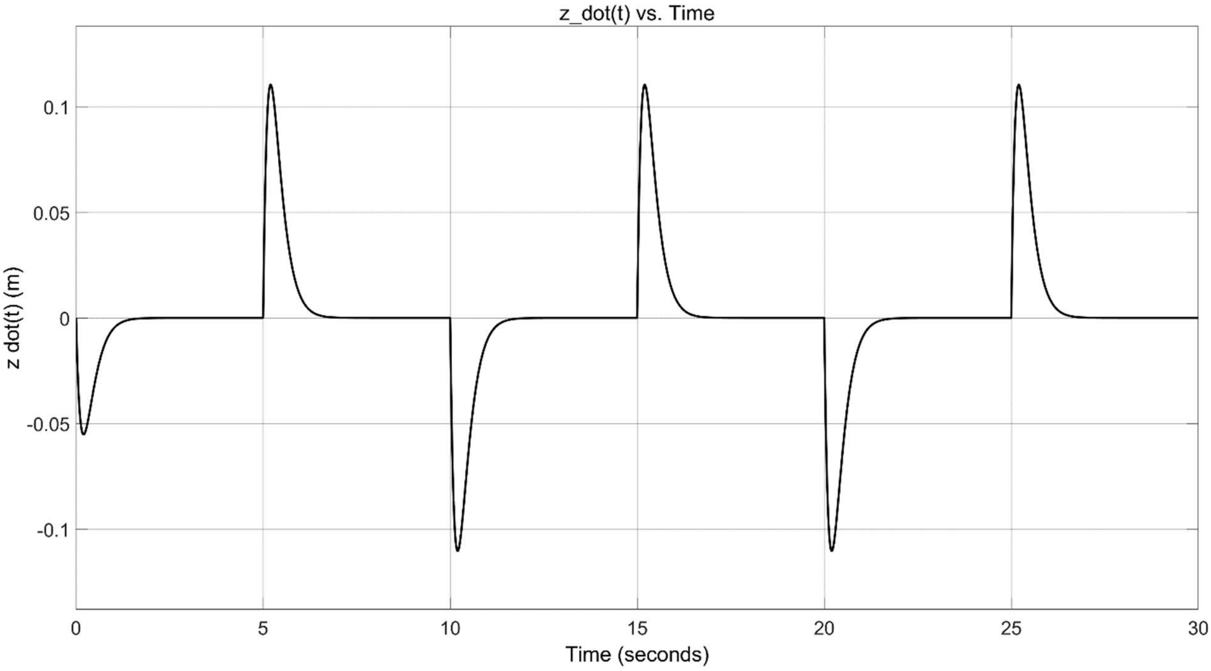

Figure 3. $\dot{z}(t)$ for eigenvalues = {-5, -5}

The settling time, where the steady state value stays under 2% of the step size, can be seen in Fig 2. Given the reference signal step size is 0.06, T_{s1} is the time at which $|z(\bar{t}+T_{s1}) - (-0.03)| \leq 0.02 * 0.06 = 1.2 * 10^{-3}$, where $\bar{t} > 0$ is an intermediate step in the square wave from high to low. We are looking for the time difference between $\bar{t}$ and the time at which z(t) = -0.03 + 0.0012 = -0.0288. In Fig 2, these two times are labelled using the vertical red lines. Their horizontal difference is the settling time: $\boxed{T_{s1} = 1.17s}$.

To analyze the effect of eigenvalues on settling time and motor force, we re-run the code by assigning the eigenvalues to be {-2, -2}. The new settling time is $\boxed{T_{s2} = 2.92s}$, as Fig 4 shows. Like the previous part, we used the difference between the two vertical red lines to find the settling time. Furthermore, Fig 5 shows $\dot{z}(t)$ for eigenvalues = {-2, -2}. Compared with Fig 3, we see the peak velocity is lower for eigenvalues of {-2, -2}.

The ratio can be computed as follows: T_{s1}/T_{s2} = 1.17/2.92 ≈ 0.40. This corresponds to the ratio of the eigenvalues: $\lambda_2/\lambda_1 = -2/-5 = 0.4$. This is reasonable as the eigenvalue tells us the rate of decay of the initial error towards 0. If one eigenvalue is double the other, it will decay twice as fast. Using the fact that the ratio of settling times should be the same as the inverse ratio of decay rate, a system with eigenvalues 0.4 times that of another will decay 1/0.4 = 2.5 times as fast. This is also evident from the modal decomposition $\dot{z} = \Delta z = \begin{bmatrix} e^{\lambda t} & 0 \\ 0 & e^{\lambda t} \end{bmatrix} z = e^{\lambda t} z$. Let's say the eigenvalue and settling time for system 1 and system 2 are λ_1, T_{s1} and λ_2, T_{s2}, respectively. At the settling time, assuming both have the same initial condition, we must have $\lambda_1 * T_{s1} = \lambda_2 * T_{s2}$, or $\frac{\lambda_1}{\lambda_2} = \frac{T_{s2}}{T_{s1}}$.

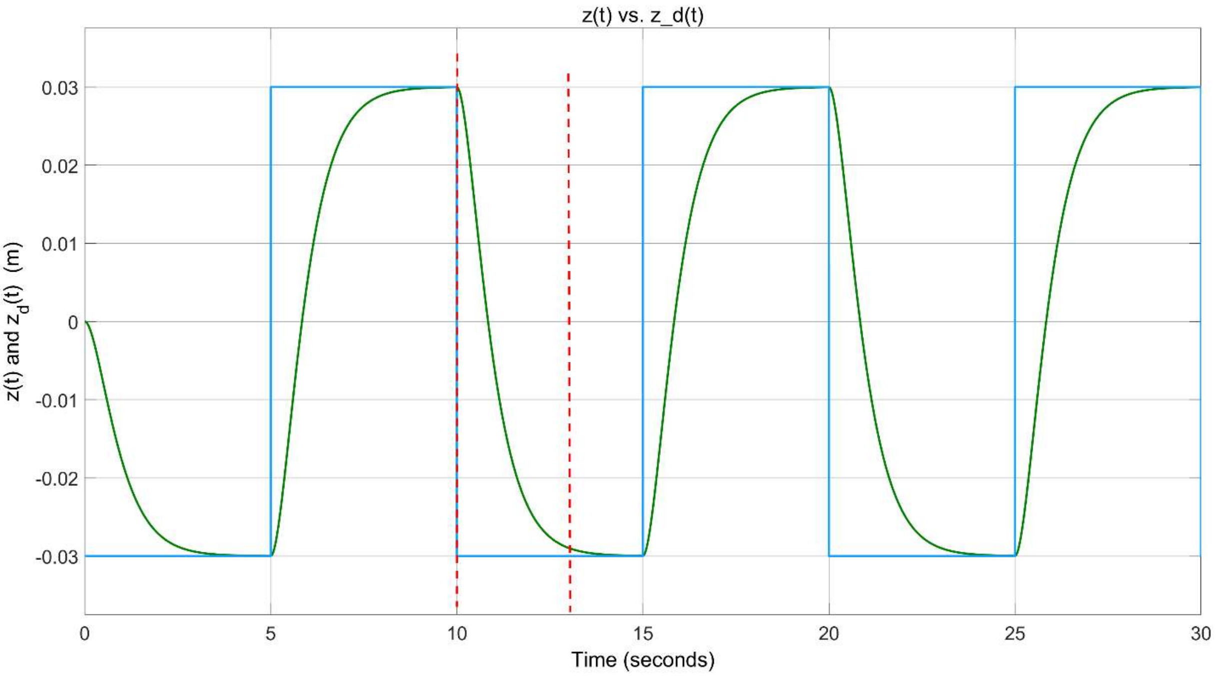

Figure 4. z(t) vs. z_d(t) for eigenvalues = {-2, -2}

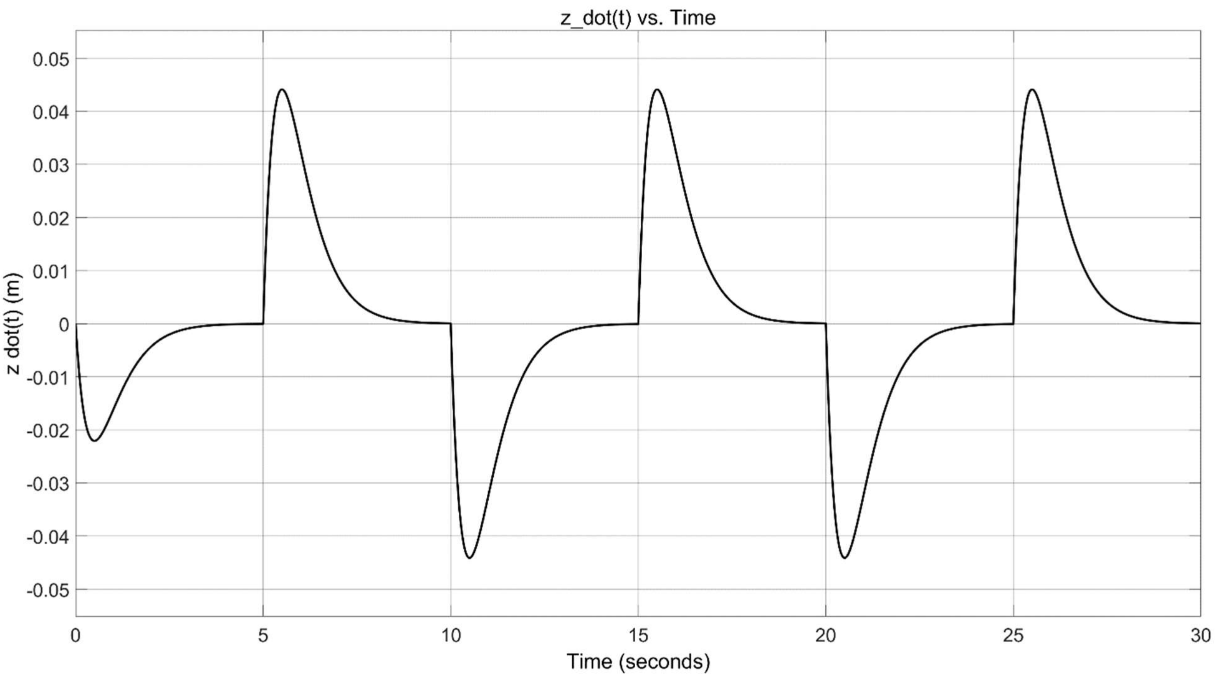

Figure 5. $\dot{z}(t)$ for eigenvalues = {-2, -2}

Output 2 - Output Feedback Controller Simulation

With p_feedback = {-2, -2} and p_observer = {-20, -20}, the gain K and L are as follows:

$$K = \begin{bmatrix} -1.90 & 4.18 \end{bmatrix}$$

$$L = \begin{bmatrix} 27.21 \\ 52.01 \end{bmatrix}$$

The relevant matrices are:

$$A_{ctrl} = \begin{bmatrix} -27.21 & 1 \\ -56.02 & -4 \end{bmatrix}$$

$$B_{ctrl} = \begin{bmatrix} 27.21 & 0 & 0 \\ 52.02 & 4 & -8.79 \end{bmatrix}$$

$$C_{Ctrl} = \begin{bmatrix} -1.90 & 4.18 \end{bmatrix}$$

$$D_{ctrl} = \begin{bmatrix} 0 & 1.90 & -4.18 \end{bmatrix}$$

The plots from the three output Simulink scopes with p_feedback = {-2, -2} and p_observer = {-20, -20} are shown here.

First, we show the peak value of the voltage magnitude |u(t)| and the settling time T_{s1} of z(t). The peak value of |u(t)| is around $\boxed{0.273V}$, as shown in Fig 6.

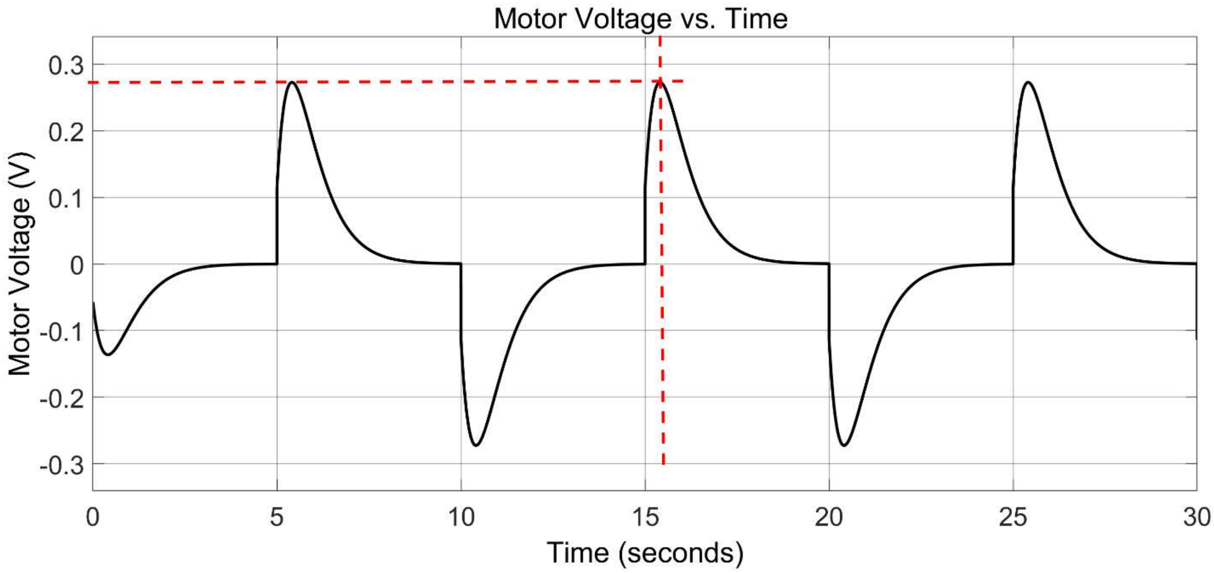

Figure 6. Motor voltage vs. Time with p_feedback = {-2, -2} and p_observer = {-20, -20}

From Fig 7, we can estimate the settling time T_{s1} is around $\boxed{2.92s}$. We used the same method discussed in Output 1 to find the settling times in this section. We show the estimation error $\tilde{x}$ vs. time in Fig 8.

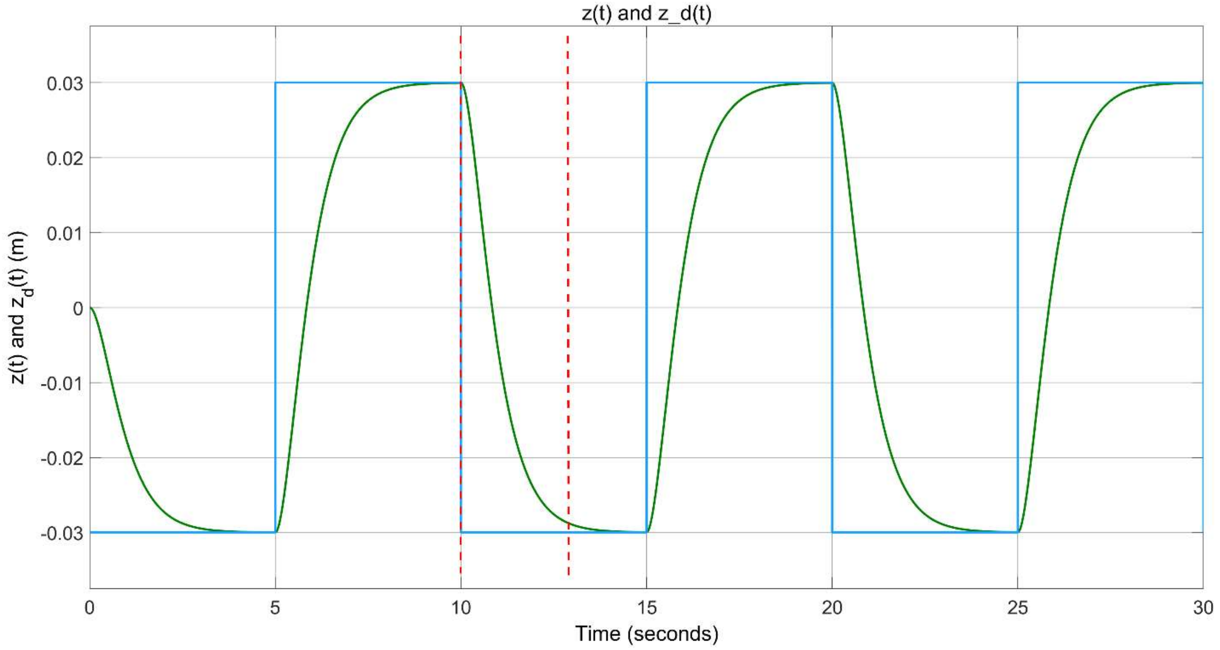

Figure 7. z(t) vs. z_d (t) with p_feedback = {-2, -2} and p_observer = {-20, -20}

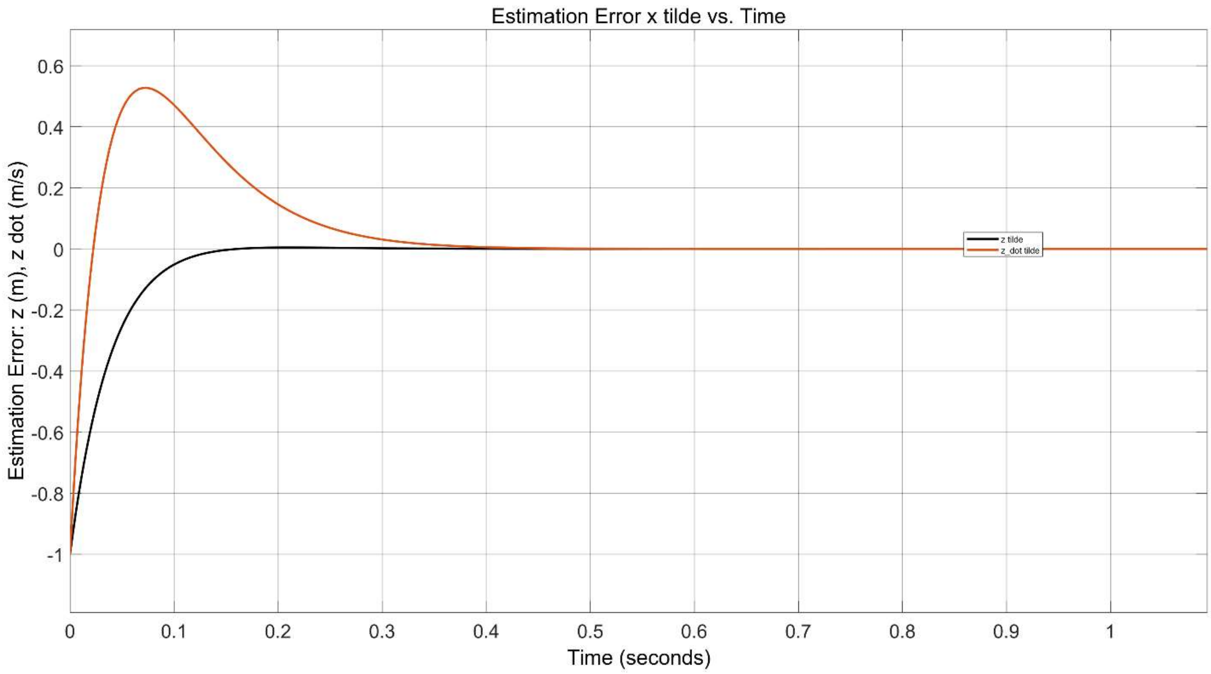

Figure 8. Estimation Error vs. Time for p_feedback = {-2, -2} and p_observer = {-20, -20}

Now we repeat the simulation by changing p_feedback = {-5, -5} and keeping p_observer = {-20, -20} and plot the tracking error z(t) vs. z_d (t) and the voltage u(t). The new peak motor voltage magnitude is $\boxed{0.78V}$, as shown in Fig 9. The new settling time is $\boxed{1.17s}$, as shown in Fig 10.

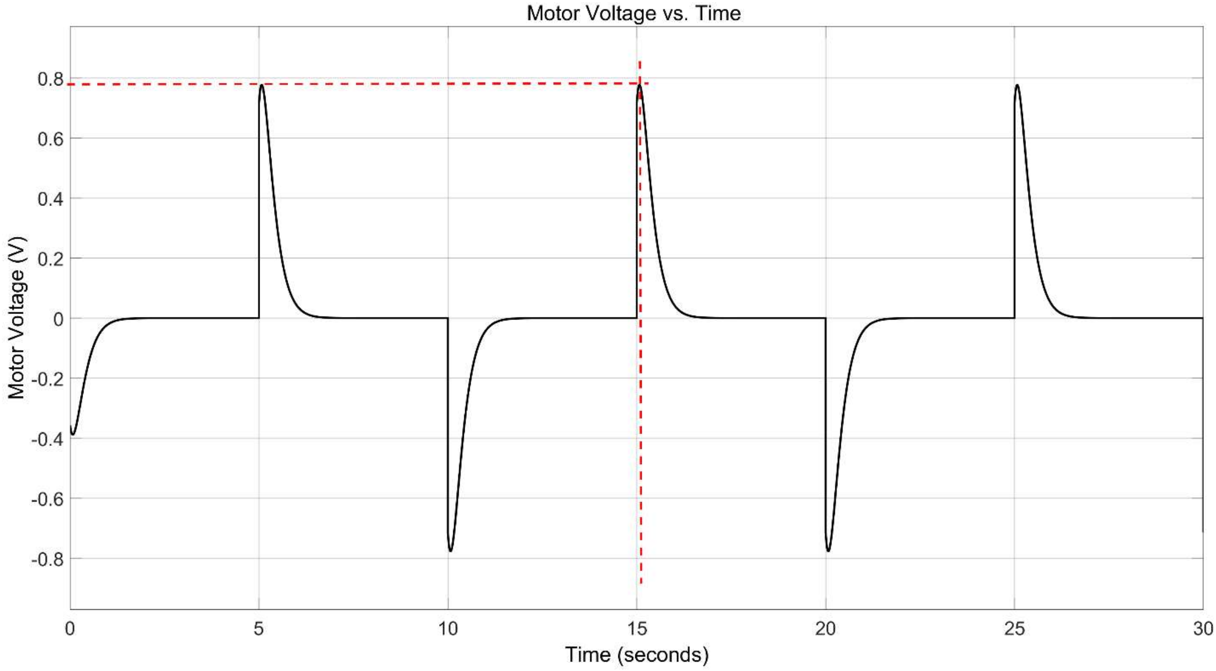

Figure 9. Motor voltage u(t) vs. time for p_feedback = {-5, -5} and p_observer = {-20, -20}

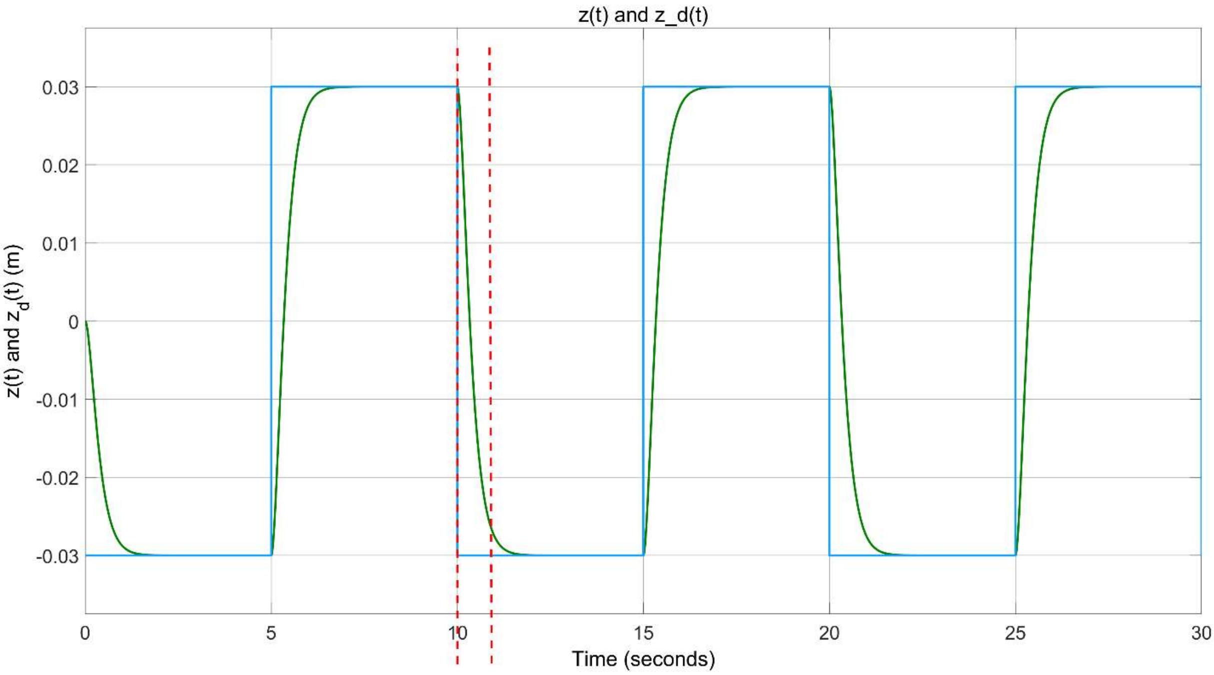

Figure 10. z(t) vs. $z_d(t)$ for p_feedback = {-5, -5} and p_observer = {-20, -20}

Changing the p_observer to {-10, -10} with p_feedback = {-2, -2}, we get Fig 11 which illustrates the effects on the state estimation error $\tilde{x}$.

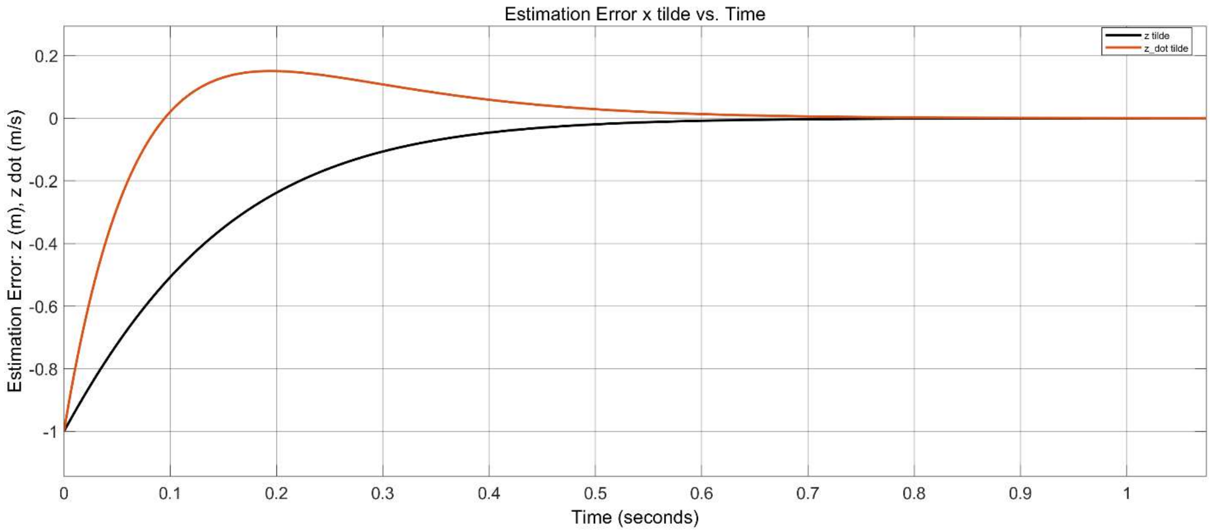

Figure 11. Estimation Error $\tilde{x}$ vs. Time for p_feedback = {-2, -2} and p_observer = {-10, -10}

Comparing Fig 7 and Fig 10 clearly shows that changing p_feedback from {-2, -2} to {-5, -5} results in a faster decay in tracking error (faster convergence to 0). We can also reach this conclusion by comparing settling times alone: 1.17s < 2.92s. This matches what we expect; negative eigenvalues that are larger in magnitude correspond to exponential terms in the error that decay faster, which means the system tracks the reference signal faster. Indeed, if we have a perfect observer, this situation reduces to the setup in Output 1, where we observed that a larger eigenvalue magnitude leads to faster decay and thus better tracking.

Comparing Fig 6 and Fig 9, the peak value of the motor voltage is 0.78V when p_feedback = {-5, -5} and 0.27V when p_feedback = {-2, -2}. The peak value of motor voltage is higher for the larger eigenvalue magnitude case; this makes sense as a faster decay requires a larger electromagnetic force from the motor. Mathematically, force is proportional to motor voltage and acceleration, so the higher acceleration required for faster tracking implies a higher voltage.

Comparing Fig 8 and Fig 11 also shows that changing p_observer from {-10, -10} to {-20, -20} results in faster state tracking. This is only noticeable if we set the initial condition of the estimation to be nonzero. In the original setup with 0 initial observation errors, both elements of $\tilde{x}$ oscillate around values extremely close to 0 (in the order of 10^{-18}), which makes no distinct difference once we change p_observer to {-20, -20} (the tracking errors are so close to 0 that it does not make sense to compare them). Therefore, in our experiment, we perturb the initial estimation of the position to 1m and velocity to 1m/s, which produces Fig 8 and 11. However, to generate all the other plots in this report, we revert to an initial estimate of [0, 0] for consistency. Since the observer eigenvalues are the eigenvalues of the matrix relating $\dot{\tilde{x}}$ to $\tilde{x}$, larger magnitude means faster convergence of $\tilde{x}$ to 0, which obviously means $\hat{x}$ approaches x faster, since $\tilde{x} = x - \hat{x}$. From Fig 8 with p_observer = {-20, -20}, we see both position and velocity approach 0 around 0.4s, whereas in Fig 11 with p_observer = {-10, -10}, both position and velocity converge to 0 around 0.8s. In addition, we also note a more negative p_observer leads to a higher velocity overshoot beyond 0 in Fig 8 (around 0.5 m/s) compared with Fig 11 (around 0.15 m/s).

Output 3 - Output Feedback Controller Experiment

Our final tuned eigenvalues that give the best performance are p_feedback = {-50, -50} and p_observer = {-50, -50}. The main idea behind our tuning process is to make the eigenvalues as negative as physically realizable; we want to minimize steady state error and settling time while keeping the motion feasible for the motor to achieve. From the previous two sections, we note that for a more negative p_feedback, the settling time for z(t) decreases which makes the position converge to reference position faster. For a more negative p_observer, the estimation error $\tilde{x}$ to converges to 0 faster, and the tracking error would improve in this case.
During the physical experiment, we started off with a relatively large negative eigenvalue (around -10) and gradually decreased them to improve the steady-state tracking error and the settling time of the system. The eigenvalues were decreased until the motor limits were reached and the improvement in the system performance is no longer noticeable. In this experiment, we kept the eigenvalues for feedback and observer to be real numbers.

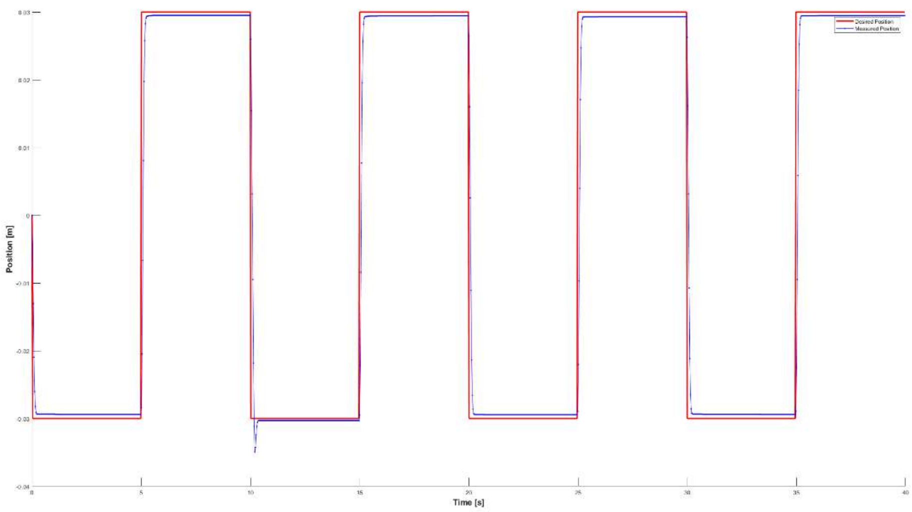

Figure 12. z(t) vs. $z_d(t)$ in Physical Experiment with p_feedback = {-50, -50} and p_observer = {-50, -50}

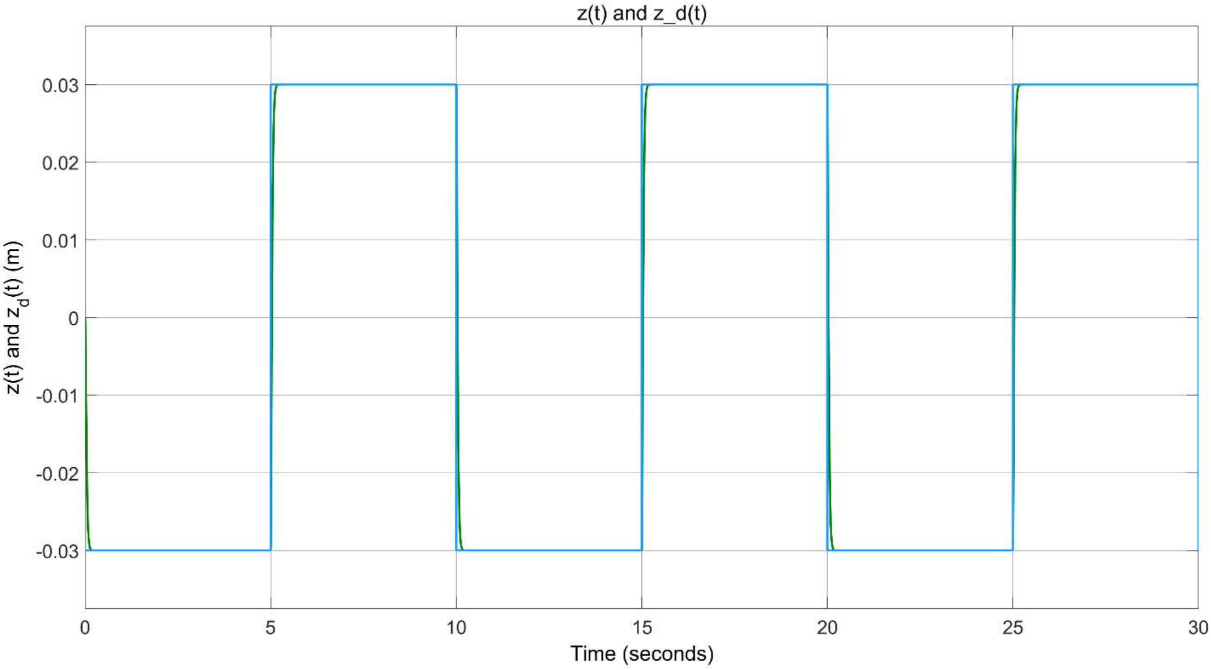

Figure 13. z(t) vs. $z_d(t)$ in Simulation p_feedback = {-50, -50} and p_observer = {-50, -50}

Qualitatively, our physical experiment in Fig 12 and simulation results in Fig 13 are very similar. Furthermore, they conform to our expectations; we expect that as the magnitude of the poles increases, both the observer and state feedback components of the output feedback controller will perform better, i.e., the respective errors will converge to zero faster. Indeed, in both theory (simulation) and practice (physical experiment), the output signal very closely matches the reference square wave. Steady state error in the simulation is 0. Steady state error in the physical experiment is near 0 (0.0005 m). The settling times of the physical and simulation are 0.129 s and 0.117 s. These results are significantly shorter (1 order of magnitude) than those presented in Outputs 1 or 2, so the results are satisfactory.

The observed differences between the simulated and physical experiments can arise due to several sources of error. First, the track was not perfectly level, resulting in a non-zero gravitational force in the horizontal direction. Second, there are physical constraints on the motor voltage input; the overshoot at around 10s in Fig 12 may be caused by a jerk in the motor output due to an excess in input voltage beyond limits. There may also be electric and mechanical noises in the physical motors. Finally, simplifications were made in our plant model; we assume that the friction is viscous, the transient dynamics of the motor were ignored, and we assume that the constants provided for an arbitrary motor represents the motor used in our experiment.