ECE557 Lab 4: Square Wave Tracking of a Cart-Pendulum Robot

Group: PRA04 – Group 6

Name 1: Aoran Jiao

Name 2: Hshmat Sahak

Name 3: Richard Marchelletta

Introduction

The purpose of this lab is to design an output feedback controller to make a cart-pendulum robot track a square wave while balancing a rod. The controller is analyzed in both simulation and experiment using eigenvalue assignment and LQR techniques. After tuning the parameters of the best controller on the physical system, the output integral feedback controller is tested experimentally, and its performance is compared to the standard controller.

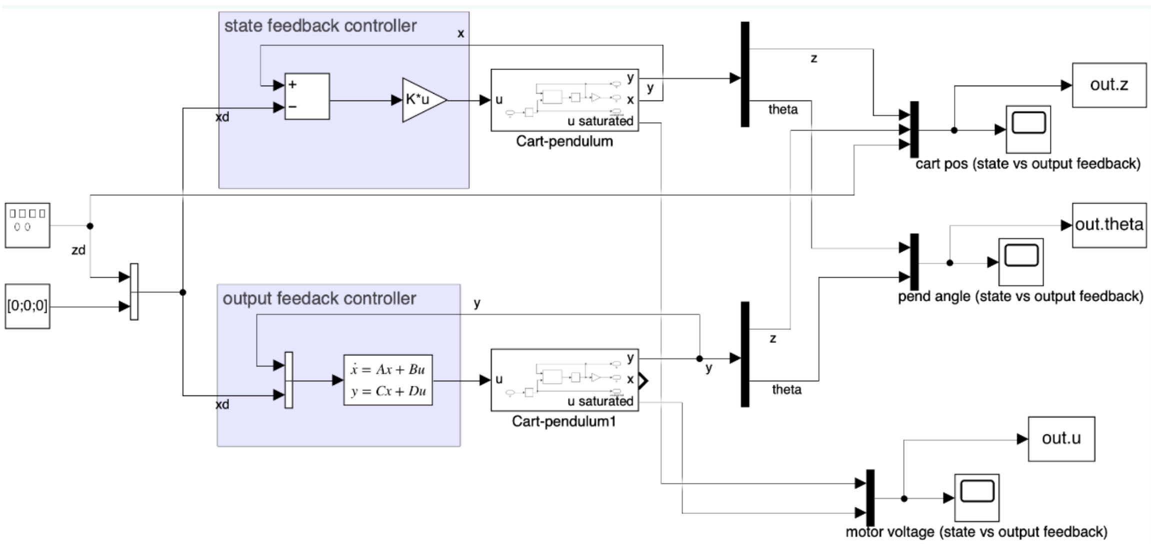

Figure 1. Block Diagram of Closed Loop System

Figure 1. describes the block diagram. For the linearized model,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -8.82 & 1.89 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -26.70 & 35.41 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1.45 \\ 0 \\ 4.39 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and the linearized model has the state equations:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

The input to the linearized and nonlinear system is the feedback control u = -K*x and the output is the state which consists of cart position, cart velocity, pendulum angle, and pendulum angle velocity.

Output 1 – Simulation Comparison of State and Output Feedback Controllers

K_place = [-86.04 -50.21 119.63 21.68]

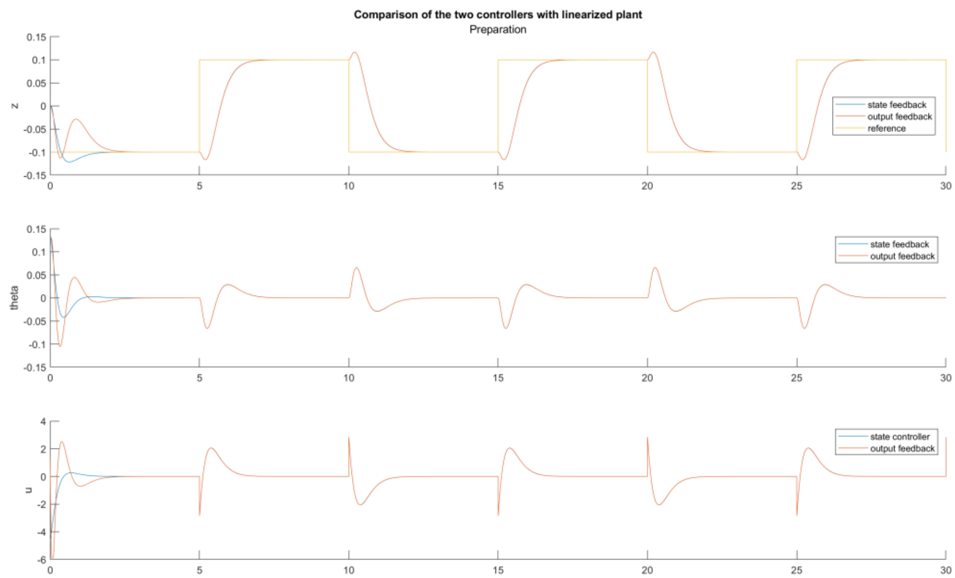

Figure 2. Comparison of the two controllers with linearized plant with -10 eigenvalues

Setting the eigenvalues of A+BK to -5 and A-LC to -10 results in a noticeable difference between the state and output feedback for t< 5s. During this time, the state estimation error by the observer is large because our initial estimate is off. However, with time, the estimate of state becomes better, so it turns out the observer just needs time to reach zero estimation error. Indeed, after t=5s, the state and observer plots are identical for all graphs.

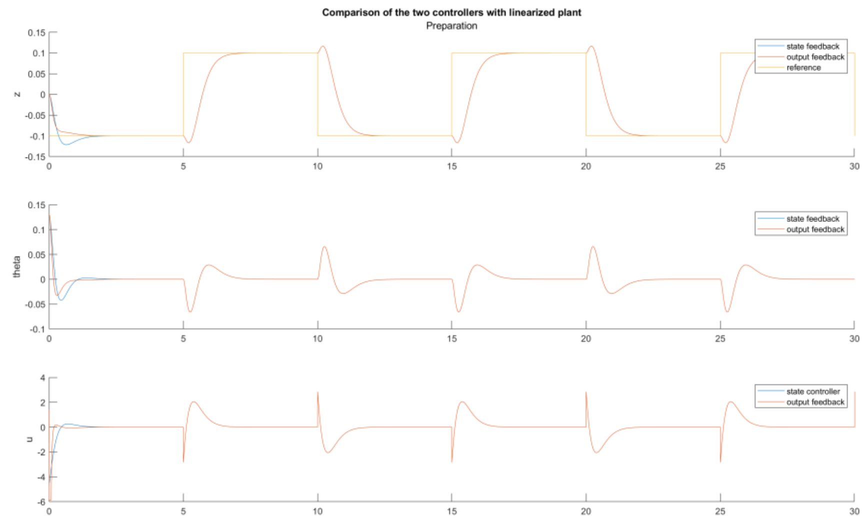

Figure 3. Comparison of the two controllers with linearized plant with -40 eigenvalues

When the eigenvalues of A-LC are moved to -40, we observe an improvement in the state estimation error for early t. The output feedback more accurately tracks the state feedback, and in turn, more accurately tracks the reference. This is because the state estimation error is reduced in our observer – a negative eigenvalue larger in magnitude corresponds to faster error decay to 0.

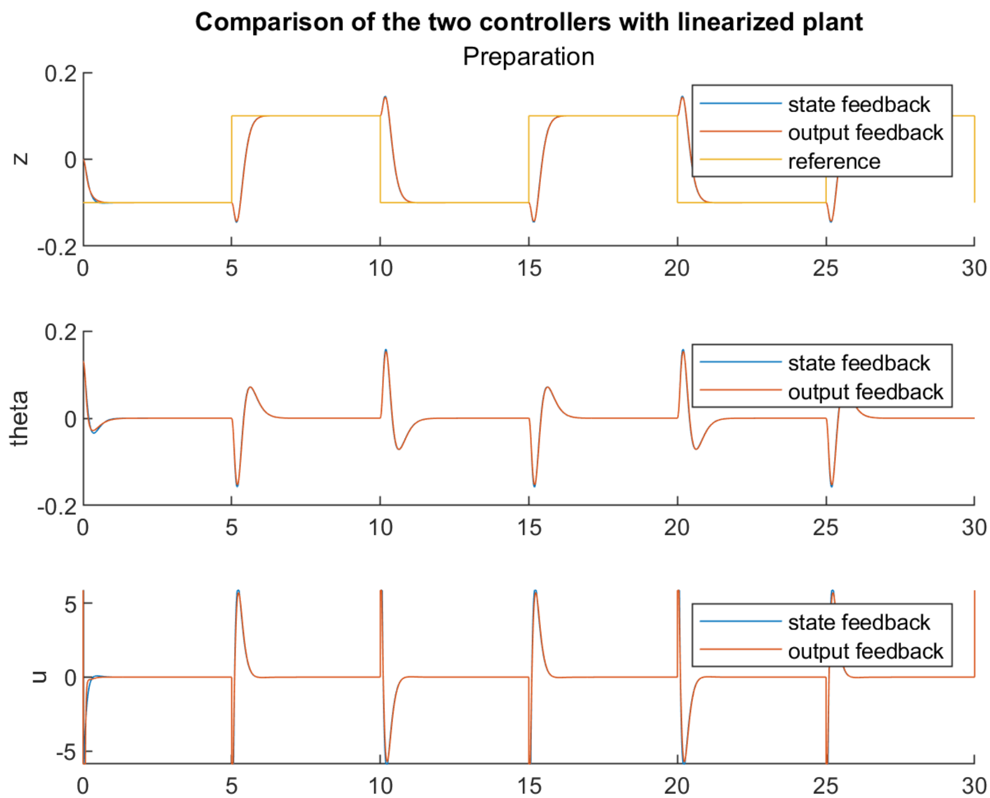

Figure 4. Tuned for required Ts with pole placement, p_desired = [-7.81, -7.82, -7.79, -7.78], tuned K_place = [-86.04 -50.21 119.63   21.68] with a Ts = 0.92s

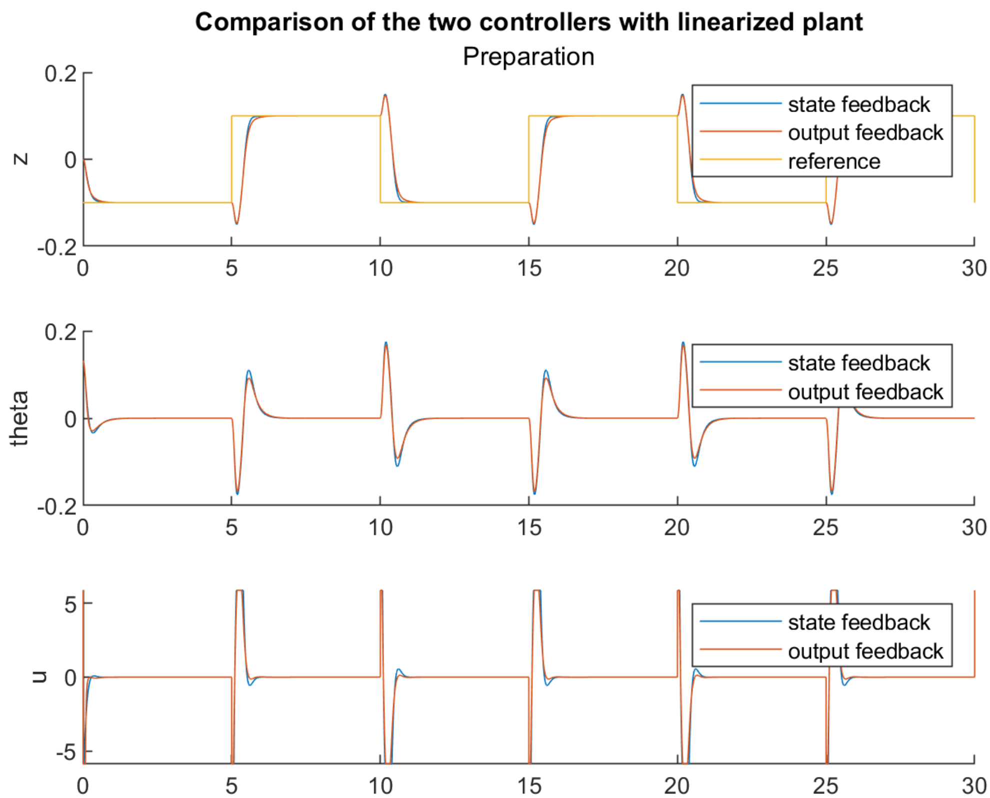

Figure 5. Tuned for required Ts with LQR, q1 = 2000; q2 = 0.5; R = 0.2; K_lqr = [-100.00 -56.13 129.01 23.44] with a Ts = 0.94s

The goal for tuning K was to reduce the settling time without excessively exceeding +- Ulim. In theory, if Ulim was inf, we would be able to make the pole placement of K sufficiently negative such that the state tracks the reference perfectly. In practice, due to the limitations of the actuators, Ulim restricts how well we can track the reference. For this reason, we made the eigenvalues around -10.8, which gives us a good compromise between settling time and saturation time. The tuned K meet the specification of a) a setting time less than 1 second, and b) the control signal u(t) does not display excessive saturation. As shown in Figure 4, we tuned the system for required Ts using pole placement method, with p_desired = [-7.81, -7.82, -7.79, -7.78], and tuned K_place = [-86.04 -50.21 119.63 21.68] with a Ts = 0.92s < 1s. As show in Figure 5, we tuned the system for the required Ts with LQR, with q1 = 2000, q2 = 0.5, R = 0.2, K_lqr = [-100.00 -56.13 129.01 23.44] with a Ts = 0.94s. The time for saturations for both these methods are relatively low (less than 0.2 s), therefore, we can conclude that our design in simulation meet the specified requirements.

In terms of the saturation time, T_{sat}, we observe the controller's time spent during saturation is 0s (u does not reach the saturation limit) in the first two scenarios (in Figure 2 and 3) with conservative A+BK eigenvalues at -5. However, the time spent during saturation for the later two cases (Figure 4 and 5) with a more aggressive eigenvalue for both pole assignment and LQR method is around 0.2 s in Figure 5 and 0.04s in Figure 4. The time it takes to reach the saturation control limit is around 0.1 s.

Output 2 – Physical Experimentation of Output Feedback Controller

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q1 = 8;
q2 = 5;
R = 0.2;
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Our group found that tuning the LQR parameters was better than pole placement. This is because we had more control over the importance of individual state variables (position, angle, control input), and could tailor our trial-and-error process to reflect this freedom. Specifically, our trial-and-error procedure was to first find Q and R parameters that balanced the rod, and then improve the parameters such that the cart tracked the square wave as accurately as possible. We were successful in balancing the rod; however, we were not able to fully optimize the square wave tracking due to the limited time during the lab session, combined with noises and imperfections in the physical setup.

By investigating the behavior of the cart-pendulum system during the experiment, we were able to decide what specific LQR parameters to tune. First, we prioritized the rod balancing by increasing the weighting associated with the theta state signal. Second, we prioritized the reference signal tracking by increasing the weighting for the cart position state signal. Finally, we did not want the cart to act too aggressively to achieve the square wave tracking, so we increased the value of R to restrict the voltage of the motor. Although Figure 10 does not shown an asymptotic tracking behavior, we could approximate the settling time using the interval between 15 to 20 s. Using the 2% rule, we estimate the setting time to be around 4s.

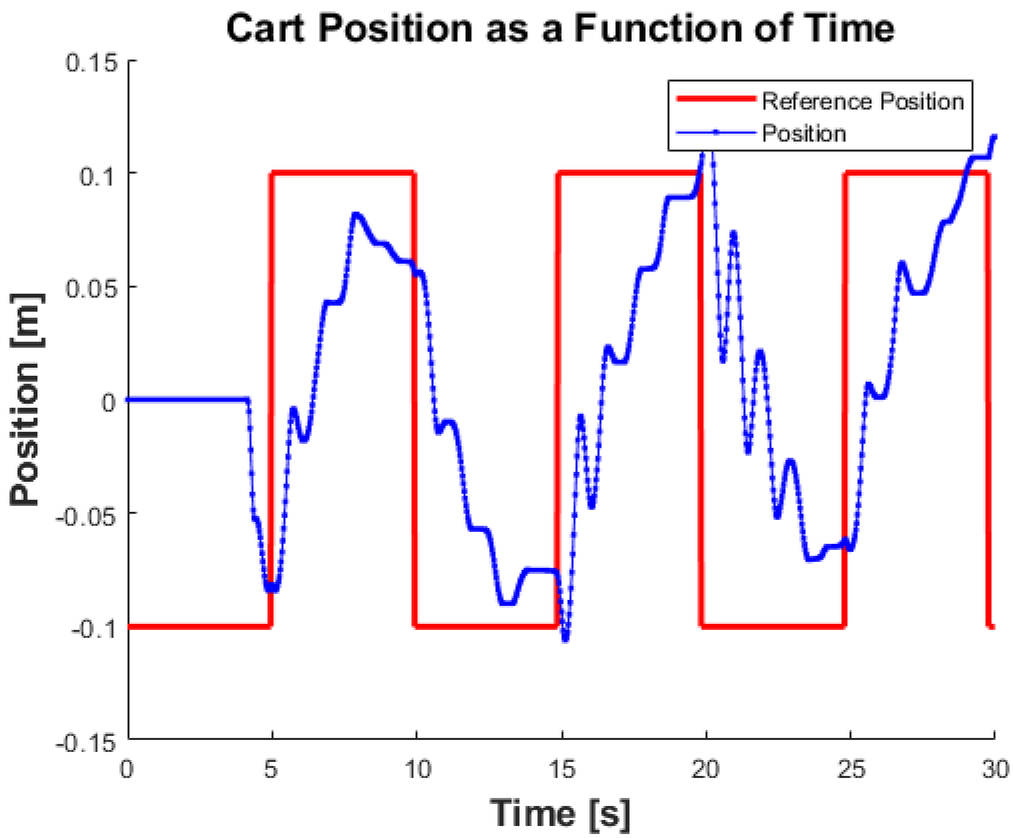

Figure 6. Output 2 reference vs. real cart position as a function of time

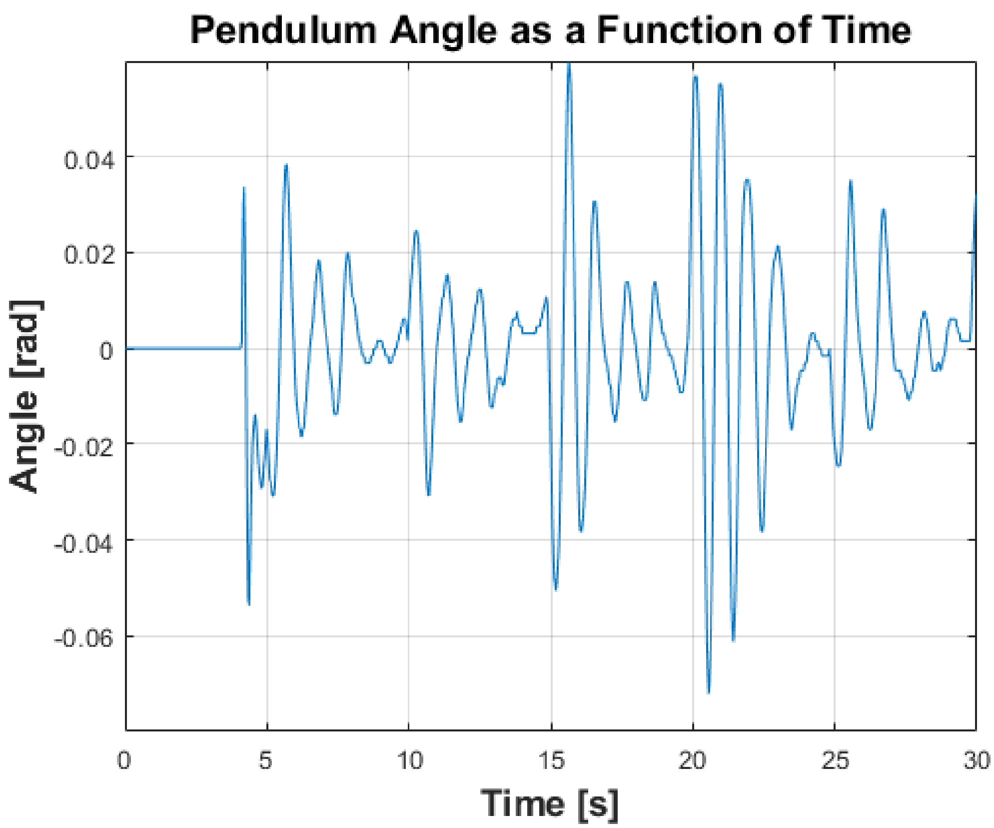

Figure 7. Pendulum angle as a function of time

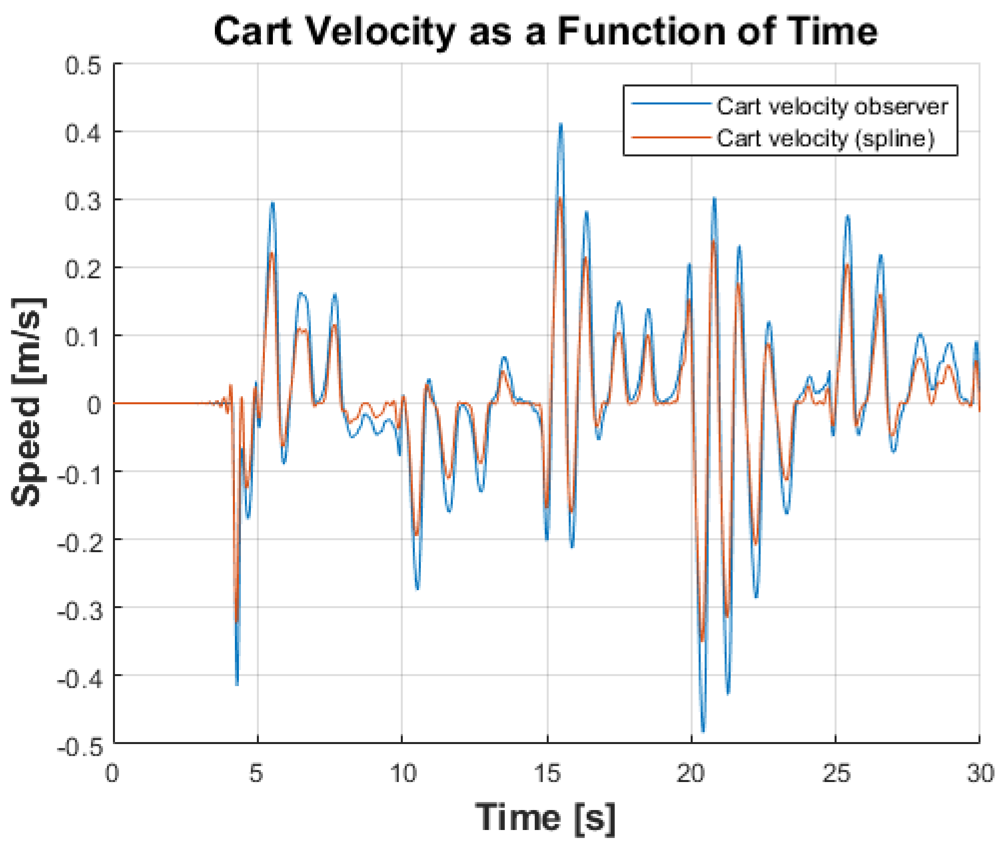

Figure 8. Cart velocity as a function of time

From the experimental results above, we pay particular attention to the pendulum angle. We see that the output feedback controller sufficiently balances the rod by keeping the pendulum angle close to 0. Since the pendulum angle is more sensitive to position (due to gravity), we only want to optimize the position *given* the rod can be balanced.

Next, we notice the cart position as a function of time. The triangular behaviour of the cart position in Figure 6 suggests that the output feedback controller is attempting to track the square wave reference; however, it does not conform to our expectations. We expect two key differences. First, the position curve should be smoother. We see a jaggedness in the response, explained by the motor accelerating too quickly left to cover distance, then sharply to the right to balance the rod. It repeats this behaviour till it reaches the desired position. The curvature of the position curve doesn't exhibit the anticipated asymptotic behavior, partly due to its jaggedness which takes away from the overall exponential curve.

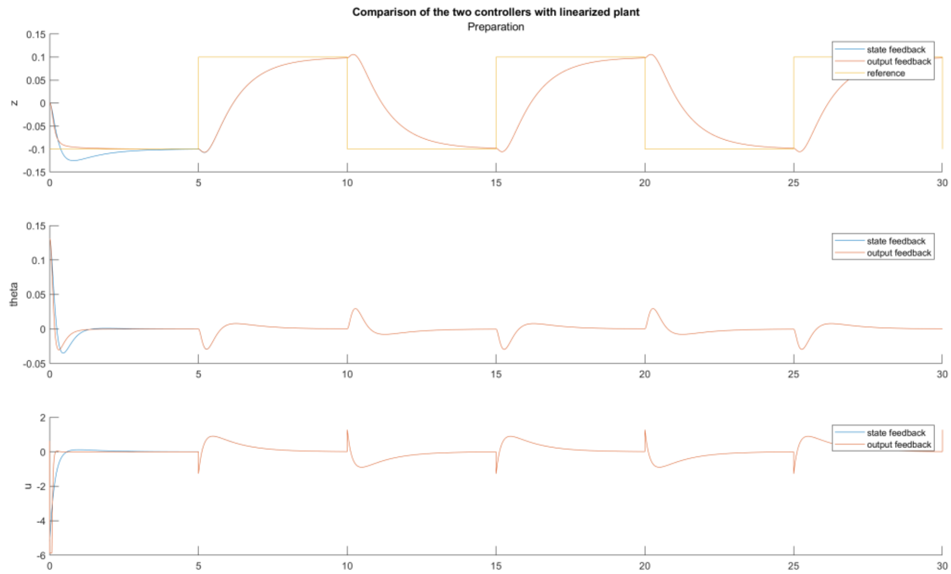

Figure 9. Output 2 simulation results

Figure 9 provides the simulation plots of our state variables over time for the same controller parameters used in our experiments. The main difference in our results is in the jaggedness mentioned earlier. A comparison of theta suggests that the jaggedness occurs as the cart works to balance the rod. In simulation, this process is a lot smoother. As show in the Figure 9 plot for position, the cart is displaced opposite the direction of desired motion a bit, which displaces the pendulum. Then, it follows a smooth motion to go to the desired destination while recovering the pendulum angle so that its 0 at the end. In the physical experiment, instead of moving back, the forwards; we repeat the process of moving forward the back to recover, till we reach the desired position, resulting in the jaggedness observed. Similarly, the position curve of the simulation shows a smooth exponential shape in simulation. Jaggedness aside, the experimental position has a similar shape, settling time, and overshoot.

The start and end positions of the motion are not exactly aligned with the square wave; this can be partially credited to our table not being perfectly balanced. We expect that we can solve our main problem of jaggedness by further tuning the LQR parameters. Specifically, we want to make penalize the controller more (increase R) so that we obtain more smooth motions from the motor. However, it is not as simple as that; we would also lower the weight on the angle parameter to allow it to make a single relatively large deviation in angle at the beginning rather than multiple small deviations. Overall, we want to tune the parameters to get good settling time, not have too much control effort, and make sure the angle is near 0 at the end of each period.

Output 3 – Output Feedback Control with Integral Action

Q = [0.2 0 0 0 0;
0 0 0 0 0;
0 0 50 0 0;
0 0 0 0 0;
0 0 0 0 3];
R = 0.05;

The trial-and-error procedure used for the integral action controller was similar to the output feedback controller. The values obtained for the optimal Q and R are shown above. The first two weights in the Q matrix are position and angle, and the third additional weight is the integral of the error term (z_d - z). Our general strategy is to first balance the weights between the first two LQR weights – the position and angle. Like before, we want to put enough weight on minimizing theta because we want the system to balance well while moving. Therefore, we select a relatively high weight on theta. However, we need the system to track the square wave at the same time, so we gradually increase the weight for position and the control input weight R. In addition, we want to the control signal to be "aggressive" enough so that the cart could overcome the friction on the track. Therefore, we start with the given value of R = 0.01 and gradually increase it by 0.01 till we get a good result. Finally, we want to use the integral term to mitigate the steady state error on top of the proportional controller. Therefore, we start with a weight of 1 and gradually increase the weight on the error integral term. Note that due to the limit time during the lab, there could still be room of improvement for further tuning to achieve better results.

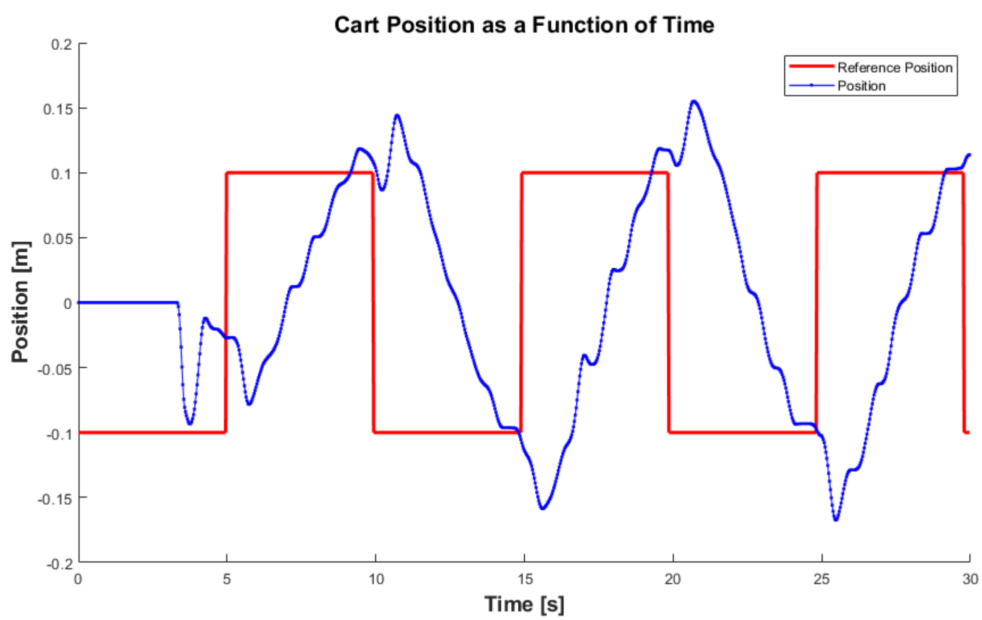

Figure 10. Reference vs. Real cart position as a function of time

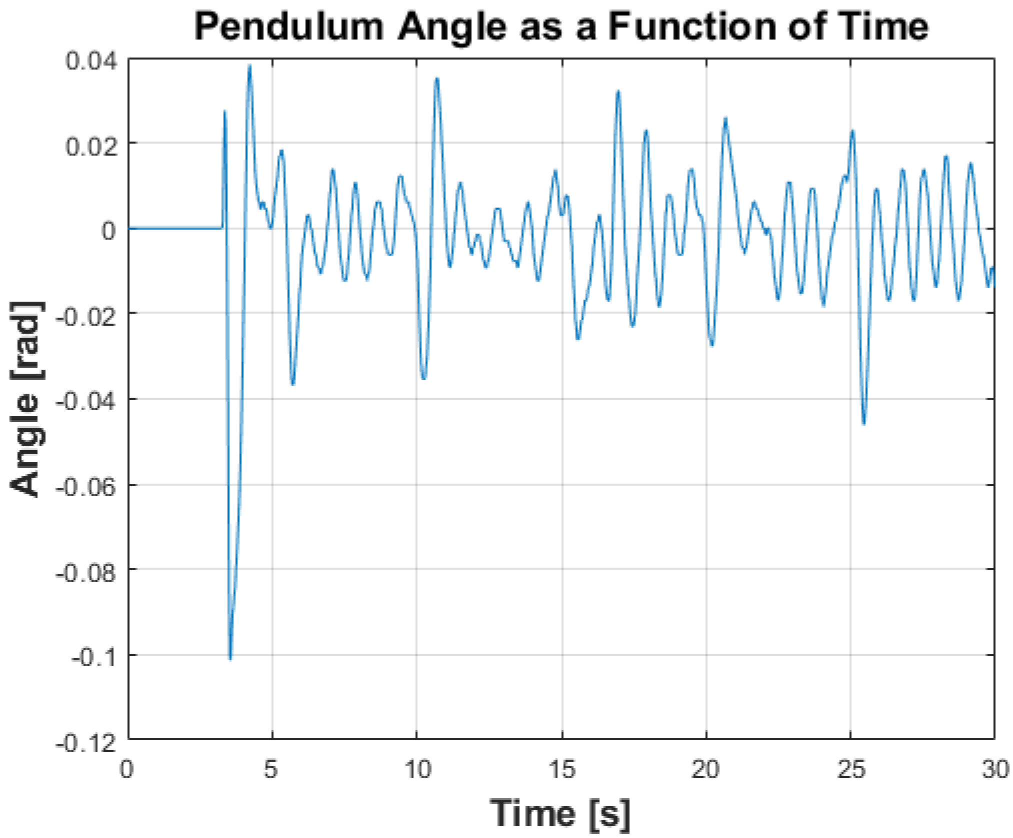

Figure 11. Pendulum angle as a function of time

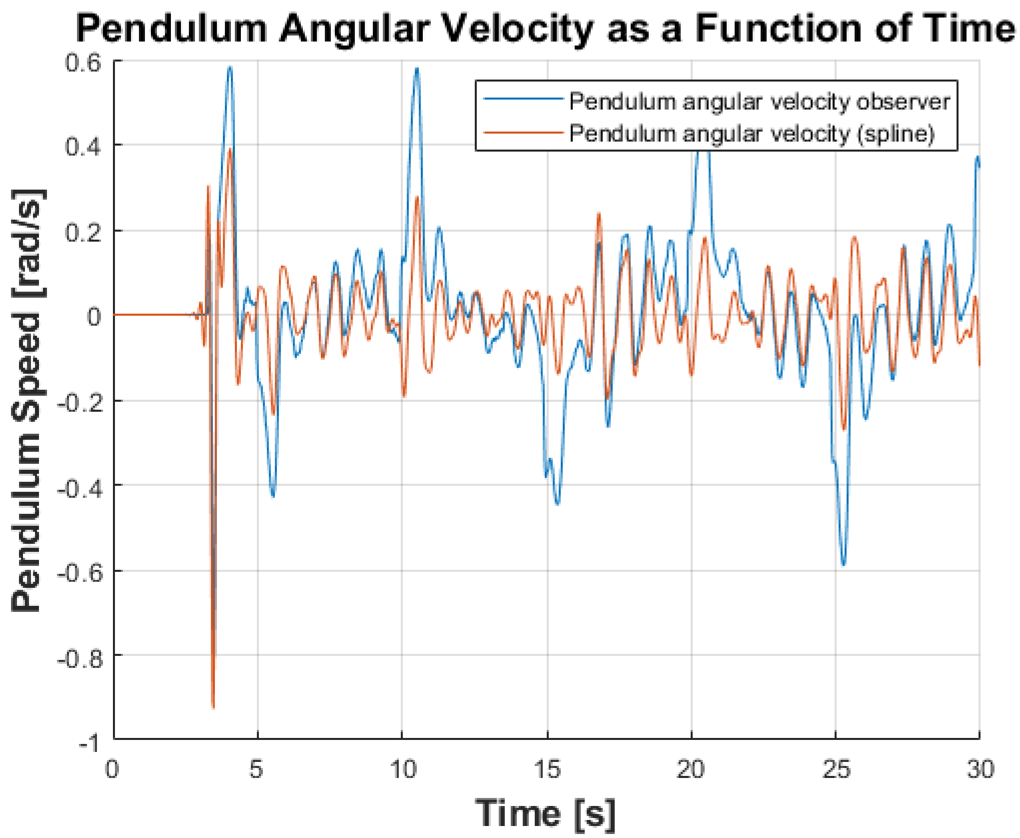

Figure 12. Pendulum angular velocity as a function of time

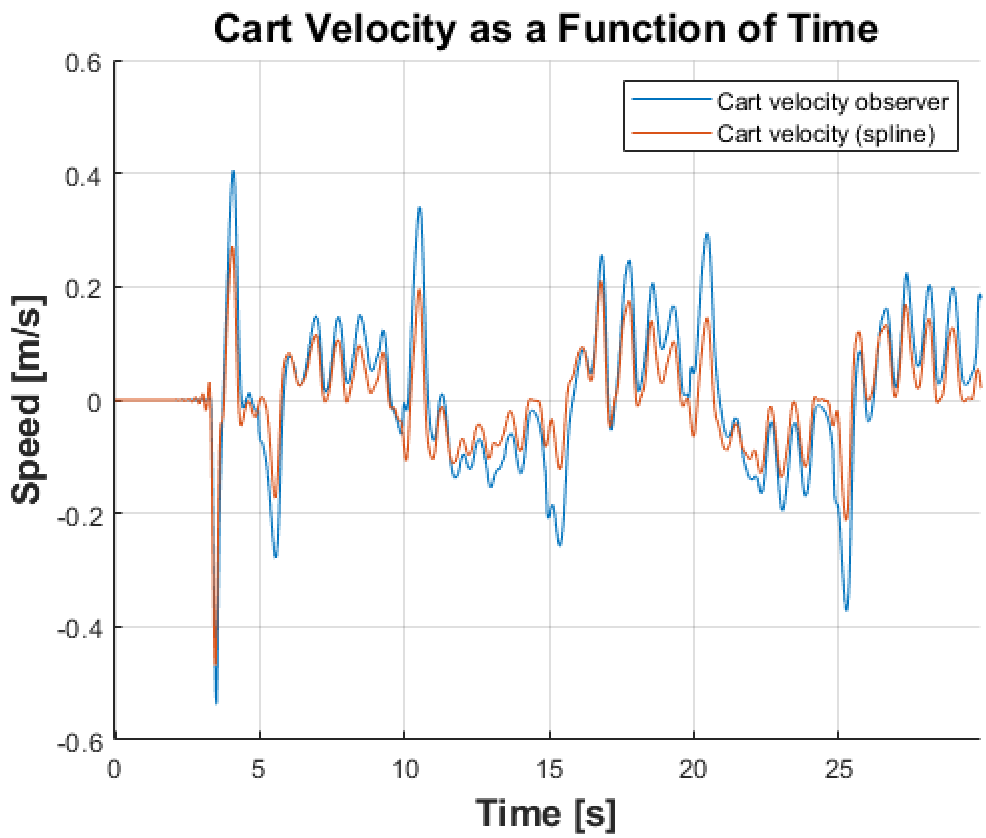

Figure 13. Cart velocity as a function of time

The experimental plots obtained for the integral action controller are shown in Figures 10 and 11. Overall, the behaviour is not satisfactory; the settling time is much larger than the required 1 second mentioned in the lab document preparation section. The results do not meet our expectations that integral controller should improve results; however, this could be due to insufficient tuning and unmodelled dynamics like friction and the table being unlevel.

Before comparing results with and without integral action, we first note that the integral term should theoretically reduce steady state error at the cost of an overshoot. This is because we penalize steady state error more and more over time, so we encourage the controller to make sure that the position hits the desired target; after which the motor voltage being applied on a continuous scale causes a slight overshoot before the motor pushes the cart the other way. These general phenomena are depicted in Figure 10. Like in the previous part, the settling time is not obvious due to the jagged performance of the controller as well as the overshoot.

The LQR cost function already integrates the (squared) tracking error, so we can think of the cost here an integrating twice. Indeed, there are scenarios where introducing an additional integral term (commonly seen in PID controllers) might still be beneficial. While LQR minimizes the integral of the error in its optimization process, introducing an additional integral term can further refine the controller's behavior to achieve more precise tracking or compensation for unmodeled dynamics and disturbances, enhancing the overall control performance.

Comparing the results to the standard output feedback controller, we see that the integral action does improve the performance, especially regarding the jaggedness observation in the position and angle plots. With reference to Figure 10, the jaggedness is still present in the position curve, but is much less than what was observed for the output feedback controller. Consequently, the theta plot in Figure 11 shows a similar decrease in magnitude and frequency of oscillation. These observations could result from putting more importance on position tracking (via the integral term), which puts less importance on getting the angle right, and so the controller does not try fixing the angles during its motion.