ESC103F Engineering Mathematics and Computation: Lab #1

Pre-Lab

Consider the integral that will be used as the basis for Exercises 1 and 2:

$$\int_0^3 \sqrt{x+1}\,dx$$

Find the exact value for the integral using an analytical approach. This will be used as a basis for comparison with your numerical approaches in Exercise 1 and 2.

Exercise 1

Write a MATLAB program that produces a **midpoint approximation** with the interval [0,3] subdivided into *n* evenly spaced subintervals. Your code should be designed to work for an arbitrary *n*.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the numerical approximations to the integral in the upper plot using the subplot command. Also plot a horizontal solid line at the exact value.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the upper bound on the absolute value of the midpoint approximation error, E_M, in the lower plot using the subplot command. Also plot the absolute value of the actual error $|E_M|$ using a solid line.

Label all axes and add legends to both plots as appropriate.

Given information:

$$M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$

where M_n is the midpoint approximation, with

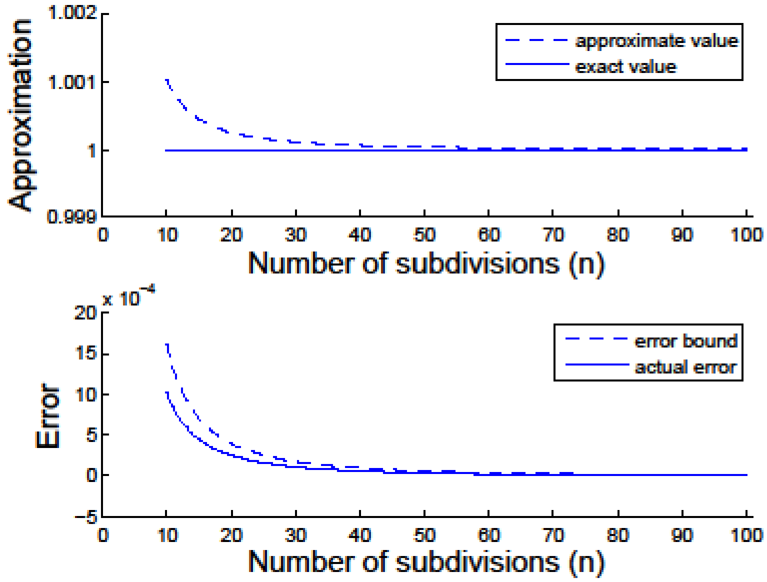
$$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$$

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$|E_M| = \left|\int_0^3 f(x)dx - M_n\right| \leq \frac{(3-0)^3 K}{24n^2}$$

K on the right-hand side of the above expression is the least upper bound on the absolute value of the second derivative of the function over the interval, i.e. $|f''(x)| \leq K$ over $0 \leq x \leq 3$.

To help guide you in the development of your plots for Exercises 1 and 2, here is a sample plot for illustrative purposes of the results obtained for a different integral, $\int_0^{\pi/2} \cos x \, dx$:



Exercise 2

Write a MATLAB program that produces a **trapezoidal approximation** with the interval [0,3] subdivided into *n* evenly spaced subintervals. Your code should be designed to work for an arbitrary *n*.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the numerical approximations to the integral in the upper plot using the subplot command. Also plot a horizontal solid line at the exact value.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the upper bound on the absolute value of the trapezoidal approximation error, E_T, in the lower plot using the subplot command. Also plot the absolute value of the actual error $|E_T|$ using a solid line.

Label all axes and add legends to both plots as appropriate.

Given information:

$$T_n = \sum_{i=1}^{n} \left(\frac{f(x_{i-1}) + f(x_i)}{2}\right) \Delta x$$

where T_n is the trapezoidal approximation, with

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$|E_T| = \left| \int_0^3 f(x)dx - T_n \right| \le \frac{(3-0)^3 K}{12n^2}$$

K on the right-hand side of the above expression is the least upper bound on the absolute value of the second derivative of the function over the interval, i.e. $|f''(x)| \le K$ over $0 \le x \le 3$.

Testing your code for Exercise 2

In MATLAB, find a built-in function for performing trapezoidal numerical integration and use this function to check your answers in Exercise 2.