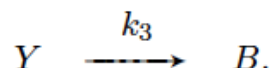
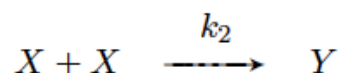
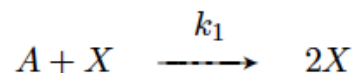


## Homework 3

**Due Thursday 11/03/16 (9am)**

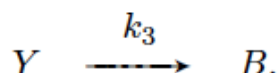
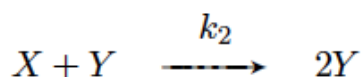
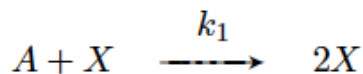
### 4.8.6 Global dynamics from local stability analysis.

a) Consider the chemical reaction network with mass-action kinetics:



Assume that  $[A]$  and  $[B]$  are held constant.

- Write a differential equation model describing the concentrations of  $X$  and  $Y$ .
  - Verify that the system has two steady states.
  - Determine the system Jacobian at the steady states and characterize the local behavior of the system near these points.
  - By referring to the network, provide an intuitive description of the system behaviour starting from any initial condition for which  $[X] = 0$ .
  - Sketch a phase portrait for the system that is consistent with your conclusions from (iii) and (iv).
- b) Repeat for the system



In this case, you'll find that the non-zero steady-state is a center: it is surrounded by concentric periodic trajectories.

#### 4.8.8 Linearization.

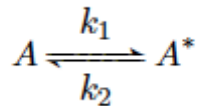
Consider the simple reaction system  $\rightarrow S \rightarrow$ , where the reaction rates are

$$\text{production: } V_0 \qquad \text{consumption: } \frac{V_{\max}[S]}{K_M + [S]}.$$

- a) Write the differential equation that describes the dynamics in  $s = [S]$ . Find the steady state. Next, approximate the original system by linearizing the dynamics around the steady state. This approximation takes the form of a linear differential equation in the new variable  $x(t) = s(t) - s^{\text{ss}}$ . b) Take parameter values  $V_0 = 2$ ,  $V_{\max} = 3$ , and  $K_M = 1$  and run simulations of the nonlinear and linearized systems starting at initial conditions  $[S] = 2.1$ ,  $[S] = 3$ , and  $[S] = 12$ . Comment on the discrepancy between the linear approximation and the original nonlinear model.

#### 4.8.13 Sensitivity analysis: reversible reaction.

Consider the reversible reaction with mass-action rate constants as shown.



Let  $T$  be the total concentration of  $A$  and  $A^*$ .

- a) Solve for the steady-state concentration of  $A^*$  and verify that an increase in  $k_1$  leads to an increase in  $[A^*]^{\text{ss}}$ .  
b) Use parametric sensitivity analysis to determine whether the steady state concentration of  $A^*$  is more sensitive to a 1% increase in  $T$  or a 1% increase in  $k_1$ . Does the answer depend on the values of the parameters?