Homework 3
Due Thursday 11/03/16 (9am)

4.8.6 Global dynamics from local stability analysis.
a) Consider the chemical reaction network with mass-action kinetics:

$$A + X \xrightarrow{k_1} 2X$$

$$X + X \xrightarrow{k_2} Y$$

$$Y \xrightarrow{k_3} B.$$

Assume that [A] and [B] are held constant.
i) Write a differential equation model describing the concentrations of X and Y.
ii) Verify that the system has two steady states.
iii) Determine the system Jacobian at the steady states and characterize the local behavior of the system near these points.
iv) By referring to the network, provide an intuitive description of the system behaviour starting from any initial condition for which [X] = 0.
v) Sketch a phase portrait for the system that is consistent with your conclusions from (iii) and (iv).
b) Repeat for the system

$$A + X \xrightarrow{k_1} 2X$$

$$X + Y \xrightarrow{k_2} 2Y$$

$$Y \xrightarrow{k_3} B.$$

In this case, you'll find that the non-zero steady-state is a center: it is surrounded by concentric periodic trajectories.

4.8.8 Linearization.

Consider the simple reaction system → S →, where the reaction rates are

$$\text{production:}\quad V_0 \qquad\qquad \text{consumption:}\quad \frac{V_{\max}[S]}{K_M + [S]}.$$

a) Write the differential equation that describes the dynamics in s = [S]. Find the steady state. Next, approximate the original system by linearizing the dynamics around the steady state. This approximation takes the form of a linear differential equation in the new variable $x(t) = s(t) - s^{ss}$. b) Take parameter values V0 = 2, V_{max} = 3, and K_M = 1 and run simulations of the nonlinear and linearized systems starting at initial conditions [S] = 2.1, [S] = 3, and [S] = 12. Comment on the discrepancy between the linear approximation and the original nonlinear model.

4.8.13 Sensitivity analysis: reversible reaction.

Consider the reversible reaction with mass-action rate constants as shown.

$$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A^*$$

Let T be the total concentration of A and A^*.

a) Solve for the steady-state concentration of A^* and verify that an increase in k_1 leads to an increase in $[A^*]^{ss}$.

b) Use parametric sensitivity analysis to determine whether the steady state concentration of A^* is more sensitive to a 1% increase in T or a 1% increase in k_1. Does the answer depend on the values of the parameters?