

## BEBI5009 Homework1

**Due 10/20/2016:** before class( 9:10am)

1. Complete the following table. Assume mass action kinetics for all reaction mechanisms. (20 point)

	Interaction graph	Rate equation scheme	ODE
a)		$\begin{array}{c} \xrightarrow{k_1} A \\ A \xrightarrow{k_2} \end{array}$	
b)			
c)			$\begin{aligned} \frac{d[A]}{dt} &= -k_1[A][B] + k_{-1}[C] \\ \frac{d[B]}{dt} &= -k_1[A][B] + k_{-1}[C] \\ \frac{d[C]}{dt} &= k_1[A][B] - (k_{-1} + k_2)[C] \\ \frac{d[D]}{dt} &= k_2[C] \end{aligned}$
d)			
e)		$A + A \xrightarrow{k_1} C$	

2. Problem sets from the textbook:

### 2.4.7 Network Modelling. (35 points)

a) Consider the closed reaction network in the figure 2.16 with reaction rates  $v_i$  as indicated. Suppose that the reaction rates are given by mass action as  $v_1 = k_1[A][B]$ ,  $v_2 = k_2[D]$  and  $v_3 = k_3[C]$ .

i) Construct a differential equation model for the network. Use moiety conservations to reduce your model to three differential equations and three algebraic equations.

ii) Solve for the steady-state concentrations as functions of the rate constants and the initial concentrations. (Note, because the system is closed, some of the steady-state concentrations are zero.)

iii) Verify your result in part (ii) by running a simulation of the system from initial conditions (in mM) of  $([A], [B], [C], [D], [E], [F]) = (1, 1, 1, 2, 0, 0)$ . Take rate constants  $k_1 = 3/\text{mM}/\text{sec}$ ,  $k_2 = 1/\text{sec}$ ,  $k_3 = 4/\text{sec}$ .

b) Next consider the open system in Figure 2.17 with reaction rates  $v_i$  as indicated. Suppose that the reaction rates are given by mass action as  $v_0 = k_0$ ,  $v_1 = k_1[A][B]$ ,  $v_2 = k_2[D]$ ,  $v_3 = k_3[C]$ ,  $v_4 = k_4[E]$ , and  $v_5 = k_5[F]$ .

i) Construct a differential equation model for the network. Identify any moiety conservations in the network.

ii) Solve for the steady state as a function of the rate constants and the initial concentrations.

iii) Verify your result in (ii) by running a simulation of the system from initial conditions (in mM) of  $([A], [B], [C], [D], [E], [F]) = (1, 1, 1, 2, 0, 0)$ . Take rate constants  $k_0 = 0.5 \text{ mM}/\text{sec}$ ,  $k_1 = 3/\text{mM}/\text{sec}$ ,  $k_2 = 1/\text{sec}$ ,  $k_3 = 4/\text{sec}$ ,  $k_4 = 1/\text{sec}$ ,  $k_5 = 5/\text{sec}$ .

iv) Given the initial conditions and rate constants in part (iii), why would there be no steady state if we take  $k_0 = 5 \text{ mM}/\text{sec}$ ?

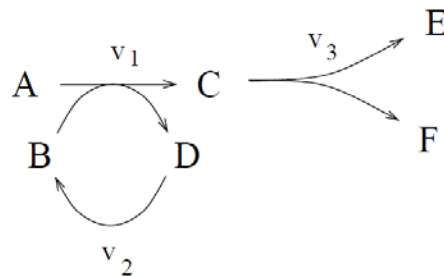


Figure 2.16: Closed reaction network for Problem 2.4.7(a).

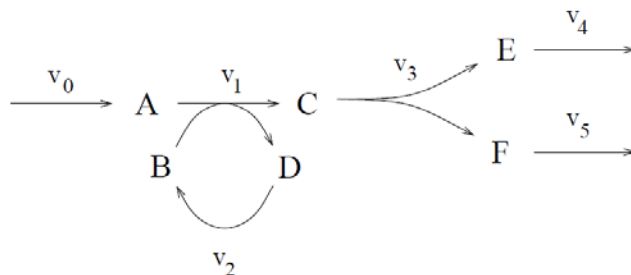
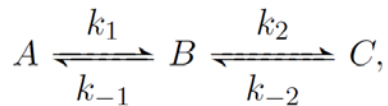


Figure 2.17: Open reaction network for Problem 2.4.7(b).

## 2.4.8 Rapid equilibrium approximation. (25 points)

Consider the closed system:

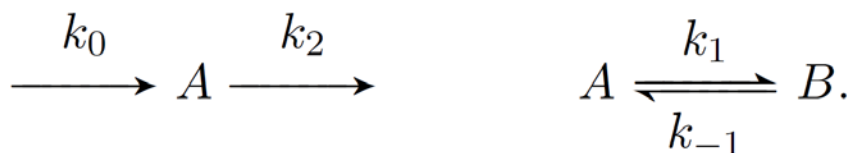


with mass-action rate constants as shown. Suppose the rate constants are (in min<sup>-1</sup>)  $k_1 = 0.05$ ,  $k_2 = 0.7$ ,  $k_{-1} = 0.005$ , and  $k_{-2} = 0.4$ .

- Construct a differential equation model of the system. Simulate your model with initial conditions (in mM) of  $A(0) = 1.5$ ,  $B(0) = 3$ ,  $C(0) = 2$ . Plot the transient and steady-state behaviour of the system. You may need to make two plots to capture all of the dynamics (i.e. two different window sizes).
- It should be clear from your simulation in part (a) that the system dynamics occur on two different time-scales. This is also apparent in the widely separated rate constants. Use a rapid equilibrium assumption to reduce your description of the system to two differential equations (describing one of the original species and one combined species pool) and two algebraic equations (describing the contents of the combined pool).
- Run a simulation of your reduced model in part (b) to compare with the simulation in part (a). Verify that the simulation of the reduced system is in good agreement with the original, except for a short initial transient. (Note, you will have to select initial conditions for the reduced system so that the initial total concentration is in agreement with part (a), and the rapid equilibrium condition is satisfied at time  $t = 0$ .)

#### 2.4.9 Quasi-steady-state approximation. (25points)

Consider the reaction network:



Suppose the mass-action rate constants are (in min<sup>-1</sup>)  $k_0 = 1$ ,  $k_1 = 11$ ,  $k_{-1} = 8$ , and  $k_2 = 0.2$ .

- Construct a differential equation model of the system. Simulate your model with initial conditions  $A(0) = 6$  mM,  $B(0) = 0$  mM. Plot the transient and steady-state behaviour of the system. You may need to make two plots to capture all of the dynamics (i.e. two different window sizes).
- It should be clear from your simulation in part (a) that the system dynamics occur on two different time-scales. This is also apparent in the widely separated rate constants. Use a quasi steady-state assumption to reduce your description of the system by replacing a differential equation with an algebraic equation.
- Run a simulation of your reduced model in part (b) to compare with the simulation in part (a). Verify that the simulation of the reduced system is a good approximation to the original at steady state, but not over the initial transient. (Note, you will have to select initial conditions

for the reduced system so that the total concentration is in agreement with part (a), and the quasi-steady state condition is satisfied at time  $t = 0$ , as in Exercise 2.2.4.)