General equilibrium analysis of the Eaton–Kortum model of international trade[☆]

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Abstract

We study a variation of the Eaton–Kortum model, a competitive, constant-returns-to-scale multicountry Ricardian model of trade. We establish existence and uniqueness of an equilibrium with balanced trade where each country imposes an import tariff. We analyze the determinants of the cross-country distribution of trade volumes, such as size, tariffs and distance, and compare a calibrated version of the model with data for the largest 60 economies. We use the calibrated model to estimate the gains of a world-wide trade elimination of tariffs, using the theory to explain the magnitude of the gains as well as the differential effect arising from cross-country differences in pre-liberalization tariff levels and country size.
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Keywords: General equilibrium; Ricardian trade theory; Trade volume; Tariff policy

1. Introduction

Eaton and Kortum (2002) have proposed a new theory of international trade, an economical and versatile parameterization of the models with a continuum of tradeable goods that Dornbusch et al. (1977) and Wilson (1980) introduced many years ago. In the theory, constant-returns producers in different countries are subject to idiosyncratic productivity shocks. Buyers of any good search over producers in different countries for the lowest price, and trade assigns production of any good to the most efficient producers, subject to costs of transportation and other impediments. The gains from trade are larger, the larger is the variance of individual productivities, which is the key parameter in the model.

The model shares with those of the "new trade theory" the important ability to deal sensibly with intra-industry trade: trade in similar categories of goods between similarly endowed countries.[1] But unlike the earlier theory, the Eaton–Kortum (2002) model is competitive, involving no fixed costs and no monopoly rents. Of course, fixed costs and monopoly rents are present in reality, but theories based on competitive behavior are much simpler to calibrate and permit the use of a large body of general equilibrium theory to help in analysis.

One aim of this paper is to restate the economic logic of a variation of the Eaton–Kortum model of trade in a particular general equilibrium context. In the next section, we will introduce the basic ideas using a closed economy with a production technology of the Eaton–Kortum type. In Section 3, we define an equilibrium with balanced trade in a world with many countries, each one imposing import tariffs. Section 4 gives sufficient conditions for this equilibrium to exist, and addresses the problem of determining whether this equilibrium is unique and of finding an algorithm to compute it.

A second goal of the paper is to find out whether the cross-country distribution of trade volumes generated by a model of this type is consistent with the behavior of volumes in the data. In Section 5, we calibrate some of the main parameters of the theory. Section 6 discusses some instructive special cases that are simple enough to work out by hand. Using estimates from Section 5, we examine the implications of these special cases of the theory for the volume of trade and the way that trade volume behaves as a function of size, and compare these implications to data on total GDP and trade volumes for the 60 largest economies. Sections 7 and 9 go over the same ground numerically with more realistic assumptions. In these two sections we apply the algorithm described in Section 4, calibrate the model to the observed distribution of GDPs and the relative prices of tradeables to non-tradeable goods, and introduce heterogeneity in transportation costs and tariff rates.

Our normative goal is to use the quantitative theory to estimate the welfare gains from hypothetical trade liberalizations. Comparisons between free trade and autarchy are carried out in Sections 6. Section 8 studies the optimal tariff policy of a small economy. In Section 8 we also calculate the effects of a world-wide liberalization in which every country's tariffs are set to zero. We use the theory to explain the magnitude of the average gains of trade, as well as differential effects arising from cross-country differences in pre-liberalization tariff levels and country size. Section 9 describes the effects on these estimates

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[1]See, for example, Ethier (1979, 1982), Krugman (1979), Helpman (1981), and the Helpman and Krugman (1985) monograph. Baxter (1992) argues that competitive, Ricardian models are equally capable of dealing realistically with intra-industry trade.

of calibrating the model to fit relative price differences across countries. Conclusions are contained in Section 10.

2. Preferences, technology, and closed economy equilibrium

The Eaton–Kortum model is Ricardian, with a continuum of goods produced under a constant-returns technology. The new idea is a two-parameter probabilistic model that generates the input requirements for producing each good. It will be useful to introduce this model of a technology in the simpler context of a single, closed economy before turning to the study of a model of a world of n countries in Section 3.

We develop a purely static model in which labor is the only primary (non-produced) factor of production. There are L consumers, and each is endowed with one unit of labor. All production is subject to constant returns, and we conduct the entire analysis of the closed economy in per capita terms. There is a single, produced final good, c, which is the only good valued by consumers. We use c for utility as well.

A continuum of intermediate goods are produced, and these goods affect production symmetrically via a Spence–Dixit–Stiglitz (SDS) aggregate. Total factor productivity (TFP) levels vary across goods. As in Eaton and Kortum (2002), we model the *inverses* of these TFP levels ("costs") as random variables, independent across goods, with a common density ϕ. Since intermediate goods differ only in their costs, in this sense, it is convenient to name each intermediate good by its cost draw, $x > 0$, and to speak of "good x."

Let $q(x)$ be production of intermediate good x, and denote the production of the SDS aggregate by q. Then

$$q = \left[\int_0^\infty q(x)^{1-1/\eta} \phi(x) \, dx \right]^{\eta/(\eta-1)}. \tag{2.1}$$

The labor endowment is allocated over final goods production, s_f, and production of the intermediates, $s(x)$, $x > 0$:

$$s_f + \int_0^\infty s(x) \phi(x) \, dx \leqslant 1. \tag{2.2}$$

The intermediates q_f and $q_m(x)$ are allocated over the same uses:

$$q_f + \int_0^\infty q_m(x) \phi(x) \, dx \leqslant q.$$

The production technologies are Cobb–Douglas. For final goods,

$$c = s_f^\alpha q_f^{1-\alpha}. \tag{2.3}$$

For intermediate good x,

$$q(x) = x^{-\theta} s(x)^\beta q_m(x)^{1-\beta}. \tag{2.4}$$

Throughout we follow Eaton–Kortum (2002) and assume that the density ϕ is exponential with parameter λ: $x \sim \exp(\lambda)$. These x draws are then amplified in percentage terms by the parameter θ.[2] (The random variables $x^{-\theta}$ then have a Frechet distribution.)

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[2] We are using θ for the parameter that Eaton and Kortum call $1/\theta$, so that in this paper a larger θ means a larger variance in individual productivities.

It is important to emphasize that these cost draws x are economy-wide effects. Anyone is free to produce any specific good, and every producer of that good has access to the same production technology (2.4), with the same stochastic intercept $x^{-\theta}$, as other producers do. Since (2.4) is a constant-returns technology the number of firms producing any good will be indeterminate, but whatever that number is, no single producer has any market power and all prices will be set at marginal cost, equivalent to minimum unit cost. This is true for final goods production as well.

Denote the wage rate by w, the price of final goods by p, the prices of individual tradeables by $p(x)$, and the price of the intermediate aggregate by p_{m}. Cost minimizing behavior of all producers will ensure that equilibrium prices satisfy

$$p = \alpha^{-\alpha}(1 - \alpha)^{-1+\alpha}w^\alpha p_{\mathrm{m}}^{1-\alpha}, \tag{2.5}$$

$$p(x) = Bx^\theta w^\beta p_{\mathrm{m}}^{1-\beta}, \tag{2.6}$$

where

$$B = \beta^{-\beta}(1 - \beta)^{-1+\beta},$$

and

$$p_{\mathrm{m}} = \left(\lambda \int_0^\infty e^{-\lambda x} p(x)^{1-\eta} \, dx \right)^{1/1-\eta}. \tag{2.7}$$

We can use (2.6) and (2.7) to solve for p_{m} as a multiple of the wage. We have

$$p_{\mathrm{m}} = \left[\lambda \int_0^\infty e^{-\lambda x} (Bx^\theta w^\beta p_{\mathrm{m}}^{1-\beta})^{1-\eta} \, dx \right]^{1/(1-\eta)}$$

$$= Bw^\beta p_{\mathrm{m}}^{1-\beta} \lambda^{-\theta} \left[\int_0^\infty e^{-z} z^{\theta(1-\eta)} \, dz \right]^{1/(1-\eta)}, \tag{2.8}$$

using the change of variable $z = \lambda x$. We write $A(\theta, \eta)$, or sometimes just A, for

$$A(\theta, \eta) = \left[\int_0^\infty e^{-z} z^{\theta(1-\eta)} \, dz \right]^{1/(1-\eta)}.$$

The integral in brackets is the Gamma function $\Gamma(\xi)$, evaluated at the argument $\xi = 1 + \theta(1 - \eta)$. Convergence of the integral requires

$$1 + \theta(1 - \eta) > 0, \tag{2.9}$$

which we assume to hold throughout this paper.[3]

In terms of A, (2.8) implies

$$p_{\mathrm{m}} = (AB)^{1/\beta} \lambda^{-\theta/\beta} w. \tag{2.10}$$

Substituting from (2.10) back into (2.5) and (2.6) then yields the prices of individual intermediate goods

$$p(x) = A^{(1-\beta)/\beta} B^{1/\beta} x^\theta \lambda^{-\theta(1-\beta)/\beta} w, \tag{2.11}$$

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[3]If η were too large to satisfy (2.9), the integral in (2.8) would not converge. Economically, this would mean unbounded production of the tradeable aggregate, as labor is concentrated on goods where x is near zero (where $x^{-\theta}$ is very high). Changes in the parameter η will affect the units in which tradeables are measured, and hence relative prices that depend on these units. The allocation of labor and materials between the two sectors is independent of the value of η. (See footnote 4.)

and of final goods

$$p = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha}(AB)^{(1-\alpha)/\beta}\lambda^{-\theta(1-\alpha)/\beta}w. \tag{2.12}$$

Notice that all these prices, p, p_m, and $p(x)$, are different multiples of the wage rate w. This *is* a labor theory of value: everything is priced according to its labor content.

Given these solutions for equilibrium prices, equilibrium quantities are readily calculated, using the familiar Cobb–Douglas constant share formulas. We turn, in the next section, to a model of international trade, in which the intermediate goods are tradeable (at a cost) and neither labor nor final goods can move. This fact will completely alter the determination of the price p_m of the tradeable, intermediate aggregate, but within each country, *nothing else will change*. The pricing functions (2.5) and (2.6) will continue to obtain, and the calculation of quantities, given prices, will be the same as in this closed economy section.

3. General equilibrium

We turn to a version of the Eaton–Kortum (2002) model, and consider an equilibrium in a world of n countries, all with the structure described in Section 2, in which trade is balanced. Let total labor endowments be $L = (L_1, \ldots, L_n)$, where L_i is the total units of labor in i, measured in efficiency units. Labor is not mobile. The exponential distributions that define each country's technology have the parameters $\lambda = (\lambda_1, \ldots, \lambda_n)$. We use $w = (w_1, \ldots, w_n)$ for the vector of wages in the individual countries. Preferences and the technology parameters θ, β, α, and η, are common to all countries. The structure of production in each country is exactly as described in Section 2, except that now intermediate goods are traded—and will now be called *tradeables*—subject to transportation costs and tariffs.

Transportation costs are defined in physical, "iceberg" terms: we assume that one unit of any tradeable good shipped from j to i results in κ_{ij} units arriving in i. Interpreting the terms κ_{ij} as representing costs that are proportional to distance, it is natural to assume that $\kappa_{ij} > 0$, $\kappa_{ij} \leqslant 1$, with equality if $i = j$, $\kappa_{ij} = \kappa_{ji}$ for all i, j, and

$$\kappa_{ij} \geqslant \kappa_{ik}\kappa_{kj} \quad \text{for all } i, j, k. \tag{3.1}$$

We also want to consider tariffs that distort relative prices but do not entail a physical loss of resources. In practice, trade barriers take many forms, but here we consider only flat rate tariffs levied by country i on goods imported from j, and where the proceeds are rebated as lump sum payments to the people living in i. Define ω_{ij} to be the fraction of each dollar spent in i on goods made in j that arrives as payment to a seller in j.

In the closed economy analysis of Section 2 we exploited the assumptions of competition and constant returns to solve for all equilibrium prices as multiples of the wage w, with coefficients depending only on the technology. With this done, we can then calculate equilibrium quantities. This same two-stage procedure can be applied to the case of many countries, though of course each stage is more complicated.

A new notation for the commodity space is needed. Let $x = (x_1, \ldots, x_n)$ be the vector of technology draws for any given tradeable good for the n countries. We refer to "good x", as before, but now $x \in \mathbf{R}_+^n$. Assume that these draws are independent across countries,

so that the joint density of x is

$$\phi(x) = \left(\prod_{i=1}^{n} \lambda_i \right) \exp\left\{ -\sum_{i=1}^{n} \lambda_i x_i \right\}.$$

Use $q_i(x)$ for the *consumption* of tradeable good x in country i, and q_i for consumption in i of the aggregate,

$$q_i = \left[\int q_i(x)^{1-1/\eta} \phi(x)\, dx \right]^{\eta/(\eta-1)}.$$

(Here $\int$ denotes integration over $\mathbf{R}_+^n$.) Let $p_i(x)$ be the prices paid for tradeable good x by producers in i. Let

$$p_{mi} = \left[\int p_i(x)^{1-\eta} \phi(x)\, dx \right]^{1/(1-\eta)} \tag{3.2}$$

be the price in i for a unit of the aggregate.

The tradeable good $x = (x_1, \ldots, x_n)$ is available in i at the unit prices

$$B x_1^\theta w_1^\beta p_{m1}^{1-\beta} \frac{1}{\kappa_{i1}\omega_{i1}}, \ldots, B x_n^\theta w_n^\beta p_{mn}^{1-\beta} \frac{1}{\kappa_{in}\omega_{in}},$$

which reflect both production costs (labor and intermediate inputs) and transportation and tariff costs. All producers in i buy at the same, lowest price:

$$p_i(x) = B \min_j \left[\frac{w_j^\beta p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} x_j^\theta \right]. \tag{3.3}$$

Note that without assumption (3.1), the right side of (3.3) would not necessarily be the least cost way of obtaining good x in country i.

The price index p_{mi} of tradeables in i must be calculated country by country. We derive an expression for p_{mi} from (3.2) and (3.3). The derivation uses two well-known properties of the exponential distribution:

$$x \sim \exp(\lambda) \quad \text{and} \quad k > 0 \quad \Rightarrow \quad kx \sim \exp\left(\frac{\lambda}{k} \right) \tag{3.4}$$

and

$$\begin{aligned} & x \text{ and } y \text{ independent}, \quad x \sim \exp(\lambda), \ y \sim \exp(\mu), \\ & \text{and} \quad z = \min(x, y) \quad \Rightarrow \quad z \sim \exp(\lambda + \mu). \end{aligned} \tag{3.5}$$

From (3.2), we have

$$p_{mi}^{1-\eta} = \int p_i(x)^{1-\eta} \phi(x)\, dx, \tag{3.6}$$

and note that the right side is the expected value of the random variable $p_i(x)^{1-\eta}$. From (3.3)

$$p_i(x)^{1/\theta} = B^{1/\theta} \min_j \left[\frac{w_j^{\beta/\theta} p_{mj}^{(1-\beta)/\theta}}{(\kappa_{ij}\omega_{ij})^{1/\theta}} x_j \right].$$

Property (3.4) implies that $z_j \equiv w_j^{\beta/\theta} p_{mj}^{(1-\beta)/\theta} (\kappa_{ij}\omega_{ij})^{-1/\theta} x_j$ is exponentially distributed with parameter

$$\psi_{ij} = \left(\frac{w_j^\beta p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} \right)^{-1/\theta} \lambda_j, \tag{3.7}$$

and property (3.5) implies that $z \equiv \min_j z_j$ is exponentially distributed with parameter $\sum_{j=1}^n \psi_{ij}$. Applying (3.4) again, this proves that $p_i(x)^{1/\theta}$ is exponentially distributed with parameter

$$\mu = B^{-1/\theta} \sum_{j=1}^n \psi_{ij}.$$

It then follows from (3.6) that

$$p_{mi}^{1-\eta} = \mu \int_0^\infty u^{\theta(1-\eta)} e^{-\mu u} \, du.$$

Using the change of variable $z = \mu u$, we have that

$$p_{mi}^{1-\eta} = \mu^{-\theta(1-\eta)} \int_0^\infty e^{-z} z^{\theta(1-\eta)} \, dz$$
$$= \mu^{-\theta(1-\eta)} A^{1-\eta},$$

where $A = A(\theta, \eta)$ is the constant defined in Section 2. Then

$$p_{mi}(w) = AB \left(\sum_{j=1}^n \psi_{ij} \right)^{-\theta} \equiv AB \left(\sum_{j=1}^n \left(\frac{w_j^\beta p_{mj}(w)^{1-\beta}}{\kappa_{ij}\omega_{ij}} \right)^{-1/\theta} \lambda_j \right)^{-\theta}, \tag{3.8}$$

$i = 1, \ldots, n$.

We view (3.8) as n equations in the prices $p_m = (p_{m1}, \ldots, p_{mn})$, to be solved for p_m as a function of the wage vector w. It is the same formula as (7) and (9) in Eaton and Kortum (2002). Notice that the country identifier i appears on the right side of (3.8) only via the parameters κ_{ij} and ω_{ij}.

Next we calculate the tradeables expenditure shares for each country i: the fraction D_{ij} of country i's per capita spending $p_{mi}q_i$ on tradeables that is spent on goods from country j. Economically, the total spending in i on goods from j is just

$$p_{mi}q_i D_{ij} = \int_{\mathbf{B}_{ij}} p_i(x) q_i(x) \phi(x) \, dx,$$

where $\mathbf{B}_{ij} \subset \mathbf{R}_+^n$ is the set on which j attains the minimum in (3.3). Using (3.2) and (3.3), this integral can be evaluated to obtain the expression (3.10) (below) for D_{ij}.

One can also show that the D_{ij} will simply be the probabilities that for a particular good x, the low price vendors for buyers in i are sellers in j. These probabilities can be calculated directly, using a third fact about exponential distributions:

x and y independent, $x \sim \exp(\lambda)$ and $y \sim \exp(\mu)$

$$\Rightarrow \quad \Pr\{x \leqslant y\} = \frac{\lambda}{\lambda + \mu}. \tag{3.9}$$

From (3.3) we have

$$D_{ij} = \Pr\left\{\frac{w_j^\beta p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} x_j^\theta \leqslant \min_{k \neq j}\left[\frac{w_k^\beta p_{mk}^{1-\beta}}{\kappa_{ik}\omega_{ik}} x_k^\theta\right]\right\}$$

$$= \Pr\left\{\left(\frac{w_j^\beta p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}}\right)^{1/\theta} x_j \leqslant \min_{k \neq j}\left[\left(\frac{w_k^\beta p_{mk}^{1-\beta}}{\kappa_{ik}\omega_{ik}}\right)^{1/\theta} x_k\right]\right\}.$$

By (3.4), the random variable on the left of the inequality is exponential with parameter ψ_{ij}. By (3.4) and (3.5), the random variable on the right is exponential with parameter $\sum_{k \neq j} \psi_{ik}$, and the two are obviously independent. Thus (3.9) implies

$$D_{ij} = \frac{\psi_{ij}}{\sum_{k=1}^n \psi_{ik}} = (AB)^{-1/\theta}\left(\frac{w_j^\beta p_{mj}(w)^{1-\beta}}{p_{mi}(w)\kappa_{ij}\omega_{ij}}\right)^{-1/\theta} \lambda_j. \qquad (3.10)$$

Note that $\sum_j D_{ij} = 1$.

We now impose trade balance. Under this assumption, the dollar payments for tradeables flowing into i from the rest of the world must equal the payments flowing out of i to the rest of the world. Firms in i spend a total of $L_i p_{mi} q_i$ dollars on tradeables, including both transportation costs and tariff payments. Of this amount,

$$L_i p_{mi} q_i \sum_{j=1}^n D_{ij}\omega_{ij}$$

reaches sellers in all countries (including i itself). The rest is collected in taxes, and rebated as a lump sum to consumers in i.

Buyers in j spend a total of $L_j p_{mj} q_j D_{ji}$ dollars for tradeables from i, but of this total only

$$L_j p_{mj} q_j D_{ji}\omega_{ji}$$

reaches sellers in i. The rest remains in j, as rebated tax receipts. Trade balance then requires that the condition

$$L_i p_{mi} q_i \sum_{j=1}^n D_{ij}\omega_{ij} = \sum_{j=1}^n L_j p_{mj} q_j D_{ji}\omega_{ji} \qquad (3.11)$$

must hold. Notice that the term $L_i p_{mi} q_i D_{ii}\omega_{ii}$—country i's spending on home-produced tradeables—appears on both sides of (3.11). Cancelling thus yields the usual definition of trade balance: payments to foreigners equal receipts from foreigners.

Our strategy for constructing the equilibrium in this world economy draws on the analysis of a single, closed economy in Section 2. As in that section, we first note that all prices in all countries can be expressed in terms of wages. In the present case, wages are a vector $w = (w_1, \ldots, w_n)$ and we express the coefficients ψ_{ij} as functions of w and p_m, and then use the n equations (3.8) to solve for the prices $p_m = (p_{m1}, \ldots, p_{mn})$ as a function $p_m(w)$ of wages. This problem is the subject of Theorem 1 in the next section. The impact of the rest of the world on the behavior of individual producers in i is entirely determined by these prices p_{mi}. With tradeable prices expressed as functions of wages (and of the tax rates and other parameters involved in (3.8)), (3.10) expresses the expenditure shares D_{ij} as functions of wages and tax rates, too. Then (3.11) can be viewed as an equation in wages w and the vector q of tradeables consumption per capita.

We first consider the implications of trade balance in the absence of taxes: $\omega_{ij} = 1$ for all i, j. In this case, the trade balance condition (3.11) reduces to

$$L_i p_{\mathrm{m}i} q_i = \sum_{j=1}^{n} L_j p_{\mathrm{m}j} q_j D_{ji}. \tag{3.12}$$

The share of tradeables in the production of the final good is, in this case,

$$1 - \alpha = \frac{L_i p_{\mathrm{m}i} q_{\mathrm{f}i}}{L_i w_i},$$

using the fact that in the absence of indirect taxes, GDP equals national income: $L_i p_i c_i = L_i w_i$. The share of tradeables in the production of tradeables is

$$1 - \beta = \frac{L_i p_{\mathrm{m}i}(q_i - q_{\mathrm{f}i})}{L_i p_{\mathrm{m}i} q_i},$$

using the fact—trade balance, again—that the value of tradeables produced must equal the value of tradeables used in production. These two share formulas together imply

$$\beta L_i p_{\mathrm{m}i} q_i = (1 - \alpha) L_i w_i. \tag{3.13}$$

Applying this fact to both sides of the trade balance condition (3.12) and cancelling, we obtain

$$L_i w_i = \sum_{j=1}^{n} L_j w_j D_{ji}(w), \quad i = 1, \dots, n. \tag{3.14}$$

We do not need to restrict the transportation cost parameters κ_{ij} to reduce (3.11) to (3.14) because the effects of these costs are entirely captured in (3.8).

Theorem 2 in the next section provides conditions that ensure that (3.14) has a solution w, but since our objective is to be able to analyze the effects of changes in tariff policies, we cannot stop with this special case. Nor can we make use of the share formulas to simplify (3.11) in the general case: the presence of indirect business taxes breaks the equality of GDP and national income that we used to derive (3.14).

To deal with the general case, we need to start from (3.11), not (3.12), and we need to adapt the share formulas to incorporate taxes. This adaptation is primarily an accounting exercise, based on the national income and product accounts for a country i.

Table 1 provides the accounts for country i, viewed as a three sector economy. All entries are in dollars. The left side gives value added in each sector; the right side gives

Table 1

Value added in services	Labor income in services
$L_i p_i c_i - L_i p_{\mathrm{m}i} q_{\mathrm{f}i}$	$L_i w_i s_{\mathrm{f}i}$
Value added in tradeables	Labor income in tradeables
$\sum_j L_j p_{\mathrm{m}j} q_j D_{ji} \omega_{ji} - L_i p_{\mathrm{m}i} q_{\mathrm{m}i}$	$L_i w_i (1 - s_{\mathrm{f}i})$
Value added in importing	Indirect business taxes
$L_i p_{\mathrm{m}i} q_i - L_i p_{\mathrm{m}i} q_i \sum_j D_{ij} \omega_{ij}$	$L_i p_{\mathrm{m}i} q_i \sum_{j \neq i} D_{ij}(1 - \omega_{ij})$
GDP	Total labor income *plus* indirect taxes
$L_i p_i c_i$	$L_i w_i + L_i p_{\mathrm{m}i} q_i \sum_{j \neq i} D_{ij}(1 - \omega_{ij})$

factor payments (labor payments plus indirect business taxes). Two of these sectors are services (final goods) and tradeables. The third is an importing sector, in which firms buy tradeable goods from both home and foreign producers, pay import duties to their own government, and resell the goods to home producers. This is a constant-returns, free-entry activity, so of course selling prices must be marked up exactly to cover the taxes. If $\omega_{ij} = 1$, the entries for this sector would be zero in both columns. To verify that the three sector value-added terms sum to GDP, one needs to use the trade balance condition (3.11).

Based on these accounts, the derivation of (3.14) from (3.11) can be paralleled, and it can be shown that (3.11) implies

$$L_i w_i (1 - s_{\mathrm{f}i}) = \sum_{j=1}^{n} L_j \frac{w_j (1 - s_{\mathrm{f}j})}{F_j} D_{ji} \omega_{ji}, \tag{3.15}$$

where

$$s_{\mathrm{f}i} = \frac{\alpha[1 - (1 - \beta)F_i]}{(1 - \alpha)\beta F_i + \alpha[1 - (1 - \beta)F_i]} \tag{3.16}$$

and

$$F_i = \sum_{j=1}^{n} D_{ij} \omega_{ij}. \tag{3.17}$$

Here $s_{\mathrm{f}i}$ is labor's share in the production of final goods and F_i is the fraction of country i's spending on tradeables that reaches producers (as opposed to the home government). Both are functions of w as are the shares D_{ij}. Details are provided in Appendix A.

We view solving these equations as finding the zeros of an excess demand system $Z(w)$:

$$Z_i(w) = \frac{1}{w_i} \left[\sum_{j=1}^{n} L_j \frac{w_j (1 - s_{\mathrm{f}j}(w))}{F_j(w)} D_{ji}(w) \omega_{ji} - L_i w_i (1 - s_{\mathrm{f}i}(w)) \right]. \tag{3.18}$$

We sum up this section in the following definition.

Definition. An *equilibrium* is a wage vector $w \in \mathbf{R}_{++}^n$ such that $Z_i(w) = 0$ for $i = 1, \ldots, n$, where the functions $p_{\mathrm{m}i}(w)$ satisfy (3.8), the functions $D_{ij}(w)$ satisfy (3.10), the functions $F_i(w)$ satisfy (3.17), and the functions $s_{\mathrm{f}i}(w)$ satisfy (3.16).

Given an equilibrium wage vector w and the tradeable aggregate prices $p_{\mathrm{m}i}(w)$ in each country i, all other equilibrium prices and quantities can be calculated as we have described in Section 2.

4. Existence, uniqueness, and computation of equilibrium

The economy we analyze is specified by the technology parameters α, β, η, and θ, common to all countries, the country-specific populations and technology levels $L = (L_1, \ldots, L_n)$ and $\lambda = (\lambda_1, \ldots, \lambda_n)$, the transportation parameters $[\kappa_{ij}]$, and the tax parameters $[\omega_{ij}]$. All these numbers are strictly positive. Moreover, we impose:

Assumptions (A). (A1) $\alpha, \beta < 1$,
   (A2) $1 + \theta(1 - \eta) > 0$, and for some numbers $\underline{\kappa}$ and $\underline{\omega}$,
   (A3) $0 < \underline{\kappa} \leqslant \kappa_{ij} \leqslant 1$ and $0 < \underline{\omega} \leqslant \omega_{ij} \leqslant 1$.

Under these assumptions, we study the existence and uniqueness of solutions to the excess demand system (3.18). Before turning directly to these issues, Theorem 1 characterizes the function $p_{\mathrm{m}}(\cdot) : \mathbf{R}_{++}^n \to \mathbf{R}_+^n$ relating tradeable goods prices to wage rates, defined implicitly by Eqs. (3.8). Then Theorem 2 shows that the excess demand system (3.18) satisfies the sufficient conditions for a theorem on the existence of equilibrium in an n-good exchange economy. Theorem 3 gives one set of assumptions that implies that this solution is unique.

Theorem 1. *Under Assumptions* (A), *for any* $w \in \mathbf{R}_{++}^n$ *there is a unique* $p_{\mathrm{m}}(w)$ *that satisfies* (3.8). *For each* i, *the function* $p_{\mathrm{m}i}(w)$ *is*

(i) *continuously differentiable on* $\mathbf{R}_{++}^n$,
(ii) *homogeneous of degree one*,
(iii) *strictly increasing in* w,
(iv) *strictly decreasing in the parameters* κ_{ij} *and* ω_{ij}, *and*
(v) *satisfies the bounds*

$$\underline{p_{\mathrm{m}}}(w) \leqslant p_{\mathrm{m}i}(w) \leqslant \overline{p_{\mathrm{m}}}(w),$$

for all $w \in \mathbf{R}_{++}^n$, *where*

$$\overline{p_{\mathrm{m}}}(w) = \left(\frac{AB}{\underline{\kappa\omega}}\right)^{1/\beta} \left(\sum_{j=1}^n w_j^{-\beta/\theta} \lambda_j\right)^{-\theta/\beta} \tag{4.1}$$

and

$$\underline{p_{\mathrm{m}}}(w) = (AB)^{1/\beta} \left(\sum_{j=1}^n w_j^{-\beta/\theta} \lambda_j\right)^{-\theta/\beta}. \tag{4.2}$$

The *proof* of Theorem 1 is provided in Appendix B.

To prove that an equilibrium exists, we will apply an existence theorem for an exchange economy with n goods to the demand system $Z(w)$ defined in (3.18).

Theorem 2. *Under Assumptions* (A) *there is a* $w \in \mathbf{R}_{++}^n$ *such that*

$$Z(w) = 0.$$

Proof. We verify that $Z(w)$ has the properties

(i) $Z(w)$ is continuous,
(ii) $Z(w)$ is homogeneous of degree zero,
(iii) $w \cdot Z(w) = 0$ for all $w \in \mathbf{R}_{++}^n$ (Walras's Law),
(iv) for $k = \max_j L_j > 0$, $Z_i(w) > -k$ for all $i = 1, \ldots, n$ and $w \in \mathbf{R}_{++}^n$, and
(v) if $w^m \to w^0$, where $w^0 \neq 0$ and $w_i^0 = 0$ for some i, then

$$\max_j \{Z_j(w^m)\} \to \infty. \tag{4.3}$$

Then the result will follow from Proposition 17.C.1 of Mas-Colell et al. (1995, p. 585).

(i) The continuity of $p_{\mathrm{m}i}$ is implied by part (i) of Theorem 1. The continuity of the functions D_{ij} is then evident from (3.10). The functions F_i defined in (3.17) are continuous,

and are uniformly bounded from below by $\underline{\omega}$. The functions s_{fi} defined in (3.16) are continuous. The continuity of Z then follows from (3.18).

(ii) From Theorem 1, p_{mi} is homogeneous of degree one. Then (3.10) implies that the D_{ij} are homogeneous of degree zero, and it is immediate that F_i, s_{f_i}, and Z_i all have this property.

(iii) To verify Walras's Law, restate (3.18) as

$$w_i Z_i = \sum_{j=1}^{n} L_j w_j (1 - s_{fj}) \frac{1}{F_j} D_{ji} \omega_{ji} - L_i w_i (1 - s_{fi})$$

and sum over i to get

$$\sum_{i=1}^{n} w_i Z_i = \sum_{i=1}^{n} \sum_{j=1}^{n} L_j w_j (1 - s_{fj}) \frac{1}{F_j} D_{ji} \omega_{ji} - \sum_{i=1}^{n} L_i w_i (1 - s_{fi})$$

$$= \sum_{j=1}^{n} L_j w_j (1 - s_{fj}) \sum_{i=1}^{n} \frac{1}{F_j} D_{ji} \omega_{ji} - \sum_{i=1}^{n} L_i w_i (1 - s_{fi}) = 0$$

using (3.14).

The proofs of parts (iv) and (v) are in Appendix A.   □

We next establish a sufficient condition for the equilibrium be unique. To do so, we add to Assumptions (A) the assumption that the import duties ω_{ij} levied by country i are uniform over all source countries j, so that we write $\omega_{ij} = \omega_i$ for $i \neq j$ and $\omega_{ii} = 1$. Then we have:

Theorem 3. *If Assumptions (A) hold, if $\omega_{ij} = \omega_i$ for all $i \neq j$, and if*

$$(\underline{\kappa \omega})^{2/\theta} \geqslant 1 - \beta, \tag{4.4}$$

$$\alpha \geqslant \beta, \tag{4.5}$$

and

$$1 - \underline{\omega} \leqslant \frac{\theta}{\alpha - \beta}, \tag{4.6}$$

there is exactly one solution to $Z(w) = 0$ that satisfies $\sum_{i=1}^{n} w_i = 1$.

Proof. In Appendix B, we use the results from Theorem 1, (iii) and (v), to establish that Z has the gross substitute property

$$\frac{\partial Z_i(w)}{\partial w_k} > 0 \quad \text{for all } i,k, \ i \neq k \quad \text{for all } w \in \mathbf{R}_{++}. \tag{4.7}$$

(Since Z is homogeneous of degree zero, (4.7) will imply that

$$\frac{\partial Z_i(w)}{\partial w_i} > 0 \quad \text{for all } i \text{ for all } w \in \mathbf{R}_{++}.)$$

Then the result will follow from Proposition 17.F.3 of Mas-Colell et al. (1995, p. 613).   □

One can see that (4.4) and (4.6) are satisfied if the tariff and transportation costs are small enough. For the parameter values for α, β, and θ proposed in Section 5, condition (4.4) is satisfied if tariffs and transportation cost are both less than 2.5%. Condition (4.5) is

easily satisfied for the benchmark calibration presented later on. Of course, conditions (4.4)–(4.6) are sufficient, not necessary, conditions for the gross substitute property to obtain. For the case of two countries, it can be shown that (4.6) alone is sufficient. Our numerical experience also confirms that the gross substitute property holds under much wider conditions than (4.4)–(4.6), including quite high tariff and transportation costs.

We finish this section by discussing the algorithm that we use to compute equilibrium. The gross substitute property applied in Theorem 3 suggests the use of a discrete time analogue of the continuous time tatonnement process. Let Δ_w be defined as

$$\Delta_w = \left\{ w \in \mathbf{R}_+^n : \sum_{i=1}^n w_i L_i = 1 \right\}.$$

Then we define the function T, mapping Δ_w into itself as follows:

$$T(w)_i = w_i(1 + v Z_i(w)/L_i), \quad i = 1, \dots, n, \tag{4.8}$$

where v is an arbitrary constant satisfying $v \in (0, 1]$. Notice that $Z_i(w)/L_i$ is country i's labor excess demand per unit of labor, so the operator T prescribes that the percentage increase in country i's wage be in proportion to a scaled version of country i's excess demand. To see that $T : \Delta_w \to \Delta_w$, note first that $T(w)_i \geqslant 0$ if $1 + v Z_i(w)/L_i \geqslant 0$, since Z_i is bounded below by $-L_i$ by part (iv) of Theorem 2. Note second that for any $w \in \Delta_w$

$$\sum_{i=1}^n T(w)_i L_i = \sum_i w_i \left(1 + v \frac{Z_i}{L_i}\right) L_i = \sum_{i=1}^n w_i L_i + v \sum_{i=1}^n w_i Z_i(w) = 1,$$

where the last equality uses Walras' Law. To calculate $T(w)$ numerically, one first needs to calculate $p_m(w)$, the solution to (3.8). We used an algorithm based on the contraction property used to prove Theorem 1.

This function T is closely related to a continuous time version of the tatonnement process. To see this, interpret T dynamically as giving the value $T(w)$ to $w(t + v)$ whenever $w(t)$ takes the value w. Then (4.8) becomes

$$\frac{1}{w_i(t)} \frac{w_i(t + v) - w_i(t)}{v} = \frac{Z_i(w(t))}{L_i}.$$

Letting $v \to 0$, we obtain

$$\frac{d \log w_i(t)}{dt} = \frac{Z_i(w(t))}{L_i}. \tag{4.9}$$

If Z satisfies the gross substitute property, the differential (4.9) converges globally to the unique equilibrium wage. A proof can be constructed by showing that $\mathscr{L}(w) = \max_i \{Z_i(w)/L_i\}$ is a Lyapounov function for this system. In our computational experiments, we found that setting the parameter v of (4.9) equal to one always produced monotone convergence, in the sense of sequences with decreasing Lyapounov functions $\mathscr{L}(w)$.

5. Calibration

The quantitative exercises presented below require estimated values for the parameters η, α, β, and θ, which are assumed to be common across economies, the matrices $[\kappa_{ij}]$ and

$[\omega_{ij}]$ that describe transportation costs and tariff policies, the endowments $L = (L_1, \ldots, L_n)$, and the technology parameters $\lambda = (\lambda_1, \ldots, \lambda_n)$.

The value of the substitution parameter η used in forming the tradeables aggregate does not affect any of the results reported below. [4] (See footnote 3 on this counterintuitive result.)

For α and β, we use 0.75 and 0.5, respectively. The theory divides production into two categories: tradeables and non-tradeables (final goods). We classify agriculture, mining, and manufacturing as the ''tradeable'' sectors. In the theory β is the share of labor in the total value of tradeables produced, and α is the fraction s_{fi} of employment in the non-tradeable sector. To calibrate the parameters α and β, we think of the primary factor L_i as ''labor-plus-capital'' or perhaps as ''equipped labor'', and identify $w_i L_i$ with total value added, not just compensation of employees. Thus we use data on value added, as well as on the distribution of employment and capital to calibrate α and β.

Here we discuss the evidence used for our choice of α. Using the BEA input–output tables for the U.S. for 1996–1999, the value-added share of agriculture, mining, and manufacturing sectors was about 0.2 which corresponds to $\alpha = 0.8$. Using employment shares would yield $\alpha = 0.82$, while using fixed capital shares yields $\alpha = 0.73$. The United Nations Common Database for 1993 reports value added in the tradeable sectors averaging around 0.3 for the OECD countries, and levels ranging to 0.5 and higher for poorer countries. The OECD input–output tables for 1990 imply $\alpha = 0.72$ for the OECD countries. In short, 0.75 seems a reasonable value for α in the industrialized world, although it overstates the importance of non-tradeables for economies that are still substantially pre-industrial. We use $\alpha = 0.75$ throughout this paper.

We now turn to the evidence used for the calibration of β. Using the BEA input–output tables for the U.S. for 1996–1999, the ratio of value added in manufacturing to the total value of production in this sector was about 0.38. This figure can be compared to the UN (UNIDO Industrial Statistics database) estimate of a world average value of 0.38 in manufacturing for 1998. The OECD input–output table for 1990 gives an average of 0.38 in agriculture, mining, and manufacturing. Since the labor share in most services is higher, including tradeable services in total tradeables would require a higher value of β. In the World Development Indicators data for 1996–1999, trade in goods was about 77% of total trade (exports plus imports over two) in goods and services for the U.S. and it was about 80% on average for the countries listed in Table 2. Using the U.S. input–output table for 1997, the average of the ratio of value added to gross product across all sectors, weighted by the share of each sector in U.S. exports, is 0.5. Based on these considerations, we use $\beta = 0.5$ throughout this paper.

We use values of θ in the range $[0.1, 0.25]$, with 0.15 being our preferred value based on the following information. The parameter θ describing the variability of the national idiosyncratic component to productivity is central in quantitative applications of the theory. The Eaton and Kortum specification implies a set of aggregate demands for goods produced by different countries. Demand functions of the same form can be derived from the assumption that each country has a CES utility function where goods produced in different countries appear as differentiated commodities, as in the Armington aggregation specification. The connection between these two parameterizations, based on the bilateral

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[4]The parameter η does not affect the expenditure shares D_{ij}, and so does not affect the variables F_i and s_i, and so does not affect equilibrium wages.

Table 2
Data and simulation results

Country name	Size GDP as % of world GDP	Trade volume Imports/GDP	Relative price. Consumption/ mach. & equipt.	Tariff (in %) Mean across goods	Welfare gain of eliminating tarrifs, in percent	Per capita GDP PPP adjusted U.S. = 1
	Y_i [1]	V_i [2]	P_i [3]	$100 \times (1 - \Omega_i)$ [4]	[5]	Y_i [6]
United States	27.99	0.10	1.37	5.40	0.15	1.00
Japan	15.69	0.08	1.65	5.48	0.25	0.83
Germany	7.35	0.27	1.48	5.86	0.41	0.76
ROW	5.25	0.32	0.70	11.49*	0.20	0.10
France	4.86	0.24	1.55	5.86	0.52	0.73
UK	4.30	0.28	1.28	5.86	0.56	0.70
Italy	3.85	0.25	1.27	5.86	0.59	0.71
China	2.86	0.22	0.70 (*)	18.58	0.47	0.10
Brazil	2.29	0.09	1.00	13.73	0.42	0.23
Canada	2.06	0.33	1.30	5.10	0.84	0.82
Spain	1.93	0.26	1.35	5.86	0.79	0.56
Mexico	1.40	0.29	0.72	14.26	0.56	0.26
India	1.34	0.13	0.70 (*)	33.44	2.79	0.07
Australia	1.32	0.17	1.13	5.30	0.94	0.76
Netherlands	1.31	0.48	1.51	5.86	0.89	0.75
Russia	1.09	0.28	0.48	12.47	0.63	0.24
Argentina	0.94	0.11	1.02	12.40	0.66	0.39
Switzerland	0.91	0.31	1.70	0.68*	1.56	0.88
Belgium	0.85	0.73	1.61	5.86	0.99	0.80
Sweden	0.80	0.39	1.61	5.86	1.00	0.71
Austria	0.72	0.35	1.57	5.86	1.02	0.79
Turkey	0.61	0.25	0.62	12.20	0.75	0.21
Indonesia	0.59	0.32	0.60	9.88	0.81	0.10
Denmark	0.58	0.35	1.50	5.86	1.05	0.82
Hong Kong	0.52	1.39	1.70	0.00	1.74	0.76
Norway	0.50	0.36	1.67	4.04	1.24	0.90
Thailand	0.49	0.49	0.64	18.00	0.91	0.21
Poland	0.47	0.27	0.65	15.55	0.81	0.25
Saudi Arabia	0.47	0.36	0.70(*)	12.48	0.78	0.37
South Africa	0.47	0.24	0.70 (*)	8.30	0.92	0.29
Finland	0.41	0.34	1.39	5.86	1.10	0.70
Greece	0.40	0.19	1.08	5.86	1.10	0.48
Portugal	0.36	0.34	0.97	5.86	1.11	0.50
Israel	0.32	0.39	1.65	7.55	1.00	0.60
Iran	0.32	0.21	0.48	4.90	1.21	0.18
Colombia	0.31	0.19	0.70 (*)	11.70	0.85	0.20
Venezuela, RB	0.29	0.24	0.71	12.28	0.84	0.20
Malaysia	0.29	1.00	0.70 (*)	9.18	0.93	0.26
Singapore	0.29	1.62	2.02	0.16	1.78	0.67
Ireland	0.26	0.63	1.29	5.86	1.14	0.72
Egypt	0.25	0.22	0.22	27.60	2.05	0.11
Philippines	0.25	0.48	0.67	11.22	0.88	0.12
Chile	0.23	0.29	0.82	10.25	0.91	0.28
Pakistan	0.20	0.18	0.76	39.90	5.16	0.06
New Zealand	0.19	0.26	1.29	4.84	1.25	0.61

Table 2 (*continued*)

Country name	Size GDP as % of world GDP	Trade volume Imports/GDP	Relative price. Consumption/ mach. & equipt.	Tariff (in %) Mean across goods	Welfare gain of eliminating tarrifs, in percent	Per capita GDP PPP adjusted U.S. = 1
	Y_i [1]	V_i [2]	P_i [3]	$100 \times (1 - \Omega_i)$ [5] [4]		Y_i [6]
Peru	0.18	0.16	0.72	13.30	0.88	0.15
Czech Republic	0.18	0.60	0.52	7.04	1.09	0.43
Algeria	0.16	0.27	0.70 (*)	24.60	1.63	0.16
Hungary	0.15	0.46	0.52	13.42	0.89	0.35
Ukraine	0.15	0.47	0.24	10.20	0.94	0.12
Bangladesh	0.14	0.15	0.68	21.30	1.27	0.05
Romania	0.12	0.31	0.38	16.03	0.95	0.21
Morocco	0.11	0.30	0.50	30.77	2.68	0.11
Nigeria	0.11	0.41	0.61	24.06	1.58	0.03
Vietnam	0.08	0.25	0.28	15.15	0.94	0.06
Belarus	0.08	0.62	0.30	12.63	0.92	0.20
Kazakhstan	0.07	0.40	0.37	1.15*	1.71	0.16
Slovak Rep.	0.06	0.64	0.43	6.88	1.14	0.32
Tunisia	0.06	0.45	0.40	31.17	2.77	0.18
Sri Lanka	0.05	0.41	0.57	7.67*	1.09	0.10
Simple average	1.67	0.370	0.94	11.26	1.12	0.41
Weighted average		0.210		7.62	0.50	

Sources: [1]: Share in world GDP. GDP in current dollars. From WDI 2002 cdRom, average 1994–2000. [2]: 0.5*(exports + imports)/GDP, all in current dollars. From WDI 2002 cdRom, average 1994–2000. [3]: Price of machinery and equipment relative to consumption, from benchmark PWT year 1996. Average of other countries for ROW. The countries marked with (*) are not in the 1996 PWT benchmark, so we use the average for the other countries. [3] displays the reciprocal of this number. [4]: Average 1996–2000 ad valorem tariff rate, simple average across products, from Dollar and Kraay, using World Bank database "Data on Trade and Imports Barriers", when available. When it is not available, indicated as *, import duties/imports from WDI 2002. Average of other 59 countries for ROW. [5]: Calculations described in Section 8, Figs. 6 and 7. [6]: Penn World Table average 1994–2000. The weighted averages are GDP weighted for trade volume and welfare gains, and weighted by imports for tariffs.

gravity formula, is $\theta = 1/(\sigma - 1)$, where σ is the Armington elasticity. Eaton and Kortum (2002, Sections 3–5) estimate θ using bilateral trade data as well as prices of individual goods. They obtain estimates in the range 0.08–0.28, with their preferred value being $\theta = 0.12$. Anderson and van Wincoop (2004) survey analogous bilateral, gravity-type estimates of the Armington substitution elasticity. They conclude, based on several studies, that a reasonable range is $\sigma \in [5, 10]$, which corresponds to $\theta \in [0.11, 0.25]$. Based on these findings, we report numerical experiments based on θ values of 0.1, 0.15, and 0.25. [5]

For many simulations we use a value of $\kappa = 0.75$, applied symmetrically to pairs i, j with $i \neq j$, based on the following information. Gravity equation estimates often estimate

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[5]More recently, Broda and Weinstein (2005, Table 5) have estimated import weighted elasticities (σ) for the U.S. of 10.35 for the period 1972–1988 and of 5.36 for the period 1990–2001.

jointly θ as well as trade cost (transportation costs plus tariffs and other artificial barriers). For instance, the Eaton and Kortum trade cost estimates corresponding to $\theta = 0.12$ range from 28% for neighboring countries up to 66% for distant pairs. Such high estimated "border effects" are a common feature of estimates obtained using gravity equations, as summarized by Anderson and van Wincoop (2004) and Feenstra (2004, Chapter 5). Such indirect statistical evidence, using distance measures, the presence or absence of common borders, and the like, can support κ values as low as 0.65. Anderson and van Wincoop (2004) also report direct evidence of transportation costs—freight charges—for the U.S. on the order of 4% using trade weights and 11% simple averaging. Considering imputed time costs on cargo in transit may add 9%. Such direct estimates applied to the world at large support an estimate of $\kappa = 0.9$. Our choice of $\kappa = 0.75$ for the symmetric case is a compromise between these two types of estimates.[6]

Some direct evidence on tariff costs is given in column (4) of our Table 2, described below. They range from 5% or less for wealthy economies (which account for almost all trade) to as high as 40% for some poor ones. Values like this show up in many studies. Most experts think that non-tariff barriers are at least as important, but they are hard to quantify. Anderson and van Wincoop (2004) review evidence from OECD countries, where non-tariff barriers are estimated to be equivalent to an 8% tariff. For many simulations we assume the value $\omega = 0.9$ applied symmetrically to all foreign suppliers.

Neither the endowments $L = (L_1, \ldots, L_n)$ nor the technology parameters $\lambda = (\lambda_1, \ldots, \lambda_n)$ can be observed directly, and the problem of inferring their values from characteristics we can observe will be a focus of Sections 7 and 9. Here we simply describe the limited, aggregate data set we use for this and other purposes.

We use the 2002 WDI cdRom to assemble a cross-section of the 59 economies with the largest total GDPs. These countries and the variables measured for each are listed in Table 2. We also include a residual, rest-of-world category (with 5% of world GDP), treated as the 60th economy. For each country, five variables are recorded, along with the utility gains from a simulated tariff reform that will be described in Section 8.

Column (1) is total GDP, denoted by $Y = (Y_1, \ldots, Y_n)$. These are IMF-based nominal values, converted to U.S. dollars at market exchange rates (where available). They are not on a purchasing power parity basis. In the table they are expressed as fractions of total world GDP. These flows and all the import and export flows that we used were averaged over the years 1994–2000 in order to reduce the importance of trade imbalances and year to year fluctuations, about which the theory evidently has nothing to say.

Column (2) is trade volume, denoted by $V = (V_1, \ldots, V_n)$, defined as the average of the values of imports and exports, also from the 2002 WDI cdRom, divided by GDP. Both imports and exports are defined to include services as well as goods.

Column (3) reports the ratio of the consumption goods deflator for each country to an index of the prices of machinery and equipment, from the 1996 benchmark year in the Penn World Table. We will use them as observations $P = (P_1, \ldots, P_n)$ on the prices $(p_1/p_{m1}, \ldots, p_n/p_{mn})$ in the theory.

Column (4) lists estimates of average 1996–2000 import tariff rates for each country. These are unweighted averages of ad valorem tariffs applied to different commodities. They are available in the World Bank database "Data on Trade and Import Barriers," and

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[6]In regressions reported in Section 7 we use pair-specific values of κ_{ij} based on estimates of the effect of the distance between countries i and j.

are described in Dollar and Kraay (2004). (For the three countries for which we do not have tariff data, indicated by asterisks, we substituted tariffs from a second source: ratios of import duties to imports, from WDI 2002. For the residual ROW, we used the average of the rest of the column.)

Column (5) contains simulation results that are discussed in Section 8. Column (6) is a 1994–2000 average of per capita GDP, on a purchasing power basis, from the Penn World Table. This last series is not used in any of the calculations reported below, but will aid in interpreting some of the results.

6. Examples

The algorithm proposed in Section 4 makes it easy to compute equilibria with many countries, differing arbitrarily, but it will be instructive to work through some examples first that are simple enough to solve by hand. We derive the predictions of special cases of the theory for the behavior of trade volumes and the gains from trade, as measured by the effects of changes in trade on real consumption.

For future reference, we start with the derivation of some useful formulas for trade volumes and gains, under the assumption—used also in Theorem 3—that tariffs are uniform: $\omega_{ij} = \omega_i$ for all $i \neq j$. We first derive expressions for the value of imports I_i and the volume of trade v_i, defined as the ratio of the value of imports to GDP. The value of imports I_i is the fraction of tradeable expenditures bought abroad,

$$I_i = L_i p_{\mathrm{m}i} q_i \sum_{j \neq i} D_{ij}.$$

Specializing the price and share formulas we used in Section 3 to deriving the basic equilibrium condition (3.14), we find that for this uniform tariff case,

$$v_i = \frac{I_i}{L_i p_i c_i} = \frac{(1 - \alpha)}{\beta(D_{ii}/(1 - D_{ii}) + \omega_i) + 1 - \omega_i}. \tag{6.1}$$

Notice that $(1 - \alpha)/\beta$ is an upper bound for v_i.

Using this formula, we consider first the case of costless trade: $\kappa_{ij} = \omega_{ij} = 1$, all i, j. This is the analogue of the zero-gravity case analyzed in Section 4.4 of Eaton and Kortum (2002). We solve for each country's wages w_i, the prices of non-tradeable goods relative to tradeables $p_i/p_{\mathrm{m}i}$, the shares in world GDP $L_i w_i / \sum_j L_j w_j$, and the volume of trade v_i, all as functions of the parameters L_i and λ_i. With costless trade every country buys the intermediate inputs from the same lowest cost producer, so the $p_{\mathrm{m}i}$ are the same for all countries with the common value

$$p_{\mathrm{m}} = (AB)^{1/\beta} \left(\sum_{j=1}^{n} w_j^{-\beta/\theta} \lambda_j \right)^{-\theta/\beta}.$$

Inserting this information into (3.10) yields

$$D_{ij}(w) = \left(\sum_{k=1}^{n} w_k^{-\beta/\theta} \lambda_k \right)^{-1} w_j^{-\beta/\theta} \lambda_j. \tag{6.2}$$

Notice that the expenditure shares D_{ij} do not depend on the identity i of the importer. With $\omega = 1$, (3.16) and (3.17) imply $s_{\mathrm{f}i} = \alpha$, and the excess demand functions

(3.18) can now be written as

$$Z_i(w) = \left(\frac{1-\alpha}{\beta}\right)\left(\sum_{j=1}^{n} \frac{D_{ij}(w)}{w_i} L_j w_j - L_i\right).$$

Equating $Z_i(w)$ to 0 and applying (6.2), we solve for equilibrium wages

$$w_i = \left(\pi \frac{\lambda_i}{L_i}\right)^{\theta/(\beta+\theta)}, \tag{6.3}$$

where the parameter π, which does not depend on i, will be set by whatever normalization we choose for w. (Compare to Eaton and Kortum (2002, equation (22)).)

Setting $\pi = 1$, total GDP for country i is

$$L_i w_i = L_i^{\beta/(\beta+\theta)} \lambda_i^{\theta/(\beta+\theta)}, \tag{6.4}$$

a geometric mean of productivity in tradeables λ_i and labor in efficiency units L_i. Notice that if $\theta = 0$, so that there is no variation of productivities, then country i's GDP $L_i w_i$ is simply L_i. The price of the final non-tradeable goods relative to tradeable goods in country i is given by

$$\frac{p_i}{p_m} = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha} \left(\frac{\lambda_i}{L_i}\right)^{\alpha\theta/(\beta+\theta)} p_m^{-\alpha}, \tag{6.5}$$

so that countries with high productivity λ_i in tradeables have a high relative price of non-tradeables.

We use this example to compute gains of trade by comparing the real value of final consumption in the costless-trade case with the one in autarchy. To this end, denote by c_{i0} and p_{m0} the real consumption and price of aggregate tradeable in the costless-trade case, and likewise use c_i and p_{mi} for the autarchy case. Normalizing country i's wage $w_i = 1$ in both cases, we have that

$$\frac{c_{i0}}{c_i} = \left(\frac{p_{mi}}{p_{m0}}\right)^{1-\alpha}. \tag{6.6}$$

Using (6.3) and (6.4) into the expression for $p_{m0} = p_m$ derived above, and using (2.10) for p_{mi} we obtain

$$\log\left(\frac{c_{i0}}{c_i}\right) = (1-\alpha)\frac{\theta}{\beta}\log\left(\frac{\sum_{j=1}^{n} w_j L_j}{w_i L_i}\right), \tag{6.7}$$

where $w_j L_j$ denotes country j's GDP in the costless-trade equilibrium.

The volume of trade v_i in the costless trade case is given by

$$v_i = \frac{1-\alpha}{\beta}(1-D_{ii}) = \frac{1-\alpha}{\beta}\left(1 - \frac{L_i w_i}{\sum_j L_j w_j}\right). \tag{6.8}$$

Our second example explores a different special case. We next study a symmetric equilibrium with equal sized countries $L_i = L = 1$, identical technologies, $\lambda_i = \lambda$, and uniform transportation costs and tariffs, described by

$$\kappa_{ij} = \kappa \quad \text{and} \quad \omega_{ij} = \omega \quad \text{if } i \neq j$$

and $\kappa_{ii} = \omega_{ii} = 1$. In these circumstances, there will be a common equilibrium wage $w_i = w$, all i. We normalize it to $w = 1$. Everyone will face the same tradeables price p_{m}, and (3.8) can be solved for

$$p_{\mathrm{m}} = \frac{(AB)^{1/\beta}}{(1 + (n-1)(\kappa\omega)^{1/\theta})^{\theta/\beta} \lambda^{\theta/\beta}}. \tag{6.9}$$

The price of final goods is given by (2.12), which with $w = 1$ and p_{m} given by (6.9) yields

$$p = \frac{\alpha^{-\alpha}(1-\alpha)^{-1+\alpha} (AB)^{(1-\alpha)/\beta}}{[1 + (n-1)(\kappa\omega)^{1/\theta}]^{\theta(1-\alpha)/\beta} \lambda^{\theta(1-\alpha)/\beta}}. \tag{6.10}$$

With $w = 1$, (3.10) implies

$$D_{ij} = \frac{(\kappa\omega)^{1/\theta}}{1 + (n-1)(\kappa\omega)^{1/\theta}}$$

for $i \neq j$. Then applying (6.1), the imports/GDP ratio v equals

$$v = \frac{1-\alpha}{\beta} \frac{(n-1)(\kappa\omega)^{1/\theta}}{1 + (1 + \beta\omega - \omega)(n-1)(\kappa\omega)^{1/\theta}/\beta}. \tag{6.11}$$

Nominal GDP per capita in this example is

$$pc = 1 + I(1 - \omega) = 1 + vpc(1 - \omega),$$

with v given by (6.11), and consumption, or utility, per unit of labor, is given by

$$c = \frac{1}{1 - v(1 - \omega)} \frac{1}{p}. \tag{6.12}$$

We calculate the utility gain from eliminating a tariff ω. Denote by c, I, and p the levels of consumption, imports, and the consumption price corresponding to a tariff ω and by c_0, I_0, and p_0 the values corresponding to the case of no tariff: $\omega = 1$. Then using (6.10)

$$\frac{c_0}{c} = [1 - v(1 - \omega)] \left(\frac{1 + (n-1)\kappa^{1/\theta}}{1 + (n-1)(\kappa\omega)^{1/\theta}} \right)^{\theta(1-\alpha)/\beta}. \tag{6.13}$$

We use Eqs. (6.11) and (6.13) to derive an expression for the gain $\Lambda \equiv \log(c_0/c)$ of going from pure autarchy, $\omega = 0$, to costless trade, $\kappa = \omega = 1$. Specializing (6.12) to the autarchy-costless-trade comparison, we get

$$\Lambda \equiv \log(c_0/c) = \frac{\theta(1-\alpha)}{\beta} \log(n). \tag{6.14}$$

In the last section we argued that the values $\alpha = 0.75$ and $\beta = 0.5$ are empirically reasonable, at least for the high income countries. Using the value 0.15 for θ, (6.14) then implies the gain estimate

$$\Lambda = (0.075) \log(n).$$

We can think of n in (6.14) as the ratio of world GDP to the home country's, so that taking values from Table 2, $n = 3.6 \cong 1/0.28$ for the United States, $n = 6.2$ for Japan, and $n = 170$ for Denmark. In percentage terms, this formula implies benefits of 10% of consumption for the U.S., 14% for Japan, and 38% for Denmark. These are fantasy

calculations—even ideally free trade is not costless trade—but they give useful upper bounds for the magnitude of gains we will be discussing in the rest of the paper.

Next we return to the case of positive transportation costs and tariffs, retaining symmetry. The properties of the volume and welfare gain functions defined in (6.11) and (6.12)–(6.14) are illustrated in Figs. 1 and 2. In both figures we used the values $\alpha = 0.75$, $\beta = 0.5$, $\kappa = 0.75$ and $\omega = 0.9$. In Fig. 1, the values of θ are varied, as shown. In Fig. 2, $\theta = 0.15$.

Fig. 1 shows the gains of eliminating a tariff corresponding to $\omega = 0.9$ for three values of θ. As expected, the gains from eliminating a 10% tariff are far smaller than the gains (6.14) of moving from autarchy. Notice too that the gains in Fig. 1 are not always decreasing in the size of the country: this is due to the effect of the revenue from the tariff. As formula (6.13) makes clear, there are two effects of eliminating a tariff (setting $\omega = 1$). One is to reduce the price of the final, non-tradeable consumption good (that is, to increase p/p_0). The other is that tariff revenues are lost. These effects have opposing effects on welfare, and are stronger if n is large (i.e. if countries are small). The first effect must dominate— eliminating the tariff must be welfare improving—but the welfare gain need not decrease monotonically in n. We also computed the gains from eliminating a tariff for different ω values (not shown in Fig. 1). Holding θ fixed at 0.15, the difference in the welfare gains from eliminating a 30% versus a 20% tariff is smaller than the difference between
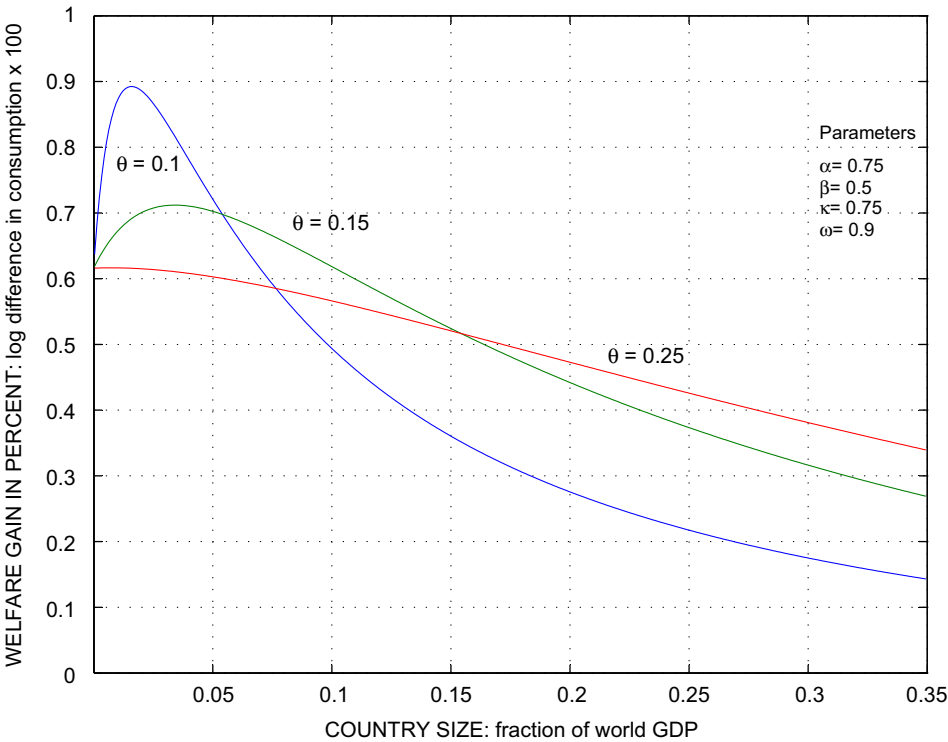

Fig. 1. Welfare gains from eliminating a 10% tariff. Three values of dispersion θ.
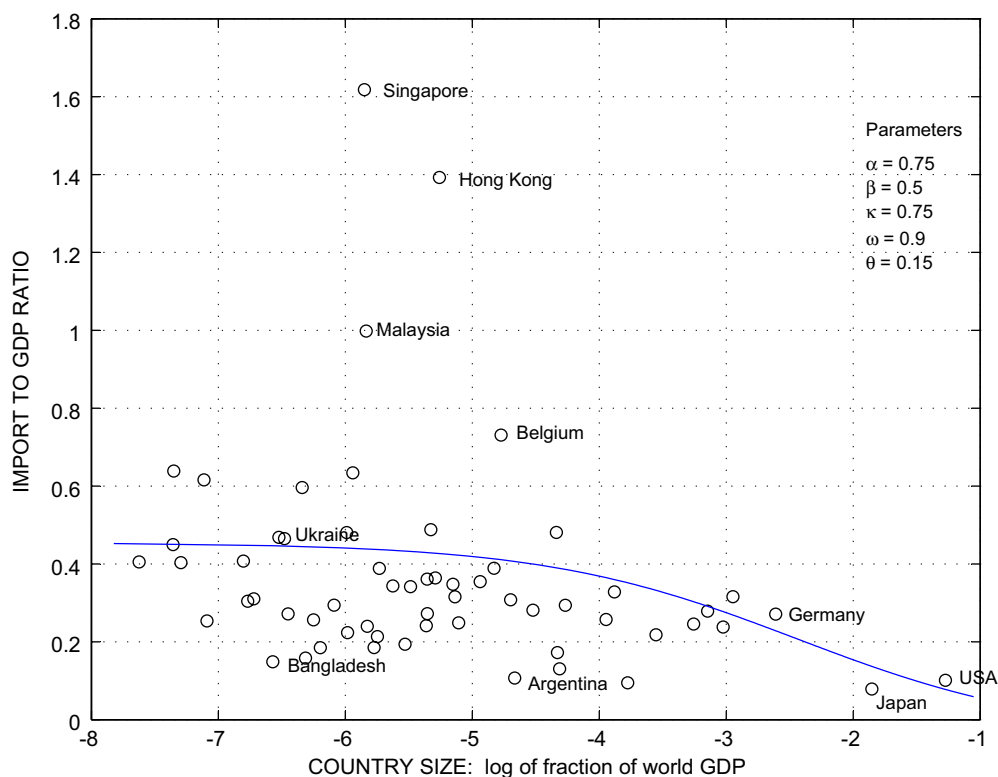
Fig. 2. Volume of trade.

eliminating a 40% versus a 30% tariff. For countries that are 5% of world GDP or smaller, the gains from eliminating a 30% tariffs are about 6%.

Fig. 2 plots the relation between the volume of trade, (6.11), measured as the ratio of import value to GDP, and the size of the economy, measured as the log of the economy's GDP relative to the world total. The volume of trade is decreasing in size, and is bounded above by the ratio of $(1 - \alpha)$ to $(1 - (1 - \beta)\omega)$, which equals 0.45 for our benchmark parameter values and $\omega = 0.9$. Small economies have trade volumes that nearly attain the bound. We have also experimented varying the value of ω between 0.7 and 1 (not shown in Fig. 2). The effects of variations in ω are large for economies of all sizes: for an economy that is 5% of world GDP, trade volume increases from 10% to 27%, as tariffs are decreased from 30% to 10%.

Figs. 1 and 2 refer to symmetric world economies, where all economies have the same size and technology levels. These are *not* cross-sections. The scatter of points in Fig. 2 are GDPs, Y_i, and import to GDP ratios, V_i, for 60 large countries: columns (1) and (2) in Table 2. These data *are* a cross-section. But the continuous curve on the picture is calculated for a symmetric world, with a 10% tariff ($\omega = 0.9$) just as in Fig. 1.

The data and all of the parameter values used to compute the theoretical curve in Fig. 2 have all been discussed in Section 5. No adjustments have been made to fit the curve to the data. The theoretical curve reproduces the negative relation between trade volume and size

in the data, but implies a slightly higher average trade volume than the average implied by the data.

In a world with very different national policies toward trade one would not expect equal trade volumes at each GDP level, even if the assumptions underlying the construction of Fig. 2 were correct: the points should not lie on the theoretical curve. If the theory were accurate, the rich economies with more or less free trade—roughly, the OECD—should be near the curve, and the protectionist economies should fall below it by varying amounts.

There are also four striking outliers in the figure: Hong Kong, Singapore, Malaysia, and Belgium, with trade volumes much higher than others', and much higher than our theoretical upper bound. It is a characteristic of port cities that a high volume of goods passes through, counted as imports when they enter and exports when they leave. Countries in which such ports are important would appear as "low β" countries in our parameterization, so it is possible that relaxing the assumption that β is uniform across economies would yield a better fit of the volume-size curves in Figs. 2 and 4, below.[7]

7. Volume of trade

The algorithm described in Section 4 lets us replace the theoretical curve in Fig. 2, based on an assumption of symmetry, with the volume predictions of the general theory, calibrated to fit the *actual* distribution of economies by size. In addition, the general theory lets us incorporate other kinds of international differences—for example, differences in tariff policies—into the trade volume predictions. We do this in this section, in two ways.

Once the assumption of identical countries is dropped there is no reason for equilibrium wages to be equal, and if they are not, observed GDPs Y cannot be taken as direct observations on equipped labor endowments L. Even without tariff distortions, Y_i will be the product $w_i L_i$, and neither w nor L can be directly inferred from observations on Y. What can be done about this depends on what other data are used.

The simplest calibration method uses the theory to infer w and L from the data on Y only. To do this, it seems a natural starting point to think of the parameter λ_i in any country as *proportional* to that country's equipped labor endowment L_i. That is, we assume that if country 1 has twice the labor endowment of country 2, country 1 will also have twice as many "draws" from the distribution of productivities. With exponentially distributed productivities, this means $\lambda_1 = 2\lambda_2$, and in general, that the vector λ is proportional to the endowment vector L. This assumption of uniform *ratios* λ_i/L_i surely has more appeal than assuming uniform *levels* λ_i. In the latter case, there would be enormous diseconomies of size: Denmark would be the low cost producer of as many goods as the United States is, but with its much smaller workforce, Danish wage rates

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[7]There is some evidence supporting this interpretation of the outliers in Fig. 2 as low-β port cities. The UN Statistics Division (Commodity Trade Statistics Database: COMTRADE) collects data on re-exports of goods—exports of goods that have been imported with no local value added—for 50 countries, 10 of which are in our 60 country data set. Of the four outlying high-volume countries in Fig. 2, only Hong Kong has re-export data. Hong Kong reports that goods re-exports averaged about 85% of total goods exports during 1994–1999. Since goods exports were about 86% of total exports, removing re-exports from total exports would lead to a reduction in the estimated trade volume in Hong Kong from 1.4 to $1.4 \times (0.14 + 0.86 \times 0.15) = 0.38$, or to about the level of the theoretical curve in Fig. 2. For the other 50 countries in the COMTRADE data set, re-exports are less than 10% of goods exports, and for most countries they are less than 1% of the total.

would be bid up to much higher levels than wages in the U.S.[8] Of course, these are not the only possibilities.

Under this assumption, the equilibrium condition

$$Z(w, L, \lambda) = 0, \qquad (7.1)$$

written so as to emphasize the dependence of the excess demand system Z on L and λ, is specialized to $Z(w, L, kL) = 0$. The choice of the constant k is just a matter of the units chosen for tradeables and labor input. We set it equal to one:

$$Z(w, L, L) = 0. \qquad (7.2)$$

A second set of equations in the variables w and L is given by the GDP-equals-national income conditions

$$L \cdot \widehat{w}(w, L) = Y, \qquad (7.3)$$

where $\widehat{w}_i(w, \lambda)$ is w_i adjusted for indirect taxes using the functions $s_{\mathrm{f}i}$ and F_i of the equilibrium wage vector defined in (3.16) and (3.17):

$$\widehat{w}_i(w, \lambda) = w_i \left[1 + \frac{(1 - s_{\mathrm{f}i}(w, \lambda))(1 - F_i(w, \lambda))}{\beta F_i(w, \lambda)} \right]. \qquad (7.4)$$

(Notice that without tariffs, $\omega_i = 1$ and $\widehat{w}_i(w, \lambda) = w_i$.) We view (7.2) and (7.3) as $2n$ equations in the pair (w, L), given the data Y.

We describe the algorithm used to solve (7.2)–(7.3). Define $w^*(\lambda, L)$ to be the function that solves (7.1). Its values can be calculated using the algorithm described in Section 4. Define φ by

$$\varphi_i(L) = \frac{Y_i}{\widehat{w}_i(w^*(L, L), L)} \Big/ \sum_{j=1}^{n} \frac{Y_j}{\widehat{w}_j(w^*(L, L), L)}. \qquad (7.5)$$

Then φ maps the n-dimensional simplex Δ_n into itself, and if L is a fixed point of φ, the pair $(w^*(L, L), L)$ satisfies (7.2)–(7.3). We located a fixed point by iterating using (7.5), applying the algorithm from Section 4 to compute w^* at each iteration, from an initial guess for L. In practice, this algorithm always converged to a fixed point.[9]

Fig. 3 displays the equilibrium wages calculated in this way as a function of size (GDP share). The benchmark parameters used in Figs. 1 and 2 were used, and the same range of θ values. The equilibrium wages increase with size, reflecting the scale economy in transportation enjoyed by larger economies. Since the technology level λ is assumed proportional to size L in the construction of the figure, λ cannot provide a second source of wage variation.

Given equilibrium wages and endowments computed in this way, the analogues to Figs. 1 and 2 are readily constructed. We were surprised to find that the new figures constructed in this way were very similar to the figures based on the assumption of equal

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[8]See the costless trade formula for wages (6.3), for example. Note that although the hypothesis $\lambda = kL$ avoids an unrealistic *diseconomy* of scale, it leaves in place an unrealistic scale *economy*. In the theory, transportation costs within an economy, no matter how large, are taken to be zero. Insofar as the parameters κ_{ij} measure the resources used in moving goods over space, this is a deficiency that can only be fixed by introducing some actual geography.

[9]Indeed, under the assumptions of Theorem 3 it can be shown that $\varphi : int(\Delta_n) \to A \subset int(\Delta_n)$, where A is closed and convex, and that φ is continuous, so that the existence of a fixed point follows from Brouwer's theorem.
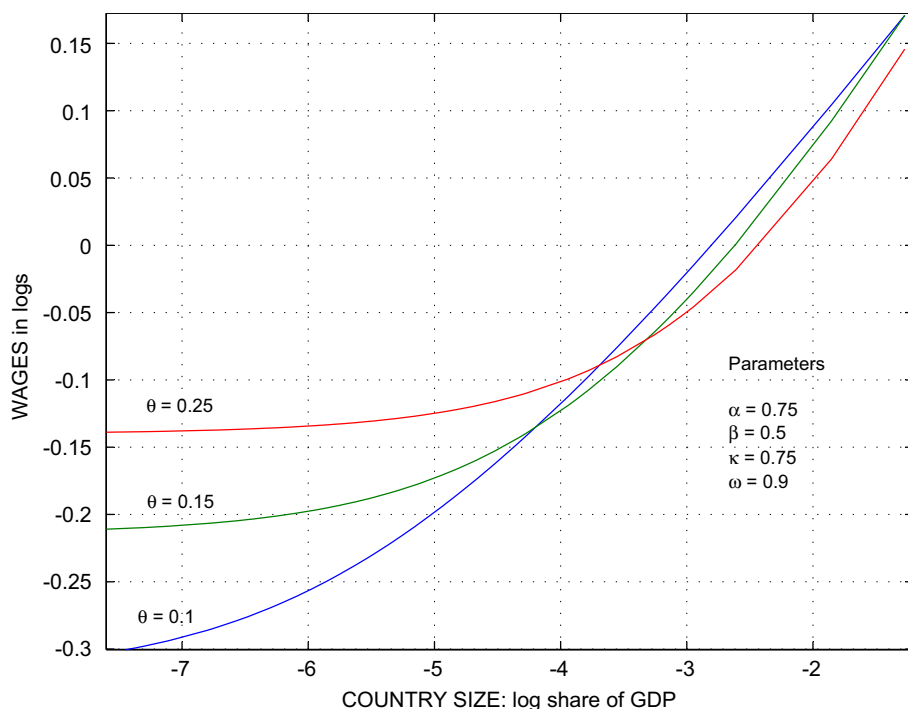
Fig. 3. Wages versus size. Three values of θ.

size countries and wage equality, even though these two sets of assumptions seem very different.

This finding is illustrated in Fig. 4. This figure is the exact analogue to Fig. 2, except that the very high volume countries have been left off so as to get a clearer picture of the others. The volume and GDP data used in both figures are the same. The theoretical curve plotted in Fig. 2, based on a symmetric model with identical countries and uniform wages, is reproduced in Fig. 4, as is a new second curve, constructed by solving the general equilibrium system with endowments L calibrated in the way we have just described. Despite the completely different computational methods used to construct them the two curves are very similar, except for the largest economies—Japan and the U.S.—where the symmetric model predicts a smaller volume than the more realistic one does. This exception is due to the effects of size on wages in the calibrated economy, shown in Fig. 3. The implication we draw from the similarity of the two curves is that even though an economy's size relative to the world economy matters for the determination of trade volume, the way the rest of the world is configured matters very little.

Neither of these curves is a particularly good fit: they pick up the effects of size on trade volume, and nothing else. Some other factors were remarked on in our discussion of Fig. 2, and other possible influences will occur to anyone. Here we examine the possible effects of tariff policies, under the assumption—also used in Sections 4 and 6—that each country i imposes a uniform tariff factor ω_i on all countries j.

We introduce tariffs simply by repeating the simulation (7.2)–(7.3) with a uniform tariff factor of $\omega = 0.9$ replaced by the vector $\Omega = (\Omega_1, \ldots, \Omega_n)$ of observed tariff factors from
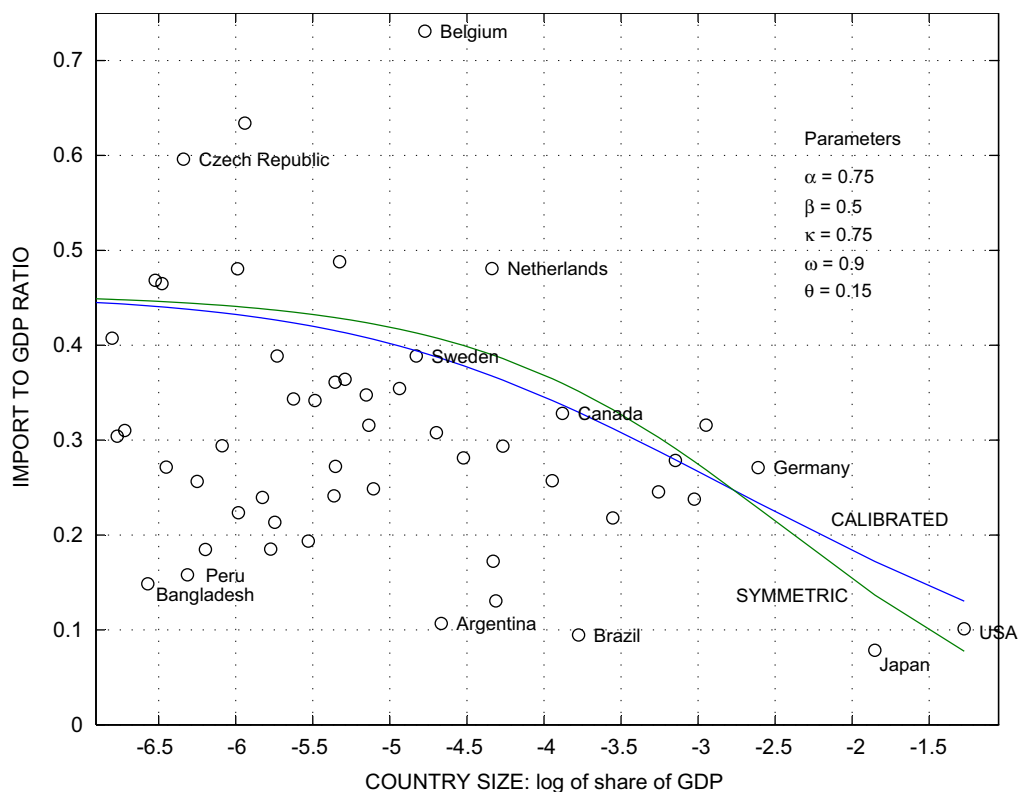
Fig. 4. Volume of trade.

the rates in column (4) of Table 2, interpreting Ω_i as the uniform tariff factor that country i imposes on all imports. To measure the impact on the fit of the model from considering the cross-country variation in tariffs, we report that the correlation coefficient between the logarithm of the trade volume in the data—those from column (2) in Table 2—and the model with the same tariff for all countries—those from the "calibrated" line of Fig. 4—is 0.49, while the same correlation coefficient using the trade volume for the model incorporating the heterogeneity in tariffs is 0.59. As a basis for comparison, we ran a regression of volume on GDP and tariff levels for the 60 countries. The results were

$$\log(\hat{V}_i) = a - (0.23)\log(Y_i) - (0.029)(100)(1 - \Omega_i) \tag{7.6}$$

with an $R^2 = (0.58)^2 = 0.34$.[10] The slope parameters in (7.6) are freely chosen to fit the data. The effect of tariffs derived from the calibrated model was not selected in any way to improve the fit, and no actual tariff data (beyond average levels) were used in the calibration. Yet the cross-sectional variation in tariffs and GDPs have essentially the same ability to fit when constrained to work through our theory as with a freely chosen regression coefficient.

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[10]We obtained only slightly different estimates when the four countries with trade volumes exceeding 0.75 were excluded from the regressions.

We also experimented by taking into account the main free trade agreements between countries in our data set and by making κ_{ij} a function of the distance between countries. In both cases the simulated trade volumes became more correlated with the trade volume from the data.

To model the effect of the free trade agreements we let $\omega_{ij} = 1$ for any two countries with a free-trade agreement, and otherwise used the tariffs described in Table 2. We considered the European Union, NAFTA, CEFTA, and Mercosur. In this case the correlation between the (log) model trade volume and the (log) trade volume in the data was 0.61.

To model the effect on distance on transportation cost we let d_{ij} be the distance between countries i and j, measured in linear miles between the capitals of the two countries, normalized so that the average distance equals 1. We let $\kappa_{ij} = \kappa \exp(-\delta_0(d_{ij} - 1))$, so that δ_0 has the interpretation of the elasticity of transportation cost with respect to distance. We use $\delta_0 = 0.3$, a number consistent with the estimates of the elasticity of freight cost to distance (see, for instance, Hummels, 2001) and set κ so that the mean bilateral value of κ_{ij} is 0.75. In this case, we found that the correlation between the (log) model trade volume and the (log) trade volume in the data was 0.62 if we let $\omega_{ij} = 0.9$ be uniform across countries, and 0.69 if we incorporate the cross-country variation in ω_{ij} as in column (4) of Table 2.

8. Gains from trade

In Section 6 we studied the gains from trade using the autarchy versus costless trade example and hypothetical tariff reductions in the context of a symmetric world economy. In this section, we incorporate differences among countries in a more realistic way, using the general version of the theory calibrated to the actual world GDP distribution and the measured tariff factors Ω used in Section 7.

Results of a specific, world-wide tariff reduction are described below and displayed in Fig. 6. But before turning to these results, it will be helpful to study the effects of unilateral tariff changes in a small economy, or to calculate the "best-response function" for a small economy, taking the tariff policies of the rest of the world as given. Studying this problem will help us to interpret the results of uniform, multilateral tariff changes.

We focus on economy 1 as the "small economy." We use the notation $L_{-1} = (L_2, L_3, \ldots, L_n)$ and $\lambda_{-1} = (\lambda_2, \lambda_3, \ldots, \lambda_n)$ to denote the parameters corresponding to countries other than 1, and similarly with w_{-1}, c_{-1}, and p_{m-1}. Assume that country 1 applies a uniform tariff ω to all its imports, and assume that all other countries apply a common tariff $\widehat{\omega}$ to country 1's exports. Our interest is in analyzing the behavior of country 1's welfare (final goods consumption) c_1 as a function of the pair $(\omega, \widehat{\omega})$.

To make precise the idea that country 1 is small, consider a sequence of world economies $\{L^r, \lambda^r\}$ with $(L^r, \lambda^r) \to (L, \lambda)$, and with $\lambda_1^r / L_1^r = k > 0$ along the sequence. Let $\{w^r, c^r, p_m^r\}$ denote the corresponding sequence of equilibrium values, and let (w, c, p_m) be the corresponding equilibrium values of the limiting economy. In Appendix C we establish that as $L_1^r \to 0$, the limiting behavior $(w_{-1}, c_{-1}, p_{m-1})$ of the other $n - 1$ economies is equal to the equilibrium of a world economy with $n - 1$ countries and endowments (L_{-1}, λ_{-1}), and that the limiting behavior of economy 1, (w_1, c_1, p_{m1}), is given by

$$w_1 = \left[\frac{\alpha + (\beta - \alpha)\omega}{\omega^{1-(1-\beta)/\theta}} k \right]^{\theta/(\theta+\beta)} \widehat{\omega}^{(1+\theta)/(\theta+\beta)} \hat{w}_1, \tag{8.1}$$
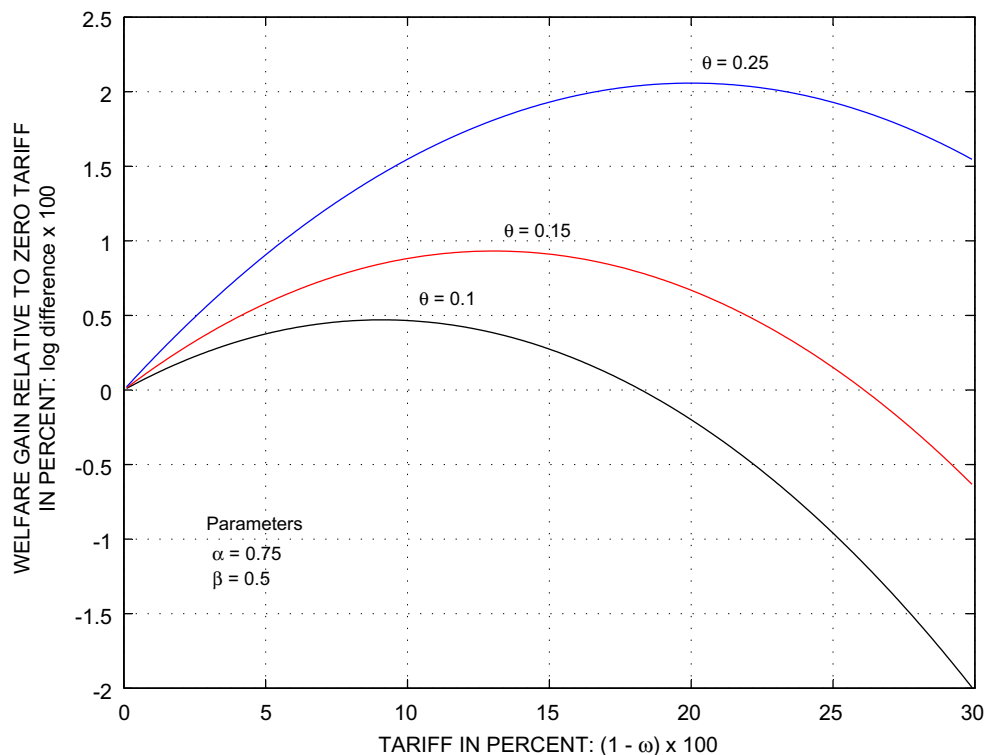
Fig. 5. Welfare effects of tariff factor ω for a small open economy. Three values of θ.

$$c_1 = \omega^{(1-\alpha)/(\theta+\beta)}[1 + (\beta - 1)\omega]$$
$$\times [\alpha + (\beta - \alpha)\omega]^{-(\alpha\theta+\beta)/(\theta+\beta)}\widehat{\omega}^{(1-\alpha)(1+\theta)/(\theta+\beta)}k^{(1-\alpha)\theta/(\theta+\beta)}\,\hat{c}_1, \qquad (8.2)$$

and

$$p_{m1} = \hat{p}_{m1}/\omega_1, \qquad (8.3)$$

where the numbers $\hat{p}_{m1}$, $\hat{w}_1$, and $\hat{c}_1$ do not depend on ω, $\widehat{\omega}$, and k.

The expression (8.2) can then be used to calculate the optimal tariff: the level of ω that maximizes utility c_1 for country 1. One can show, provided that $\beta < \alpha$, that there is a unique ω^* that maximizes c_1 and that the optimal tariff is strictly positive (that $\omega^* \in (0, 1)$) and increasing in θ. This result is quite intuitive. For small θ values, there are small differences across countries, and hence a given increase in tariffs produces a large decrease in the demands for the products of country 1. Consequently, the optimal tariff increases in θ. Fig. 5, based on (8.2), illustrates the way utility c_1 varies with ω for different θ values. The vertical axis in the figure is $\log(c_1(\omega)) - \log(c_1(1))$.

Should we be surprised at this persistence of market power as the economy becomes vanishingly small? Eq. (8.3) states that as a buyer of tradeable goods the limit economy 1 is a price-taker. The set of tradeables it produces for home use has measure zero and no effect on the pre-tax price $\hat{p}_{m1}$ of the tradeables aggregate. But under the Eaton–Kortum technology, any economy, no matter how small, has some goods which it is *extremely*

efficient at producing, and even a small country can serve a large part of the world market for these particular goods. In our case, this market power cannot be exploited by individual sellers, because others in the same economy have free access to the efficient, constant-returns technology. But as Fig. 5 illustrates, it can be exploited by the government. Since we do not permit export duties, the way to restrict supply of these goods is through import tariffs.[11]

Not surprisingly, the optimal tariff is larger for higher θ, when the elasticity of the demand for country 1's exports is lower. Indeed, the welfare gains in Fig. 5 peak at approximately $1 - \omega = \theta/(1 + \theta)$, the optimal markup charged by a monopolist facing a demand with elasticity given by $(1 + \theta)/\theta$. (See Section 5 for the interpretation of θ as the determinant of the elasticity of export's demand.) It follows from these observations that a Nash equilibrium of a world-wide tariff game involving many small countries would involve strictly positive tariff levels for every country.

We end this section with a report of the welfare benefits of a trade liberalization for a world economy calibrated to the GDP distribution and with an initial level of tariff as in column (2) of Table 2, the calibrated economy displayed in Fig. 4. In particular, we compare consumption in the calibrated equilibrium with the one that results when the observed tariff factors Ω are replaced with the free-trade factors $(1, 1, \ldots, 1)$. These gains are reported in column (4) of Table 2.[12] They are shown in Fig. 6, plotted against each country's initial tariff rate, $(1 - \Omega_i) \times 100$. Fig. 7 reports the results of the same calculation, with the welfare gains plotted against size to facilitate comparison with Fig. 1.[13]

One can see the optimal tariff structure in Fig. 5 reflected in the U-shaped pattern of gains from trade shown in Fig. 6. The figure shows the effect of a tariff reform beginning from a situation in which tariffs vary realistically cross-sectionally, and ending with all tariffs at zero. In the post-reform situation, every country would like to have its own tariff at the best response to a world of zero tariffs. Countries with initial tariffs near this level lose the most from moving their own tariffs to zero, though they still gain from others' tariff reductions (see (8.2)). Countries with very high initial tariffs gain from a reduction to the optimal tariff, but then lose some of these gains back as they continue toward zero. Countries with very low initial tariffs were never at their optimal tariff, so they only gain from others' reductions. From Fig. 7, we can see two features already present in the symmetric example of Fig. 1: first, for small countries with tariffs near 10% both figures give similar estimates of the welfare gains and, second, that the gains from trade are larger

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[11]A similar point is made in Helpman and Krugman (1989) and Gros (1987), in a context of imperfect competition. Our analysis of the optimal tariff applies only for the small open economy case, but we have numerically verified for our calibrated economy that the calculations in Fig. 5 are an excellent approximation for all but the largest economies. Compare to the Eaton and Kortum (2002, p. 1774) finding that if the U.S. were to reduce its tariffs on manufacturing goods unilaterally, it would suffer a welfare loss of about 0.0005%.

[12]The overall magnitude of these estimates is within the rather wide range of estimates of static gains from tariff elimination that other economists have obtained. For example, Anderson (2004, Table 1) reports estimates of the gains from a hypothetical "full global liberalization" carried out in 2005 that range from $254 to $2080b. (1995 dollars). Using an estimate of 2005 world GDP in 1995 dollars of $32000b. (our calculation), the implied range in percent is 0.8–6.5. Most studies are nearer the lower end of this range. Our estimates are also similar to those reported by Eaton and Kortum (2002) for the mobile labor version of their model.

[13]Incorporating distance into the transportation cost and the main free trade agreements in the modelling of $[\omega_{ij}]$ as explained in Section 7 have very small effects on patterns for the estimated welfare effects of a world trade liberalization.
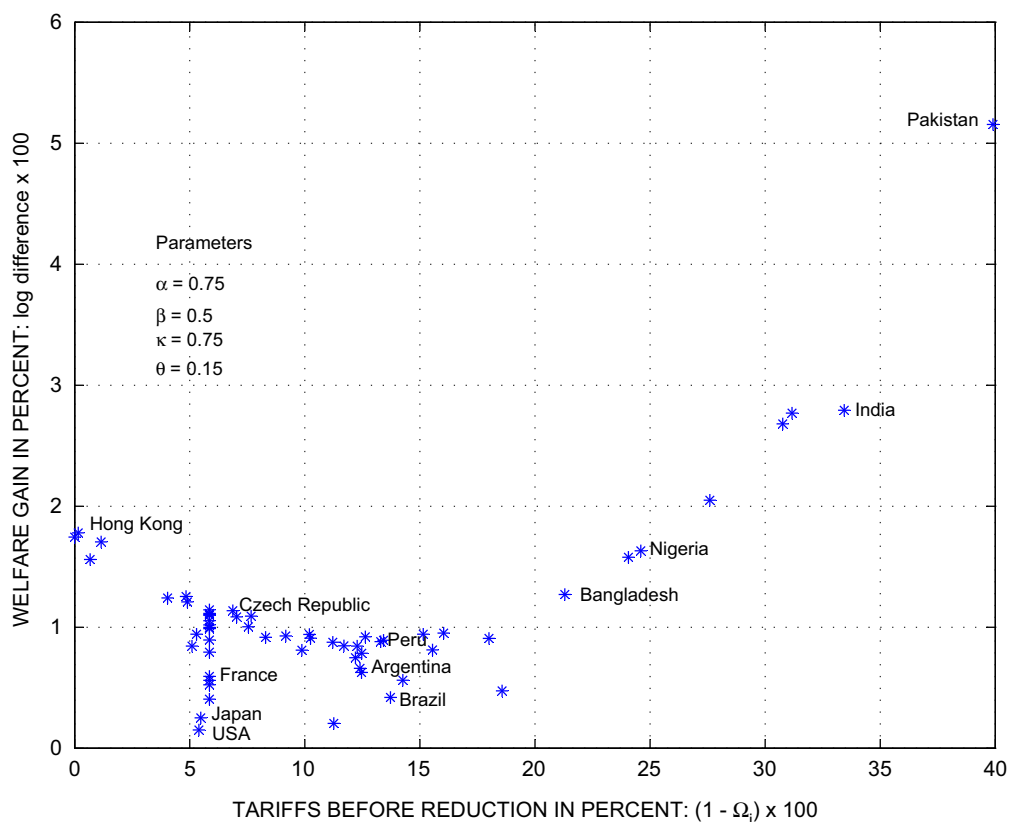
Fig. 6. Welfare gains from eliminating tariffs.

for smaller countries. Using the averages presented in Table 2, the world-wide cost of the current level of import tariffs is 0.5% of world GDP and 31% of world tariff revenues.

9. Relative productivities and relative prices

In our calibration of Section 7 we imposed the proportionality assumption $\lambda = kL$, and used data on total GDP to estimate the vector L of the countries' endowments of equipped labor. We did not quite take GDP as a *measure* of the unobserved endowments—see Fig. 3—but we came very close to doing so. In this section we refine our calibration to take into account that countries may differ in their relative productivities across tradeable and non-tradeable goods.

To understand the calibration in this section we review the effect of L_i and λ_i in the equilibrium distribution of GDPs and relative prices, which is fully characterized in the costless-trade case analyzed in Section 6. In this case, if L_i doubled and all prices were to stay unaltered, country i would produce twice as much tradeable and non-tradeable goods. But at the same prices, the demand for tradeable goods will be the same, so to keep trade balanced its exports prices must decrease to become more competitive. This is achieved by decreasing w_i by the factor $\theta/(\beta + \theta)$, as shown in (6.3). Hence the effect of doubling L_i on
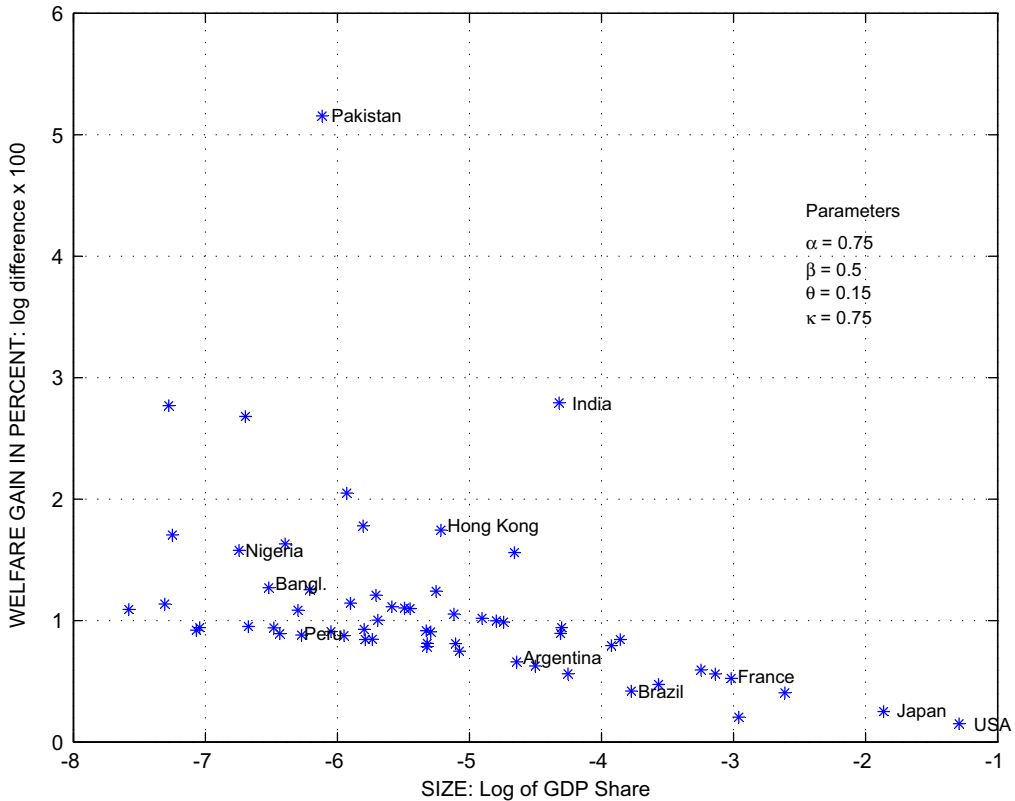
Fig. 7. Welfare gains from eliminating tariffs.

GDP is to increase it only by the factor $\beta/(\beta + \theta)$, as shown in (6.4). Given that trade is costless, the price of the tradeable aggregate is common to all countries, and thus the effect on country i's relative price of final non-tradeable goods to tradeable goods, $P_i = p_i/p_m$, depends exclusively on the effect on the price of the non-tradeable good p_i. As shown in (6.5), the relative price of the non-tradeable good decreases, since this good is produced using the tradeable aggregate and labor, and the wage has decreased.

Consider now the case where λ_i doubles. In this case, if all w's were to remain unaltered, country i would increase the number of tradeable goods for which it is the world's cheapest producer, decreasing its export prices, and hence increasing the value of the demand for its imports. To keep trade balanced, its export prices must increase, which is achieved by a higher value of w_i, as shown in (6.3). Thus, the value of country i's GDP increases, as shown in (6.4). By the same reason, as in the case of an increase in L_i, the relative price $P_i = p_i/p_m$ increases as shown in (6.5).[14] Hence since λ_i and L_i affect both GDP and the

---

[14]The idea that countries with higher productivity in tradeable goods will have a higher relative price of non-tradeables is known as the Balassa–Samuelson effect (Balassa, 1964; Samuelson, 1964).

relative prices of each country, from (6.4) and (6.5), we can solve for L_i and λ_i

$$L_i = \frac{1}{\pi} \, Y_i P_i^{-1/\alpha}$$

and

$$\lambda_i = \pi^{\beta/\theta} Y_i P_i^{\beta/(\alpha\theta)},$$

in terms of the observables, GDP Y_i and relative prices P_i, where π is a constant independent of i.

In the general case, Eq. (2.12) implies that equilibrium wages and prices satisfy

$$\frac{p_i}{p_{\mathrm{m}i}} = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha}\left(\frac{w_i}{p_{\mathrm{m}i}(w,\lambda)}\right)^\alpha$$

for all i, where the notation emphasizes that the right side can be computed as a function of w and λ. Write $\psi(w,\lambda)$ for the n-vector of right side values, and view the relative prices on the left as the theoretical counterparts to the observed relative prices P_i in column (3) of Table 2. Then we can obtain estimates of $w, L,$ and λ by solving

$$Z(w, L, \lambda) = 0, \tag{9.1}$$

$$L \cdot \widehat{w}(w, \lambda) = Y, \tag{9.2}$$

and

$$\psi(w, \lambda) = P, \tag{9.3}$$

where $\widehat{w}$ is defined in (7.4). System (9.1)–(9.3) consists of $3n$ equations to be solved for the $3n$ unknowns $w, L,$ and λ. To solve this system, we used an algorithm that parallels the one described in Section 7.

The asterisks in Fig. 8 are the equilibrium wages implied by this calculation, plotted against the log of GDP. The circles in the figure are the equilibrium wages from the analogous calculation described in Section 7, in which the ratios λ_i/L_i are constrained to equal a common value. (Neither reported calculation uses country specific-tariff data, but we have carried out versions that do: the figure is not much affected.) One can see that constraining λ_i/L_i to be constant suppresses most of the cross-country variability in equilibrium wages, relative to the case where variations in λ_i and L_i are permitted to exercise independent influences. The two log standard deviations in the upper right of the figure quantify this difference. Putting it differently, to match the disparity on the relative prices of non-tradeable to tradeable goods requires large variations on the relative productivity of tradeable to non-tradeable goods measured by λ_i/L_i, which then imply large variations in the relative wages w_i.

We find that the simulations of this section, with λ_i/L_i left free, give very similar answers on the volume of, and (not reported here) gains from, trade as do the constrained simulations of Section 7: estimation of the effects of trade policy is not closely connected to the estimation of sources of relative price differences. This is not a coincidence. To understand it, notice that in the model, volume and gains from trade depend on the level of tariffs and transportation cost, as well as on country size, as measured by total GDP. To see this in the starkest possible way, consider the costless-trade case. In this case, (6.8) and (6.7) imply that trade volume and gains of trade depend only on the country i's GDP relative to the world GDP, as given by $L_iw_i/(\sum w_jL_j)$, regardless of the combination of L_i
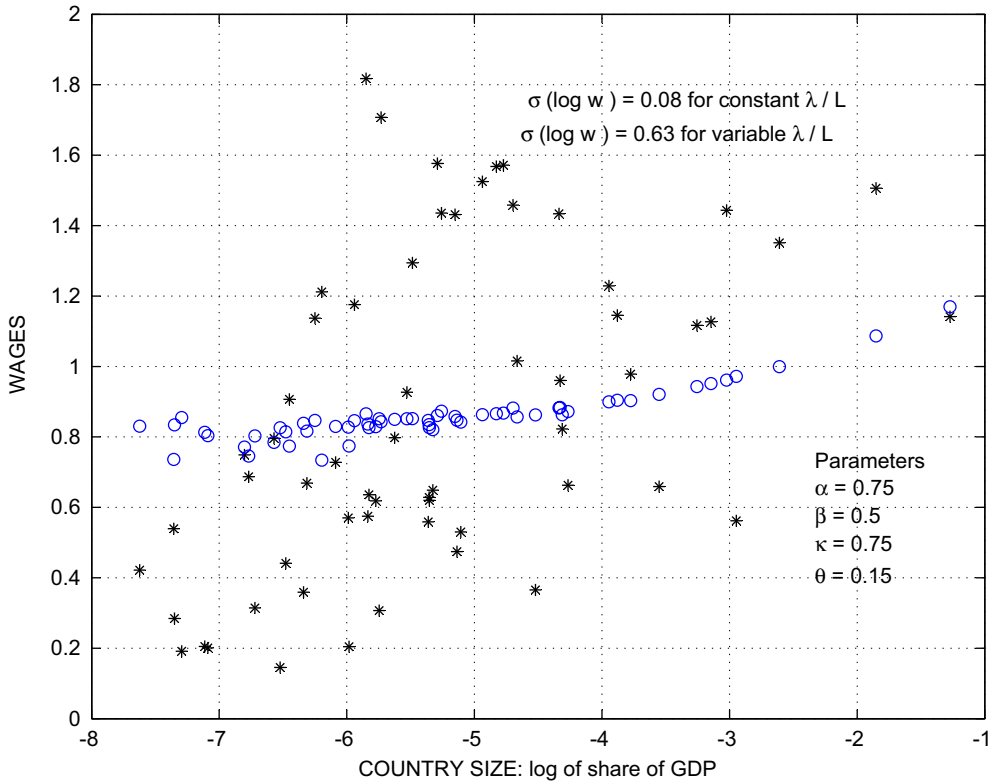
Fig. 8. Wages in the two calibrations.

and λ_i that determines country j's GDP, $Y_i = L_i^{\beta/(\beta+\theta)}\lambda_i^{\theta/(\beta+\theta)}$. Hence, as a first approximation, once we calibrate to the distribution of total GDPs, we have fixed the determinants of volume and gains of trade regardless of whether we have chosen λ and L as in Section 7 or as in this section.

We have referred to L as the countries' endowments of labor, or more generally equipped labor, but it is important to emphasize that our model does not distinguish between units of "raw" labor, and units of labor in efficiency units. Hence our procedure of calibrating to the world distribution of total GDP and relative prices is completely silent about whether differences in per-capita income are due to differences in per-capita endowments or differences in the productivity of these endowments.[15] Nevertheless, per-capita income differences do affect our calibrated values of λ_i/L_i, and hence equilibrium wages in an indirect way. As it can be seen from the data in Table 2, the observed relative prices P_i are strongly correlated with per capita GDPs. Thus, since the calibration of this section, essentially, uses cross-country variations in relative prices to infer λ_i/L_i, then it implies that the resulting equilibrium wages w_i are positively correlated with per-capita GDP.

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[15]This is to be compared with the conclusions of Parente and Prescott (2000) and Hall and Jones (1999) that differences in per-capita income are mostly attributed to TFP differences.

10. Conclusions

We think of this paper as a kind of trial run of a particular version of the Eaton and Kortum trade theory. As we formulated the theory, the problem of solving for equilibrium prices and quantities can be reduced to solving for the vector of equilibrium wages in the n countries that comprise the world economy. We have shown that such an equilibrium exists under reasonable conditions and that under somewhat tighter assumptions it will be unique. We have proposed and tested an algorithm that is essentially a tatonnement process for calculating equilibria. We have discovered that "toy versions" of the theory can provide surprisingly accurate approximations to predictions about wages, trade volumes, and gains from trade, so pencil-and-paper calculations can be used to provide inexpensive checks on quantitative conjectures and to help interpret simulation results.

For the most part, objects in the theory match up naturally to counterparts in the national income and product accounts, input–output accounts, and standard trade statistics. This makes much of the calibration easy to carry out, lets us focus attention sharply on small regions of the theory's parameter space, and facilitates interpretation of simulation results. These features are essential to successful quantitative economics.

We have kept the analysis in this paper on a strictly static basis, in order to keep complications within bounds and to understand better the connections with other trade theories. A more satisfactory treatment of physical capital is needed, in which the dynamics of capital accumulation can be examined as well as the contributions of capital to current production. Capital goods play a large role in trade, so it is natural to conjecture that tariff and other barriers have large effects on the return to investment and hence on capital accumulation and growth. We are currently exploring this topic. Another natural direction, already examined by Eaton and Kortum (1999), will be to introduce technology diffusion by introducing a law of motion for the parameters λ. Perhaps in some combination such extensions can help us to discover the long-sought theoretical link between trade and growth.

Appendix A. Derivation of (3.15) and (3.16)

We first verify (3.16), using the share formulas from both producing sectors and the trade balance condition (3.11). The share formulas in final goods production are

$$w_i s_{fi} = \alpha p_i c_i \quad \text{and} \quad p_{mi} q_{fi} = (1 - \alpha) p_i c_i,$$

implying that

$$w_i s_{fi} = \frac{\alpha}{1 - \alpha} p_{mi} q_{fi}. \tag{A.1}$$

The share formulas for tradeables production are

$$L_i w_i (1 - s_{fi}) = \beta \sum_{j=1}^{n} L_j p_{mj} q_j D_{ji} \omega_{ji} = \beta L_i p_{mi} q_i F_i \tag{A.2}$$

and

$$L_i p_{mi} q_{mi} = (1 - \beta) \sum_{j=1}^{n} L_j p_{mj} q_j D_{ji} \omega_{ji} = (1 - \beta) L_i p_{mi} q_i F_i, \tag{A.3}$$

where the second equality in each line follows from trade balance (3.12) and the definition (3.17) of F_i. From (A.3) and the fact that $q_i = q_{fi} + q_{mi}$ we have that

$$q_{fi} = q_i[1 - (1 - \beta)F_i]. \tag{A.4}$$

Then (A.1) and (A.2) imply

$$w_i s_{fi} = \frac{\alpha}{1 - \alpha} p_{mi} q_i [1 - (1 - \beta)F_i], \tag{A.5}$$

and (A.5) and (A.2) imply

$$w_i(1 - s_{fi}) = \beta p_{mi} q_i F_i. \tag{A.6}$$

Finally, eliminating $p_{mi}q_i$ between (A.5) and (A.6) and simplifying yields (3.16).

We next verify (3.15). From (A.6),

$$p_{mi} q_i = \frac{w_i(1 - s_{fi})}{\beta F_i}.$$

Inserting this expression into (3.12), (3.15) follows.

Appendix B. Proofs of Theorems 1–3

Proof of Theorem 1. It is convenient to restate (3.8) in terms of the log $\widetilde{p}_{mi} = \log(p_{mi})$ and $\widetilde{w}_i = \log(w_i)$ of prices and wages:

$$\widetilde{p}_{mi} = \log(AB) - \theta \log\left(\sum_{j=1}^{n} (\kappa_{ij}\omega_{ij})^{1/\theta} \exp\left\{-\frac{1}{\theta}[(1 - \beta)\widetilde{p}_{mj} + \beta\widetilde{w}_j]\right\}\lambda_j\right),$$

$i = 1, \ldots, n$. Define the function $f : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^n$ so that these n equations are

$$\widetilde{p}_m = f(\widetilde{p}_m, \widetilde{w}). \tag{B.1}$$

Let $S = [\xi_{ij}]$ be the $n \times n$ matrix with elements

$$\xi_{ij} = \frac{(\kappa_{ij}\omega_{ij})^{1/\theta}(p_{mj}^{1-\beta}w_j^{\beta})^{-1/\theta}\lambda_j}{\sum_{k=1}^{n}(\kappa_{ik}\omega_{ik})^{1/\theta}(p_{mk}^{1-\beta}w_k^{\beta})^{-1/\theta}\lambda_k}, \tag{B.2}$$

so that

$$\frac{\partial f_i(\widetilde{p}_m, \widetilde{w})}{\partial \widetilde{p}_{mj}} = (1 - \beta)\xi_{ij}.$$

The Jacobian of the system $\widetilde{p}_m - f(\widetilde{p}_m, \widetilde{w})$ with respect to $\widetilde{p}_m$ is then $I - (1 - \beta)S$. We note that S is a stochastic matrix ($\xi_{ij} > 0$ for all i, j and $\sum_j \xi_{ij} = 1$ for all i) and that $\beta \in (0, 1)$, so that the inverse of this Jacobian is the strictly positive matrix

$$[I - (1 - \beta)S]^{-1} = \sum_{i=0}^{\infty} (1 - \beta)^i S^i. \tag{B.3}$$

If (3.8) (equivalently (B.1)) has a differentiable solution $\widetilde{p}_{\mathrm{m}}(\widetilde{w})$, its derivatives are given by the formulas

$$\frac{\partial \widetilde{p}_{\mathrm{m}}}{\partial \widetilde{w}_k} = [I - (1 - \beta)S]^{-1}\beta\xi_k, \tag{B.4}$$

where $\xi_k = (\xi_{1k}, \ldots, \xi_{nk})$ denotes the kth column of S.

With this notation, we proceed with the proof of Theorem 1.

It is evident from the properties of the functions $f_i(\widetilde{p}_{\mathrm{m}}, \widetilde{w})$ in $w, \kappa_{ij}, \omega_{ij}$, and $p_{\mathrm{m}i}$ that the homogeneity and monotonicity properties (ii)–(iv) must hold for any solution. To verify the bounds (v), note first that if $\kappa_{ij}\omega_{ij} = a$ for all i, j for any constant a then (3.8) is solved by

$$p_{\mathrm{m}i}(w) = \left(\frac{AB}{a}\right)^{1/\beta} \left(\sum_{j=1}^{n} w_j^{-\beta/\theta}\lambda_j\right)^{-\theta/\beta}$$

for all i. This fact together with properties (iii) and (iv) implies that any solution to (3.8) must satisfy the bounds (v).

For given $w \in \mathbf{R}^n_{++}$ define the set $\mathbf{C}$ by

$$\mathbf{C} = \{z \in \mathbf{R}^n : \log(\underline{p_{\mathrm{m}}}(w)) \leqslant z_i \leqslant \log(\overline{p_{\mathrm{m}}}(w)), \text{ all } i\}.$$

Under the sup norm

$$\|z\| = \max_i |z_i|,$$

$\mathbf{C}$ is compact. We first show that $f(\cdot, \widetilde{w}) : \mathbf{C} \to \mathbf{C}$. To see this, we write $f(z, \widetilde{w}; \omega, \kappa)$ to emphasize the dependence on ω and κ. Then for any $z \in \mathbf{C}$, (ω, κ), and $\widetilde{w}$, we have

$$\log(\overline{p_{\mathrm{m}}}(w)) = f(\log(\overline{p_{\mathrm{m}}}(w)), \widetilde{w}; 1, 1) \geqslant f(z, \widetilde{w}; \omega, \kappa),$$

using the bounds (v), the monotonicity properties (iv), and the fact that f is increasing in z. Likewise,

$$\log(\underline{p_{\mathrm{m}}}(w)) = f(\log(\underline{p_{\mathrm{m}}}(w)), \widetilde{w}; \underline{\omega}, \underline{\kappa}) \leqslant f(z, \widetilde{w}; \omega, \kappa).$$

We next show that $f(\cdot, \widetilde{w})$ is a contraction on $\mathbf{C}$ by verifying the Blackwell sufficient conditions. We have already observed that $f(\cdot, \widetilde{w})$ is monotone. Let $a > 0$ and apply the mean value theorem to obtain

$$f_i(z + a) = f_i(z) + \sum_{j=1}^{n} \frac{\partial f_i}{\partial z_j}(z + a(1 - v))a$$

$$= f_i(z) + (1 - \beta)a \sum_{j=1}^{n} \xi_{ij} = f_i(z) + (1 - \beta)a,$$

using (B.2) and the fact that $\sum_j \xi_{ij} = 1$ for all i. Thus $f(\cdot, \widetilde{w})$ has the discounting property

$$f_i(z + a) \leqslant f_i(z) + (1 - \beta)a.$$

The contraction mapping theorem then implies the existence of a unique fixed point $\widetilde{p}_{\mathrm{m}}(\widetilde{w})$ for f and a unique solution $p_{\mathrm{m}}(w)$ to (3.8).

The Jacobian of system (B.1) has the inverse (B.3), so the implicit function theorem implies that $\widetilde{p}_m(\widetilde{w})$ is continuously differentiable everywhere. This completes the proof of Theorem 1. $\square$

Proof of Theorem 2. Parts (i)–(iii) of the theorem were proved in the text. We proceed with the proof of parts (iv) and (v). The lower bound (iv) on $Z_i(w)$ is implied by

$$
\begin{aligned}
Z_i(w) &= \frac{1}{w_i}\left[\sum_{j=1}^{n} L_j w_j (1 - s_{fj})\frac{1}{F_j} D_{ji}\omega_{ji} - L_i w_i (1 - s_{fi})\right] \\
&\geqslant -L_i(1 - s_{fi}) \geqslant -L_i.
\end{aligned}
$$

To prove part (v), suppose that $\{w^m\}$ is a sequence in $\mathbf{R}_{++}^n$, that $w^m \to w^0 \neq 0$, and that $w_i^0 = 0$ for some i. We need to verify that (4.3) holds for this sequence. For any $w \in \mathbf{R}_{++}^n$, we have

$$
\begin{aligned}
\max_k Z_k(w) &= \max_k \left[\sum_{j=1}^{n} L_j \frac{w_j}{w_k}(1 - s_{fj})\frac{1}{F_j} D_{jk}\omega_{jk} - L_k(1 - s_{fk})\right] \\
&\geqslant \max_k \sum_{j=1}^{n} L_j \frac{w_j}{w_k}(1 - s_{fj}) D_{jk}\omega_{jk} - \max_k L_k \\
&\geqslant \max_{k,j} L_j \frac{w_j}{w_k}(1 - s_{fj}) D_{jk}\omega_{jk} - \max_k L_k.
\end{aligned}
$$

By Assumption (A), $\omega_{jk} \geqslant \underline{\omega}$, implying in turn that the functions F_i take values in $[\underline{\omega}, 1]$. Then (3.16) implies that the shares $1 - s_{fj}$ are uniformly bounded away from zero. Thus (4.3) will be proved if it can be shown that

$$
\max_{k,j} \frac{w_j^m}{w_k^m} D_{jk}(w^m) \to \infty \tag{B.5}
$$

for the wage sequence $\{w^m\}$. From (3.10) we have, for any w

$$
D_{jk}(w) \geqslant (AB)^{-1/\theta}(\underline{\kappa\omega})^{1/\theta}\lambda_k \left(\frac{p_{mk}}{w_k}\right)^{\beta/\theta}\left(\frac{p_{mj}}{p_{mk}}\right)^{1/\theta}. \tag{B.6}
$$

Using (3.8) directly,

$$
\left(\frac{p_{mj}}{p_{mk}}\right)^{1/\theta} = \left[\sum_{r=1}^{n}\left(\frac{w_r^{\beta}p_{mr}^{1-\beta}}{\kappa_{jr}\omega_{jr}}\right)^{-1/\theta}\lambda_r\right]^{-1}\sum_{r=1}^{n}\left(\frac{w_r^{\beta}p_{mr}^{1-\beta}}{\kappa_{kr}\omega_{ir}}\right)^{-1/\theta}\lambda_r \geqslant (\underline{\kappa\omega})^{1/\theta}. \tag{B.7}
$$

Using the lower bound established in Theorem 1, (v), we have

$$
\left(\frac{p_{mk}}{w_k}\right)^{\beta/\theta} \geqslant w_k^{-\beta/\theta}(AB)^{1/\theta}\left(\sum_{r=1}^{n} w_r^{-\beta/\theta}\lambda_r\right)^{-1}. \tag{B.8}
$$

It follows from (B.6)–(B.8) that

$$
\frac{w_j}{w_k} D_{jk}(w) \geqslant (\underline{\kappa\omega})^{2/\theta}\lambda_k \left(\sum_{r=1}^{n}\left(\frac{w_k}{w_r}\right)^{\beta/\theta}\lambda_r\right)^{-1}\frac{w_j}{w_k}
$$

and therefore that for all w,

$$\max_{k,j} \frac{w_j}{w_k} D_{jk}(w) \geqslant (\underline{\kappa\omega})^{2/\theta} \left(\min_k \lambda_k \right) \left(\sum_{r=1}^n \lambda_r \right)^{-1} \frac{\max_j w_j}{\min_k w_k}.$$

Since $w^m \to w^0 \neq 0$ with $w_i^0 = 0$ for some i

$$\max_j w_j^m \to \max_j w_j^0 > 0 \quad \text{and}$$

$$\min_k w_k^m \to \min_k w_k^0 = 0.$$

This verifies (B.5) and hence (4.3) and completes the proof of Theorem 2. $\quad\square$

Proof of Theorem 3. We show that Z has the gross substitute property (4.7):

$$\frac{\partial Z_i(w)}{\partial w_k} > 0 \quad \text{for all } i,k, \ i \neq k \quad \text{for all } w \in \mathbf{R}_{++}..$$

Before calculating the derivatives, note that Z_i can be written as

$$Z_i(w) = \frac{(1-\alpha)\beta}{w_i} \left[\sum_{j=1}^n \frac{L_j w_j D_{ji}\omega_j}{\alpha + (\beta-\alpha)F_j} - \frac{L_i w_i F_i}{\alpha + (\beta-\alpha)F_i} \right],$$

using (3.16) to substitute for s_{f_j} in (3.15) and using the fact that

$$(1-\alpha)\beta F_j + \alpha[1-(1-\beta)F_j] = \alpha + (\beta-\alpha)F_j.$$

Thus we can write the derivatives in (4.7) as

$$\frac{\partial Z_i(w)}{\partial w_k} = \frac{(1-\alpha)\beta}{w_i} \sum_{j=1, j\neq i, j\neq k}^n L_j w_j \omega_j \frac{\partial}{\partial w_k}\left(\frac{D_{ji}}{\alpha + (\beta-\alpha)F_j} \right)$$

$$+ \frac{(1-\alpha)\beta}{w_i} L_k \omega_k \frac{\partial}{\partial w_k}\left(\frac{w_k D_{ki}}{\alpha + (\beta-\alpha)F_k} \right)$$

$$+ (1-\alpha)\beta L_i \frac{\partial}{\partial w_k}\left(\frac{D_{ii}-F_i}{\alpha + (\beta-\alpha)F_i} \right).$$

In the following three steps we sign each of the three terms of this derivative.

Step (ia): If (4.4) holds, then $\partial D_{ji}/\partial w_k > 0$ for all $j \neq i$. To see this, notice that direct computation gives

$$\frac{\partial D_{ji}}{\partial w_k} = \frac{D_{ji}}{w_k} \frac{1}{\theta}\left[\frac{\partial \tilde{p}_{mj}}{\partial \tilde{w}_k} - (1-\beta)\frac{\partial \tilde{p}_{mi}}{\partial \tilde{w}_k} \right].$$

We seek bounds on these derivatives. Let ξ_{ik} be the expressions defined in (B.2), related to the elasticities of the function f by

$$\frac{\partial f_i(\widetilde{p}_m, \widetilde{w})}{\partial \widetilde{p}_{mk}} = (1-\beta)\xi_{ik}.$$

We will show that

$$\underline{\xi}_k \leqslant \frac{\partial \tilde{p}_{mi}}{\partial \tilde{w}_k} \leqslant \overline{\xi}_k, \tag{B.9}$$

where $\underline{\xi}_k = \min_i \xi_{ik}$ and $\overline{\xi}_k = \max_i \xi_{ik}$. Then the vector ξ_k satisfies $\xi_k \leqslant \iota \overline{\xi}_k$, where ι is a vector of ones, and (B.4) implies

$$\frac{\partial \widetilde{p}_{\mathrm{m}}}{\partial \widetilde{w}_k} \leqslant [I - (1-\beta)S]^{-1} \beta \iota \overline{\xi}_k. \tag{B.10}$$

Now $S\iota = \iota$, since S is a stochastic matrix, implying that

$$[I - (1-\beta)S]\iota = \beta \iota \quad \text{or that} \quad [I - (1-\beta)S]^{-1} \beta \iota = \iota.$$

Thus (B.10) is equivalent to

$$\frac{\partial \widetilde{p}_{\mathrm{m}}}{\partial \widetilde{w}_k} \leqslant \iota \overline{\xi}_k.$$

This verifies the upper bound in (B.9). An analogous argument shows that

$$\frac{\partial \widetilde{p}_{\mathrm{m}}}{\partial \widetilde{w}_k} \geqslant \iota \underline{\xi}_k.$$

Resuming the main line of argument, let $\hat{\xi}_k \equiv \left(\left(w_k^\beta p_{\mathrm{m}k}^{1-\beta} \right)^{-1/\theta} \lambda_k \right) / \sum_{r=1}^n \left(w_r^\beta p_{\mathrm{m}r}^{1-\beta} \right)^{-1/\theta} \lambda_r$, so that ξ_{ik}, $\underline{\xi}_k$, and $\overline{\xi}_k$ imply

$$(\underline{\kappa}\,\underline{\omega})^{1/\theta}\, \hat{\xi}_k \leqslant \underline{\xi}_k \leqslant \xi_{ik} \leqslant \overline{\xi}_k \leqslant \frac{1}{(\underline{\kappa}\,\underline{\omega})^{1/\theta}}\, \hat{\xi}_k$$

for all i. Then (B.9) implies that

$$\frac{\partial D_{ji}}{\partial w_k} \geqslant \frac{D_{ji}}{w_k} \frac{1}{\theta} \left[\underline{\xi}_k - (1-\beta)\overline{\xi}_k \right] \geqslant \frac{D_{ji}}{w_k} \frac{1}{\theta} \hat{\xi}_k \left[(\underline{\kappa}\,\underline{\omega})^{1/\theta} - \frac{(1-\beta)}{(\underline{\kappa}\,\underline{\omega})^{1/\theta}} \right]$$

$$= \frac{D_{ji}}{w_k} \frac{1}{\theta(\underline{\kappa}\,\underline{\omega})^{1/\theta}} \hat{\xi}_k [(\underline{\kappa}\,\underline{\omega})^{2/\theta} - (1-\beta)]$$

which is positive if (4.4) holds.

Step (ib): If (4.5) holds ($\beta \leqslant \alpha$), then for $j \neq k$, $\partial(\alpha + (\beta - \alpha)F_j)/\partial w_k \leqslant 0$. To see this, notice that under the assumption of uniform tariffs

$$F_j = \omega_j + (1-\omega_j)D_{jj} = \omega_j + (1-\omega_j)(AB)^{-1/\theta} \left(\frac{p_{\mathrm{m}j}}{w_j} \right)^{\beta/\theta} \lambda_j. \tag{B.11}$$

Hence

$$\frac{\partial}{\partial w_k}(\alpha + (\beta - \alpha)F_j) = (\beta - \alpha)(1-\omega_j)\frac{\partial D_{jj}}{\partial w_k}$$

and for $j \neq k$

$$\frac{\partial D_{jj}}{\partial w_k} = (\beta/\theta)\frac{D_{jj}}{p_{\mathrm{m}j}}\frac{\partial p_{\mathrm{m}j}}{\partial w_k} > 0,$$

by (iii) in Theorem 1. Thus if (4.5) holds (if $\beta \leqslant \alpha$) then

$$\frac{\partial}{\partial w_k}(\alpha + (\beta - \alpha)F_j) = (\beta - \alpha)(1-\omega_j)(\beta/\theta)\frac{D_{jj}}{p_{\mathrm{m}j}}\frac{\partial p_{\mathrm{m}j}}{\partial w_k} \leqslant 0.$$

Clearly, (ia) and (ib) imply that

Clearly, (ia) and (ib) imply that

$$\sum_{j=1, j\neq i, j\neq k}^{n} L_j w_j \omega_j \frac{\partial}{\partial w_k}\left(\frac{D_{ji}}{\alpha+(\beta-\alpha)F_j}\right) > 0.$$

Step (ii): If (4.5) and (4.6) hold for $k\neq i$, $\partial\left(\frac{w_k D_{ki}}{[\alpha+(\beta-\alpha)F_k]}\right)/\partial w_k > 0$. To see this notice that direct computation gives

$$\frac{\partial}{\partial w_k}\left(\frac{w_k D_{ki}}{\alpha+(\beta-\alpha)F_k}\right)$$
$$= \frac{(D_{ki}+w_k\partial D_{ki}/\partial w_k)(\alpha+(\beta-\alpha)F_k)-w_k D_{ki}(\beta-\alpha)\partial F_k/\partial w_k}{[\alpha+(\beta-\alpha)F_k]^2}.$$

In step (ia) we verified that (4.4) implies that $\partial D_{ki}/\partial w_k \geqslant 0$ so that it will suffice to show that

$$\frac{D_{ki}(\alpha+(\beta-\alpha)F_k)-w_k D_{ki}(\beta-\alpha)\partial F_k/\partial w_k}{[\alpha+(\beta-\alpha)F_k]^2} \geqslant 0.$$

Using (B.11),

$$\frac{\partial F_k}{\partial w_k} = (1-\omega_k)\frac{\partial D_{kk}}{\partial w_k} = (1-\omega_k)\frac{D_{kk}}{w_k}\frac{\beta}{\theta}\left[\frac{\partial\tilde{p}_{mk}}{\partial\tilde{w}_k}-1\right] \geqslant -(1-\omega_k)\frac{1}{w_k}(\beta/\theta)$$

since $D_{kk}\leqslant 1$ and $\partial\tilde{p}_{mk}/\partial\tilde{w}_k > 0$. Thus

$$\frac{\partial}{\partial w_k}\left(\frac{w_k D_{ki}}{\alpha+(\beta-\alpha)F_k}\right) \geqslant D_{ki}\frac{(\alpha+(\beta-\alpha)F_k)+(\beta-\alpha)(1-\omega_k)(\beta/\theta)}{[\alpha+(\beta-\alpha)F_k]^2}$$
$$= \frac{D_{ki}}{[\alpha+(\beta-\alpha)F_k]}\left[1-\frac{(\alpha-\beta)(1-\omega_k)(\beta/\theta)}{[\alpha+(\beta-\alpha)F_k]}\right]$$

and since $F_k \in [\underline{\omega}, 1]$ and $\beta\leqslant\alpha$

$$\frac{\partial}{\partial w_k}\left(\frac{w_k D_{ki}}{\alpha+(\beta-\alpha)F_k}\right) \geqslant \frac{D_{ki}}{[\alpha+(\beta-\alpha)F_k]}\left[1-\frac{(\alpha-\beta)(1-\omega_k)(\beta/\theta)}{\beta}\right]$$
$$= \frac{D_{ki}}{[\alpha+(\beta-\alpha)F_k]}\left[1-\frac{(\alpha-\beta)(1-\omega_k)}{\theta}\right],$$

and thus if condition (4.6) holds, the inequality in step (ii) is verified.

Step (iii): For $k\neq i$, $\partial(D_{ii}-F_i/[\alpha+(\beta-\alpha)F_i])/\partial w_k > 0$. To see this, use (B.11) so that

$$\frac{D_{ii}-F_i}{\alpha+(\beta-\alpha)F_i} = \frac{-\omega_i(1-D_{ii})}{\alpha+(\beta-\alpha)(\omega_i+(1-\omega_i)D_{ii})}.$$

Hence

$$\frac{\partial}{\partial w_k}\left(\frac{D_{ii}-F_i}{\alpha+(\beta-\alpha)F_i}\right) = \frac{\partial}{\partial D_{ii}}\left(\frac{-\omega_i(1-D_{ii})}{\alpha+(\beta-\alpha)(\omega_i+(1-\omega_i)D_{ii})}\right)\frac{\partial D_{ii}}{\partial w_k}$$

with $\partial D_{ii}/\partial w_k = (\beta/\theta)[D_{ii}/p_{mi}\partial p_{mi}/\partial w_k] > 0$ and

$$\frac{\partial}{\partial D_{ii}} \left(\frac{-\omega_i(1 - D_{ii})}{\alpha + (\beta - \alpha)(\omega_i + (1 - \omega_i)D_{ii})} \right)$$

$$= \frac{\omega_i\beta}{[\alpha + (\beta - \alpha)(\omega_i + (1 - \omega_i)D_{ii})]^2} > 0$$

which establishes the inequality in step (iii).

This shows that Z satisfies the gross substitute property, and hence that the equilibrium is unique. □

Appendix C. Behavior of the limiting economies

We verify that as $L_1^r \to 0$, the limiting behavior $(w_{-1}, c_{-1}, p_{m-1})$ of the other $n - 1$ economies is equal to the equilibrium of a world economy with $n - 1$ countries and endowments (L_{-1}, λ_{-1}), and that the limiting behavior of economy 1, (w_1, c_1, p_{m1}), satisfies (8.1)–(8.3). We proceed under the hypothesis that $w_1 \in (0, \infty)$, which we verify later on. Then when $\lambda_1 = kL_1 = 0$, (3.8) implies

$$p_{mi} = AB \left(\sum_{j=2}^n \left(\frac{w_j^\beta p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} \right)^{-1/\theta} \lambda_j \right)^{-\theta}$$

for all i and, using the assumption that country 1 imposes a uniform tariff ω,

$$p_{m1} = \omega^{-1} AB \left(\sum_{j=2}^n \left(\frac{w_j^\beta p_{mj}^{1-\beta}}{\kappa_{1j}} \right)^{-1/\theta} \lambda_j \right)^{-\theta} = \omega^{-1}\hat{p}_{m1}.$$

The second equality defines $\hat{p}_{m1}$ and verifies (8.3).

The fraction of country j's expenditures on tradeables produced by country 1 is

$$D_{j1}(w) = (AB)^{-1/\theta} \left[\frac{p_{mj}\kappa_{j1}\omega_j}{p_{m1}^{1-\beta}w_1^\beta} \right]^{1/\theta} \lambda_1 = 0.$$

As $L_1^r \to 0$, this fraction goes to zero for each j, $\lim_{r\to\infty} D_{j1}(w^r) = 0$. Inspection of the expression for $Z_i(w)$ in (3.18) then confirms that when $\lambda_1 = kL_1 = 0$ the excess demand system $Z_{-1}(w) \equiv (Z_2(w), Z_3(w), \ldots, Z_n(w)) = 0$ does not depend on $w_1, \hat{\omega}, \omega$. The continuity of Z implies that w_{-1} solving $Z_{-1}(w_1, (w_{-1})) = 0$ is the desired limit.

The next step is to derive an expression for the limit of economy 1. We take w_{-1} as given in the previous step. For $L_1 > 0$, we have that $Z_1(w_1, w_{-1}) = 0$ is equivalent to $Z_1(w_1, w_{-1})/L_1 = 0$, so we analyze the latter expression. It can be shown, using the share formulas (A.2) and (A.3), that for $L_1 > 0$, the ratio $Z_1(w_1, w_{-1})/L_1 = 0$ is equivalent to

$$\sum_{j=2}^n L_j \frac{w_j(1 - s_j)}{F_j} (AB)^{-1/\theta} (p_{mj}\kappa_{j1})^{1/\theta} k \left(\frac{1}{p_{m1}^{1-\beta}w_1^\beta} \right)^{1/\theta} \hat{\omega}^{1+1/\theta}$$

$$= w_1^{1+\beta/\theta} \frac{(1 - \alpha)\beta F_1}{\alpha + (\beta - \alpha)F_1} \left[1 - \frac{1}{F_1} (AB)^{-1/\theta} \left(\frac{p_{m1}}{w_1} \right)^{\beta/\theta} L_1 \right].$$

As $\lambda_1 = kL_1 \to 0$, then $D_{11} \to 0$, and hence $F_1 = D_{11} + (1 - D_{11})\omega \to \omega$. Thus, taking the limit as $L_1 \to 0$ yields

$$\sum_{j=2}^{n} L_j \frac{w_j(1-s_j)}{F_j}(AB)^{-1/\theta}(p_{mj}\kappa_{j1})^{1/\theta}k\,\hat{\omega}^{(1+\theta)/\theta}$$

$$= w_1^{1+\beta/\theta}\frac{(1-\alpha)\beta\omega}{\alpha+(\beta-\alpha)\omega}(p_{m1})^{(1-\beta)/\theta}.$$

Solving for w_1 yields (8.1), where it can be seen that the factor $\hat{w}_1$ does not depend on $\omega, \hat{\omega}$, and k.

Finally, we turn to the calculation of c_1. From (6.1) we have that with uniform tariffs

$$p_1 c_1 L_1 = L_1 w_1\left[1 + \frac{(1-\alpha)(1-\omega_1)}{\alpha+(\beta-\alpha)F_1}(1-D_{11})\right],$$

and since $D_{11} \to 0$ and $F_1 \to \omega_1$ as $L_1 \to 0$ then

$$\lim_{L_1 \to 0} p_1 c_1 = w_1\left[1 + \frac{(1-\alpha)(1-\omega_1)}{\alpha+(\beta-\alpha)\omega_1}\right] = w_1\left[\frac{\beta\omega_1+(1-\omega_1)}{\alpha+(\beta-\alpha)\omega_1}\right].$$

Using the expression for p_1 in (2.12),

$$c_1 = \frac{1}{\alpha^{-\alpha}(1-\alpha)^{-1+\alpha}}\left(\frac{w_1}{p_{m1}}\right)^{1-\alpha}\left[\frac{\beta\omega+(1-\omega)}{\alpha+(\beta-\alpha)\omega}\right].$$

Using the expression for p_{m1} in (8.3) and for w_1 in (8.1),

$$c_1 = \frac{1}{\alpha^{-\alpha}(1-\alpha)^{-1+\alpha}}\left(\frac{\omega}{\hat{p}_{m1}}\right)^{1-\alpha}\left[\frac{\beta\omega+(1-\omega)}{\alpha+(\beta-\alpha)\omega}\right]$$

$$\times\left(\frac{\alpha+(\beta-\alpha)\omega}{(1-\alpha)\beta\omega}\left(\frac{\omega}{\hat{p}_{m1}}\right)^{(1-\beta)/\theta}G(w_{-1})k\hat{\omega}^{(1+\theta)/\theta}\right)^{(1-\alpha)\theta/(\theta+\beta)},$$

where

$$G(w_{-1}) = \left[\sum_{j=2}^{n} L_j \frac{w_j(1-s_j)}{F_j}(AB)^{-1/\theta}(p_{mj}\kappa_{j1})^{1/\theta}\right]^{\theta/(\theta+\beta)}.$$

Collecting terms involving $\omega, \hat{\omega}$, and k

$$c_1 = k^{(1-\alpha)\theta/(\theta+\beta)}\omega^{(1-\alpha)/(\theta+\beta)}[1+(\beta-1)\omega][\alpha+(\beta-\alpha)\omega]^{-(\alpha\theta+\beta)/(\theta+\beta)}\hat{\omega}^{(1-\alpha)(1+\theta)/(\theta+\beta)}\hat{c}_1,$$

where it can be seen that the factor $\hat{c}_1$ does not depend on $\omega, \hat{\omega}$, or k:

$$\hat{c}_1 = \frac{(\hat{p}_{m1})^{-(1-\alpha)[(1+\theta)/(\theta+\beta)]}}{\alpha^{-\alpha}(1-\alpha)^{-1+\alpha}}\left(\frac{G(w_{-1})}{(1-\alpha)\beta}\right)^{(1-\alpha)\theta/(\theta+\beta)}.$$

This completes the proof.

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