Gravity Models

ECON 871

Gravity Over Time

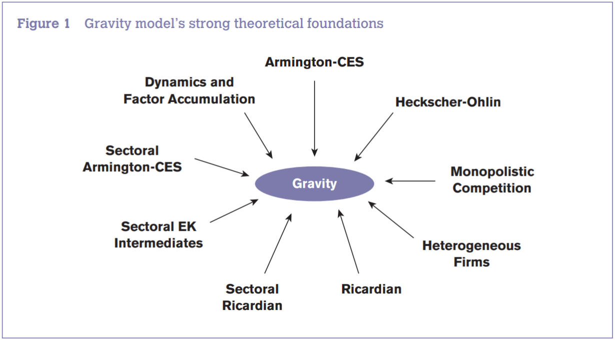
Gravity in international trade implies that countries trade in proportion to **their respective market size** (e.g., GDP) and **proximity**.[1]

- Based on Newton's Law of Universal Gravitation—initial applications to economics were purely empirical.
  Ravenstein (1885), Tinbergen (1962)
- Anderson (1979) and Bergstrand (1985) are among the first to offer a theoretical foundation for the gravity equation.
- Didn't really take off until Eaton and Kortum (2002), who derive gravity on the supply side as a Ricardian structure with inermediate goods and Anderson and Van Wincoop (2003) who derive gravity in an Armington-CES model.

[1] Sources for these notes: Head and Mayer (2014) and Yotov et al. (2016).

Gravity Over Time

Arkolakis et al. (2012) demonstrated that a large class of modeles generate *isomorphic gravity equations*, which preserves the gains from trade.



SOURCE: An Advanced Guide to Trade Policy Analysis

Simple Structural Gravity Model

Consider the following model environment:

- N countries.
- Each economy produces a variety of goods (i.e., **goods are differentiated by origin**, as in Armington (1969)).
- Fixed supply of each good, Q_i, sold at factory-gate price p_i.
- Thus, the **value of domestic production** in country i is $Y_i = p_i Q_i$.
- Country i's **aggregate expenditure** is $E_i = \phi_i Y_i$, where $\phi_i > 1$ indicates country i runs a **trade deficit** and $\phi_i < 1$ reflects a **trade surplus**. (We will take trade balance as exogenous.)

Simple Structural Gravity Model

On the demand side, consumer preferences are homothetic, identical across countries, and given by a CES utility function for country *j*:

$$U_j = \left[\sum_i \alpha_i^{\frac{1-\sigma}{\sigma}} c_{ij}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- $\sigma > 1$ is the elasticity of substitution among different varieties (countries).
- $\alpha_i > 0$ is an exogenous taste parameter.
- c_{ij} denotes consumption of varieties from country *i* in country *j*.

Simple Structural Gravity Model

Consumers maximize utility subject to a standard budget constraint:

$$\sum_i p_{ij} c_{ij} = E_j$$

- Here, $p_{ij} = p_i t_{ij}$ are *delivered prices* from i to j.
  - p_i are factory-gate prices in the country of origin, i.
  - $t_{ij} \geq 1$ are bilateral trade costs between i and j.
- Customary to think of the t_{ij}'s as iceberg trade costs.

Simple Structural Gravity Model

Solving the consumer's optimization problem yields the expenditures on goods shipped from origin i to destination j as:

$$X_{ij} = \left[\frac{\alpha_i p_i t_{ij}}{P_j}\right]^{1-\sigma} E_j$$

- X_{ij} are trade flows from exporter i to destination j.
- P_j is the CES price index in j:

$$P_j = \left[\sum_i (\alpha_i p_i t_{ij})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Simple Structural Gravity Model

From the last slide, we can see that expenditure in j on goods from i:

$$X_{ij} = \left[\frac{\alpha_i p_i t_{ij}}{P_j}\right]^{1-\sigma} E_j$$

- Are **proportional to total expenditure** in the destination, E_j.
- **Inversely related to the (delivered) prices** from i to j, $p_{ij} = p_i t_{ij}$.
  - Depends on factory-gate prices and bilateral trade costs.
- Directly related to the CES price aggregator, P_j, reflecting **substitution effects** across varieties from different countries.
- Contingent on the **elasticity of stubstitution** σ_i—higher elasticity of substitution will magnify trade diversion effects.

Simple Structural Gravity Model

Last step is to impose market clearing conditions:

$$Y_i = \sum_j \left(\frac{\alpha_i p_i t_{ij}}{P_j} \right)^{1-\sigma} E_j$$

- At *delivered prices*, the value of output in country i should be equal to the total expenditure on this country's variety in all countries (including i)

Defining $Y \equiv \sum_i Y_i$, dividing by Y and rearranging, we have:

$$(\alpha_i p_i)^{1-\sigma} = \frac{Y_i / Y}{\sum_j \left(\frac{t_{ij}}{P_j} \right)^{1-\sigma} (E_j / Y)}$$

Simple Structural Gravity Model

From the last slide:

$$(\alpha_i p_i)^{1-\sigma} = \frac{Y_i/Y}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} (E_j/Y)}$$

Following Anderson and van Wincoop (2003), define the term in the denominator:

$$\Pi_i^{1-\sigma} \equiv \sum_j (t_{ij}/P_j)^{1-\sigma} E_j/Y$$

Substitute back in:

$$(\alpha_i p_i)^{1-\sigma} = \frac{Y_i/Y}{\Pi_i^{1-\sigma}}$$

Simple Structural Gravity Model

Now substitute in for $(\alpha_i p_i)^{1-\sigma}$ in our expression for X_{ij}

$$X_{ij} = \left[\frac{\alpha_i p_i t_{ij}}{P_j}\right]^{1-\sigma} E_j \rightarrow \boxed{X_{ij} = \frac{Y_i E_j}{Y}\left[\frac{t_{ij}}{\Pi_i P_j}\right]^{1-\sigma}}$$

The boxed term is the **theoretical gravity equation**. Two components:

1. A **size term:** $Y_j E_j / Y$ representing the hypothetical level of frictionless trade between i and j.
2. A **trade cost term** $(t_{ij}/\Pi_i P_j)^{1-\sigma}$, which captures the total effects of trade costs that drive a wedge between realized and frictionless trade.

Simple Structural Gravity Model

The **size term:** $Y_j E_j / Y$ provides some info on the relationship between country size and bilateral trade flows:

- Large producers will export more to all destinations.
- Bigger markets will import more from all sources.
- Trade flows between *i* and *j* will be larger the more similar in size the trading partners are.

Simple Structural Gravity Model

The **trade cost term** $(t_{ij}/\Pi_i P_j)^{1-\sigma}$, has several components:

- Bilateral trade cost between i and j, t_{ij} is typically approximated by geographic and trade policy variables (distance, tariffs, RTAs, etc.).
- Structural term P_j is an *inward multilateral resistance* term representing importer j's ease of market access.
- Structural term Π_i is *outward multilateral resistance*—exporter i's ease of market access.

Note the Resemblence

Let T_{ij}^{θ} be the inverse of the trade cost term and define $\tilde{G} \equiv 1/Y$. Our gravity equation becomes:

$$X_{ij} = \tilde{G}\frac{Y_i E_j}{T_{ij}^{\sigma}}$$

Compare this to Newton's Law of Universal Gravitation:

$$F_{ij} = G\frac{M_i M_j}{D_{ij}^2}$$

Isomorphic to EK

We just derived gravity using a demand-side approach, but it is isomorphic to the supply side approach in EK. Recall, from EK, the probability that country *i* sells a good to *j* is:

$$\pi_{ij} = \frac{T_i \left(c_i t_{ij}\right)^{-\theta}}{\Phi_j}$$

And the fraction of goods that *j* buys from *i*, π_{ij}, is also the fraction of its expenditures spent on goods from *i*:

$$X_{ij} = \frac{T_i(c_i t_{ij})^{-\theta}}{\Phi_j} E_j = \frac{T_i(c_i t_{ij})^{-\theta}}{\sum_{j=1}^{N} T_k(c_k t_{kj})^{-\theta}} E_j$$

Isomorphic to EK

Imposing market clearing conditions and substituting in for $T_i c_i^{-\theta}$, we get:

$$X_{ij} = \frac{t_{ij}^{-\theta}}{\Phi_j \left(\sum_{j=1}^{N} \frac{t_{ij}^{-\theta}}{\Phi_j} E_j \right)} Y_i E_j$$

Now, replacing Φ_j using $P_j = \gamma \Phi_j^{-\frac{1}{\theta}}$, this becomes:

$$X_{ij} = \frac{t_{ij}^{-\theta}}{\gamma^{\theta} P_j^{-\theta} \left(\sum_{j=1}^{N} \frac{t_{ij}}{P_j^{-\theta}} E_j \right)} Y_i E_j$$

Isomorphic to EK

Finally, define Π_i as:

$$\Pi_i = \left(\sum_{j=1}^{N} \left(\frac{t_{ij}}{P_j} \right)^{-\theta} \frac{E_j}{Y} \right)^{-\frac{1}{\theta}}$$

where $Y \equiv \sum_j Y_j$. And note that P_j can be expressed as:

$$P_j = \left(\sum_{j=1}^{N} \left(\frac{t_{ij}}{\Pi_i} \right)^{-\theta} \frac{Y_i}{Y} \right)^{-\frac{1}{\theta}}$$

Isomorphic to EK

Now we can write our expression for X_{ij} as follows:

$$X_{ij} = \frac{Y_i E_j}{Y} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{-\theta}$$

This is the same equation we derived in the Arminton-type model, but we have replaced the elasticity of substitution, $(1 - \sigma)$, with $-\theta$—the Frechet parameter governing variation within the distribution.

Gravity Estimation

Typical to log-linearize the structural gravity equation and add an error term $\varepsilon_{ij,t}$:

$$\ln X_{ij,t} = \ln E_{j,t} + \ln Y_{i,t} - \ln Y_t - \theta \ln t_{ij,t} - (1-\sigma) \ln P_{j,t} - (1-\sigma) \ln \Pi_{i,t} + \varepsilon_{ij,t}$$

- The terms in blue are exporter characteristics.
- The terms in red are importer characteristics.
- **Problem:** $P_{j,t}$ and $\Pi_{i,t}$ are structural parameters, with no observable counterparts.
- Two typical approaches:
  - Fixed Effects Estimation
  - Ratio-Type Estimation

Fixed-Effects Estimation

$$\ln X_{ij,t} = \ln E_{j,t} - (1-\sigma)\ln P_{j,t} + \ln Y_{i,t} - (1-\sigma)\ln \Pi_{i,t} - \ln Y_t - \theta \ln t_{ij,t} + \varepsilon_{ij,t}$$

Basic Idea: We can't observe $P_{j,t}$ and $\Pi_{i,t}$, but we can replace them with exporter-time and importer-time fixed effects.

- Several benefits to this approach:
  - Can get a consistent estimate of θ, which is usually the parameter of interest.
  - Don't have to make many strong structural assumptions.

- Importantly, the FEs will also absorb the rest of the blue/red terms (country size) and other unobservable country-specific characteristics.

$$\ln X_{ij,t} = \ln Y_t + \ln S_j + \ln M_i - \theta \ln t_{ij,t} + \varepsilon_{ij,t}$$

Fixed-Effects Estimation

There are potentially interesting trade determinants that can no longer be identified if we use importer/exporter FEs. Notably:

1. Anything that affects exporters' propensity to export to all destinations.
   - e.g., hosting an Olympics.
2. Variables that affect imports without regard to country of origin.
   - e.g., country-level average tariff rates.

There are possible ways to estimate the effects of these *monadic* variables of interest even with FEs.

Fixed-Effects Estimation

More generally, suppose we want to estimate the following (changing notation slightly):

$$\ln X_{ni} = \alpha_i + \beta V_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

- V_i is a monadic variable of interest (e.g., cost or quality of exports from country *i*).
- D_{ni} are the "dyadic" controls (trade frictions—e.g., distance, RTAs).
- α_i are the other $i - \mathit{level}$ determinants of exports (other exporter FEs).
- γ_n are the importer FEs.

A few approaches to estimating β.

Fixed-Effects Estimation

$$\ln X_{ni} = \alpha_i + \beta V_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

One-Step Approach: Combine α_i and ε_{ni} as the error term.

- Even if α_i is uncorrelated with V_i (which is unlikely), the error terms for the same exporter will be correlated.
- This will result in downward-biased standard errors of β unless standard errors are **clustered by exporter**.

Fixed-Effects Estimation

$$\ln X_{ni} = \alpha_i + \beta V_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

Two-Step Estimator: Basic idea is to do the following:

1. First, estimate the two-way fixed effects version:

$$\ln X_{ni} = S_i + \gamma_n + \delta D_{ni} + \varepsilon_{ni}$$

2. Then, regress $\ln \hat{S}_i$ on V_i.
   - EK 2002 do a version of this to estimate θ (not their baseline).
   - Gives an unbiased estimate of $\hat{\beta}$.

Fixed-Effects Estimation

Lastly, we can get a one-step estimator for both $\hat{\beta}$ and $\hat{\delta}$ by modeling α_i as:

$$\alpha_i = \alpha_0 + \alpha_1 C_i + \alpha_2 \bar{D}_i + \psi_i$$

Where:

- C_i are *i*-specific controls.
- $\bar{D}_i$ are average characteristics of each exporter: $\bar{D}_i = \frac{1}{N} \sum_n D_{ni}$
- An error term.

Subbing this in to our original estimating equation:

$$\ln X_{ni} = \alpha_0 + \alpha_1 C_i + \beta V_i + \gamma_n + \delta D_{ni} + \alpha_2 \bar{D}_i + (\psi_i + \varepsilon_{ni})$$

- Again, we have to cluster standard errors at the *i*-level because of the ψ_i.

Ratio-Type Estimation

Another solution to estimation of the θ's involves using the multiplicative structure of the gravity model to eliminate the monadic terms.

Let's rewrite the gravity equation one more time as:

$$X_{ni} = GS_i M_n \phi_{ni}$$

- S_i are the exporter characteristics.
- M_n are the importer characteristics.
- ϕ_{ni} are the bilateral relationship characteristics (trade frictions).
- G is a "gravitational constant." (Constant in the cross section.)

Ratio-Type Estimation

Eaton and Kortum (2002) normalize bilateral trade flows, X_{ni} by own-trade flows:

$$\frac{X_{ni}}{X_{nn}} = \frac{GS_i M_n \phi_{ni}}{GS_n M_n \phi_{nn}} = \left(\frac{S_i}{S_n}\right)\left(\frac{\phi_{ni}}{\phi_{nn}}\right)$$

- This removes the importer FE, M_n.
- Still have to measure S terms, presumably with substantial measurement error.

Head and Ries (2001) propose a simple solution to cancel out the exporter terms as well—multiply the above by $X_{in}X_{ii}$:

$$\left(\frac{X_{ni}}{X_{nn}}\right)\left(\frac{X_{in}}{X_{ii}}\right) = \left(\frac{S_i}{S_n}\right)\left(\frac{\phi_{ni}}{\phi_{nn}}\right) \times \left(\frac{S_n}{S_i}\right)\left(\frac{\phi_{in}}{\phi_{ii}}\right)$$

Ratio-Type Estimation

If we make the following two assumptions:

- Symmetry in bilateral trade costs: $\phi_{ni} = \phi_{in}$
- Frictionless trade inside countries $\phi_{nn} = \phi_{ii} = 1$

We end up with a simple index that Eaton et. al. (2011) call the Head-Ries Index (HRI):

$$\hat{\phi_{ni}} = \sqrt{\frac{X_{ni}X_{in}}{X_{nn}X_{ii}}}$$

Problem:

- Need a measure of own-country trade flows, X_{ii}'s.
- Can be proxied using production minus total exports, but this often generates negative observations related to measurement issues.

Ratio-Type Estimation

Caliendo and Parro (2015) estimate the trade cost elasticity using tariff data and exploiting asymmetries in protectionism as an identification strategy.

Suppose trade costs can be written:

$$\phi_{ni} = \left[(1+\tau_{ni})d_{ni}^{\delta}\right]^{\epsilon}$$

Where $d_{ni} = d_{in}$ captures all symmetric trade costs (e.g., distnace) in $X_{ni} = GS_i M_n \phi_{ni}$. We can introduce a third country, h, and then multiply X_{ni}/X_{nh}, X_{ih}/X_{hi}, and X_{hn}/X_{in}:

$$\frac{X_{ni}X_{ih}X_{hn}}{X_{nh}X_{hi}X_{in}} = \left(\frac{(1+t_{ni})(1+t_{ih})(1+t_{hn})}{(1+t_{nh})(1+t_{hi})(1+t_{in})}\right)^{\epsilon}$$

We can estimate this with bilateral tariff data.

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