ECON880: Computational Economics
Fall 2025, University of Wisconsin
Instructor: Dean Corbae

Problem Set #2 - Due Date 9/19/25

I. Consider the same environment as Huggett (1993, JEDC) except assume that there are enforceable insurance markets regarding the idiosyncratic shocks to earnings and that there are no initial asset holdings. Solve for a competitive equilibrium. What are prices? What is the allocation? (Hint: think about the planner's problem and then decentralize).

II. Now compute Huggett (1993, JEDC) with incomplete markets. The following takes you through the steps of solving a simple general equilibrium model that generates an endogenous steady state wealth distribution. The basic algorithm is to: 1) taking a price of discount bonds $q \in [0,1]$ as given, solve the agent's dynamic programming problem for her decision rule $a' = g_\theta(a,s;q)$ where $a \in A$ are asset holdings, $s \in S \subset \mathbb{R}_{++}$ is exogenous earnings, and θ is a parameter vector; 2) given the decision rule and stochastic process for earnings, solve for the invariant wealth distribution $\mu^*(A,S;q)$; 3) given μ^*, check whether the asset market clears at q (i.e.$\int_{A,S} g_\theta(a,s;q)\mu^*(da,ds;q) = 0$). If it is, we are done. If not (i.e. it is not within an acceptable tolerance), then bisect $[0,1]$ in the direction that clears the market (e.g. if $\int_{A,S} a'\mu^*(da,ds;q) > 0$, then choose a new price $\widehat{q} = q + [1-q]/2$ and go to step 1.

1. Let the parameter vector $\theta = (\beta, \alpha, S, \Pi, A)$ be given by the discount factor $\beta = .9932$, the coefficient of relative risk aversion $\alpha = 1.5$, the set of possible earnings $S = \{e, u\}$ where $e = 1$ and $u = 0.5$ are interpreted as earnings when employed (normalized) and unemployed respectively, the markov process for earnings $\Pi(s' = e|s = e) = 0.97$ and $\Pi(s' = u|s = u) = 0.5$ (calibrated using duration of unemployment data of 2 quarters and an average unemployment rate of 6%)[1], and the space of asset holdings be given by the compact set $A = [-2, 5]$. Define the operator T on the space of bounded functions on $A \times S$ (bounded by virtue of the fact that $A \times S$ is compact) by

$$(Tv)(a,s;q) = \max_{(c,a')\in\Gamma(a,s;q)} \frac{c^{1-\alpha}-1}{1-\alpha} + \beta \sum_{s'\in S} \Pi(s'|s)v(a',s';q)$$

where

$$\Gamma(a,e;q) = \{(c,a') \in \mathbb{R}_+ \times A : c + qa' \le s + a\}.$$

Starting with a guess for $v(a,s;q)$, call it v^0, use the operator T to define mappings T^n where $T^1v^0 = Tv^0$, $T^2v^0 = T(Tv^0)$, just iterations on the value

[1] For the duration number D and replacement rate b, see

http://www.oecd.org/dataoecd/28/9/36965805.pdf#search=%22net%20replacement%20rates%20and%20unemployment %20insurance%20benefit%20duration%20in%2026%20oecd%20countries%202004%22 function until $\sup_a |\frac{T^{n+1}v^0 - T^n v^0}{T^n v^0}| < \varepsilon$ for all s for arbitrarily small ε. The fact that the operator T can be shown to be a contraction mapping assures you that you'll eventually arrive near the unique fixed point on the computer (if not, you've got a computational mistake). One way to think about doing this is to make a grid on A of N points (call it $\widetilde{A}$). Let v_0 be something simple like a constant function or even a the zero vector and form a $2 \times N \times N$ dimensional array in which the decision rule will live. Then for each e in the first dimension of the array and each a in the second dimension, evaluate all possible a' in the third dimension via $u(s + a - qa') + \beta \sum_{s' \in S} \Pi(s'|s)v(a', s'; q)$.Since there are a finite number of evaluations, a maximum will exist and the index of the optimal choice in the third dimension, call it $\iota^*(a, s)$, gives the mapping $g_q : \widetilde{A} \times S \to \widetilde{A}$. An even quicker way is to make use of the fact that *if* v is concave, then since $u(e + a - qa')$ is decreasing at a faster rate than $\beta \sum_{s' \in S} \Pi(s'|s)v(a', s'; q)$ is increasing as we raise a', it is likely that there is an interior maximum so you can simply evaluate whether the next point in the grid in the third dimension yields a higher value than the current one; if it is lower, then stop.

2. Given the decision rule g_q from step 1, define the operator T^* on the space of probability measures $\Lambda(\widetilde{A} \times S)$ by

$$(T^*\mu)(\widetilde{A}_0, S_0) = \sum_{(a',s') \in \widetilde{A}_0 \times S_0} \left\{ \sum_{(a,s) \in \widetilde{A} \times S} \chi_{\{a' = g_q(a,s)\}} \Pi(s'|s)\mu(a, s; q) \right\}.$$

where $\chi_{\{a'=g_q(a,s)\}}$ is an indicator function that picks out combinations of (a, s) which map to a given a' and $\widetilde{A}_0 \times S_0 \subset \widetilde{A} \times S$. Starting with a guess for $\mu(a, s; q)$, call it μ^0 (a good one might be $\mu^0_{i,j} = \frac{a_j - \underline{a}}{\overline{a} - \underline{a}} \Pi^*_i$ where $\underline{a} = -2$ and $\overline{a} = 30$) use the operator T^* to define mappings T^{*n} where $T^{*1}\mu^0 = T^*\mu^0$, $T^{*2}\mu^0 = T^*(T^*\mu^0)$, just iterations on the wealth distribution until $\sup_a |\frac{T^{*n+1}\mu_0 - T^{*n}\mu^0}{T^{*n}\mu^0}| < \varepsilon$ for all s for arbitrarily small ε. The fact that the transition function $Q((a, s), \widetilde{A}_0 \times S_0) = \sum_{(a',s') \in \widetilde{A}_0 \times S_0} \chi_{\{a'=g_q(a,s)\}} \Pi(s'|s)$ can be shown to satisfy certain monotonicity and "mixing" properties (namely that there is always a strictly positive probability of going downtimes when at the upper bound and uptimes when at the lower bound) assures you that you'll eventually arrive near the unique fixed point $\mu^*_q = T^*\mu^*_q$ on the computer. With a discrete state space represented by the $2 \times N$ dimensional arrray, the process of distributing weight on points (a, s) [which are just indices (i, j)] to points (a', s') [which are just indices $(\iota^*(i, j), j')$] is just a series of do loops on i, j, and j'. Specifically, for a given pair (i', j'), $\mu'(\iota', j') = \sum_{i,j} \pi(j, j') * \chi_{\{\iota^*(i,j)=i'\}}\mu(i, j)$ where μ' and μ are $N \times 2$ arrays and π is the 2×2 transition matrix. On my website is a program to help implement the T^* operator called Qsample.m.

3. Embed the functions associated with steps 1 and 2 into a program to calculate excess demand or supply of assets. Specifically, given μ^*, check whether the asset market clears at q (i.e.$ED(q) \equiv \int_{A,S} g_\theta(a, s; q)\mu_\theta^*(da, ds; q) = 0$).[2] If $ED(q) = 0$, stop. If $ED(q) > 0$ raise q and if $ED(q) < 0$ lower q, and go to step 1.[3]

4. After finding fixed points of the T and T^* operators, answer the following questions:

a. Plot the policy function $g(a, s)$ over a for each s to verify that there exist $\widehat{a}$ where $g(\widehat{a}, s) < \widehat{a}$ as in Figure 1 of Huggett. (Recall this condition establishes an upper bound on the set A necessary to obtain an invariant distribution).

b. What is the equilibrium bond price? Plot the cross-sectional distribution of wealth for those employed and those unemployed on the same graph.

c. Plot a Lorenz curve.What is the gini index for your economy? Compare them to the data. For this problem set, define wealth as current earnings (think of this as direct deposited into your bank, so it is your cash holdings) plus net assets. Since market clearing implies aggregate assets equal zero, this wealth definition avoids division by zero in computing the Gini and Lorenz curve.

III. To assess the question about the welfare gains associated with moving from incomplete to complete markets, compute consumption equivalents using the following formulas. In particular, you are to answer the following specific question: what fraction of consumption would a person in a steady state of the incomplete markets environment be willing to pay (if positive) or have to be paid (if negative) in all future periods to achieve the allocation of the complete markets environment?

For each (a, s) we compute $\lambda(a, s)$ such that

$$\begin{aligned} W^{FB} &= E\left[\sum_{t=0}^{\infty}\beta^t \frac{[(1+\lambda(a,s))c_t(a,s)]^{1-\alpha}-1}{1-\alpha}|(a,s)\right] \\ &= (1+\lambda(a,s))^{1-\alpha}E\left[\sum_{t=0}^{\infty}\beta^t\frac{c_t(a,s)^{1-\alpha}}{1-\alpha}\right] - \frac{1}{(1-\alpha)(1-\beta)} \\ &= (1+\lambda(a,s))^{1-\alpha}\left[v(a,s)+\frac{1}{(1-\alpha)(1-\beta)}\right] - \frac{1}{(1-\alpha)(1-\beta)} \end{aligned}$$

[2] Recall that if $a < 0$, households are supplying bonds (i.e. debt just like the government supplies debt) while if $a > 0$ households are demanding bonds (i.e. just like saving that government debt). Thus, when $ED(q) > 0$, prices need to rise (interest rates fall) to choke off too much saving while if $ED(q) < 0$, prices need to fall (interest rates rise) to choke off too much borrowing.

[3] Sometimes pure bisection to revise q' adjusts too quickly. In that case, you can use the following algorithm:

```
set adjustment_step=0.01*q (the factor could be even smaller for better precision)
If ( abs(excess_demand) > 10^(-6) .and. excess_demand >0 ) then
q'=q + adjustment_step
else if ( abs(excess_demand) > 10^(-6) .and. excess_demand <0 ) then
q'=q - adjustment_step
end if
```

or

$$\lambda(a,s) = \left[\frac{W^{FB} + \frac{1}{(1-\alpha)(1-\beta)}}{v(a,s) + \frac{1}{(1-\alpha)(1-\beta)}} \right]^{1/(1-\alpha)} - 1$$

where $v(a,s)$ is the value function from the incomplete markets economy. Then the economywide welfare gain is given by

$$WG = \sum_{(a,s)\in A\times S} \lambda(a,s)\mu(a,s).$$

a. Plot $\lambda(a,s)$ across a for both $s = e$ and $s = u$ in the same graph.

b. What is W^{FB}? What is $W^{INC} = \sum_{(a,s)\in A\times S} \mu(a,s)v(a,s)$? What is WG?

c. What fraction of the population would favor changing to complete markets? That is, $\sum_{(a,s)\in A\times S} \mathbf{1}_{\{\lambda(a,s)\geq 0\}}(a,s)\mu(a,s)$.