

Fall25 ECON880 PSET1

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1 DP in an Infinite Horizon

We assume that households have log preferences (β), and the production technology satisfies

$$Y_t = Z_t K_t^\alpha \quad (1.1)$$

with the parameters $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$. The technology shock Z follows a two-state Markov chain with transition matrix:

$$\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix} \quad (1.2)$$

and the two states of technology are $Z \in \{Z_g, Z_b\}$. The dynamic programming problem you are to solve is:

$$V(K, Z) = \max_{K'} \log \left(ZK^\alpha + (1 - \delta)K - K' \right) + \beta \mathbf{E} \left[V(K', Z') \right] \quad (1.3)$$

where to save on notation any variable $X_t = X$ and $X_{t+1} = X'$.

Exercise 1.1 (VFI). Solve the stochastic dynamic programming problem via value function iteration over a discrete grid on K of 1000 linearly spaced grid points over $[0.01, 90]$. Specifically, let $K_i \in \{0.01, 0.1001, \dots, 90\}$. What are the differences in computing time between the un-parallelized versus parallelized versions for both deterministic and stochastic cases?

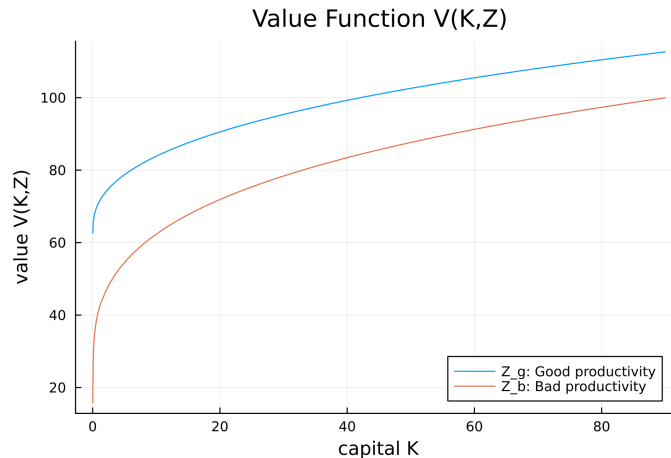
Suggested Solution. I got the parallelized versions ran *slower* than the un-parallelized versions in both deterministic and stochastic cases. My guess is that the model is too simple in a way that parallelization introduces an overhead issue.

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Exercise 1.2 (Value Function). Plot the value function over K for each state Z . Is it *increasing* (i.e. is $V(K_{i+1}, Z) \geq V(K_i, Z)$ for $K_{i+1} > K_i$)? Is it *concave* (in the sense that $V(K_{i+1}, Z) - V(K_i, Z)$ is decreasing)?

Suggested Solution. In Figure 1 below, we see that the value functions are increasing and concave in K for both states of technology Z . \square

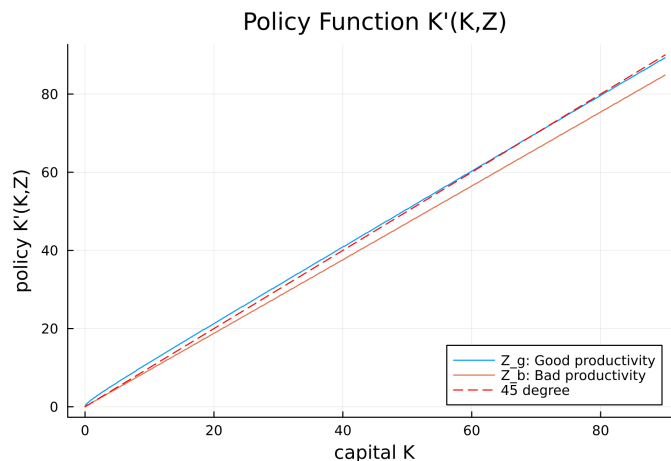
Figure 1: Value Function



Exercise 1.3 (Decision Rule / Policy Function). Plot the decision rules $K'(K, Z)$ for each state Z including a 45 degree line. Is the decision rule increasing/decreasing in K and Z (i.e. is $K'(K_{i+1}, Z) \geq K'(K_i, Z)$ for $K_{i+1} > K_i$ and is $K'(K, Z_g) \geq K'(K, Z_b)$)?

Suggested Solution. In Figure 2 below, the decision rules are both increasing in capital level K (positive slopes) and technology state Z ($K'(K, Z_g)$ lies above $K'(K, Z_b)$). \square

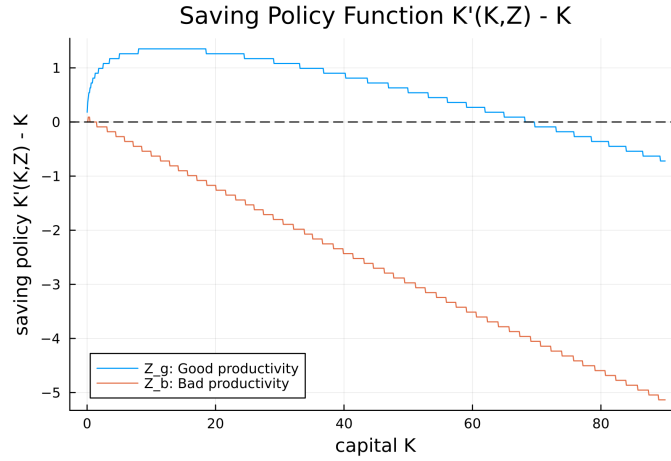
Figure 2: Policy Function



Exercise 1.4 (Saving / Policy Function Changes). Is saving increasing in K and Z (to see this, plot the change in the decision rule $K'(K, Z) - K$ across K for each possible exogenous state Z)? Provide intuition in terms of the marginal benefit versus marginal cost of increasing capital holdings. What in the environment might lead to saving at low levels of capital and dissaving at high levels of capital?

Suggested Solution. In Figure 3, we notice that the saving schedule increases at lower levels of capital but then decreases at higher levels of capital if producer starts with good productivity. While in the opposite case where the producer starts with bad productivity, the producer always borrows (i.e., dissaving) for all levels of capital.

Figure 3: Policy Function Changes



In our DP formulation, the consumption is given by $C = ZK^\alpha + (1 - \delta)K - K'$. With log utility, the consumption–saving decision should be shaped by:

$$\underbrace{\frac{1}{C}}_{\text{MC of } \nearrow K} = \beta \mathbb{E}_{Z'} \underbrace{\left[\frac{1}{C'} \left(\alpha Z'(K')^{\alpha-1} + (1 - \delta) \right) \right]}_{\text{MB of } \nearrow K}, \quad (1.4)$$

① For producer that starts with good productivity (Z_g), the transition matrix suggests that she is more likely to stay in good state, and the MB of increasing capital is high at low levels of capital (should $\searrow C$ for $\nearrow K$; saving). Then, as levels of capital increases, the MB diminishes so that she starts to dissave (should $\searrow K$ for $\nearrow C$; dissaving).

② For producer that starts with bad productivity (Z_b), the transition matrix suggests that she is more likely to stay in bad state, and in that case the MB of increasing capital would be lower in every period due to $Z_b < 1$. Then there is no saving motive at all levels of capital.