Fall25 ECON880 PSET2

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1 Huggett (1993, JEDC)

Exercise 1.1 (Competitive EQM)**.** Consider the same environment as Huggett (1993) except assume that there are enforceable insurance markets regarding the idiosyncratic shocks to earnings and that there are no initial asset holdings. Solve for a competitive equilibrium. What are prices? What is the allocation? (Hint: think about the planner's problem and then decentralize)

Suggested Solution. (I follow the notations in slides.) The environment is as follows:

- Population: unit measure of HH.
- Preferences: $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right]$, where U is continuously differentiable, strictly concave, and bounded.
- Endowment: $s_t \in \mathcal{S} = \{e, u\}$ (earning shocks, which are i.i.d. across HH)
  - Earns $y^e = 1$ if employed; earns $y^u = 0.5$ if unemployed
  - Markov employment process: $\mathcal{P} = \begin{bmatrix} \pi(e|e) & \pi(u|e) \\ \pi(e|u) & \pi(u|u) \end{bmatrix} = \begin{bmatrix} 0.97 & 0.03 \\ 0.5 & 0.5 \end{bmatrix}$.[1]
  - We can invariant dist by $\Pi\mathcal{P} = \Pi \implies \Pi = \{\pi_e, \pi_u\} = \{0.94, 0.06\}$
- Asset: non-contingent bonds $a_t \in A$ with price of next-period bonds q_t (for a_{t+1}); borrowing constraint $\underline{a} \leq 0$
- Other: discount factor $\beta = 0.9932$; relative risk aversion $\alpha = 1.5$

(Cont'd on next page!)

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[1] Use duration of unemployment data of 2 quarters and an average unemployment rate of 6%

Planner's problem. SP maximizes the expected utility of the HH by choosing consumption allocation $\{c_t^e, c_t^u\}_{t=0}^{\infty}$ subject to the resource constraint (RC):

$$\max_{\{c_t^e, c_t^u\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\underbrace{U(c_t^e)\pi_e}_{\text{EU employed}} + \underbrace{U(c_t^u)\pi_u}_{\text{EU unemployed}} \right] \text{ s.t. [RC] } \pi_e c_t^e + \pi_u c_t^u \leq \pi_e y^e + \pi_u y^u =: \mathbf{y} \tag{1.1}$$

$$\implies \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[U(c_t^e)\pi_e + U(c_t^u)\pi_u + \lambda_t \left[\mathbf{y} - \pi_e c_t^e - \pi_u c_t^u\right]\right] \tag{1.2}$$

Then, we take FOCs:

$$[c_t^e]: \qquad 0 = \beta^t \pi_e U'(c_t^e) - \lambda_t \pi_e \tag{1.3}$$

$$[c_t^u]: \qquad 0 = \beta^t \pi_u U'(c_t^u) - \lambda_t \pi_u \tag{1.4}$$

$$\implies U'(c_t^e) = U'(c_t^u) \implies c_t^e = c_t^u \tag{1.5}$$

$$\xRightarrow{\text{plug in [RC]}} \pi_e c_t^e + \pi_u c_t^e = c_t^e = \mathbf{y} = \underbrace{0.94 \times 1}_{\pi_e y^e} + \underbrace{0.06 \times 0.5}_{\pi_u y^u} = 0.97 = c_t^u \tag{1.6}$$

So the SP allocation is $c_t^e = c_t^u = \mathbf{c}^{sp} = 0.97$ for employed and unemployed HH $\forall t$. □

HH problem. In the decentralized economy, each HH chooses consumption and non-contingent bonds $\{c, a'\}$ (recursive form) to solve:

$$\max_{c,\, a'} U(c) + \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) U(a', s') \text{ s.t. } c + qa' = y(s) + a \tag{1.7}$$

$$\implies \max_{a'} U(y(s) + a - qa') + \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) U(a', s') \tag{1.8}$$

We take F.O.C.:

$$[a']: \quad 0 = -qU'(c) + \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) U'(c') \cdot 1 \tag{1.9}$$

$$\implies \quad q = \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) \frac{U'(c')}{U'(c)} = \beta \mathbb{E}\left[\frac{U'(c')}{U'(c)} \middle| s \right] \tag{1.10}$$

Note that, to have the same allocation as in SP problem, we need $c = c' = \mathbf{c}^{sp} (= 0.97)$ $\forall s, s' \in \mathcal{S}$. So the price of next-period bonds is pinned down:

$$q = \beta \mathbb{E}\left[\frac{U'(c')}{U'(c)} \middle| s \right] = \beta \frac{\cancel{U'(\mathbf{c}^{sp})}}{\cancel{U'(\mathbf{c}^{sp})}} = \beta = 0.9932 \tag{1.11}$$

Budget constraint for agents (employed/unemployed) has that:

$$s = e: \quad c^e + a^e = y^e \quad \Longrightarrow a^e = y^e - \mathbf{c}^{sp} = 1 - 0.97 = 0.03 \ (> 0 : \textit{save}) \tag{1.12}$$
$$s = u: \quad c^u + a^u = y^u \quad \Longrightarrow a^u = y^u - \mathbf{c}^{sp} = 0.5 - 0.97 = -0.47 \ (< 0 : \textit{borrow}) \tag{1.13}$$

Finally, we check market clearing condition for bonds:

$$a^e \pi_e + a^u \pi_u = 0.03 \times 0.94 + (-0.47) \times 0.06 = 0 \quad \checkmark \tag{1.14}$$

$\square$

2 Endogeneous steady state wealth distribution

The following takes you through the steps of solving a simple general equilibrium model that generates an endogenous steady state wealth distribution:

Step 1. We first take a price of discount bonds $q \in [0, 1]$ as given (assume that $\beta < q \leq 1$), then solve the agent's dynamic programming problem for her decision rule $a' = g(a, s; q)$. Define the operator T on the space of bounded functions on $A \times S$ (bounded by virtue of the fact that $A \times \mathcal{S}$ is compact) by

$$(Tv)(a, s; q) = \max_{(c,a') \in \Gamma(a,s;q)} \left\{ u(c) + \beta \sum_{s' \in \mathcal{S}} \Pi(s'|s) \, v(a', s'; q) \right\}, \tag{2.1}$$

where

$$\Gamma(a, s; q) = \{(c, a') \in \mathbb{R}_+ \times A : c + qa' \leq s + a\} \tag{2.2}$$

is the set of feasible consumption-asset choices given state (a, s) and bond price q. Essentially, we solve for the value and policy functions given q in this step.

Step 2. Given the decision rule $a' = g(a, s; q)$ in **Step 1.**, we next compute the invariant structure of wealth distribution. So, we take the decision rule g_q as given, define the operator T^* on the space of probability measures $\Lambda(\tilde{A} \times \mathcal{S})$ by

$$(T^* \mu)(\tilde{A}_0, S_0) = \sum_{(a',s') \in \tilde{A}_0 \times S_0} \left\{ \sum_{(a,s) \in \tilde{A} \times \mathcal{S}} \chi_{\{a' = g_q(a,s)\}} \, \Pi(s'|s) \, \mu_q(a, s) \right\}. \tag{2.3}$$

Here, $\chi_{\{a'=g_q(a,s)\}}$ is an indicator function that picks out combinations of (a, s) which map to a given a', and $\tilde{A}_0 \times S_0 \subseteq \tilde{A} \times S$. Starting with a guess for $\mu(a, s; q)$, call it μ^0, use the operator T^* to define mappings T^{*n} where $T^{*1}\mu^0 = T^*\mu^0$, $T^{*2}\mu^0 = T^*(T^*\mu^0)$, etc. Thus, in this step, we are computing the invariant distribution $\mu(a, s; q)$ as the fixed point of T^*.

Step 3. Finally, we check whether the bond market clears. Embed the functions associated with **Step 1.** and **2.** into a program to calculate the excess demand or supply of assets. Specifically, given the invariant distribution $\mu(a, s; q)$ from **Step 2.**, check whether the asset market clears at the bond price q we started with in **Step 1.**, that is,

$$\mathrm{ED}(q) \equiv \int_{A,\mathcal{S}} g_q(a, s)\, \mu_q^*(da, ds) = 0. \tag{2.4}$$

- If $\mathrm{ED}(q) = 0$, the market clears and we are done.
- If $\mathrm{ED}(q) > 0$, we need a higher bond price to make saving less attractive (raise q), and
- If $\mathrm{ED}(q) < 0$, we need a lower bond price to make borrowing less attractive (lower q).

Repeat **Steps 1.** and **2.** until the bond market clears. The equilibrium bond price is then denoted by q^{**}.

Remark. Here is an overview of the main functions in my `2 functions.jl` file:

① `Solve_HH(prim, res)` → Solves T operator for given q

② `Solve_Invariant(prim, res)` → Solves T^* operator for distribution

③ `Update_q(res, tol)` → Checks market clearing and updates q

For asset space, I choose $A = [-2, 2.5]$ with 1000 grid points, where 2.5 aligns with the upper bound in Dean's slides. And, I guess an initial bond price to be $q_0 = \beta + (1 - \beta)/8$, which is artificially higher than the complete market benchmark $q = \beta$.

Exercise 2.1 (Policy function $g(a, s)$)**.** Plot the policy function $g(a, s)$ over a for each s to verify that there exists $\hat{a}$ where $g(\hat{a}, s) < \hat{a}$ as in *Figure 1 of Huggett*. (Recall this condition establishes an upper bound on the set A necessary to obtain an invariant distribution).

Suggested Solution. In Figure 1, I plot the policy function $g(a, s)$ over a for each employment state s. There exists $\hat{a}$ where $g(\hat{a}, s) < \hat{a}$ for both employed and unemployed states. Specifically, the thresholds are $\hat{a}_{\text{employed}} = 1.1261$ and $\hat{a}_{\text{unemployed}} = -2.0$ (in dashed lines).

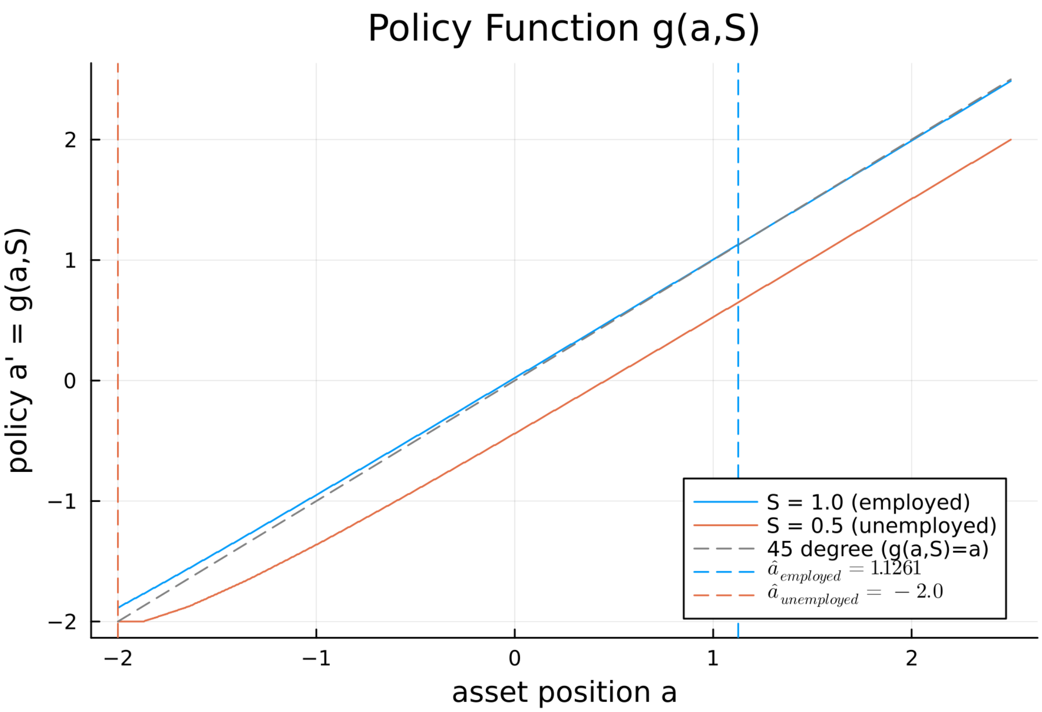

Figure 1: Policy Function

Remark. The employed agents should save, and the unemployed agents should dissave (i.e., borrow). □

Exercise 2.2 (Equilibrium bond price q^{**}). What is the equilibrium bond price? Plot the cross-sectional distribution of wealth for those employed and those unemployed on the same graph.

Suggested Solution. In this incomplete market, the equilibrium price of bonds is $q^{**} = 0.9942462299971491 \approx 0.99425$ (slightly higher than complete market benchmark $q = \beta$). The cross-sectional distribution of wealth for those employed and those unemployed is plotted in Figure 2.

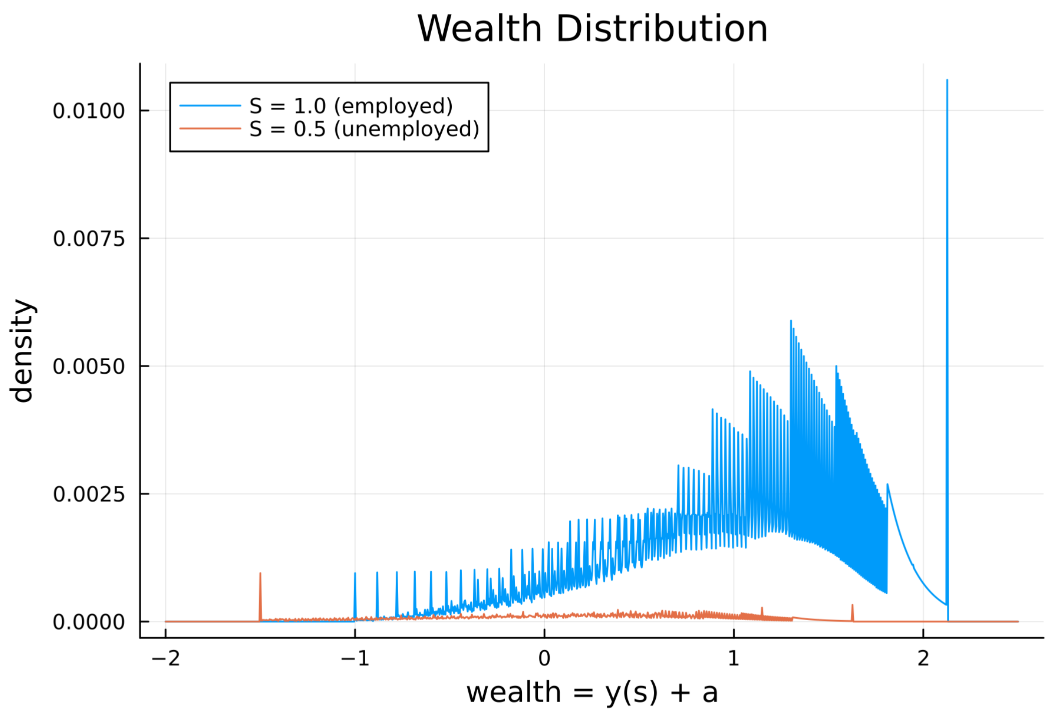

Figure 2: Wealth Distribution

Remark. The wealth of employed agents peak at 2.1261 and that of unemployed peak at -1.5. In general, the employed agents hold more wealth than the unemployed agents, as expected. $\square$

Exercise 2.3 (Lorenz curve and Gini coefficient)**.** Plot a Lorenz curve.What is the gini index for your economy? Compare them to the data. For this problem set, define wealth as current earnings (think of this as direct deposited into your bank, so it is your cash holdings) plus net assets. Since market clearing implies aggregate assets equal zero, this wealth definition avoids division by zero in computing the Gini and Lorenz curve

Suggested Solution. My calculated Gini coefficient is 0.38525, which is not too far from the ballpark of empirical estimates of wealth inequality in the U.S. (between 0.4 and 0.5 in year 2024). The Lorenz curve is plotted in Figure 3.

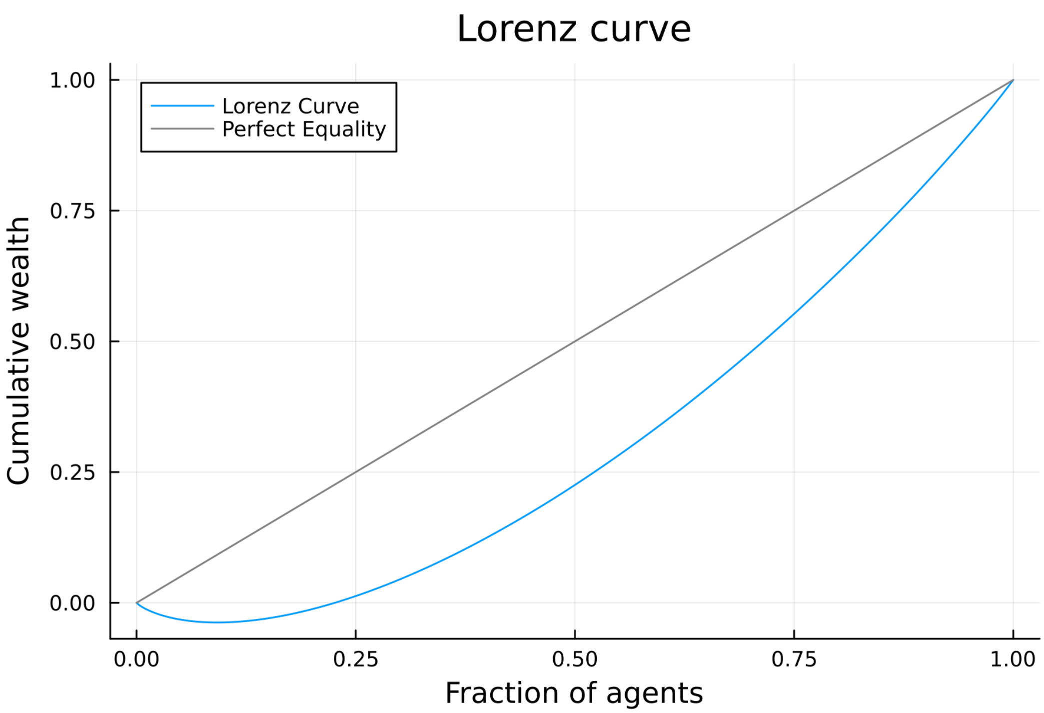

Figure 3: Lorenz Curve

Remark. The Lorenz curve is negative in the first quantile because, in the model, the sum of the wealth of agents in that quantile is less than zero. □

3 Welfare analysis of complete vs. incomplete markets

To assess the question about the welfare gains associated with moving from incomplete to complete markets, compute **consumption equivalents** using the following formulas. For each (a, s) we compute $\lambda(a, s)$ such that

$$W = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{\left[(1+\lambda(a,s))c_t(a,s)\right]^{1-\alpha} - 1}{1-\alpha} \middle| (a,s)\right] \tag{3.1}$$

$$= (1+\lambda(a,s))^{1-\alpha}\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t(a,s)^{1-\alpha}}{1-\alpha}\right] - \frac{1}{(1-\alpha)(1-\beta)} \tag{3.2}$$

$$= (1+\lambda(a,s))^{1-\alpha}\left[v(a,s) + \frac{1}{(1-\alpha)(1-\beta)}\right] - \frac{1}{(1-\alpha)(1-\beta)}, \tag{3.3}$$

$$\iff \lambda(a,s) = \left[\frac{W + \frac{1}{(1-\alpha)(1-\beta)}}{v(a,s) + \frac{1}{(1-\alpha)(1-\beta)}}\right]^{1/(1-\alpha)} - 1 \tag{3.4}$$

where $v(a, s)$ is the value function from the incomplete markets economy.

Exercise 3.1 (Consumption equivalents $\lambda(a, s)$). Plot $\lambda(a, s)$ across a for both $s = e$ and $s = u$ in the same graph.

Suggested Solution. Consumption equivalents $\lambda(a, s)$ are plotted in Figure 4. □

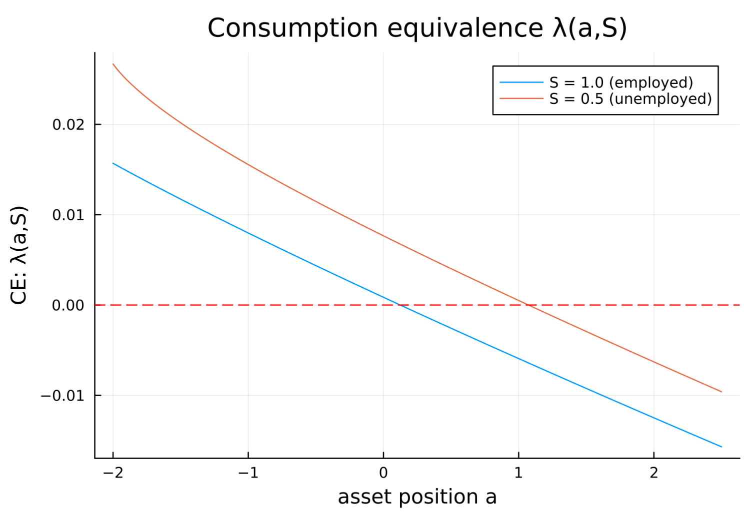

Figure 4: Consumption Equivalents by Asset and Employment states

Exercise 3.2 (Welfare in two markets and welfare gains/losses)**.** The welfare in complete market (henceforce W^{FB}) is:

$$W^{FB} \quad = \quad \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \frac{(c^{FB})^{1-\alpha}-1}{1-\alpha}\right] \tag{3.5}$$

$$\xrightarrow[\text{stationary}]{c^{FB}} \sum_{t=0}^{\infty}\beta^t \frac{(c^{FB})^{1-\alpha}-1}{1-\alpha} = \frac{(c^{FB})^{1-\alpha}-1}{(1-\alpha)(1-\beta)}. \tag{3.6}$$

The economywide welfare gain (or loss) from switching back to complete market is given by

$$WG = \sum_{(a,s)\in A\times S} \lambda(a,s)\mu(a,s). \tag{3.7}$$

Then,

① What is W^{FB}?

② What is W^{INC}, where

$$W^{INC} = \sum_{(a,s)\in A\times \mathcal{S}} \mu(a,s)v(a,s) \tag{3.8}$$

③ What is WG, the welfare gain (or loss) from switching back to complete markets?

Suggested Solution. From my Julia terminal, W^{FB} (rounded to 5 digits) is -4.25252. W^{INC} (rounded to 5 digits) is -4.45153. This aligns to class slides, *"Aggregate welfare is higher in the complete markets economy than the incomplete markets economy (no surprise)"*. The welfare gain of switching to complete markets (rounded to 5 digits) is 0.00134. □

Exercise 3.3 (Switching back or not)**.** What fraction of the population would favor changing to complete markets? That is,

$$\sum_{(a,s)\in A\times S} \mathbb{1}\{\lambda(a,s)>0\}\mu(a,s). \tag{3.9}$$

Suggested Solution. From my Julia terminal, the fraction of agents that favor switching to complete markets (rounded to 5 digits) is 0.54145. This is highlighted by the $\lambda(a,s)\geq 0$ (i.e., above red dashed line) in Figure 4. □

References

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, *17*(5), 953–969. https://doi.org/10.1016/0165-1889(93)90024-M