# Fall25 ECON880 PSET2

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## 1 Huggett (1993, JEDC)

> **Exercise 1.1** (Competitive EQM)**.** Consider the same environment as Huggett (1993) except assume that there are enforceable insurance markets regarding the idiosyncratic shocks to earnings and that there are no initial asset holdings. Solve for a competitive equilibrium. What are prices? What is the allocation? (Hint: think about the planner's problem and then decentralize)

**Suggested Solution.** (I follow the notations in slides.) The environment is as follows:

- Population: unit measure of HH.
- Preferences: $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right]$, where $U$ is continuously differentiable, strictly concave, and bounded.
- Endowment: $s_t \in \mathcal{S} = \{e, u\}$ (earning shocks, which are i.i.d. across HH)
  - Earns $y^e = 1$ if employed; earns $y^u = 0.5$ if unemployed
  - Markov employment process: $\mathcal{P} = \begin{bmatrix} \pi(e|e) & \pi(u|e) \\ \pi(e|u) & \pi(u|u) \end{bmatrix} = \begin{bmatrix} 0.97 & 0.03 \\ 0.5 & 0.5 \end{bmatrix}$.[1]
  - We can invariant dist by $\Pi\mathcal{P} = \Pi \implies \Pi = \{\pi_e, \pi_u\} = \{0.94, 0.06\}$
- Asset: non-contingent bonds $a_t \in A$ with price of next-period bonds $q_t$ (for $a_{t+1}$); borrowing constraint $\underline{a} \leq 0$
- Other: discount factor $\beta = 0.9932$; relative risk aversion $\alpha = 1.5$

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[1] Use duration of unemployment data of 2 quarters and an average unemployment rate of 6%

**Planner's problem.** SP maximizes the expected utility of the HH by choosing consumption allocation $\{c_t^e, c_t^u\}_{t=0}^{\infty}$ subject to the resource constraint (RC):

$$\max_{\{c_t^e, c_t^u\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ \underbrace{U(c_t^e)\pi_e}_{\text{EU employed}} + \underbrace{U(c_t^u)\pi_u}_{\text{EU unemployed}} \right] \text{ s.t. [RC] } \pi_e c_t^e + \pi_u c_t^u \leq \pi_e y^e + \pi_u y^u =: \mathbf{y} \tag{1.1}$$

$$\implies \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ U(c_t^e)\pi_e + U(c_t^u)\pi_u + \lambda_t \left[ \mathbf{y} - \pi_e c_t^e - \pi_u c_t^u \right] \right] \tag{1.2}$$

Then, we take FOCs:

$$[c_t^e]: \quad 0 = \beta^t \pi_e U'(c_t^e) - \lambda_t \pi_e \tag{1.3}$$

$$[c_t^u]: \quad 0 = \beta^t \pi_u U'(c_t^u) - \lambda_t \pi_u \tag{1.4}$$

$$\implies U'(c_t^e) = U'(c_t^u) \implies c_t^e = c_t^u \tag{1.5}$$

$$\xRightarrow{\text{plug in [RC]}} \pi_e c_t^e + \pi_u c_t^e = c_t^e = \mathbf{y} = \underbrace{0.94 \times 1}_{\pi_e y^e} + \underbrace{0.06 \times 0.5}_{\pi_u y^u} = 0.97 = c_t^u \tag{1.6}$$

So the SP allocation is $c_t^e = c_t^u = \mathbf{c}^{sp} = 0.97$ for employed and unemployed HH $\forall t$. $\square$

**HH problem.** In the decentralized economy, each HH chooses consumption and non-contingent bonds $\{c, a'\}$ (recursive form) to solve:

$$\max_{c,\, a'} U(c) + \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) U(a', s') \text{ s.t. } c + qa' = y(s) + a \tag{1.7}$$

$$\implies \max_{a'} U(y(s) + a - qa') + \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) U(a', s') \tag{1.8}$$

We take F.O.C.:

$$[a']: \quad 0 = -qU'(c) + \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) U'(c') \cdot 1 \tag{1.9}$$

$$\implies \quad q = \beta \sum_{s' \in \mathcal{S}} \pi(s'|s) \frac{U'(c')}{U'(c)} = \beta \mathbb{E} \left[ \frac{U'(c')}{U'(c)} \middle| s \right] \tag{1.10}$$

Note that, to have the same allocation as in SP problem, we need $c = c' = \mathbf{c}^{sp} (= 0.97)$ $\forall s, s' \in \mathcal{S}$. So the price of next-period bonds is pinned down:

$$q = \beta \mathbb{E} \left[ \frac{U'(c')}{U'(c)} \middle| s \right] = \beta \frac{\cancel{U'(\mathbf{c}^{sp})}}{\cancel{U'(\mathbf{c}^{sp})}} = \beta = 0.9932 \tag{1.11}$$

Budget constraint for agents (employed/unemployed) has that:

$$s = e: \quad c^e + a^e = y^e \quad \Longrightarrow a^e = y^e - \mathbf{c}^{sp} = 1 - 0.97 = 0.03 \; (> 0 : \textit{save}) \tag{1.12}$$

$$s = u: \quad c^u + a^u = y^u \quad \Longrightarrow a^u = y^u - \mathbf{c}^{sp} = 0.5 - 0.97 = -0.47 \; (< 0 : \textit{borrow}) \tag{1.13}$$

Finally, we check market clearing condition for bonds:

$$a^e \pi_e + a^u \pi_u = 0.03 \times 0.94 + (-0.47) \times 0.06 = 0 \quad \checkmark \tag{1.14}$$

□

# 2 Endogeneous steady state wealth distribution

**Exercise 2.1** (Incomplete market). aaaa

## References

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, *17*(5), 953–969. https://doi.org/10.1016/0165-1889(93)90024-M