Financial Market Incompleteness and Inequality©

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Questions

We will address the following questions:

1. If financial markets are incomplete (e.g. the only available asset is a non-contingent bond with a borrowing limit), how much does earnings inequality account for wealth inequality?
2. How much would people be willing to pay for “better” (more complete) financial markets?
3. If borrowing constraints become tighter (as they have in the recent crisis), how much does wealth inequality change? What are the welfare effects of tighter constraints?
4. How much does a doubling of unanticipated unemployment duration affect wealth inequality and welfare?

Complete vs. Incomplete Markets

What does macroeconomics have to say about wealth inequality?

- With complete markets, not much. Even if everyone experiences idiosyncratic earnings shocks, complete asset markets perfectly smooth income and consumption so we focus on a "representative agent". The wealth distribution is indeterminate.

- With incomplete markets, agents cannot perfectly insure earnings fluctuations and the wealth distribution is determinate.

- This latter framework is known in macro as "heterogeneous agent" models developed theoretically by Bewley thirty years ago and made quantitative by Aiyagari, Imrohoroglu, and Huggett twenty years ago.

Intro to Methodology

- To answer these questions, we will use the simplest possible structural general equilibrium heterogeneous agent model where people receive persistent idiosyncratic shocks to their earnings and can only insure through noncontingent assets subject to borrowing constraints.

- The basic methodology is to input an exogenous earnings shock process into a heterogeneous agent model to obtain endogenous asset decision rules and associated wealth distribution.

- We can then compute summary statistics of the wealth distribution like the Gini coefficient to analyze positive and normative questions about inequality.

A Quantitative Inequality Question

- Question: How much of wealth inequality as measured by the Gini coefficient can be explained by individual specific earnings shocks in a simple incomplete markets model?
- Key things to note about the question:
  - it is quantitative ("How much").
  - it requires us to compare data moments to model moments.
- Answer: The wealth Gini in the data $=0.8$ while the model wealth Gini$=0.38$. Hence, the simplest incomplete financial markets model can account for roughly 50% of wealth inequality.

Solving the Model

- Start with an underlying *exogenous* earnings process y_{t+1} which follows a finite state Markov Process. These shocks are iid (Unit measure of) across agents.

- Solve the model via dynamic programming for a precautionary savings decision rule $a_{t+1} = g(a_t, y_t; \theta)$ as a function of parameters θ.

- Given incomplete markets and idiosyncratic earnings shocks, even if everyone starts at the same wealth level, their wealth holdings will differ next period.

- In the limit, this fanning out generates an endogenous cross-sectional wealth distribution $\mu(a_t, y_t; \theta)$.

Model Moments vs. Data Moments

- Once you have these two endogenous functions ($g(a_t, y_t; \theta)$ $\mu(a_t, y_t; \theta)$), you can generate many different model moments to compare with the data moments.

- Choose parameters θ so that the model moments match certain data moments (e.g. average long run unemployment and real interest rates).

- The (overidentified) model is then "tested" by how well it does on other untargeted moments like the wealth Gini coefficient.

Lecture Outline

1. Inequality Data
2. A Parsimonious Quantitative Model
3. Experiments
   - 3.1 Assessing the Welfare Costs of Market Incompleteness
   - 3.2 Tighter Financial Constraints
   - 3.3 Longer Duration of Unemployment
4. Directions for Future Research
   - 4.1 Endogenize Borrowing Constraint
   - 4.2 Endogenize Earnings and Mobility
   - 4.3 Endogenize Redistribution

Preview of Results: Parameterization

We use the following parameterization (where a model period is one quarter) in our benchmark incomplete markets model:

for annual: $\times 4$

β	α	$y(e)$	$y(u)$	$\pi(e,e)$	$\pi(u,u)$	$\underline{a}$
0.994	1.5	1	0.5	0.97	0.5	-2

exogenous borrowing.

where

the model (off-the-shelves)

- β is the discount rate (outside)
- α is the coefficient of relative risk aversion (outside)
- $y(e)$ is earnings if employed (normalized)
- $y(u)$ is earnings if unemployed (outside)
- $\pi(e,e)$ is probability of staying employed (outside)
- $\pi(u,u)$ is probability of staying unemployed (outside)
- $\underline{a}$ is the borrowing constraint (i.e. twice income) (inside)

(*) this is just a simple 2-point distribution

Preview of Results: Accounting

Results from steady state of incomplete markets benchmark:

	Data	Bench
Unemployment (target) %	5.66	5.66
Real Interest (target) %	2.00	2.00
Wealth Gini (untargeted)	0.80	0.38
CE(employed) %	*	0
CE(unemployed) %	*	0
CE %	*	0

where CE denotes the Consumption Equivalent welfare measure.

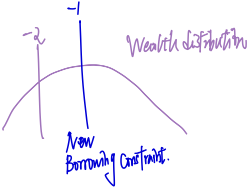
- Comparing the benchmark with data, we see that the incomplete markets model is only able to account for roughly half of the wealth inequality in the data.

Preview of Results: Counterfactual Financial Distress

(exogenous)

Experiment 2: Cut borrowing limits in half.

	Data	Bench	$\underline{a} = -1$	
Unemployment (target) %	5.66	5.66	?	No change.
Real Interest (target) %	2.00	2.00	?	↓ (to offset the saving motive)
(inequality) Wealth Gini (untargeted)	0.80	0.38	?	↓ (more likely; r ↓ makes ppl less willing to save)
CE(employed) %	*	0	?	
CE(unemployed) %	*	0	?	
CE %	*	0	?	



Preview of Results - Counterfactual Financial Distress

Experiment 2: Cut borrowing limits in half.

$\mu(a, 1)$ employment (e)

$\mu(a, 0.5)$

	Data	Bench	$\underline{a} = -1$
Unemployment (target) %	5.66	5.66	5.66
Real Interest (target) %	2.00	2.00	0.82
Wealth Gini (untargeted)	0.80	0.38	0.18
CE(employed) %	*	0	0.1987
CE(unemployed) %	*	0	0.2048
CE %	*	0	0.1991

- Positive Effects: Households' precautionary savings rise, thereby lowering interest rates and inequality roughly in half.

- Normative Effects: "What fraction of consumption would people in a steady state of an economy where borrowing is limited to 1 times quarterly employed earnings be willing to pay in all future periods to achieve the allocation of an economy where borrowing is limited to 2 times quarterly employed earnings?
  - Answer: 2/10 of one percent.

Preview of Results - Counterfactual Unemployment Distress

Experiment 3: Double the unemployment spell.

Increase P of being unemployed

	Data	Bench	$\underline{a} = -1$	$\pi(u, u) = 0.75$
Unemployment (target) %	5.66	5.66	5.66	? $\uparrow$
Real Interest (target) %	2.00	2.00	0.82	guess ? $\downarrow$
Wealth Gini (untargeted)	0.80	0.38	0.18	guess ? $\uparrow$
CE(employed) %	*	0	0.1987	?
CE(unemployed) %	*	0	0.2048	?
CE %	*	0	0.1991	?

($\downarrow k$ since unemployed $\uparrow$ to bill borrowing)

Preview of Results - Counterfactual Unemployment Distress

Experiment 3: Double the unemployment spell.

	Data	Bench	$\underline{a} = -1$	$\pi(u, u) = 0.75$
Unemployment (target) %	5.66	5.66	5.66	10.71
Real Interest (target) %	2.00	2.00	0.82	0.94
Wealth Gini (untargeted)	0.80	0.38	0.18	0.49
CE(employed) %	*	0	0.1987	3.1204
CE(unemployed) %	*	0	0.2048	4.1448
CE %	*	0	0.1991	3.2765

- Positive Effects: Households' precautionary savings rise, thereby lowering interest rates by roughly half. Longer spells of unemployment raise wealth inequality by about a quarter.
- Normative Effects: "What fraction of consumption would people in a steady state of an economy where the average duration of unemployment is one year be willing to pay in all future periods to achieve the allocation of an economy where the average duration is 2 quarters?
    - Answer: Over 3%.

[Data]: PSID
Survey Consumer Finance
Compustat
WRDS

Inequality Data

Main References for this section:

USC

1. Budria Rodriguez, S., J. Diaz Gimenez, V. Quadrini, V. Rios-Rull. 2002. “Updated Facts on the U.S. Distributions of Earnings, Income, and Wealth”, *Federal Reserve Bank of Minneapolis Quarterly Review*, Summer, p. 2-35, (BDQR uses 1998 SCF)

Penn.

2. Diaz Gimenez, J., A. Glover, and V. Rios-Rull. 2011. “Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update”, *Federal Reserve Bank of Minneapolis Quarterly Review*, February, p. 2-35. (DGR uses 2007 SCF)

Data Sources

Survey cross-section but not survey same HH.

1. Survey of Consumer Finances (SCF): A cross section of detailed balance sheet, pension, and income characteristics of US families. Conducted by NORC at Chicago every three years starting in 1992. Sample size is 4,500 and over-represents the rich. Since the SCF is not a panel, can't follow a household over time and hence cannot track mobility.

   not a panel

2. Panel Study of Income Dynamics (PSID): A panel of individuals (and their family) that includes income sources and amounts, employment, housing, education, age, but less detailed wealth information. Conducted by the University of Michigan. After 1997 it was conducted every two years. Sample size has grown from 4,800 in 1968 to 9,000 in 2009. Since it's a panel, it can be used to assess economic mobility.

Definitions

- Household: A person or a couple who live together and all the other people who live in the same household who are financially dependent on them.
  - The SCF considers the male of a couple to be the head of the household in every case.
  - In single households, the financially independent person of either sex is considered to be the head of the household.

- Cohort: A group with a common defining observable characteristic (e.g. age).

Definitions

exogeneous to the model. (outside)

- Earnings: wages and salaries plus a fraction (0.86) of business income (entrepreneurial income from professional practices not from capital, etc).

these smooth out volatility $\Rightarrow \sigma_{inc} \leq \sigma_{earning}$

- Income: all kinds of revenue before taxes but includes government and private transfers. Includes earnings, interest income, dividends, capital gains/losses from sale of stocks/bonds/real estate, unemployment compensation, income from social security and pensions, child support, food stamps and other welfare assistance, inheritances, disability compensation, etc.

endogeneous in the model (we solve for this)

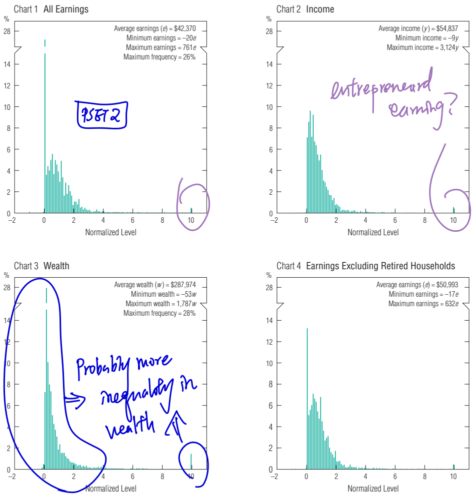
- Wealth: net worth includes value of financial (checking, money markets , bonds, stocks, investment accounts, cash value of life insurance, pension plans) and real assets (residences, vehicles) net of debts (mortgages, home equity loans, credit card debt, loans).

Histograms

Charts 1–4

U.S. Distributions of Earnings, Income, and Wealth

With Levels Normalized by the Mean*



*The first and last observations represent the frequencies of households with, respectively, less than –1 times and more than 10 times the corresponding averages.

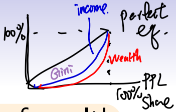
Source: 1998 Survey of Consumer Finances

Histograms

Charts 1-4 (BDQR): Histograms where the levels have been normalized by the mean, so 1 on the horizontal axis represents the fraction of households at mean earnings $42,370.

- The first and last observations represent the frequencies of households with, respectively, less than -2 times and more than 10 times the corresponding averages.
- Note the differences in ranges (min vs. max).
- Chart 4 displays Earnings excluding retirees. A large share of households have zero labor earnings.

For Gini Coeff, we need to construct Lorenz Curve first.

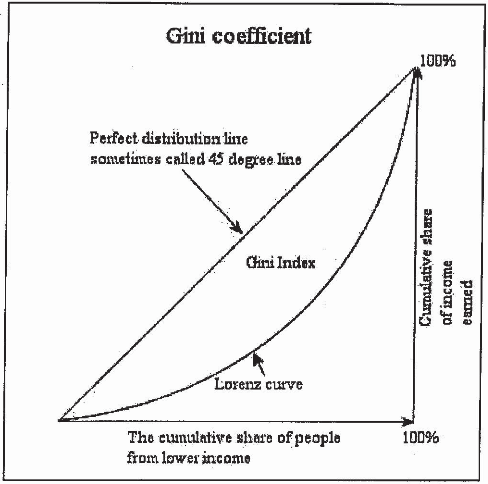


Definitions

- Lorenz Curve: The cumulative distribution function of wealth (or income, etc.). e.g. "the bottom 20% of all households have 10% of the total wealth".
  - A perfectly equal wealth distribution would be one in which every person has the same wealth. In this case, the bottom N% of society would always have N% of the wealth (i.e. a 45° line or the "line of perfect equality").
  - By contrast, a perfectly unequal distribution would be one in which one person has all the wealth and everyone else has none. In that case, the curve would be at $y = 0$ for all $x < 100\%$, and $y = 100\%$ when $x = 100\%$.

- Gini coefficient (a measure of concentration): The area between the line of perfect equality and the observed Lorenz curve (denoted A), as a percentage of the area between the line of perfect equality and the line of perfect inequality (A+B, where B is the area under the Lorenz curve).
  - Hence, perfect equality $=0$, perfect inequality$=1$.

Inequality Measures

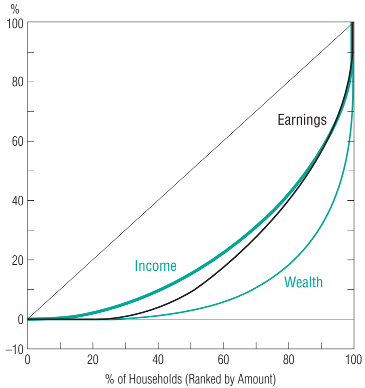


Lorenz Curve

Chart 5

The Lorenz Curves for the U.S. Distributions of Earnings, Income, and Wealth

What % of All Households Have
What % of All Earnings, Income, or Wealth



Source: 1998 Survey of Consumer Finances

Inequality Measures

Table 8

Changes in Concentration

median is poorer than average.

CV scores: σ / μ

	Gini Indexes				Mean-to-Median Ratios				Coefficients of Variation			
	E	*I*	*W*	*N-H-W*	*E*	*I*	*W*	*N-H-W*	*E*	*I*	*W*	*N-H-W*
2007	0.636	0.575	0.816	0.881	1.72	1.77	4.61	10.45	3.60	4.32	6.02	7.60
1998	0.611	0.548	0.800	0.861	1.56	1.62	3.95	7.64	2.82	3.56	6.47	7.93
%△	4.1	4.9	2.0	2.3	10.2	9.3	16.7	36.8	27.7	21.3	–7.0	–4.2

Table 9

Changes with Respect to the Medians

	50–30 Ratios				90–50 Ratios			
	E	*I*	*W*	*N-H-W*	*E*	*I*	*W*	*N-H-W*
2007	2.77	1.68	4.54	4.73	3.41	3.00	7.55	15.73
1998	2.80	1.71	4.00	4.54	3.18	2.87	6.88	12.56
%△	–1	–2	13	4	7	4	10	25

Earnings Statistics

Table 4 on Earnings (DGR):

- The earnings poorest tend to have sizable business losses but hold almost twice sample average wealth (e.g. unlucky entrepreneurs).
- Many of the earnings poor are retirees (who also have sizeable wealth holdings).
- Earnings richest tend to be 46-65 years old, college educated, self-employed and married.
- Earnings rich are similar but a bit younger. All part of the hump shaped earnings. Bottom line, age is an important observable characteristic.

Earnings Statistics

Table 4

Earnings Partition of the 2007 SCF Sample (Gini Index = 0.636)

	Bottom (%)			Quintiles					Top (%)			All
%	0–1	1–5	5–10	1st	2nd	3rd	4th	5th	90–95	95–99	99–100%	0–100
					Averages (x 10^3 2007 USD)							
Earnings	−9.1	0.0	0.0	−0.5	13.4	37.2	66.4	202.5	149.9	264.8	1,191	63.8
Income	71.8	27.1	29.3	30.4	26.5	44.3	74.0	242.6	173.7	321.6	1,553	83.6
Wealth	1,026	309.3	317.8	359.0	199.6	200.4	328.2	1,690	1,094	2,618	12,197	555.4
	? retire? entre losses?											
					Shares of Total Sample (%)							
Earnings	−0.1	0.0	0.0	−0.1	4.2	11.7	20.8	63.5	11.7	16.6	18.7	100.0
Income	0.9	1.3	1.8	7.3	6.3	10.6	17.7	58.1	10.4	15.4	18.6	100.0
Wealth	1.8	2.2	2.9	12.9	7.2	7.2	11.8	60.9	9.9	18.9	22.0	100.0
					Income Sources (%)							
Labor	1.8	0.0	0.0	0.2	45.2	76.6	82.9	66.6	77.4	63.3	49.1	64.3
Capital	78.6	12.5	16.8	25.1	8.6	5.2	3.7	11.5	8.6	12.2	17.7	10.2
Business	−16.7	0.0	0.0	−2.0	6.5	8.4	7.9	19.6	10.3	22.0	31.9	13.9
Transfers	34.8	83.2	78.3	73.4	35.7	8.1	4.5	1.8	2.9	1.6	1.2	10.3
Other	1.6	4.4	4.8	3.2	4.1	1.7	1.0	0.6	0.8	0.9	0.0	1.2
					Age (%)							
Under 31	2.8	5.4	3.2	3.1	26.0	23.1	14.2	6.1	5.8	2.5	0.1	14.5
31–45	7.9	3.6	13.3	7.82	24.2	37.5	38.4	36.2	34.4	31.8	22.5	28.8
46–65	51.9	20.0	25.6	25.1	29.9	32.9	44.1	53.5	54.9	57.5	68.9	37.1
Over 65	37.5	70.9	57.9	64.0	19.9	6.5	3.3	4.2	4.9	8.2	8.5	19.6
Average (years)	62.6	69.7	66.8	68.5	46.8	42.9	44.5	47.4	47.8	50.1	52.8	50.0
					Education (%)							
Dropouts	24.7	24.0	26.6	26.1	20.3	13.9	5.1	2.3	3.1	0.3	0.3	13.5
High school	30.8	41.8	37.6	39.3	40.9	35.5	31.4	17.2	14.6	10.3	4.5	32.9
Some college	19.9	12.9	16.7	15.3	20.4	23.1	18.3	14.6	14.6	9.1	6.9	18.4
College	24.6	21.3	19.2	19.3	18.4	27.5	45.2	65.9	67.7	80.3	88.3	35.3
					Employment Status (%)							
Workers	2.7	0.6	1.6	1.4	58.6	81.5	81.6	76.4	75.9	61.1	42.3	59.9
Self-employed	17.3	0.3	1.1	1.7	11.3	9.4	11.6	18.2	18.3	30.4	47.8	10.5
Retired	53.7	75.0	65.4	68.9	14.4	3.6	3.2	3.5	2.7	7.7	8.4	18.7
Nonworkers	26.4	24.2	31.9	28.0	15.6	5.6	3.6	1.8	3.0	0.8	1.5	10.9
					Marital Status (%)							
Married	48.4	28.6	32.8	33.1	42.8	54.3	75.0	88.8	91.2	89.0	95.9	58.8
Single w/ dependents	16.0	17.7	16.6	16.6	30.7	22.9	10.0	4.9	3.7	5.0	0.6	17.0
Single w/o dependents	35.6	53.7	50.7	50.2	26.5	22.8	15.0	6.3	5.1	6.0	3.5	24.2
Family size	1.83	1.6	1.7	1.6	2.3	2.5	2.7	3.0	3.0	2.9	3.2	2.4
					Marital Status Excluding Retired Widows							
Single w/ dependents	8.9	10.8	13.5	11.7	30.3	22.8	10.0	4.6	3.7	4.0	0.6	15.9
Single w/o dependents	24.2	26.1	28.7	25.6	23.8	22.8	14.9	6.3	5.1	6.0	3.3	18.7
Family size	2.00	1.68	1.80	1.70	2.52	2.69	2.84	3.07	3.11	2.93	3.18	2.56

Wealth Statistics

Table 6 on Wealth (DGR):

- The wealth poorest have avg net worth -$79K and tend to be young earning 40K. For the young, the debt is student loans. Some of the wealth poorest are retirees who have outlived their savings.
- Many of the wealth poor (with avg net worth -$5K) are high school dropouts (twice the sample avg).
- Wealth richest hold 34 times the sample avg coming from an even split between labor, capital and business sources. Many are self-employed (5 times sample avg).
- Wealth rich are similar.
- The share of the bottom quintile of the wealth distribution is -0.2 (i.e. we will see something like this in the model Lorenz curve).

Wealth Statistics

Table 6

Wealth Partition of the 2007 SCF Sample (Gini Index = 0.816)

	Bottom (%)			Quintiles					Top (%)			All
	0–1	1–5	5–10	1st	2nd	3rd	4th	5th	90–95	95–99	99–100	0–100
						Averages (x 10^3 2007 USD)						
Earnings	35.5	31.9	15.7	22.1	34.4	47.4	62.0	153.2	104.6	254.1	764.3	63.8
Income	38.4	37.8	21.8	27.5	40.5	56.5	74.2	219.2	137.9	347.6	1,323	83.6
Wealth	−79.0	−13.6	−0.9	−5.3	29.7	123.6	312.3	2,316	1,233	3,710	18,653	555.4
						Shares of Total Sample (%)						
Earnings	0.6	2.0	1.2	6.9	10.8	14.9	19.4	48.0	8.2	15.9	12.0	100.0
Income	0.5	1.8	1.3	6.6	9.7	13.5	17.8	52.5	8.3	16.6	15.8	100.0
Wealth	−0.1	−0.1	−0.0	−0.2	1.1	4.5	11.2	83.4	11.1	26.7	33.6	100.0
						Income Sources (%)						
Labor	85.6	83.5	72.4	78.9	81.2	78.6	77.1	51.4	58.6	54.7	30.2	64.3
Capital	0.0	−0.0	0.0	0.1	0.5	1.0	2.7	18.3	7.9	17.8	33.7	10.3
Business	8.1	1.2	−0.3	1.9	4.2	6.2	7.5	21.4	20.1	21.4	32.0	13.9
Transfers	3.7	12.1	22.3	15.5	12.0	12.4	12.1	8.2	12.6	5.5	3.6	10.3
Other	2.7	3.3	5.5	3.7	2.0	1.8	0.7	0.7	0.9	0.7	0.6	1.2
						Age (%)						
Under 31	47.3	44.6	29.0	36.0	22.6	8.6	3.8	1.5	0.4	1.9	3.0	14.5
31–45	38.8	32.7	38.3	32.1	36.5	32.8	25.7	17.0	19.6	13.1	7.9	28.8
46–65	13.9	16.9	24.1	22.1	28.3	35.5	45.6	53.9	50.1	57.7	57.7	37.1
Over 65	0.0	5.8	8.6	9.8	12.6	23.1	24.8	27.6	29.8	27.4	31.4	19.6
Average (years)	34.2	36.6	41.8	40.8	44.2	52.0	55.3	57.9	58.7	58.0	59.4	50.0
						Education (%)						
Dropouts	6.9	12.3	34.3	25.0	42.5	14.4	8.0	4.3	3.26	1.9	1.2	13.5
High school	13.2	23.4	33.7	34.1	19.9	35.2	33.8	18.6	13.9	10.2	6.1	32.9
Some college	23.7	29.2	21.5	22.1	41.6	18.9	17.4	13.4	14.5	10.3	7.1	18.4
College	56.2	35.2	10.5	18.8	29.9	31.5	40.8	63.7	68.4	77.6	85.6	35.3
						Employment Status (%)						
Workers	73.2	70.6	53.2	61.2	71.5	61.0	59.7	46.3	43.2	30.8	28.4	59.9
Self-employed	6.1	1.8	1.2	4.2	5.4	8.1	10.6	24.0	24.3	44.7	48.6	10.5
Retired	3.3	4.1	8.8	9.1	10.9	22.9	23.9	26.7	29.6	23.2	21.8	18.7
Nonworkers	17.3	23.5	36.9	25.6	12.2	8.1	5.9	2.9	2.8	1.4	1.2	10.9
						Marital Status (%)						
Married	51.2	41.2	30.6	38.3	51.3	63.8	65.9	74.7	74.3	82.5	90.6	58.8
Single w/ dependents	22.7	35.8	35.8	32.4	22.4	13.1	9.6	7.5	7.3	3.5	1.6	17.0
Single w/o dependents	26.1	23.1	33.6	29.3	26.3	23.0	24.4	17.8	18.4	14.1	7.8	24.2
						Marital Status Excluding Retired Widows						
Single w/ dependents	22.7	35.8	35.8	32.3	21.3	11.5	8.6	5.8	5.3	3.5	1.6	15.9
Single w/o dependents	26.1	20.5	29.4	25.0	23.0	16.2	17.0	12.2	11.3	10.5	6.9	18.7
Family size	2.77	2.55	2.54	2.51	2.64	2.64	2.48	2.54	2.52	2.63	2.63	2.56

Inequality and Age

Table 11 and Figure 2A,B (DGR) on Age:

- Hump shaped earnings, but even conditioning on age, there is a lot of inequality within a cohort.
- Table 19 shows there have not been big changes over the last 10 years.

Inequality and Age

Table 11

Age Partition of the 2007 SCF Sample

if no uncertainty of income/earning
⇒ CV should be 0's.

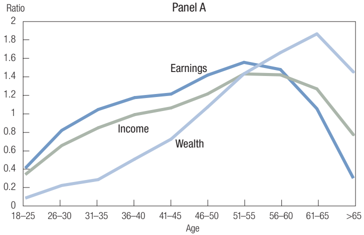
	Averages			Income Sources (%)					Gini Indexes			Coefficients of Variation				
Age	*E*	*Y*	*W*	*L*[d]	*K*[e]	*B*[f]	*Z*[g]	*O*[h]	*E*[a]	*Y*[b]	*W*[c]	*E*[a]	*Y*[b]	*W*[c]	*H* (%)[i]	Size[j]
–25	25.9	28.2	44.7	88.9	0.5	3.6	3.4	3.6	0.44	0.39	1.12	0.84	0.75	12.09	6.8	2.46
26–30	52.3	54.6	121.2	91.8	0.9	4.5	1.4	1.4	0.42	0.39	0.88	0.82	0.78	5.38	7.7	2.80
31–35	66.8	70.8	156.7	85.9	1.1	9.7	2.0	1.2	0.45	0.43	0.78	1.67	1.70	3.94	8.9	3.31
36–40	75.1	82.8	280.7	82.2	4.5	9.8	2.0	1.5	0.47	0.46	0.76	2.50	3.91	5.26	9.4	3.43
41–45	77.6	88.9	401.8	73.3	6.4	16.1	2.9	1.3	0.53	0.53	0.79	2.24	3.11	6.71	10.5	3.11
46–50	90.7	101.6	595.7	77.4	5.6	13.7	2.1	1.2	0.53	0.54	0.77	2.48	3.55	4.94	11.2	2.89
51–55	99.6	119.9	797.5	69.2	10.8	16.0	2.9	1.0	0.61	0.61	0.79	2.90	3.50	4.58	10.3	2.52
56–60	94.6	119.1	925.9	66.1	10.7	15.5	6.9	0.8	0.63	0.60	0.77	3.21	3.84	4.55	8.2	2.15
61–65	67.4	106.3	1039.5	47.6	15.5	18.3	17.4	1.3	0.75	0.64	0.79	6.08	6.36	4.62	7.5	2.03
66+	19.0	64.6	809.0	15.7	25.8	15.9	41.9	0.7	0.91	0.64	0.78	11.96	5.68	5.92	19.6	1.66
Total	63.8	83.6	555.4	64.3	10.2	13.9	10.3	1.2	0.64	0.57	0.82	3.60	4.32	6.02	100.0	2.56

[a]Earnings; [b]income; [c]wealth; [d]labor; [e]capital; [f]business; [g]transfers; [h]other; [i]percentage number of households per group;
[j]average number of persons per primary economic unit.

Inequality and Age

Figure 2

Average Earnings, Income, and Wealth (Panel A); Gini Indexes (Panel B); Income Sources (Panel C); and Coefficients of Variation (Panel D) for 10 Age Cohorts



Inequality and Education

Tables 12 and 20 (DGR) on Education:

- Higher education means higher earnings (college premium is 5 times more than dropouts and 2 times more than some college) and there is more earnings inequality among dropouts than higher educated.
- Table 20 shows there has been a large increase in proportions who are becoming more educated between 1998 and 2007.

Inequality and Education

Table 12

Education Partition of the 2007 SCF Sample

	Averages			Income Sources (%)					Gini Indexes			Coefficients of Variation				
Education	E	Y	W	L[d]	K[e]	B[f]	Z[g]	O[h]	E[a]	Y[b]	W[c]	E[a]	Y[b]	W[c]	H (%)[i]	Size[j]
Dropouts	20.5	31.3	142.9	57.1	3.0	9.8	27.9	2.1	0.66	0.45	0.78	1.86	1.47	4.31	13.5	2.69
High school	39.1	50.8	251.6	66.1	4.3	12.7	15.4	1.5	0.59	0.45	0.74	3.84	3.89	5.11	32.9	2.60
Some college	51.0	67.8	366.3	64.9	9.8	11.9	11.5	1.9	0.56	0.50	0.81	5.30	5.85	7.09	18.4	2.45
College	110.1	142.4	1095.1	64.2	12.9	15.2	6.9	0.8	0.59	0.57	0.78	2.68	3.47	4.66	35.3	2.54
Total	63.8	83.6	555.4	64.3	10.2	13.9	10.3	1.2	0.64	0.57	0.82	3.60	4.32	6.02	100.0	2.56

[a]Earnings; [b]income; [c]wealth; [d]labor; [e]capital; [f]business; [g]transfers; [h]other; [i]percentage number of households per group;
[j]average number of persons per primary economic unit.

Inequality and Financial Status

Tables 23 and 25 (DGR) on Financial Status:

- SCF asks respondents if they filed for bankruptcy (1% of sample) and if they are delinquent by 2 months or more (5% of sample).
- Not surprisingly they are poor.
- Fraction of bankrupts was higher in 1998, due to changes in bankruptcy law in 2006.
- Important for thinking about the borrowing constraint ($\underline{a} \leq a_{t+1}$) we impose in the model economy.

Inequality and Financial Status

Table 23

Late and Timely Payers in 1998 and 2007

Delinquent.

	Late Payers			Timely Payers		
	2007	1998	△	2007	1998	△
Earnings, Income, and Wealth						
Earnings	32,738	39,904	–18.0	65,630	57,603	13.9
Income	38,471	43,646	–11.9	86,212	72,884	18.3
Wealth	117,848	75,078	57.0	580,938	378,873	53.3
Sources of Income						
Labor	79.5	83.8	–4.3	64.0	67.9	–3.9
Capital	0.4	1.0	–0.6	10.5	9.0	1.5
Business	6.6	9.0	–2.4	14.1	13.0	1.1
Transfers	10.7	5.0	5.7	10.3	8.9	1.4
Other	2.9	1.4	1.5	1.1	1.2	–0.1
Education						
Dropouts	16.4	18.5	–2.1	13.4	16.3	–2.9
High school	33.8	36.1	–2.3	32.8	31.6	1.2
College	49.8	45.4	4.4	53.8	52.1	1.7
Employment Status						
Workers	66.1	67.5	–1.4	59.6	58.7	0.9
Self-employed	6.7	13.9	–7.2	10.7	11.1	–0.4
Retired	5.5	2.3	3.2	19.4	20.0	–0.6
Nonworkers	21.7	16.4	5.3	10.3	10.3	0.0
Marital Status						
Married	51.7	53.1	–1.4	59.2	58.9	0.3
Single	48.3	46.9	1.4	40.8	41.1	–0.3
Single w/ dependents	27.9	25.8	2.1	16.4	15.9	0.5
Single w/o dependents	20.4	21.1	–0.7	24.4	25.2	–0.8
Other Features	less debt portion			more debt portion		
Debt-to-income ratio	2.08	1.17	77.8%	1.14	0.83	37.3%
Debt-to-wealth ratio	0.68	0.68	0.0%	0.17	0.16	6.3%
Debt	80,033	51,227	56.2%	98,063	60,353	62.5%
Age	42.4	41.0	1.4	50.5	49.2	1.3
Household size	3.0	3.0	0.0	2.5	2.6	–0.1

* if one has good credit history has more access to borrowing
But this puts up debt ratio & make a "timely payer" more likely to become late payers.

Inequality and Financial Status

Table 25

Bankrupt and Solvent Households in 1998 and 2007

	Bankrupt Households			Solvent Households		
	2007	1998	△	2007	1998	△
Earnings, Income, and Wealth						
Earnings	35,469	42,737	–17.0%	64,084	56,790	12.8%
Income	40,792	46,009	–11.3%	83,983	71,581	17.3%
Wealth	89,884	60,707	48.1%	559,790	366,033	52.9%
Sources of Income						
Labor	84.8	93.5	–8.7	64.2	68.2	–4.0
Capital	0.1	0.5	–0.4	10.3	8.8	1.5
Business	2.5	–0.7	3.2	14.0	13.0	1.0
Transfers	9.8	4.6	5.2	10.3	8.8	1.5
Other	2.7	2.1	0.6	1.2	1.2	0.0
Education						
Dropouts	9.9	8.7	1.2	13.6	16.6	–3.0
High school	38.1	48.9	–10.8	32.8	31.6	1.2
College	52.0	42.4	9.6	53.6	51.8	1.8
Employment Status						
Workers	75.5	79.1	–3.6	59.8	58.8	1.0
Self-employed	5.8	5.4	0.4	10.5	11.4	–0.9
Retired	7.8	2.7	5.1	18.8	19.2	–0.4
Nonworkers	10.9	12.8	–1.9	10.9	10.6	0.3
Marital Status						
Married	51.0	49.6	1.4	58.9	58.7	0.2
Single	49.0	50.4	–1.4	41.1	41.3	–0.2
Single w/ dependents	32.1	34.2	–2.1	16.9	16.2	0.7
Single w/o dependents	16.9	16.2	0.7	24.2	25.1	–0.9
Retired widows	0.00	0.00	0.00	4.56	4.38	0.18
Other Features						
Debt-to-income ratio	1.94	1.38	40.6%	1.16	0.83	39.8%
Debt-to-wealth ratio	0.88	1.04	–15.4%	0.16	0.16	0.0%
Debt	79,201	63,357	25.0%	97,237	59,742	63.5%
Age	44.0	41.3	2.7	50.1	48.9	1.2
Household size	3.1	3.2	–0.1	2.6	2.6	0.0

Definitions

(Markov)

- Mobility Matrix: Each element $a_{i,j}$ denotes the probability that an individual initially in group i will end up in group j. The sum of all the elements of each row is one.
- We will input an exogenous simple 2x2 earnings mobility matrix into our model to generate an endogenous wealth distribution.

Inequality and Mobility

Table 27

PSID **Transition Matrices for Earnings, Income, and Wealth Quintiles, 2001–7**

	1st	2nd	3rd	4th	5th
Earnings Mobility					
1st	0.73	0.22	0.03	0.01	0.00
2nd	0.13	0.47	0.30	0.07	0.03
3rd	0.06	0.19	0.42	0.27	0.06
4th	0.05	0.07	0.17	0.48	0.24
5th	0.04	0.05	0.08	0.16	0.68
Income Mobility					
1st	0.65	0.21	0.08	0.04	0.02
2nd	0.21	0.45	0.22	0.09	0.02
3rd	0.07	0.21	0.40	0.25	0.07
4th	0.04	0.09	0.22	0.42	0.23
5th	0.02	0.03	0.08	0.21	0.66
Wealth Mobility					
1st	0.62	0.23	0.11	0.03	0.01
2nd	0.27	0.41	0.23	0.08	0.02
3rd	0.07	0.26	0.39	0.21	0.06
4th	0.03	0.08	0.23	0.45	0.21
5th	0.01	0.03	0.04	0.23	0.70

American dream

$\sum_i row_i = 1$

(high persistent)

Without adjusting retirees

Drop from top

Inequality and Mobility

Table 28

Transition Matrices for Earnings, 2001–7: A Closer Look

	1st	2nd	3rd	4th	5th
Heads 35–45 in 2001					
1st	0.65	0.28	0.05	0.01	0.01
2nd	0.23	0.47	0.25	0.04	0.01
3rd	0.07	0.16	0.49	0.20	0.08
4th	0.01	0.07	0.17	0.54	0.21
5th	0.03	0.02	0.05	0.21	0.70
Positive Earnings in Both 2000 and 2006					
1st	0.56	0.30	0.08	0.03	0.02
2nd	0.25	0.39	0.23	0.10	0.03
3rd	0.10	0.17	0.42	0.24	0.08
4th	0.04	0.10	0.20	0.45	0.21
5th	0.05	0.04	0.07	0.18	0.66

Inequality and Mobility

Tables 27 and 28 (DGR).

- There is a substantial amount of mobility across all variables: except for the lowest earnings quintile and the highest wealth quintile, at least one-third of the households leave that quintile after six years.
- Attempt to rid results of life cycle effects (in particular retirees) and find even more upward mobility from lowest quintile.
- These types of transition matrices have important implications for existence (of invariant wealth distribution) theorems (i.e. mixing conditions)

A Parsimonious Quantitative Incomplete Markets Model

Main Reference: Huggett, M. 1993. "The risk-free rate in heterogeneous-agent incomplete-insurance economies", *Journal of Economic Dynamics and Control*, 17, p. 953-69.

- These notes summarize an even simpler environment than Huggett's; a G.E. exchange economy where idiosyncratic shocks to a household's employment opportunities (i.e. their mobility between employed and unemployed states) are smoothed via a noncontingent real bond.

- The equilibrium objects are the borrowing/saving decision $\frac{1}{q}$ rule, the discount bond price (the inverse of which is the gross real interest rate), and the cross-sectional distribution of wealth and earnings.

- From DGR (p.5) "One of the hardest tasks that any theory of inequality faces is to account for both tails of the distributions simultaneously".

Environment: Population and Preferences

- Population: unit measure of households.

uncertainty. Infinite horizons

- Preferences: $E_0\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right]$. Assume $U : \mathbb{R}_+ \to \mathbb{R}$ is ctsly differentiable, strictly increasing, strictly concave, and bounded (the latter for technical reasons to apply results from Stokey and Lucas) ⊗.

otherwise wealth not well-defined

⊗ Check Ch7-13
Ch3-5.

Environment: Endowments

- Endowments: In any period t, HHs face two earnings shocks $s_t \in S = \{e, u\}$ where e denotes employed and u denotes unemployed.
  - These shocks are i.i.d. across agents. no correlation between shock to i & shock to j (free from joint distribution)
  - The employment process is Markov with transition matrix denoted $\pi(s'|s) = prob(s_{t+1} = s'|s_t = s)$.
  - If employed, hh earns $y(e) = 1$ (a normalization).
  - If unemployed, receives $y(u) = b < 1$.

Environment: Asset Market Structure

- Sequence of one-period, non-contingent discount bonds with borrowing constraint $\underline{a} \leq 0$. Define $A = \{a_t \in \mathbb{R} : a_t \geq \underline{a}\}$.

  $a_t < 0$: borrowing

- Hhs enter period with assets a_t and purchase next period assets a_{t+1} at price q_t. $\frac{1}{q_t} = 1 + r_t$ q_t lives in $(0,1]$ $\Rightarrow$ with assume $\beta/q < 1$ q lives in $(\beta, 1]$

- Since there is no aggregate uncertainty and we will be looking for a steady state equilibrium, we will assume $q_t = q$.

Equity premium puzzle

- Assume that $\beta < q$ (something that must be verified in equilibrium). $\Rightarrow \beta/q < 1 \Rightarrow \beta(1+r_t) < 1$ (in representative world $\beta(1+r_t) = 1$)

- In this simple environment, the borrowing constraint $a_{t+1} \geq \underline{a}$ is taken as exogenous.

large $r^e - r^f$: high r^e : high RRA (E-Z pref), with CRRA that means 70 sth (too high) RRA. ; make earning process (long run risk, rare disaster)

or low r^f : idiosyncratic earning uncertainty $\Rightarrow$ want to save $\Rightarrow$ lower interest rate

Environment: Strong Assumptions

- For simplicity we have made the following strong assumptions
  - Exogenous Earnings (Labor Econ)
  - Exogenous Borrowing Constraints (Macro)
  - No Redistribution (PE)
- Relax these assumptions in Directions for Future Research

Parameterization

6 Parameters: $\beta, U(\cdot), b, \pi(e|e), \pi(u|u), a$. $\Rightarrow$ need at least 6 moments to just-identified.

- Suppose the model period is one quarter.
- Preference parameters taken from outside the model:
  - The utility function is given by

$$U(c_t) = \frac{c_t^{1-\alpha} - 1}{1 - \alpha}$$

    where $\alpha = 1.5$
  - $\beta = 0.994$.
- Employed earnings normalized to 1 and data on replacement rates pins down $b = 0.5$.

$D = \frac{1}{1-\pi(u|u)} \Rightarrow D - D\pi(u|u) = 1$

$\boxed{\pi(u|u) = 1 - \frac{1}{D}}$

Parameterization

- Data on duration of unemployment pins down $\pi(u|u)$ by $D = 1/(1-\pi(u|u))$ where D is in model units (e.g. 2 quarters).
  - This implies $\pi(u|u) = 1 - 1/D$.
  - If $D = 2$ quarters (slightly over what it was in postwar data), then $\pi(u|u) = \frac{1}{2}$.

unemployment $P(\text{unem}/\text{em})$ employment

- Data on average unemployment $\overline{U}$ pins down $\pi(e|e)$.
  - Law of Motion of unemployment
  - From $\overline{U}' = \pi(u|u)\overline{U} + \pi(u|e)(1-\overline{U}) \Longrightarrow$ in the long run where $\overline{U}' = \overline{U}$, then $\pi(u|e) = \frac{(1-\pi(u|u))\overline{U}}{1-\overline{U}}$.
  - Thus if $\overline{U} = 5.66\%$ (roughly what was in postwar data),then $\pi(u|e) = \frac{0.0566}{2*0.9434} = 0.03 \Longrightarrow \pi(e|e) = 0.97$.
- $(1, b, \pi(e|e), \pi(u|u))$ make up the "earnings mobility" matrix for our sparsely parameterized model.

Parameterization

- Finally, $\underline{a}$ is targeted to match 2% real interest rate in U.S. over postwar period.

PSGT2

Recursive Eq: Household's Problem

- The individual's problem can be written in terms of the (DP) operator $T : \mathcal{C}(S \times A) \to \mathcal{C}(S \times A)$ as:

2-state Markov.

$$(Tv)(s, a; q) = \max_{a' \in \Gamma(s,a)} U(y(s) + a - qa') + \beta \overbrace{\sum_{s' \in S} \pi(s'|s) v(s', a')} \tag{1}$$

a contraction

where

$T(v) = v \rightsquigarrow$ fixed point problem!

$$\Gamma(s, a; q) = \left\{ a' : \underline{a} \leq a' \leq \frac{y(s) + a}{q} \right\}$$

and $\mathcal{C}(S \times A)$ denotes the space of continuous, bounded functions.

- A solution to this problem is a decision rule $a' = g(s, a; q)$.

Recursive Eq: Cross-Sectional Wealth Distribution

- Since hhs differ in their employment histories, they will in general differ in their asset holdings.
- We will describe economywide assets and employment via a probability measure, μ.
- Think of $\mu(S_0, A_0; q)$ as the fraction of the population with shocks in the set S_0 and asset holdings in the set A_0 when the price is q.

Recursive Eq: Cross-Sectional Wealth Distribution

a' works like a transition

- The decision rule $g(s, a; q)$ and the shock process π induce a law of motion for the distribution of agents $\mu' = T^*\mu$ written in terms of the (TF) operator $T^* : \Xi(S \times A, \underbrace{\mathcal{P}(S \times A)}_{\text{space for fixed point}})$ $\to \Xi(S \times A, \mathcal{P}(S \times A))$ given by:

probability space

$$(T^*\mu)(S_0, A_0; q) \tag{2}$$

WANT: continuous in q

$$= \int_{S_0, A_0} \left\{ \int_{S,A} \mathbf{1}_{\{a'=g(s,a;q) \in A_0\}}(s, a)\pi(s'|s)\underbrace{\mu(ds, da; q)}_{\text{mass of grid points.}} \right\} ds' da'$$

Need to know properties of $g(\cdot)$

where:

- $\Xi(S \times A, \mathcal{P}(S \times A))$ denotes the space of probability measures defined on the measurable space $(S \times A, \mathcal{P}(S \times A))$.
- $\mathbf{1}_{\{a'=g(s,a;q)\in A_0\}}(s, a)$ is an indicator function that is 1 if the statement $\{a' = g(s, a; q) \in A_0\}$ is true for (s, a) and zero otherwise.

$\mathbf{1}\{$in available actions$\}$

Recursive Eq: Definition

Definition

A recursive steady state equilibrium is an allocation (c, a'), a price q^*, and an invariant distribution μ^* such that

1. For given q^*, (c, a') solves HH optimization $v = Tv$ in (1).
2. μ^* is a stationary probability measure (i.e. $\mu^* = T^*\mu^*$ in (2)).
3. Given μ^*, goods and asset markets clear

$$\int_{S,A} \left[c(s, a; q^*) - y(s)\right] d\mu^* = 0 \tag{3}$$

$$\int_{S,A} g(s, a; q^*) d\mu^* = 0. \tag{4}$$

Employment spells and Asset dynamics

- There are potentially many different (s, a) which map via T^* to the same a'. For example, you could have agents with low assets but high income (who save, thereby choosing $a' > a$) or agents with high assets and low income (who dissave, thereby choosing $a' < a$) both choosing the same a'.
- To see how the mapping (2) works, *suppose* that:
  - $A = \{-\alpha, 0, \alpha\}$ where α is sufficiently small.
  - the decision rule is given by a nondecreasing mapping:

$-\alpha$: borrowing

α: saving

employed

unemployed

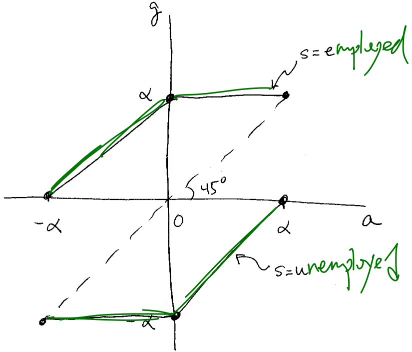
(s, a)	$g(s, a)$
$(e, -\alpha)$	0
$(e, 0)$	α
(e, α)	α
$(u, -\alpha)$	$-\alpha$
$(u, 0)$	$-\alpha$
(u, α)	0

Table: Decision Rule g

$\boxed{\text{WTS}}$ $g(s,a)$ non decreasing function in a

Employment spells and Asset dynamics - cont.

Thus, when unemployed, the person borrows or dissaves and when employed, the person saves as in Figure 1



Employment spells and Asset dynamics - cont.

Then the law of motion for the distribution is given by

Table μ'

(s', a')	$(T^*\mu)(s', a')$
$(e, -\alpha)$	$\pi(e\|u)\mathbf{1}_{\{-\alpha=g(u,-\alpha)\}}\mu(u,-\alpha) + \pi(e\|u)\mathbf{1}_{\{-\alpha=g(u,0)\}}\mu(u,0)$
$(e, 0)$	$\pi(e\|e)\mathbf{1}_{\{0=g(e,-\alpha)\}}\mu(e,-\alpha) + \pi(e\|u)\mathbf{1}_{\{0=g(u,\alpha)\}}\mu(u,\alpha)$
(e, α)	$\pi(e\|e)\mathbf{1}_{\{\alpha=g(e,0)\}}\mu(e,0) + \pi(e\|e)\mathbf{1}_{\{\alpha=g(e,\alpha)\}}\mu(e,\alpha)$
$(u, -\alpha)$	$\pi(u\|u)\mathbf{1}_{\{-\alpha=g(u,-\alpha)\}}\mu(u,-\alpha) + \pi(u\|u)\mathbf{1}_{\{-\alpha=g(u,0)\}}\mu(u,0)$
$(u, 0)$	$\pi(u\|e)\mathbf{1}_{\{0=g(e,-\alpha)\}}\mu(e,-\alpha) + \pi(u\|u)\mathbf{1}_{\{0=g(u,\alpha)\}}\mu(u,\alpha)$
(u, α)	$\pi(u\|e)\mathbf{1}_{\{\alpha=g(e,0)\}}\mu(e,0) + \pi(u\|e)\mathbf{1}_{\{\alpha=g(e,\alpha)\}}\mu(e,\alpha)$

Employment spells and Asset dynamics - cont.

- For example, take the second row which is the mass of people employed at the beginning of next period with zero assets ($\mu'(e,0)$).

Check

$\mu'(e,0)$	$\pi(e\|e)\mathbf{1}_{\{0=g(e,-\alpha)\}}\mu(e,-\alpha)+\pi(e\|u)\mathbf{1}_{\{0=g(u,\alpha)\}}\mu(u,\alpha)$

- It is arrived from:
  - the mass employed this period with borrowings ($\mu(e,-\alpha)$) who stay employed ($\pi(e|e)$) and save ($\mathbf{1}_{\{0=g(e,-\alpha;q)\}}$) (i.e. the first row in Table g), and
  - the mass of people who are unemployed this period with positive assets ($\mu(u,\alpha)$) who become employed ($\pi(e|u)$) and dissave ($\mathbf{1}_{\{0=g(u,\alpha;q)\}}$) in order to consumption smooth (the last row in Table g).

Existence of Savings Decision Rules

- A standard reference for the existence of a unique solution to the fixed point problem $v^* = Tv^*$ in (1) and associated policy function $g^*(s, a; q)$ via a contraction mapping argument is given by Lucas and Stokey when $U(\cdot)$ is bounded.

- Theorem 1 (Huggett). For $q > 0$, $\underline{a} + b - q\underline{a} > 0$, $T^n v_0$ converges uniformly to a unique solution $v^* = Tv^*$ in (1). unique fixed pt. $v^*(s, a; q)$ is strictly increasing, strictly concave, and continuously differentiable in a. The decision rule $g^*(s, a)$ is continuous, nondecreasing in a, and strictly increasing in a for $g(s, a; q) > \underline{a}$.

Existence of Savings Decision Rules - cont.

With a bounded $U(\cdot)$,

- existence of a unique v follows as direct consequence of S-L Theorem 9.6,
- v is increasing (concave, differentiable) follows from Theorem 9.7 (Theorem 9.8, 9.10),
- singlevaluedness and continuity of g follows from Theorem 9.8,
- $g(s, a)$ is increasing in a follows from concavity of v (see my notes on Huggett for a proof by contradiction using the the Euler equation which establishes this).

Existence of a Stationary Distribution

- Def. A probability measure μ^* is invariant under T^* if it is a fixed point $\mu^* = T^*\mu^*$ of the T^* operator defined in (2).

- Theorem 2 (Huggett). If the conditions of Theorem 1 hold, ① $\beta < q$ (ie. people are impatient and borrowing rates are not too high), and ② $\pi(e|e) > \pi(e|u)$ (i.e. the probability of staying employed is higher than becoming employed), then there exists a unique invariant measure given q.

$\frac{1}{q} = 1+r$

$\boxed{\beta(1+r) < 1}$

③ exists stationary distribution?

Existence of a Stationary Distribution

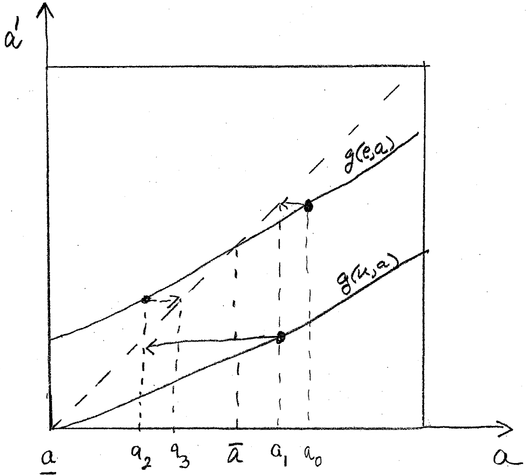
- One of the big issues is that for μ to be invariant, the distribution cannot fan out.

- While asset holdings are bounded below by $\underline{a}$, they are not necessarily bounded above. transient. ⊛ basically we need a irreducible MC ?

- It is impossible for μ to be invariant if there is mass being put on higher and higher a.

- One of the important parts of Huggett's proofs is to show that agents never accumulate savings beyond an endogenously determined upper bound $\overline{a}$. See Figure 2.

Existence of a Stationary Distribution - cont.



Start outside set, always come back in, never go out again

Existence of a Stationary Distribution - cont.

- The result for why agents don't increase savings beyond $\overline{a}$ relies on concavity.
- The marginal benefit of savings is $\partial v(s', a')/\partial a'$ while the marginal cost is $q \cdot U'(y(s) + a - qa')$.
- Hence increasing savings increases expected future marginal benefits at a decreasing rate (i.e. $\partial^2 v(s', a')/\partial a'^2 < 0$) while it increases marginal costs at an increasing rate (i.e. $-q^2 \cdot U''(y(s) + a - qa') > 0$).

Some Measure Theory Jargon

Definition

Let$(X, \mathcal{X})$ be a measurable space. A <u>transition function</u> is a function $Q : X \times \mathcal{X} \to [0, 1]$ such that: (i) for each $x \in X$, $Q(x, \cdot)$ is a probability measure on $(X, \mathcal{X})$; (ii) for each $X_0 \in \mathcal{X}$, $Q(\cdot, X_0)$ is a measurable function.

- For any probability measure μ on $(X, \mathcal{X})$, define the operator T^* by

$$(T^*\mu)(X_0) = \int_X Q(x, X_0)\mu(dx), \forall X_0 \in \mathcal{X}. \tag{5}$$

  Since for each $X_0 \in \mathcal{X}$, $Q(\cdot, X_0)$ is bounded (in $[0, 1]$) and measurable, then $T^*\mu$ is well defined (i.e. the "integral" exists) .

Measure Theory applied to Huggett

- In terms of the Huggett model, (6) is just another way to write (2). To see this,

$$
\begin{aligned}
& (T^*\mu)(S_0, A_0; q) \\
= & \int_{S_0, A_0} \left\{ \int_{S,A} \mathbf{1}_{\{a'=g(s,a;q)\in A_0\}}(s,a)\pi(s'|s)\mu(ds, da; q) \right\} ds' da' \\
= & \int_{S,A} Q((s,a),(S_0, A_0))\mu(ds, da; q)
\end{aligned}
$$

where the second equality simply rearranges the integrals and

$$
Q((s,a),(S_0,A_0)) = \int_{S_0,A_0} \mathbf{1}_{\{a'=g(s,a;q)\in A_0\}}(s,a)\pi(s'|s)ds'da'.
$$

- Continuity of g helps ensure measurability of Q (i.e. continuous functions are measurable).

Back to Existence of a Stationary Distribution

Using these definitions, a simpler (than Huggett's) proof of existence, uniqueness, and continuity of the invariant measure in q can be constructed by verifying the assumptions of certain theorems in Stokey and Lucas.

- Existence and uniqueness of invariant distribution is in Theorem 12.12 in S-L (p. 382).
  - Here the assumptions on the transition function include: (i) the Feller Property (i.e. the operator preserves boundedness and continuity), (ii) Monotonicity, and (iii) mixing properties.
- Continuity of invariant distribution with respect to parameters (can treat q parametrically) is in Theorem 12.13 in S-L (p. 384).
  - Here the assumptions are that the operator satisfies Theorem 12.12 and continuity of $g(s, a; q)$ in q.

Check.

▸ Measure Theory

Algorithm to Compute a Steady State Equilibrium

- To compute an equilibrium for a given set of parameters θ (see class website for some parts of the code):
  1. Taking $q \in [0,1]$ as given, solve the agent's dynamic programming problem for her decision rule $a' = g_\theta(a, s; q)$ (i.e. solve for a fixed point of the T operator);
  2. Given $g_\theta(a, s; q)$ and the stochastic process for earnings, solve for the invariant wealth distribution $\mu^*_\theta(A, S; q)$ (i.e. solve for a fixed point of the T^* operator);
  3. Given μ^*, check whether the asset market clears at q (i.e.$ED(q) \equiv \int_{A,S} g_\theta(a, s; q)\mu^*_\theta(da, ds; q) = 0$). If $ED(q) = 0$, stop. If $ED(q) > 0$ raise q and if $ED(q) < 0$ lower q, and go to step 1.
- Once you have $g_\theta(a, s; q)$ and $\mu^*_\theta(a, s; q)$ you can compute model moment counterparts to put into an outer simulated method of moment routine to estimate θ.
- Steps 1 and 2 are in the inner loop, 3 in middle loop, and SMM *would be* the outer loop of the nested Fixed Point.

Positive Results: Value Functions

Figure 3 plots the value function across assets of employed and unemployed agents.

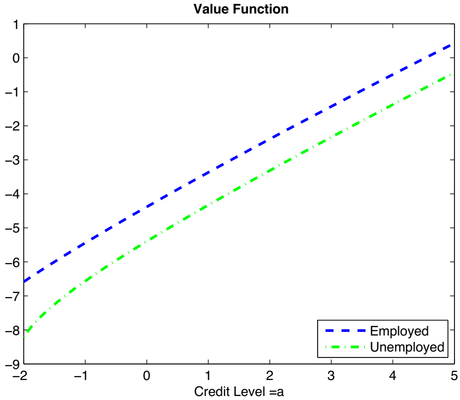


Figure: Value Functions

Positive Results: Decision Rules

The decision rules in Figure 4 are consistent with the fact that there exists $\overline{a} = 1.0381$ where $g(\overline{a}, e) = \overline{a}$ which establishes an upper bound on the the invariant distribution).

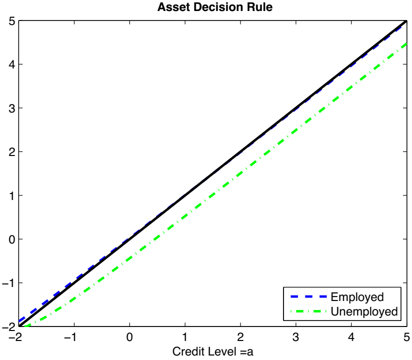


Figure: Decision Rules

Positive Results: Cross-Sectional Distribution

The the steady state wealth distribution in Figure 5 uses wealth defined as cash from direct deposit of $y(s)$ plus net bond holdings a (spike at $y(e) + \overline{a}$). Spikes arise because some fraction of households stuck on the borrowing constraint receive an employment opportunity and save.

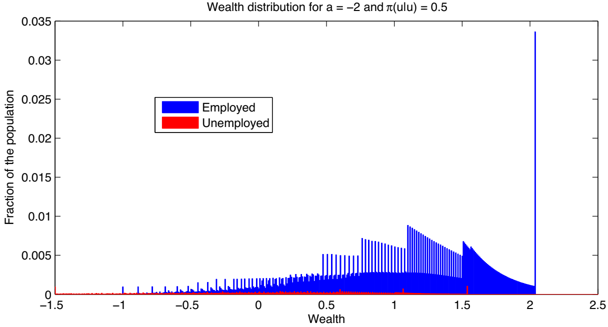


Figure: Cross-sectional Wealth Distribution

Positive Results: Gini Coefficient

- The Gini coefficient with respect to total wealth for this economy is 0.3821.
- Figure 6 shows the Lorenz curve. The Lorenz curve is negative because in the model the sum of the wealth of agents in the first quantile is less than zero, as in Table 6 of DGR.

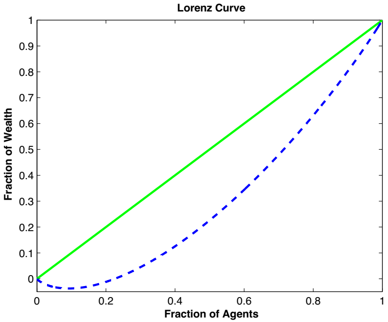


Figure: Lorenz Curve

⊛ Consumption equivalent.

Welfare Calculations

- Once we have a structural model which is consistent with data under a given parameterization, we can conduct counterfactuals to assess the welfare benefits or costs of certain changes to the environment or policy.

Q

- In particular, we can answer questions of the following variety: what fraction of consumption would a person in state (s, a) of a steady state of the incomplete markets environment be willing to pay (if positive) or have to be paid (if negative) in all future periods to achieve the utility associated with the counterfactual allocation of consumption W?

⊛

PSET 2

: known

: want to back out.

Welfare Calculations - cont.

time-invariant

For each (s, a) we compute the consumption equivalent $\lambda(s, a)$ such that

Consumption Equiv.

Compute from DP

in state (s, a)

$$W = E\left[\sum_{t=0}^{\infty} \beta^t \frac{[(1+\lambda(s,a))c_t(s,a)]^{1-\alpha} - 1}{1-\alpha} \Big| (s,a)\right]$$

not depend on t

pull this out.

$$= (1+\lambda(s,a))^{1-\alpha}\left[v(s,a) + \frac{1}{(1-\alpha)(1-\beta)}\right] - \frac{1}{(1-\alpha)(1-\beta)}$$

get this

known parameter

or

$$\Rightarrow \boxed{\lambda(s,a) = \left[\frac{W + \frac{1}{(1-\alpha)(1-\beta)}}{v(s,a) + \frac{1}{(1-\alpha)(1-\beta)}}\right]^{1/(1-\alpha)} - 1}$$

where $v(s, a)$ is the value function from the incomplete markets economy.

Welfare Calculations - cont.

- Then the economywide welfare gain is given by

$$WG = \sum_{(s,a)\in S\times A} \lambda(s,a)\underset{\substack{\uparrow \\ \text{distribution}}}{\mu(s,a)}.$$

- We can also use $\lambda(s,a)$ to calculate what fraction of the population would prefer the counterfactual allocation W.

- That is given by $\sum_{(a,s)\in A\times S} \mathbf{1}_{\{\lambda(a,s)\geq 0\}}(a,s)\mu(a,s)$.

Counterfactual 1: The Welfare Cost of Incomplete Financial Markets

- Consider the same environment as Huggett (1993, JEDC) except assume that there are complete enforceable insurance markets regarding the idiosyncratic shocks to earnings and that all agents start without any assets.
- With complete markets (CM) and locally non-satiated preferences, the first and second basic welfare theorems hold, meaning we can solve the Pareto-optimal utilitarian planner's problem for allocations and decentralize by setting asset prices to support the allocations as a competitive equilibrium (done all the time in the RBC framework).

The Welfare Cost of Incomplete Financial Markets

- At time zero the utilitarian planner solves:

$$W^{CM} = \max_{\{c_t^e, c_t^u\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\mu U(c_t^e) + (1-\mu) U(c_t^u))$$

subject to the resource constraints

$$\mu c_t^e + (1-\mu) c_t^e = \overbrace{\mu \cdot y(e)}^{\text{from employed}} + \overbrace{(1-\mu) \cdot y(u)}^{\text{from not unemployed.}}, \forall t$$

where μ and $(1-\mu)$ represent the fractions of the population employed and unemployed respectively (given at time t by the time 0 distribution and the transition matrix Π) and c_t^e and c_t^u are the consumption of the employed and unemployed respectively at time period t.

The Welfare Cost of Incomplete Financial Markets

- The first order conditions for the planner with respect to consumption at time t are

$$\begin{aligned} \mu U'(c_t^e) + \lambda_t \mu &= 0 \\ (1-\mu)U'(c_t^u) + \lambda_t(1-\mu) &= 0 \end{aligned}$$

  where λ_t is the multiplier on the period t resource constraint.

- These equations imply

$$U'(c_t^e) = U'(c_t^u) \Rightarrow c_t^e = c_t^u = \mu y^e + (1-\mu) y^u$$

  for all t.

- Thus

$$W^{CM} = U\left(\mu y^e + (1-\mu) y^u\right)/(1-\beta).$$

The Welfare Cost of Incomplete Financial Markets

- Now onto the "normative" numbers.
- The present value of future utility in the complete markets case is given by $W^{CM} = -4.8192$.
- The comparable "aggregate" welfare measure in the incomplete markets economy is given by

$$W^{INC} = \sum_{(a,s) \in A \times S} \mu(a,s) v(a,s) = -5.0432.$$

- Thus, aggregate welfare is higher in the complete markets economy than the incomplete markets economy (no surprise).

The Welfare Cost of Incomplete Financial Markets

i.e., how much do they need to pay / get compensated to go from incomplete to complete mkt.

- A better measure would answer the "what fraction of consumption would a person in a steady state of the incomplete markets environment be willing to pay (if positive) or have to be paid (if negative) in all future periods to achieve the allocation of the complete markets environment?"

- To answer this question, we compute and plot consumption equivalents $\lambda(a, s)$ in Figure 7.

- The figure shows not everyone is necessarily made better off in this experiment; wealthy people can enjoy a short run higher level of consumption by dissaving.

The Welfare Cost of Incomplete Financial Markets

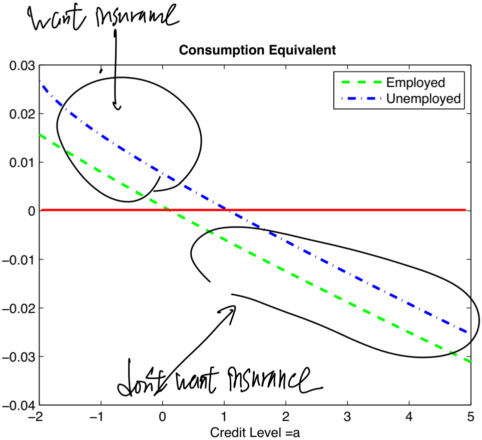


Figure: Consumption Equivalents Welfare Gain from Complete Markets

The Welfare Cost of Incomplete Financial Markets

- In the aggregate, the gains to poor people outweigh the losses to wealthy people (a consequence of risk aversion). In particular, we find $WG = 0.00133$ (i.e. a little over 1/10 of one percent). (⊛ Related to Lucas's "welfare gain from stabilization" (very small!))

→ Maybe not a bad idea to be in incomplete mkt

- Further, we can actually calculate the fraction of the population who is made weakly better off switching to complete markets (i.e. $\sum_{(a,s)\in A\times S} \mathbf{1}_{\{\lambda(a,s)\geq 0\}}(a,s)\mu(a,s)$).

- If this number is greater than 1/2, the policy would be voted in.

- For our case, the fraction who would vote to move to complete markets is 54%.

The Welfare Cost of Tighter Borrowing Constraints

- In the recent financial crisis, lenders have tightened borrowing constraints.
- What happens if the borrowing limit is cut in half?
- Positive Effects: Households' precautionary savings rise, thereby lowering interest rates and inequality roughly in half. With even stricter borrowing constraints, real interest rates become negative.

Counterfactual.

- Normative Effects: What fraction of consumption would a person of type s in a steady state of an economy where borrowing is limited to 1 times quarterly employed earnings be willing to pay in all future periods to achieve the allocation of an economy where borrowing is limited to 2 times quarterly employed earnings? Answer: 2/10 of one percent. (double)

The Welfare Cost of Tighter Borrowing Constraints

	Data	Bench	$\underline{a} = -1$	$\underline{a} = -0.5$
Unemployment Rate (targeted)	5.66	5.66	5.66	5.66
Real Interest Rate (%) (targeted)	2.00	2.00	0.82	-1.53%
Wealth Gini (untargeted)	0.80	0.38	0.18	0.08
CE(employed)	*	0	0.1987%	0.5050%
CE(unemployed)	*	0	0.2048%	0.5169%
CE	*	0	0.1991%	0.5057%

The Welfare Cost of Tighter Borrowing Constraints

Figure 8 plots the new cross-sectional wealth distribution, which is more compressed at both ends (lower real rate discourages saving).

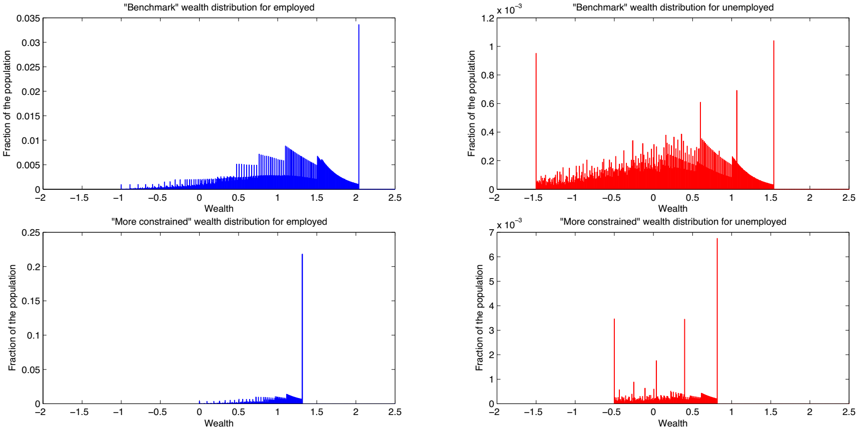


Figure: Wealth Distribution with Tighter Borrowing Constraints

The Welfare Cost of Longer Unemployment Spells

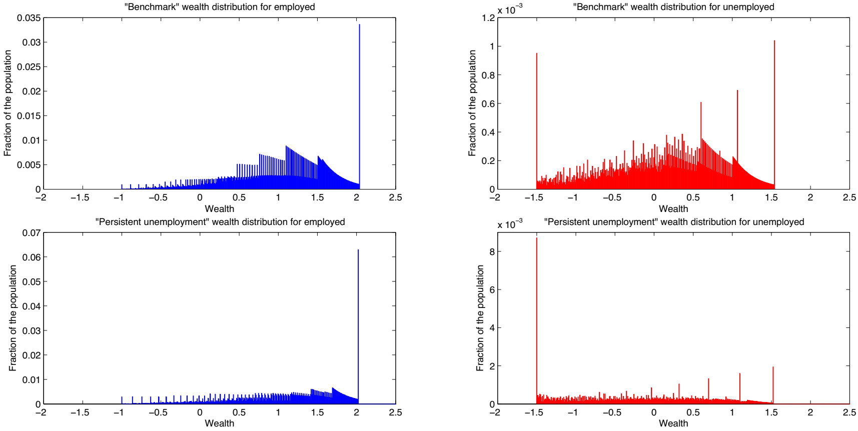
- In the recent financial crisis, the duration of unemployment has risen.
- What happens if the duration of unemployment is doubled from 2 quarters to 1 year?
- Positive Effects: Households' precautionary savings rise, thereby lowering interest rates by roughly half. Longer spells of unemployment raises wealth inequality by about a quarter.
- Normative Effects: What fraction of consumption would a person of type s in a steady state of an economy where the average duration of unemployment is one year be willing to pay in all future periods to achieve the allocation of an economy where the average duration is 2 quarters? Answer: Over 3%.

The Welfare Cost of Longer Unemployment Spells

	Data	Bench	$\pi(u, u) = 0.75$
Unemployment Rate (targeted)	5.66	5.66	10.71
Real Interest Rate (%) (targeted)	2.00	2.00	0.94
Wealth Gini (untargeted)	0.80	0.38	0.49
CE(employed)	*	0	3.1204%
CE(unemployed)	*	0	4.1448%
CE	*	0	3.2765%

The Welfare Cost of Longer Unemployment Spells

Figure 9 plots the new cross-sectional wealth distribution, which shows more mass at the tails, accounting for the higher Gini coefficient.

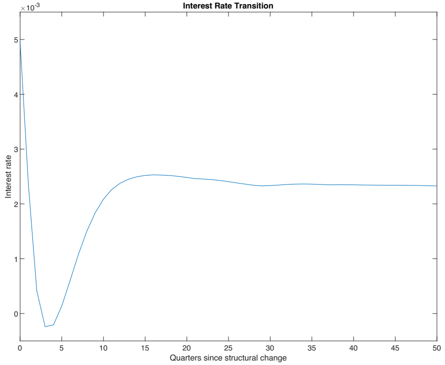


The Welfare Cost of Longer Unemployment Spells

- The previous results compared two steady states.
- As we saw for the steady state, ceteris paribus households will save more when the duration of unemployment is higher.
- But if they don't anticipate that the unemployment duration will rise, they will start with too few savings and will not jump to the new level of savings since they would prefer to consumption smooth along the way.
- Thus, we need to compute a transition from the steady-state cross-sectional distribution associated with $\pi(u, u) = 0.50$ to $\pi(u, u) = 0.75$.
- We cannot use the decision rules from the new steady state since they depend on the steady state q, so we need to compute decision rules using the path of q. You will learn this in PS4.

The Welfare Cost of Longer Unemployment Spells

- Figure 10 summarizes the quarterly interest rate transition path to the new steady state.
- Interest rates drop immediately in order to clear the bond market; the excess demand for savings raises the bond price (thereby lowering the interest rate).



⊗ MIT shock: unanticipated shocks. at steady state.

The Welfare Cost of Longer Unemployment Spells

- Figure 11 summarizes inequality along the transition path to the new steady state.

Figure: Gini Coefficient Transition to New $\pi(u, u) = 0.75$ Steady State

Measure Theory

This material draws from Stokey and Lucas Chapters 7,8,11, and 12.

Definition

Let X be a set and $\mathcal{X}$ a collection of subsets of X. Then $\mathcal{X}$ is a $\underline{\sigma-\text{algebra}}$ if (i) $\varnothing, X \in \mathcal{X}$; (ii)$\mathcal{X}$ is closed under complementation (i.e.$X_0 \in \mathcal{X} \Longrightarrow X_0^C \in \mathcal{X}$); (iii) $\mathcal{X}$ is closed under countable union (as well as countable intersection since $\{X_n\}_{n=1}^{\infty} \in \mathcal{X} \Longrightarrow \cup_{n=1}^{\infty} X_n \in \mathcal{X}$ and $\cap_{n=1}^{\infty} X_n = \left(\cup_{n=1}^{\infty} X_n^C\right)^C$).

Measure Theory

Definition

Let $X \subset \mathbb{R}^n$ be any set and $\mathcal{X}$ a Borel sigma algebra (the smallest collection of open/closed,etc.subsets of X). A measure is a function $\mu : \mathcal{X} \to \mathbb{R}$ such that: (i)$\mu(\varnothing) = 0$; (ii) $\mu(X_0) \geq 0, \forall X_0 \in \mathcal{X}$; (iii) if $\{X_n\}_{n=1}^{\infty}$ is a countable, disjoint collection of subsets of $\mathcal{X}$, then $\mu\left(\cup_{n=1}^{\infty} X_n\right) = \sum_{n=1}^{\infty} \mu(X_n)$.

Measure Theory

Definition
$(X, \mathcal{X}, \mu)$ is a measure space.

Definition
If $\mu(X) = 1$,then μ is a probability measure and $(X, \mathcal{X}, \mu)$ is a probability space.

Definition
Given a measurable space $(X, \mathcal{X})$,a function $f : X \to \mathbb{R}$ is measurable with respect to X if $\{x \in X : f(x) \leq k\} \in \mathcal{X}, \forall k \in \mathbb{R}$ (that is, the inverse image of the function is an element of the measurable space).

- Measurability is a weak concept (e.g. continuous functions are measurable).

Measure Theory

Definition

Let$(X, \mathcal{X})$ be a measurable space. A <u>transition function</u> is a function $Q : X \times \mathcal{X} \to [0, 1]$ such that: (i) for each $x \in X$, $Q(x, \cdot)$ is a probability measure on $(X, \mathcal{X})$; (ii) for each $X_0 \in \mathcal{X}$, $Q(\cdot, X_0)$ is a measurable function.

- The interpretation is that $Q(x, X_0)$ is the probability that next period's state is in the set X_0 given the current state is x. In terms of the Huggett model, you can think of the transition function as

$$Q(x, X_0) = \int_{S_0, A_0} \mathbf{1}_{\{a'=g(s,a;q)\in A_0\}}(s, a)\pi(s'|s)ds'da'$$

  from (2) where $X = S \times A$ which is composed of both the exogenous law of motion given by π and the endogenous law of motion given by g.

Measure Theory

- For any probability measure μ on $(X, \mathcal{X})$, define the operator T^* by

$$(T^*\mu)(X_0) = \int_X Q(x, X_0)\mu(dx), \forall X_0 \in \mathcal{X}. \tag{6}$$

  Since for each $X_0 \in \mathcal{X}$, $Q(\cdot, X_0)$ is bounded (in $[0, 1]$) and measurable, then $T^*\mu$ is well defined (i.e. the "integral" exists) .

- Interpret that $(T^*\mu)(X_0)$ is the probability that the state next period lies in the set X_0 if the current state is drawn from the probability measure μ. That is, $T^*\mu$ is the probability measure over the state next period if μ is the probability measure over the current state.

Measure Theory

- In terms of the Huggett model, (6) is just another way to write (2). To see this,

$$
\begin{aligned}
&(T^*\mu)(S_0, A_0; q) \\
&= \int_{S_0, A_0} \left\{ \int_{S,A} \mathbf{1}_{\{a'=g(s,a;q)\in A_0\}}(s,a)\pi(s'|s)\mu(ds, da; q) \right\} ds' da' \\
&= \int_{S,A} \left\{ \int_{S_0, A_0} \mathbf{1}_{\{a'=g(s,a;q)\in A_0\}}(s,a)\pi(s'|s) ds' da' \right\} \mu(ds, da; q)
\end{aligned}
$$

  where the second equality simply rearranges the integrals.

Measure Theory

Theorem

(S-L, 8.2) Let $\Xi(X, \mathcal{X})$ be the space of probability measures. The operator T^* defined in (6) maps the space of probability measures on $\Xi(X, \mathcal{X})$ into itself (i.e. $T^* : \Xi(X, \mathcal{X}) \to \Xi(X, \mathcal{X})$).

Proof.

Must show that for any $\mu \in \Xi(X, \mathcal{X})$, $T^*\mu$ satisfies the properties of a probability measure. In particular, (iii) Let $\{X_n\}$ be a disjoint sequence in $\mathcal{X}$ with $X = \cup_{n=1}^{\infty} X_n$. Then

$$\sum_{n=1}^{\infty} (T^*\mu)(X_n) = \sum_{n=1}^{\infty} \left[\int_X Q(x, X_n)\mu(dx) \right]$$

$$= \int_X \left[\sum_{n=1}^{\infty} Q(x, X_n) \right] \mu(dx) = \int_X Q(x, X)\mu(dx) = (T^*\mu)(X)$$

where the 2nd line uses Monotone Convergence Theorem and 3rd uses the fact that $Q(x, \cdot)$ is a probability measure. $\square$

Now we are prepared to define our object of interest (see page 317 of S-L).

Definition

A probability measure μ^* is <u>invariant</u> under T^* if it is a fixed point $\mu^* = T^*\mu^*$ of the T^* operator.

Measure Theory

- Sufficient conditions to ensure that an invariant distribution exists can be established using the following definitions (this is in Section 12.4 of S-L).
- Define another operator $\widetilde{T}$ associated with Q which takes bounded, measurable functions $f : X \to \mathbb{R}$ given by:

$$(\widetilde{T}f)(x) = \int_X f(x')Q(x, dx'), \forall x \in X \tag{7}$$

  with the interpretation as being the expected value of the function next period given the current state x.
- Theorem 8.1. of S-L establishes a result similar to Theorem 8.2 above for this operator (i.e. $\widetilde{T}[B(X)] \subset B(X)$).

Definition

A transition function Q on $(X, \mathcal{X})$ has the Feller property if the operator $\widetilde{T}$ in (7) maps the space of of bounded, continuous functions on X into itself (i.e. $\widetilde{T}[C(X)] \subset C(X)$).

- This is a stronger property (due to requirement of continuity) than in Theorem 8.1 since $C(X) \subset B(X)$ and is associated with stability.

Definition

A transition function Q on $(X, \mathcal{X})$ is monotone if the operator $\widetilde{T}$in (7) has the property that for every nondecreasing function $f : X \to \mathbb{R}$, the function $\widetilde{T}f$ is also nondecreasing.

Measure Theory

- A simple proof of Theorem 2 (what Huggett does) is to apply Theorems 12.12 and 12.13 of Stokey and Lucas.
- To apply Stokey and Lucas Theorem 12.12 (Existence of a unique invariant measure), we need to establish that
  - the transition function Q satisfies the Feller property (which basically says that the operator associated with Q maps the space of continuous functions into itself),
  - is monotone (the associated operator maps the space of monotone functions into itself),
  - and satisfies the following "mixing" condition. In particular, there exists $z \in [z_{\min}, z_{\max}]$, $\varepsilon > 0$, and $N \geq 1$ such that $Q^N(z_{\min}, [z, z_{\max}]) \geq \varepsilon$ and $Q^N(z_{\max}, [z_{\min}, z]) \geq \varepsilon$ where $Q^1(z, \widetilde{Z}) = Q(z, \widetilde{Z})$ and $Q^{i+1}(z, \widetilde{Z}) = \int_Z Q^i(z', \widetilde{Z}) Q(z, dz')$, $i = 1, 2, ...$ That is, $Q^i(z, \widetilde{Z})$ is the probability of going from z to $\widetilde{Z}$ in n periods.

Measure Theory

To apply Stokey and Lucas Theorem 12.13 (Continuity of the measure w.r.t. parameters, in this case q), we need to establish that the transition function $Q_\theta\ (X, \mathcal{X})$ for parameter vector $\theta \in \Theta$ on a compact set X, satisfies:

- if $\{(x_n, \theta_n)\}$ is a sequence in $X \times \Theta$ converging to (x_0, θ_0), the sequence $\{Q_{\theta_n}(x_n, \cdot)\}$ in the space of probability measures on $(X, \mathcal{X})$ converges weakly to $Q_\theta(x_0, \cdot)$ (continuity of $g(s, a; q)$ in q a.e. is important).
- and for each θ, T_θ^* has a unique fixed point (this was the object of Theorem 12.12) μ_θ^*.

Then if $\{\theta_n\}$ is a sequence in Θ converging to θ_0, then the sequence $\{\mu_{\theta_n}^*\}$ converges weakly to μ_θ^*.

- This latter result is important for establishing the existence of an equilibrium price q^* such that the asset market clears (i.e. necessary for continuity of the excess demand function).

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