Principle of Microeconomics
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Midterm Cheat Sheet

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1 Preference

a Check [SM] is satisfied: $\forall x_1, x_2 > 0$, $MU_1 > 0$, $MU_2 > 0$.
Check [SC] is satisfied: $\frac{\partial |MRS|}{\partial x_1} < 0$ (diminishing in x_1) or $\frac{\partial |MRS|}{\partial x_2} > 0$ (increasing in x_2).

b After checking [SM]&[SC], while both holding, we can link these features to our optimal choice problem.

2 Choices: Find x_1^m, x_2^m

a The maximization problem is: $\underset{x_1,x_2}{max}\, u(x_1, x_2)$, $s.t.\ p_1x_1 + p_2x_2 = I$.
Suppose the utility function $u(x_1, x_2)$ holds [SM]&[SC]:
The Lagrange is: $\underset{x_1,x_2}{max}\, \mathcal{L} = u(x_1, x_2) + \lambda(I - p_1x_1 - p_2x_2)$. By F.O.C, we obtain $\frac{\partial\mathcal{L}/\partial x_1}{\partial\mathcal{L}/\partial x_2} = |MRS| = \frac{p_1}{p_2}$.
And then plug this relation back in given Budget Constraint: $x_1^* = x_1(p_1, p_2, I) = x_1^m; x_2^* = x_2(p_1, p_2, I) = x_2^m$.

b Whether satisfies "Law of Demand": check $\varepsilon_1 < 0 \Leftrightarrow \frac{\partial x_1}{\partial p_1} < 0$.

3 Elasticity

a $\varepsilon_1 = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$, if $\varepsilon_1 < 0$: Ordinary good $\Leftrightarrow$ satisfies L.O.D. Otherwise, Giffen good.

b $\varepsilon_{ij} = \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}$. Suppose x_i, x_j are ordinary goods: If $\varepsilon_{ij} > 0 \Leftrightarrow$ Substitutes. Otherwise, Complements.

c $\varepsilon_{iI} = \frac{\partial x_i}{\partial I}\frac{I}{x_i}$, if $\varepsilon_{iI} > 0 \Leftrightarrow$ Normal good. Otherwise, Inferior good.

4 Sellers & Incentives

The seller's problem has three parts: **[1] Production, [2] Costs(Oppor.Cost), [3] Revenue.** We assume the market structure here is "**Perfectly Competitive**", which implies that both suppliers and demanders are "price taker" and suppliers has free cost of entering and exiting the market.

Ultimate goal for firms: Profit Maximization π.

a. Production Function :

$$y = AF(k,n)$$

$$\begin{cases} MPL = AF_n(k.n) = \frac{\partial AF(k,n)}{\partial n}, \\ MPK = AF_k(k.n) = \frac{\partial AF(k,n)}{\partial k}. \end{cases} \tag{1}$$

Now considering costs, a firm's profit(dividends) be like:

$$d = AF(k,n) - wn$$

From all above, we can observe: **[1]** FOC: $MPL \geq 0$, $MPK \geq 0$ **[2]** SOC: Marginal Product diminishes (n ↑ ⇒ MPL ↓, k↑ ⇒ MPK ↓) ⇔ **"Law of Diminishing Returns"** **[3]** A firm's marginal gain is MPL, while its marginal cost is w.

b. Costs of Production :

If a firm uses inputs, it must consider the production costs.
In the short-run, Total Cost can be represented as the sum of Variable Cost and Fixed Cost:

$$TC = VC + FC$$

We can divide the TC equation by quantity Q, obtaining:

$$\frac{TC}{Q} = \frac{VC}{Q} + \frac{FC}{Q} \Rightarrow ATC = AVC + AFC$$

where **ATC** stands for Average Total Cost, and so on. **Graph: U-shaped.** (Expl.: The first half of ATC is similar to AFC since Q is yet small. While Q is large enough, the other half of ATC is more alike to AVC.)

Now we further introduce the idea of Marginal Cost (**MC**), which measuring the additional costs induced by additional Q. We then mathematically define:

$$MC = \frac{\partial TC}{\partial Q} = \frac{\Delta TC(Q)}{\Delta Q} = \frac{\Delta VC}{\Delta Q} + \frac{\Delta FC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}$$

Apparently, we're calculating how the marginal, or changing, units of output (ΔQ) affect the change in total cost (ΔTC). It is worth noting that FC is a constant, implying Fixed Cost is literally FIXED and won't vary with the change in quantity of output: $\frac{\Delta FC}{\Delta Q} = 0$.

Another crucial idea is that **MC will always intersect ATC at ATC's minimum point.** Mathematically, this statement can be easily verified:

Proof. Since the extremum occurs when $FOC = 0$, let

$$\frac{dATC}{dQ} = 0 \Rightarrow \frac{d(\frac{TC}{Q})}{dQ} = \frac{(MC \cdot Q) - TC}{Q^2} = \frac{MC - ATC}{Q} = 0$$

Because ATC is U-shaped, we have the minimum when $MC = ATC$. ∎

c. Revenue & Profits :

Total Revenue (**TR**), Average Revenue (**AR**), Marginal Revenue (**MR**):

$$\begin{cases} TR = P \cdot Q \\ AR = \frac{TR}{Q} = P \\ MR = \frac{\Delta TR}{\Delta Q} = P \end{cases} \tag{2}$$

Thus, we have our first relation: $MR = AR = P$.

Now consider the ultimate Profit Maximization problem:

$$max\, \pi = TR - TC$$

yielding $FOC = 0$:

$$\Rightarrow \frac{\Delta \pi}{\Delta Q} = \frac{\Delta TR}{\Delta Q} - \frac{\Delta TC}{\Delta Q} = MR - MC = 0$$

So, a firm optimizes its profit <u>when output Q^* satisfies</u> the relation: $MR = MC$. Additionally, based on MR=P, we can conclude the final relation that maximize profit: $\underline{P = MR = MC}$. ■

$$\Rightarrow max\, \pi = TR - TC = (P \cdot Q^*) - (ATC \cdot Q^*) = (P - ATC) \cdot Q^*$$

d. Shutdown :

A short-run decision of temporarily stop producing if $\underline{TR < VC}$ $\Leftrightarrow$ **Shutdown point**: $P = P^{exit} = AVC$, i.e., shutdown when $P = SMC \leq AVC = P^{exit}$. (SMC: Short-run MC)

$\rightarrow$ *Why shutdown only when $TR < VC$?* Consider 2 cases:
(1) continue to produce: $\pi_1 = TR - TC = TR - (VC + FC) = (TR - VC) - FC$
(2) shutdown: $\pi_s = TR - TC = TR - (VC + FC) = 0 - (0 + FC) = -FC$
$\forall_{TR} | TR < VC, \pi_s > \pi_1 \Rightarrow$ better shutdown. ■

Based on the previous inference, we know <u>the Short-run Supply Curve is the upper segment of SMC which is above AVC</u>.