Midterm Cheat Sheet

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Hsien-Chen Chu (T09303304)

1 Preference

a Check [SM] is satisfied: $\forall x_1, x_2 > 0$, $MU_1 > 0$, $MU_2 > 0$.
Check [SC] is satisfied: $\frac{\partial |MRS|}{\partial x_1} < 0$ (diminishing in x_1) or $\frac{\partial |MRS|}{\partial x_2} > 0$ (increasing in x_2).

b After checking [SM]&[SC], while both holding, we can link these features to our optimal choice problem.

2 Choices: Find x_1^m, x_2^m

a The maximization problem is: $\underset{x_1,x_2}{max}\, u(x_1, x_2)$, $s.t.\ p_1x_1 + p_2x_2 = I$.
Suppose the utility function $u(x_1, x_2)$ holds [SM]&[SC]:
The Lagrange is: $\underset{x_1,x_2}{max}\, \mathcal{L} = u(x_1, x_2) + \lambda(I - p_1x_1 - p_2x_2)$. By F.O.C, we obtain $\frac{\partial \mathcal{L}/\partial x_1}{\partial \mathcal{L}/\partial x_2} = |MRS| = \frac{p_1}{p_2}$.
And then plug this relation back in given Budget Constraint: $x_1^* = x_1(p_1, p_2, I) = x_1^m; x_2^* = x_2(p_1, p_2, I) = x_2^m$.

b Whether satisfies "Law of Demand": check $\varepsilon_1 < 0 \Leftrightarrow \frac{\partial x_1}{\partial p_1} < 0$.

3 Elasticity

a $\varepsilon_1 = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$, if $\varepsilon_1 < 0$: Ordinary good $\Leftrightarrow$ satisfies L.O.D. Otherwise, Giffen good.

b $\varepsilon_{ij} = \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}$. Suppose x_i, x_j are ordinary goods: If $\varepsilon_{ij} > 0 \Leftrightarrow$ Substitutes. Otherwise, Complements.

c $\varepsilon_{iI} = \frac{\partial x_i}{\partial I}\frac{I}{x_i}$, if $\varepsilon_{iI} > 0 \Leftrightarrow$ Normal good. Otherwise, Inferior good.

4 Derive SE & IE: Find x_1^h, x_2^h

a Original Choice: $e^* = (x_1^*, x_2^*) \to p_i$ changes: New Choice: $e' = (x_1', x_2')$
Derivation (Slutsky equation by Hicksian's Methods): $e'' = (x_1'', x_2'') = (x_1^h, x_2^h)$

b The minimization problem is: $\underset{x_1,x_2}{min}\, p_1x_1 + p_2x_2$, $s.t.\ \bar{u}(x_1^*, x_2^*)$. Obtain:
$x_1'' = x_1(p_1, p_2, \bar{u}) = x_1^h$
$x_2'' = x_2(p_1, p_2, \bar{u}) = x_2^h$

c Substitution Effect(SE): $x_1'' - x_1^* = x_1^h - x_1^*$
Income Effect(IE): $x_1' - x_1'' = x_1' - x_1^h$
Total Effect = SE + IE

5 Intertemporal Consumption

a The maximization problem is: $\underset{c_1,c_2}{max}\, u(c_1, c_2)$, $s.t.\ c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$.

b Directly use Lagrange: obtain (c_1^*, c_2^*)
Check: #saver: $c_1^* < I_1$ or #borrower: $c_1^* > I_1$
Basic Assumption: Both c_1, c_2 are normal goods.
Discussion: SE & IE in terms of c_1

c Comparative Static Analysis: r changes

Suppose a borrower in Period 1

$r \uparrow_{small}$(remain a borrower): $SE < 0$ (Oppor.Cost of c_1 increases $\Rightarrow$ less c_1). $IE < 0$ (for a borrower: r $\uparrow$ means real I $\downarrow$) **Results:** [1]($c_1 \downarrow, c_2?$), [2]remain a borrower, [3]utility $\downarrow$

$r \uparrow_{large}$(become a saver): STRONG $SE < 0$ (Oppor.Cost of c_1 increases $\Rightarrow$ less c_1). $IE > 0$ (already become a saver: r $\uparrow$ means real I $\uparrow$) **Results:** [1]($c_1 \downarrow, c_2 \uparrow$), [2]become a saver, [3]utility $\uparrow$

Suppose a saver in Period 1

$r \uparrow$: $SE < 0$ (Oppor.Cost of c_1 increases $\Rightarrow$ less c_1) $IE > 0$ (for a saver: r $\uparrow$ means real I $\uparrow$) **Results:** [1]($c_1?, but < I_1, c_2 \uparrow$), [2]remain a saver, [3]utility $\uparrow$

d Comparative Static Analysis: I changes

Suppose a saver in Period 1

$I_1 \uparrow$: $IE > 0$ (both normal goods). Results: [1]($c_1 \uparrow, c_2 \uparrow$), [2]remain a saver, [3]utility $\uparrow$

$I_2 \uparrow$: $IE > 0$ (both normal goods). Results: [1]($c_1 \uparrow, c_2 \uparrow$), [2]remain a saver or become a borrower, [3]utility $\uparrow$

6 Sellers & Incentives

The seller's problem has three parts: **[1] Production, [2] Costs(Oppor.Cost), [3] Revenue.** We assume the market structure here is "**Perfectly Competitive**", which implies that both suppliers and demanders are "price taker" and suppliers has free cost of entering and exiting the market.

Ultimate goal for firms: Profit Maximization.

a. Production Function

$$y = AF(k, n)$$

$$\begin{cases} MPL = AF_n(k.n) = \frac{\partial AF(k,n)}{\partial n}, \\ MPK = AF_k(k.n) = \frac{\partial AF(k,n)}{\partial k}. \end{cases} \tag{1}$$

Now considering costs, a firm's profit(dividends) be like:

$$d = AF(k, n) - wn$$

From all above, we can observe: [1] F.O.C.: $MPL \geq 0$, $MPK \geq 0$ [2] S.O.C.: Marginal Product diminishes (n ↑ ⇒ MPL ↓, k↑ ⇒ MPK ↓) ⇔ **"Law of Diminishing Returns"** [3] A firm's marginal gain is MPL, while its marginal cost is w.

b. Costs of Production If a firm uses inputs, it must consider the production costs.

In the short-run, Total Cost can be represented as the sum of Variable Cost and Fixed Cost:

$$TC = VC + FC$$

We can divide the TC equation by quantity Q, obtaining:

$$\frac{TC}{Q} = \frac{VC}{Q} + \frac{FC}{Q} \Rightarrow ATC = AVC + AFC$$

where **ATC** stands for Average Total Cost, and so on. **Graph: U-shaped.** (Expl.: The first half of ATC is similar to AFC since Q is yet small. While Q is large enough, the other half of ATC is more alike to AVC.)

Now we further introduce the idea of Marginal Cost (**MC**), which measuring the additional costs induced by additional Q. We then mathematically define:

$$MC = \frac{\Delta TC(Q)}{\Delta Q} = \frac{\Delta VC}{\Delta Q} + \frac{\Delta FC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}$$

Apparently, we're calculating how the marginal, or changing, units of output (ΔQ) affect the change in total cost (ΔTC). It is worth noting that FC is a constant, implying Fixed Cost is literally FIXED and won't vary with the change in quantity of output: $\frac{\Delta FC}{\Delta Q} = 0$.

Another crucial idea is that **MC will always intersect ATC at ATC's minimum point.** Mathematically, this statement can be easily verified:

Proof. Since the extremum occurs when $F.O.C = 0$, let

$$\frac{dATC}{dQ} = 0 \Rightarrow \frac{d(\frac{TC}{Q})}{dQ} = \frac{(MC \cdot Q) - TC}{Q^2} = \frac{MC - ATC}{Q} = 0$$

Because ATC is U-shaped, we have the minimum when $MC = ATC$. ■