Midterm Brief Review

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Hsien-Chen Chu (T09303304)

1 Principles of Econ

- a. Optimization $\rightarrow \max_{x_1,x_2} u(x_1,x_2)$, s.t. BC: $p_1x_1 + p_2x_2 = I$. Equilibrium \rightarrow results of optimization Empiricism \rightarrow verify and analyze theories and hypotheses with data
- **b.** Positive Economics \rightarrow objective, testable; actually do. Normative Economics \rightarrow subjective, preferable; ought to do. *(e.g. fairness)*
- c. The trade-off between Efficiency and Equity

2 Economic Methods

a. Models are simplified descriptions or simulations of our real world, assumptions and exogenous variables needed.

- **b.** Correlation \Rightarrow Causation.
- c. By conducting experiments, economists can measure and verify the cause-and-effect relationship. $(\rightarrow \text{Empiricism})$

3 Optimization

a. Comparative Statics the comparison of 2 different outcomes after exogenous variables change. e.g. check "variable α 's influence" on target function $v(x_1, x_2) \rightarrow \text{take } \frac{\partial v(x_1, x_2)}{\partial \alpha}$ and verify relationship.

b. Measures [1] Optimization in Levels: consider the <u>total</u> net benefit (TR - TC in a whole), [2] Optimization in Differences: consider the change of the net benefit (Δ marginal analysis)

 \Rightarrow both methods get the exact same solution.

4 Demand, Supply & Equilibrium

a. Demand [1] Changes in quantity demanded: movement along the demand curve, causing by its own-price change. [2] Shifts of the demand curve: shift in line, causing by all the other factors.

b. Supply [1] Changes in quantity supplied: movement along the supply curve, causing by its own-price change. [2] Shifts of the supply curve: shift in line, causing by all the other factors.

c. Equilibrium given demand function and supply function, the equilibrium occurs when:

$$Q_D = Q_S$$

Sovle P^* , and then we get $Q^* \Rightarrow e^*(Q^*, P^*)$.

5 Consumer & Incentive

The buyer's problem has two parts: [1] Preference(Utility), [2] BC: Budget Constraint (Prices).

a. $MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2} \Rightarrow$ **Optimal**: $\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$. (or say $\frac{MB_1}{MB_2} = \frac{p_1}{p_2}$, MB stands for Marginal Benefit)

b. Elasticity

We only introduce and use <u>Arc</u> Elasticity of Demand (Midpoint formula) to solve problems in Principle of Economics. But since the sign features of arc elasticity are actually the same as those of <u>Point</u> Elasticity, which you all might largely use in Sophomore year and beyond(assuming you're in Econ major), I will display both below and simply use Point Elasticity for further explanations.

Arc [Own-price(ε_1), Cross-price(ε_{ij}), Income(ε_{iI})]:

$$arc \varepsilon_{1} = \frac{\Delta x_{1}/[(x_{1}''+x_{1}')/2]}{\Delta p_{1}/[(p_{1}''+p_{1}')/2]} = \frac{\Delta x_{1}}{\Delta p_{1}} \frac{\bar{p}_{1}}{\bar{x}_{1}},$$

$$arc \varepsilon_{ij} = \frac{\Delta x_{i}/[(x_{i}''+x_{i}')/2]}{\Delta p_{j}/[(p_{j}''+p_{j}')/2]} = \frac{\Delta x_{i}}{\Delta p_{j}} \frac{\bar{p}_{j}}{\bar{x}_{i}},$$

$$arc \varepsilon_{iI} = \frac{\Delta x_{i}/[(x_{i}''+x_{i}')/2]}{\Delta I/[(I''+I')/2]} = \frac{\Delta x_{i}}{\Delta I} \frac{\bar{I}}{\bar{x}_{i}}.$$
(1)

Just beware that we're dealing with the Arc ones, so don't forget to use the midpoint formula.

Features (using **Point** for expl.):

- [1] $\varepsilon_1 = \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1}$. If $\varepsilon_1 < 0$: Ordinary good \Leftrightarrow satisfies Law of Demand. Otherwise, Giffen good.
- [2] $\varepsilon_{ij} = \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i}$. Suppose x_i, x_j are ordinary goods \Rightarrow If $\varepsilon_{ij} > 0$: Substitutes. Otherwise, Complements.

[3] $\varepsilon_{iI} = \frac{\partial x_i}{\partial I} \frac{I}{x_i}$. If $\varepsilon_{iI} > 0$: Normal good. Moreover, $\varepsilon_{iI} > 1$: Luxuries; $0 < \varepsilon_{iI} < 1$: Necessities. Otherwise, Inferior good.

[4] In terms of abs value $|\varepsilon_1|$: $|\varepsilon_1| > 1 \Rightarrow$ Elastic Demand (flatter); $|\varepsilon_1| < 1 \Rightarrow$ Inelastic Demand (steeper); $|\varepsilon_1| = 1 \Rightarrow$ Unit Elastic Demand (rectangular hyperbola e.g. Cobb-Douglas)

c. Total Revenue change :

On the elastic segment of demand curve, TR decreases when P increases.

$$TR(\downarrow) = P(\uparrow) \cdot Q(\downarrow_{large})$$

On the inelastic segment of demand curve, TR increases when P increases.

$$TR(\uparrow) = P(\uparrow) \cdot Q(\downarrow_{small})$$

6 Sellers & Incentives

The seller's problem has three parts: [1] **Production**, [2] **Costs(Oppor.Cost)**, [3] **Revenue**. We assume the market structure here is "**Perfectly Competitive**", which implies that both suppliers and consumers are "price takers" (*P*: exogenous) and suppliers has free cost of entering and exiting the market.

Ultimate goal for firms: <u>Profit Maximization</u> π .

a. Production Function :

For A: Product shock(exogenous), K or k: Capital(exogenous), L or n: Labor(endogenous)

$$Q = AF(K, L) \Leftrightarrow y = AF(k, n)$$

$$MPL = AF_n(k, n) = \frac{\partial AF(k, n)}{\partial n},$$

$$MPK = AF_k(k, n) = \frac{\partial AF(k, n)}{\partial k}.$$
(2)

Now considering costs, w: real wage rate, a firm's profit(dividends) can be like:

$$d = AF(k, n) - wn$$

Given $\{A, k, w\}$:

$$\max_{(n)} d = AF(k,n) - wn$$

From all above, we can observe some results: [1] FOC: $MPL \ge 0$, $MPK \ge 0$ [2] SOC: Marginal Product <u>diminishes</u> (n $\uparrow \Rightarrow MPL \downarrow$, $k\uparrow \Rightarrow MPK \downarrow$) \Leftrightarrow "Law of Diminishing Returns" [3] A firm's marginal gain is <u>MPL</u>, while its marginal cost is <u>w</u>.

b. Costs of Production :

If a firm uses inputs, it must consider the production costs. In the short-run, <u>Total Cost</u> can be represented as the sum of <u>Variable Cost</u> and <u>Fixed Cost</u>:

$$TC = VC + FC$$

We can divide the TC equation by quantity Q, obtaining:

$$\frac{TC}{Q} = \frac{VC}{Q} + \frac{FC}{Q} \Rightarrow ATC = AVC + AFC$$

where **ATC** stands for <u>Average</u> Total Cost, and so on. **Graph: U-shaped.** (Expl.: The first half of ATC is similar to AFC since Q is yet small. While Q is large enough, the other half of ATC is more alike to AVC.)

Now we further introduce the idea of Marginal Cost (MC), which measuring the additional costs induced by additional Q. We then mathematically define:

$$MC = \frac{\partial TC}{\partial Q} = \frac{\Delta TC(Q)}{\Delta Q} = \frac{\Delta VC}{\Delta Q} + \frac{\Delta FC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}$$

Apparently, we're calculating how the marginal, or changing, units of output (ΔQ) affect the change in total cost (ΔTC). It is worth noting that FC is a constant, implying Fixed Cost is literally FIXED and won't vary with the change in quantity of output: $\frac{\Delta FC}{\Delta Q} = 0$.

Another crucial point is that MC will always intersect ATC at ATC's minimum point. Mathematically, this statement can be easily verified: *Proof.* Since the extremum occurs when FOC = 0, let

$$\frac{dATC}{dQ} = 0 \Rightarrow \frac{d(\frac{TC}{Q})}{dQ} = \frac{(MC \cdot Q) - TC}{Q^2} = \frac{MC - ATC}{Q} = 0$$

Because ATC is U-shaped, we have the minimum when MC = ATC. At this point, which is the **Efficient Scale**, a firm can **minimize production cost**.

c. Revenue & Profits :

Total Revenue (**TR**), Average Revenue (**AR**), Marginal Revenue (**MR**):

$$\begin{cases}
TR = P \cdot Q \\
AR = \frac{TR}{Q} = P \\
MR = \frac{\Delta TR}{\Delta Q} = P
\end{cases}$$
(3)

Thus, we have our first relation: MR = AR = P.

Now consider the ultimate **Profit Maximization problem**:

$$max \, \pi = TR - TC$$

yielding FOC = 0:

$$\Rightarrow \frac{\Delta \pi}{\Delta Q} = \frac{\Delta TR}{\Delta Q} - \frac{\Delta TC}{\Delta Q} = MR - MC = 0$$

So, a firm optimizes its profit when output Q^* satisfies the relation: MR = MC. Additionally, based on MR=P, we can conclude the final relation that **maximize profit**: P = MR = MC.

$$\Rightarrow max \, \pi = TR - TC = (P \cdot Q^*) - (ATC \cdot Q^*) = (P - ATC) \cdot Q^*$$

d. Shutdown :

A short-run decision of temporarily stop producing if $\underline{TR} < VC \Leftrightarrow$ Shutdown point: $P = P^{exit} = AVC$, i.e., shutdown when $P = SMC \leq AVC = P^{exit}$. (SMC: Short-run MC)

 \rightarrow Why shutdown only when TR < VC ? Consider 2 cases:

- (1) continue to produce: $\pi_1 = TR TC = TR (VC + FC) = (TR VC) FC$
- (2) shutdown: $\pi_s = TR TC = TR (VC + FC) = 0 (0 + FC) = -FC$
- $\forall_{TR} | TR < VC, \pi_s > \pi_1 \Rightarrow$ better shutdown.

Based on the previous inference, we know the Short-run Supply Curve is the upper segment of SMC lying above AVC.