Principle of Microeconomics
National Taiwan University
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Final Brief Review

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Literature: Ming-Jen Lin: PoME Lecture PPTs, **ch6, 12, 14**, 2020.
I'd like to reiterate that this is merely a BRIEF review of some important(relatively mathematical) final contents, so it leaves most of the chaps, texts and graphs for you to revisit the textbook or PPTs if needed.
Important formulas and features are listed sequentially.

ch6 - Long-run Equilibrium

(a) **LTC(q): Long-run Total Cost**
Facing the cost minimization problem,

$$\min_{L,K} wL + rK \ s.t.\ q = f(L,K)$$

By Lagrange, $\frac{w}{r} = \frac{MPL}{MPK} \Rightarrow$ solve the equation of L and K, then plug-in $q = f(L,K) \Rightarrow$ get $L^*(w,r,q),\ K^*(w,r,q) \Rightarrow$ plug-in $LTC(q) = wL^* + rK^*$.

[Example]: 103-1 Final Exam #Problem Set A. 2.: Given $q = 2L^{0.5}K,\ w = 4,\ r = 8$, derive the long-run TC function.
$\rightarrow \min_{L,K} 4L + 8K \ s.t.\ q = 2L^{0.5}K \xrightarrow{Lagrange} K = L$. Thus, $L^* = \sqrt[3]{\frac{q^2}{4}} = K^*$
$\Rightarrow LTC(q) = 4L^* + 8K^* = 12\sqrt[3]{\frac{q^2}{4}}$. ■

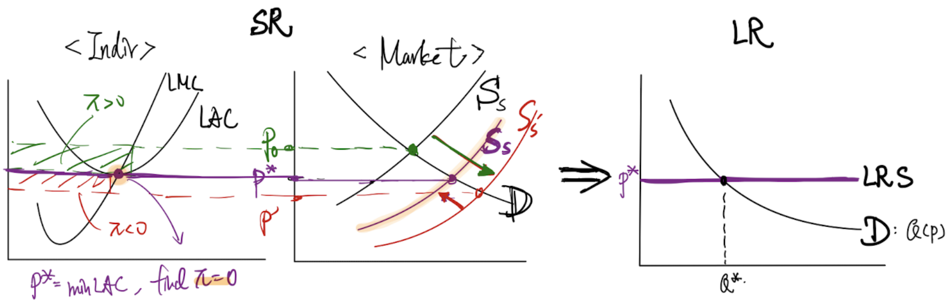
(b) **LAC: Long-run Average Total Cost**
In the long run, all factors are adjustable. Firms can always choose to produce at the efficient scale, where q_i^* makes $min\, SAC$, under different production/capital levels. Thus, LAC is the **envelope curve** of SAC $\Leftrightarrow LAC(q) \leq SAC(q)$.

(c) **Returns to Scale & Economic Scale**

i. Increasing Returns to Scale(IRS) $\Rightarrow$ Decreasing LAC: Economies of Scale
ii. Constant Returns to Scale(CRS) $\Rightarrow$ Constant LAC
iii. Decreasing Returns to Scale(DRS) $\Rightarrow$ Increasing LAC: Diseconomies of Scale

Proof of i.: IRS: $f(tL,tK) > tf(L,K)$; LAC $= \frac{LTC}{q} = \frac{wL+rK}{f(L,K)}$.
$\xrightarrow{inputs\ *t} LAC' = \frac{w(tL)+r(tK)}{f(tL,tK)} < \frac{t(wL+rK)}{tf(L,K)} = LAC$. ■

Figure 1: Long-run Decision and Equilibrium



(d) **Long-run Decision: Enter/Exit** Suppose free-entry, and consider
Profit $\pi = TR - TC = P(q) \cdot q - LAC(q) \cdot q = (P - LAC)q$

i. Enter if $\pi > 0$: $TR \geq TC \Leftrightarrow P(q) \geq LAC(q)$

ii. Exit if $\pi < 0$: $TR \leq TC \Leftrightarrow P(q) \leq LAC(q)$

When is the optimal?
$P^* = min\, LAC \Rightarrow$ Yield **0 profit** in Const-cost Industry(factor prices remain the same during expansion, forming a horizontal LR S).

LR Indiv. Supply Curve S_i:

$$LR\ S_i : q_i^*(p) = \begin{cases} P = LMC(q),\ when\ P \geq min\, LAC \\ 0,\ when\ P < min\, LAC \end{cases} \tag{1}$$

Remarks: CANNOT use horizontal summation of LR Indiv. Supply Curve to obtain LR Market Supply Curve($\sum S_i \nRightarrow S$) because the number of firms is endogenous in the LR, which is shown below. Instead, one should use the optimal condition $P^* = min\, LAC$ to solve such problem.

(e) **How to achieve LR Market Equilibrium?**
Suppose homogeneous firms,
$\Rightarrow$ Produce at $min\, LAC(q) \Rightarrow$ get $q_i^* = \arg\min_q LAC(q)$
$\Rightarrow P^* = min\, LAC(q = q_i^*)$, Q^* is determined by Demand Function
$\Rightarrow$ Number of Firms: $n = Q^*/q_i^*$.

Remarks: If heterogeneous firms, find each's $min\, LAC_i(q)$
$\Rightarrow$ LR market price $P^* =$ the smallest $min\, LAC$ amongst all firms.

[Example]: NTU 109-1 hw3 #Problem Set 6.:
$\rightarrow P^* = min\, LAC(q = q_i^* = 500) = 4 \Rightarrow Q^* = 70000 - 5000P^* = 50000$
$\Rightarrow n = Q^*/q_i^* = 100.$ ■

(f) **Sloping of LR S**: LR Supply Curve is NOT necessarily a horizontal line- it depends on the nature of the industry. $\rightarrow$ If it is a (const/increasing/decreasing) cost industry, the market supply curve is (horizontal/positive/negative) sloping. **Coverage: Sophomore Microeconomics I.**

ch12 - Monopoly

(a) **Differences between Perfectly Competitive and Monopoly?**

i. **Perfectly Competitive:** Many sellers exist in the market. All traders are "price-taker" $\Rightarrow$ P: exogenous, profit maximization $max\,\pi$: $P = MR = MC$. Identical goods are available in the market. Single firm faces a horizontal demand curve (elasticity=∞).

ii. **Monopoly:** Only one(single) seller exists in the market. The Firm is the "price-maker" $\Rightarrow$ P: endogenous, manually **set** $P > MR = MC$ **based on the Demand Function.** Single goods with no close substitutes are available. The only-one firm faces a downward sloping demand curve.

(b) **What causes Monopoly?** $\Rightarrow$ Barriers to Entry

i. Legal Market Power: Patent/Copyright/Sole Selling Agent

ii. Nature Market Power: Key Resources/Result of Economies of Scale

(c) **Profit Maximization: Monopoly**

The monopolist, unlike the indiv. perfectly competitive firm, faces a negative sloping demand curve. This, in turn, means that there is a TRADE-OFF between the PRICE it charges(Price effect) and the QUANTITY it sells(Quantity effect), so revenue loss is inevitable when expanding Q. Whether total revenue will increase or fall as a result of price cut depends on price elasticity of demand.

Where should a Monopolist produce?

Given D and TC: $max\,\pi$: $FOC = 0 \Rightarrow$ yields

$$MR = MC$$

$\Rightarrow$ get Q^* $\Rightarrow$ plug back in the Demand Function for P^*.

Remark: Maximize profit where $MR = MC$. Since $MC \geq 0 \Rightarrow MR = MC \geq 0$, it provides the critical point driving $MR = 0$, Q'. Referring this point vertically back to D, observe that the intersection point is exactly on the midpoint of D. Thus, a firm will only operate on the left segment of the demand curve: $Q \in [0, Q']$.

[Example]: NTU 104-1 hw8 #Problem Set(II) 1.: Monopolist, given $D : P = 100 - Q$, $TC = Q^2 + 16$, find $\{P^*,\ Q^*,\ \pi_i^*\}$.

$\rightarrow MR = MC \Rightarrow 100 - 2Q = 2Q,\ Q^* = 25 \Rightarrow P^* = 100 - 25 = 75$

$\Rightarrow \pi_i^* = P^* \cdot Q^* - TC(Q^*) = 75 \cdot 25 - (25^2 + 16) = 1234.$ ■

(d) **Price Discrimination**

i. First-degree(Perfect): "Eviscerate" all CS by charging different prices based on each consumer's WTP(CS$\rightarrow$PS).

ii. Second-degree: Different price set based on the characteristics of **the purchase**. [Target: goods]

iii. Third-degree: Different price set based on the characteristics of **the customers and locations**. [Target: ppl, places, **markets**]

Revisit: What if there are multiple markets(D_i) for a Monopolist?
Suppose two markets $\{D_A,\ D_B\},\ Q = Q_A + Q_B$ →Third-degree:
$\rightarrow \pi = [TR_A(Q_A) + TR_B(Q_B)] - TC(Q)$
For Market A, $max\,\pi_A : \frac{\partial \pi}{\partial Q_A} = 0 \Rightarrow [\frac{\partial TR_A(Q_A)}{\partial Q_A} + 0] - \frac{\partial TC(Q)}{\partial Q_A} = MR_A(Q_A) - \frac{\partial TC(Q)}{\partial Q} \cdot \frac{\partial Q}{\partial Q_A} = MR_A(Q_A) - MC(Q) \cdot 1 = MR_A(Q_A) - MC(Q) = 0.$
Similarly, for Market B, $MR_B(Q_B) - MC(Q) = 0$.
Thus,

$$MR_A(Q_A) = MC(Q) = MR_B(Q_B)$$

■

[Example]: NTU 104-1 hw8 #Problem Set(II) 2.(a): Monopolist, third-degree price dis., given $D_A : P_A = 100 - Q_A,\ D_B : P_B = 80 - Q_B,\ TC(Q) = 75Q = 75(Q_A + Q_B)$, find $\{P_A^*,\ P_B^*,\ Q_A^*,\ Q_B^*\}$.
$\rightarrow MR_A = 100 - 2Q_A,\ MR_B = 80 - 2Q_B,\ MC = 75$
$\Rightarrow 100 - 2Q_A = 75 = 80 - 2Q_B,\ Q_A^* = 12.5,\ Q_B^* = 2.5.$
$\Rightarrow P_A^* = 100 - 12.5 = 87.5,\ P_B = 80 - 2.5 = 77.5.$ ■

A bit more math about MR
From the derivative$\frac{dTR(Q)}{dQ}$ and the profit maximization cond.: MR=MC, obtain $MR = P(1 + \frac{1}{\varepsilon}) = MC$. Since $\varepsilon < 0$ for ordinary goods, we can also re-write: $MR = P(1 - \frac{1}{|\varepsilon|}) = MC$.
And, third-degree yields: $MR_A = MR_B \Rightarrow$

$$MR_A = P_A(1 + \frac{1}{\varepsilon_A}) = P_B(1 + \frac{1}{\varepsilon_B}) = MR_B = MC$$

$$\Rightarrow P_A(1 - \frac{1}{|\varepsilon_A|}) = P_B(1 - \frac{1}{|\varepsilon_B|}) = MC$$

Remarks: The bigger the $|\varepsilon_i|$ is, the smaller the P_i is.

[Example]: NCCU Final Exam # Problem Set 3.: Given a firm's MC is 10, and the elasticity of demand is -2. Then, the firm's profit maximizing price is?
$\rightarrow$ By $P(1 + \frac{1}{\varepsilon}) = MC \Rightarrow P(1 + \frac{1}{-2}) = 10,\ P = 20.$ ■

ch14 Oligopoly

Suppose Duopoly in the following: $n = 2,\ i \in [1, 2]$,

(a) **Cartel**
Deemed a Monopolist → solve $MR = MC \Rightarrow Q^*,\ q_i^* = \frac{Q^*}{2}$, P^* is determined by the Demand Function.

(b) **Cournot: CE of Q**
Suppose D: $P = a - bQ,\ Q = q_1 + q_2,\ MC = m$(const), then:
$\pi_1 = TR_1 - TC_1 = (a - bq_1 - bq_2)q_1 - m \cdot q_1 = (a - m)q_1 - bq_1^2 - bq_1q_2$
$\Rightarrow max\,\pi_1 : FOC = 0 \Rightarrow (a - m) - 2bq_1 - bq_2 = 0.$
Thus, we obtain the **Reaction Function** of Firm 1: $q_1 = \frac{a-m}{2b} - \frac{1}{2}q_2$.
Similarly, the Reaction Function of Firm 2: $q_2 = \frac{a-m}{2b} - \frac{1}{2}q_1$
$\Rightarrow q_i^* = \frac{a-m}{3b},\ Q^* = \frac{(a-m)n}{3b},\ P^* = a - bQ^* = a - b[\frac{(a-m)n}{3b}] = \frac{a+mn}{3}.$

(c) **Bertrand CE of P**

$\rightarrow$ Game{Reducing price for the entire market}, eq: $P^* = MC$, Q^* is determined by the Demand Function, $q_i^* = \frac{Q^*}{2}$, and $\pi_i = 0$.

Example: NTU 109-1 In-course e.g.: Duopoly:{UA vs AA}, $D: Q = 339 - P,\ Q = q_{UA} + q_{AA},\ MC = 147$, find Cartel:{$q_i^*,\ Q^*,\ P^*$}, Cournot:{Reaction Function, $q_i^*,\ Q^*,\ P^*$} and Bertrand:{$q_i^*,\ Q^*,\ P^*$}.

i. **Cartel:**$\rightarrow D: P = 339 - Q,\ MR = MC \Rightarrow 339 - 2Q = 147$
$\Rightarrow Q^* = 96,\ q_{UA} = q_{AA} = 48,\ P^* = 243.$ ■

ii. **Cournot:**$\rightarrow D: P = 339 - Q,\ \pi_{UA} = [339 - (q_{UA} + q_{AA})]q_{UA} - 147q_{UA}$.
From $FOC = 0$, obtain reaction function:

$$Reaction\ Function: \begin{cases} q_{UA} = 96 - \frac{1}{2}q_{AA} \\ q_{AA} = 96 - \frac{1}{2}q_{UA} \end{cases} \tag{2}$$

$\Rightarrow q_{UA} = q_{AA} = 64,\ Q^* = 128,\ P^* = 211.$ ■

iii. **Bertrand:**$\rightarrow P^* = MC = 147 \Rightarrow Q^* = 192,\ q_{UA} = q_{AA} = 96.$ ■