Asynchronous System – Assignment 01
Due date: 09/06/13
Tu Nguyen - 108810358

Describe an application problem that is interesting to you and that requires a distributed algorithm to solve. In particular, describe what should be computed not how to compute it. That is, describe what is given as input and what is asked as output, including any restrictions on the input and how the output is related to the input, but not how to go from the input to the output. Also, describe why this problem requires a distributed algorithm as opposed to a centralized algorithm to solve.

Solution:

Source: http://en.wikipedia.org/wiki/Induction_puzzles

Problem: The King's wise men

The King called the three wisest men in the country to his court to decide who would become his new advisor. He placed a hat on each of their heads, such that each wise man could see all of the other hats, but none of them could see their own. Each hat was either white or blue. The king gave his word to the wise men that at least one of them was wearing a blue hat - in other words, there could be one, two, or three blue hats, but not zero. The king also announced that the contest would be fair to all three men. The wise men were also forbidden to speak to each other. The king declared that whichever man stood up first and announced the color of his own hat would become his new advisor. The wise men sat for a very long time before one stood up and correctly announced the answer. What did he say, and how did he work it out?

Input:

- Each hat was either white or blue
- At least one of them was wearing a blue hat – in other words, there could be one, two or three blue hats, but not zero
- The contest would be fair to all three men

Restrict:

- Each wise man could see all of the other hats, but none of them could see their own

- The men were forbidden to speak to each other

Output:

- The man stood up first and announced the color of his own hat would become his new advisor

Why this problem requires a distributed algorithm as opposed to a centralized algorithm to solve:

- Each man only knows the color of others one, not himself, so global knowledge is not available in this situation and thus, we can't use centralized algorithm to solve this problem
- They are wisest men in the country, so they are wise enough to solve the problem if it's possible
- The contest was fair to all three men

How the output related to the input:

- Suppose that there were one blue hat. The person with that hat would see two white hats, and since the king specified that there is at least one blue hat, that wise man would immediately know the color of his hat. However, the other two would see one blue and one white hat and would not be able to immediately infer any information from their observations. Therefore, this scenario would violate the king's specification that the contest would be fair to each. So there must be at least two blue hats.
- Suppose then that there were two blue hats. Each wise man with a blue hat would see one blue and one white hat. Since they have already realized that there must be at least two blue hats, they would then immediately know that each were wearing a blue hat. However, the man with the white hat would see two blue hats and would not be able to immediately infer any information from his observations. This scenario, then, would also violate the specification that the contest would be fair to each. So there must be three blue hats.

Since there must be three blue hats, the first man to figure that out will stand up and say blue.