The Hamiltonian Brain: Efficient Probabilistic Inference with Excitatory-Inhibitory Neural Circuit Dynamics[1]

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Modeling in neuroscience - and elsewhere

Hugo Simon

22/03/2022

[1] Laurence Aitchison et al. (2016). "The Hamiltonian Brain: Efficient Probabilistic Inference with Excitatory-Inhibitory Neural Circuit Dynamics". In: *PLOS Computational Biology*.

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- Our brain operates in the face of substantial uncertainty due to ambiguity in the inputs, and inherent unpredictability in the environment.
- In such condition, reasoning under uncertainty is crucial for survival.
- The optimal way to do so is not making single-best point estimates but rather considering a whole weighted range of explanations, namely Bayesian inference.
- Some human behaviours are known to be consistent with Bayesian inference in many sensory, motor and cognitive tasks.

Figure 1: Principle of Bayesian inference

Current limitations are that

- traditional models of neural computation neglect uncertainty in favor of circuit dynamics that seek for the single best explanation of their inputs.
- they don't capture the rich dynamics of cortical responses. Neural activities in the cortex show **prominent intrinsic oscillations**, and **large transient changes** in response to stimulus onset.
- they typically violate Dales's law by having neurons with both excitatory and inhibitory outputs.

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- Markov Chain Monte Carlo (MCMC) methods are a class of methods for sampling from probability distributions.
- based on sequentially sampling new point from previous one such that the actual dynamic follows a Markov chains that have the target distribution as stationary laws.
- The higher the mixing rate, the faster the inference.

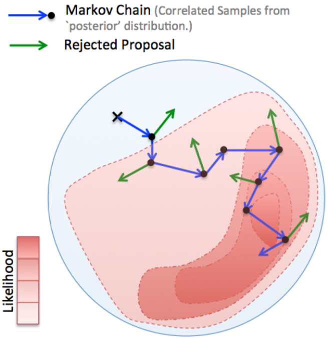


Figure 2: MCMC as Metropolis-Hastings.

- Idea: Introduce a latent space $\boldsymbol{q}$ as momentum and treat the whole as an Hamiltonian system[2].

$$\dot{\boldsymbol{q}} = \frac{\partial \mathcal{H}}{\partial \boldsymbol{p}}, \quad \dot{\boldsymbol{p}} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{q}}$$

with for instance

$$\mathcal{H}(\boldsymbol{q}, \boldsymbol{p}) = -\ln f(\boldsymbol{q}) + \frac{1}{2}\|\boldsymbol{p}\|^2_{M^{-1}},$$

- Hamiltonian dynamic proposes moves to distant states which maintain high acceptance probability due to approximate energy conserving property.

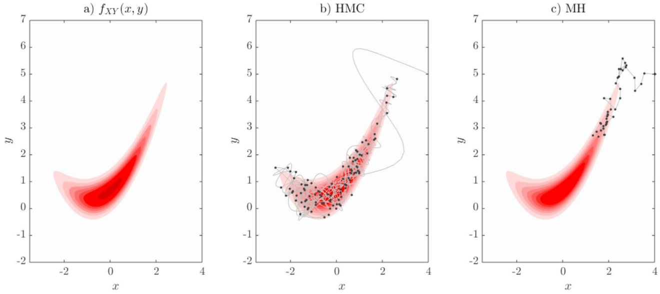


Figure 3: HMC yields higher mixing rate and acceptance ratio than MH.

Generative model defined as

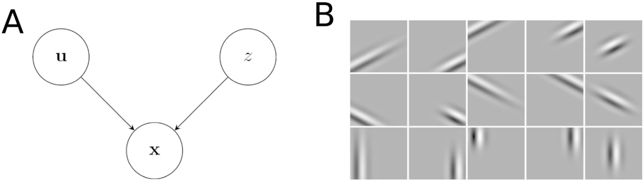


Figure 4: **A.** GSM graphical model. **B.** edge-like features $\mathbf{A}$.

$$\mathbf{u} \sim \mathcal{N}(0, C) \quad \text{where} \quad C = (1 - \sigma_{\mathbf{x}}^2)(\mathbf{A}^\top \mathbf{A})^{-1}$$
$$z \sim |\mathcal{N}(0, 1)|$$
$$\mathbf{x} \mid \mathbf{u}, z \sim \mathcal{N}\left(z\mathbf{A}\mathbf{u}, \sigma_{\mathbf{x}}^2 \mathbf{I}\right)$$

- widely used statistical model of natural images[3].
- latent variable $\mathbf{u}$ known to account for *stationary* responses of V1 neurons.
- latent variable z known to account for complex-cell activations.

[3] Martin J Wainwright et al. (1999). "Scale Mixtures of Gaussians and the Statistics of Natural Images". In: *Advances in Neural Information Processing Systems*. Vol. 12. MIT Press.

Defining the Excitatory-Inhibitory network

$$\dot{\mathbf{u}} = \frac{1}{\tau}\left[\mathbf{W}_{\mathrm{uu}}\mathbf{u} - \mathbf{W}_{\mathrm{uv}}\mathbf{v} + \frac{1}{2}\tau\rho^2\mathbf{I}_{\mathrm{input}}\right] + \rho\boldsymbol{\eta}_{\mathrm{u}}$$
$$\dot{\mathbf{v}} = \frac{1}{\tau}\left[\mathbf{W}_{\mathrm{vu}}\mathbf{u} - \mathbf{W}_{\mathrm{vv}}\mathbf{v} - \mathbf{I}_{\mathrm{input}}\right] + \rho\boldsymbol{\eta}_{\mathrm{v}}$$

where

$$\mathbf{I}_{\text{input}} = \frac{z}{\sigma_{\mathbf{x}}^2}\mathbf{A}^T(\mathbf{x} - z\mathbf{A}\mathbf{u}) - \mathbf{C}^{-1}\mathbf{u}$$

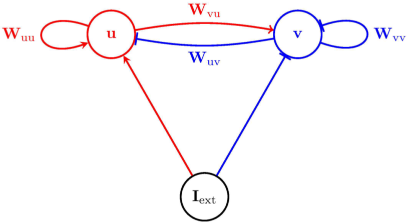


Figure 5: EI Hamiltonian network architecture.

This implements noised HMC for the GSM posterior distribution !

Face-like features

- GSM with edge-like features (Gabor filters) capture statistics of natural image but not very realist looking.
- We factorize face dataset with Multiplicative Update (MU) algorithm to get face-like features.

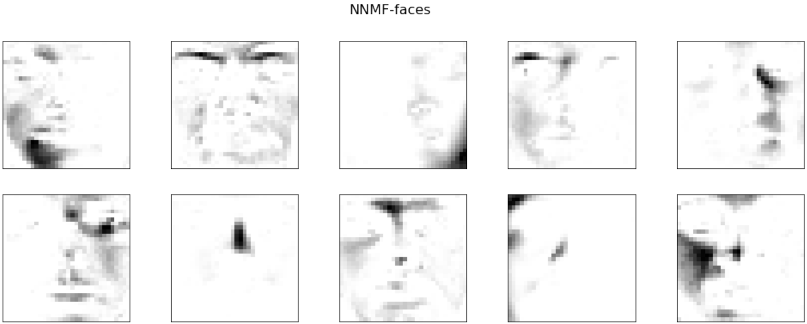


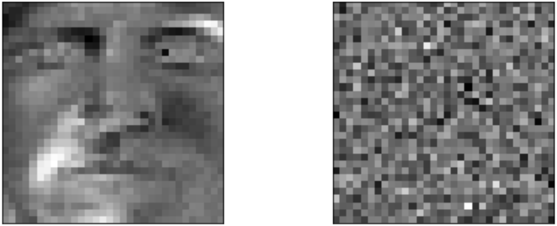
Figure 6: Face-like features obtained from NNMF of a face dataset.

Stimulus definition

- Recall

$$\mathbf{u} \sim \mathcal{N}(0, C)$$
$$z \sim |\mathcal{N}(0,1)|$$
$$\mathbf{x} \mid \mathbf{u}, z \sim \mathcal{N}\left(z\mathbf{A}\mathbf{u}, \sigma_{\mathbf{x}}^2 \mathbf{I}\right)$$

- Stimulus is when z is large, no-stimulus is when z is close to zero,



(a) Stimulus (b) No-stimulus

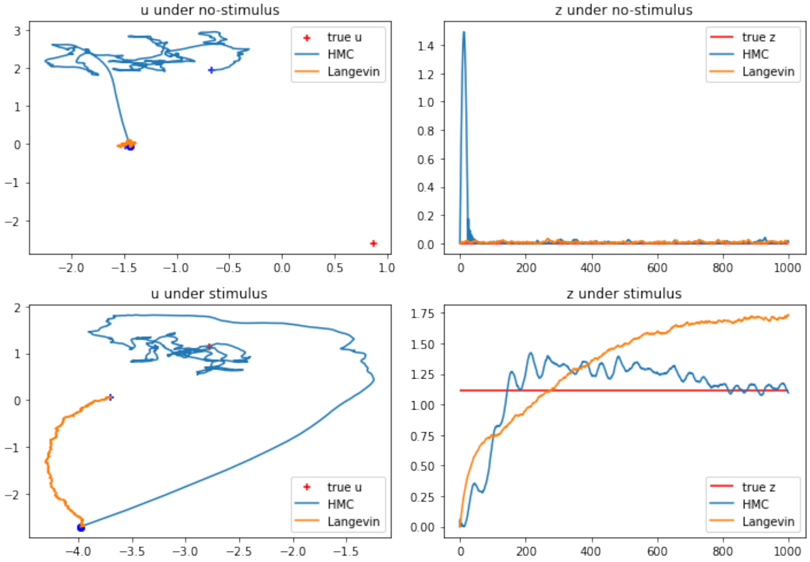
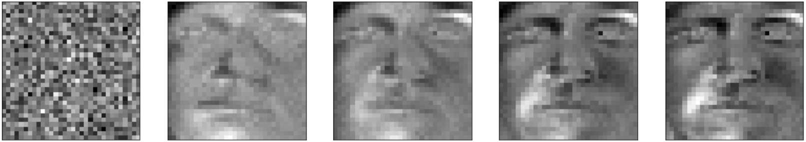


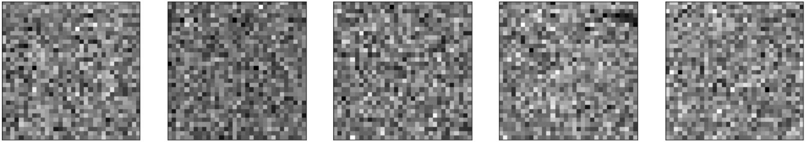
Figure 8: EI HMC and EI Langevin ($\mathbf{W}=0$) behaviours under no-stimulus and stimulus.

Stimulus reconstruction

What does reconstructed observation from inferred variables looks like ?



(a) Stimulus reconstruction among sampling.



(b) No-stimulus reconstruction among sampling.

In practice, decision is made from not a single but a distribution of samples across time, i.e. frequency estimates posterior probability.

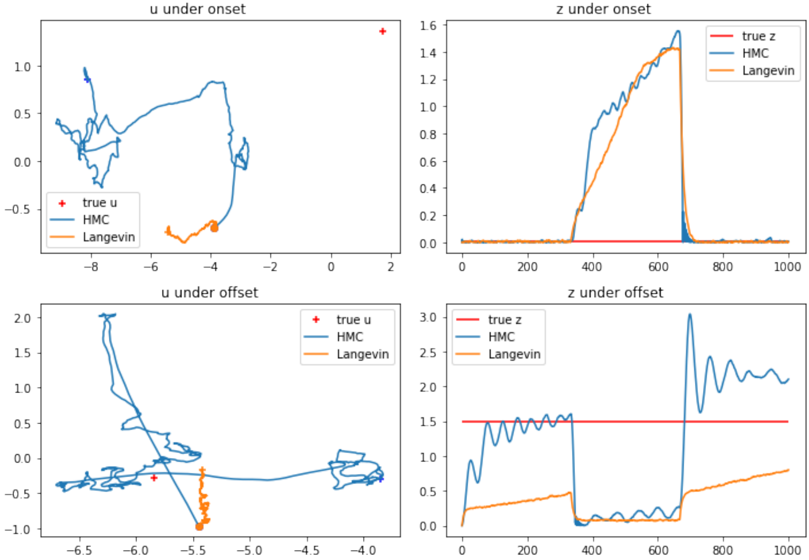
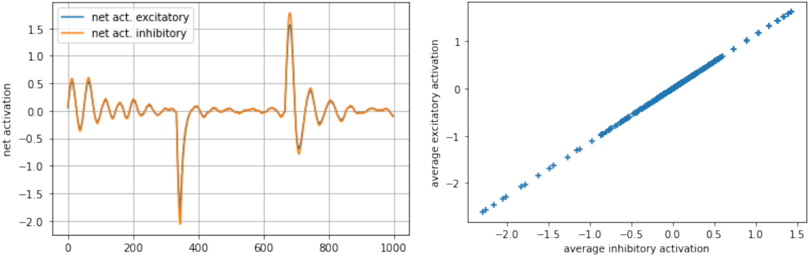


Figure 10: EI HMC and EI Langevin ($\mathbf{W}=0$) behaviours under stimulus onset and offset.

- $\mathbf{W}_{uu}\mathbf{u} - \mathbf{W}_{uv}\mathbf{v}$ and $\mathbf{W}_{vu}\mathbf{u} - \mathbf{W}_{vv}\mathbf{v}$ are respectively activation of excitatory cells and inhibitory cells.
- For stable sampling from the posterior, excitation and inhibition should be balanced.
    - For EI Langevin, $\mathbf{W}$ always 0.
    - For EI HMC, net excitation and net inhibition can be momentarily imbalanced to perform better.



(a) excitatory and inhibitory inputs among sampling.

(b) excitatory inputs vs. inhibitory inputs.

- Note transient increases in firing rates upon stimulus onset and offset.

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Conclusion

- Relatively simple and highly efficient Bayesian inference $\boxed{\checkmark}$
- Dales's principle compliant $\boxed{\checkmark}$
- Prominent intrinsic and **functionally explainable** cortical oscillations $\boxed{\checkmark}$
- Transient increase in firing rates upon stimulus onset and offset $\boxed{\checkmark}$

Perspectives

- EI HMC architecture rely on GMS model, but even with face-like features, GMS does not capture natural images notions such as object and obstruction.
- How local learning rules may be able to set up weights needed for implementing EI HMC ?

References

Aitchison, Laurence et al. (2016). "The Hamiltonian Brain: Efficient Probabilistic Inference with Excitatory-Inhibitory Neural Circuit Dynamics". In: *PLOS Computational Biology*.

Duane, Simon et al. (1987). "Hybrid Monte Carlo". In: *Physics Letters B* 195.2, pp. 216–222.

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Thank you!