Centre for
Data Analytics

Insight

Basic Modelling

Helmut Simonis

ACP Winterschool 2024

DCU NUI Galway OÉ Gaillimh UCC UCD Dublin SFI

Licence

Acknowledgments

This publication has emanated from research conducted with the financial support of Science Foundation Ireland under Grant number 12/RC/2289-P2 at Insight the SFI Research Centre for Data Analytics at UCC, which is co-funded under the European Regional Development Fund.
A version of this material was developed as part of the ECLiPSe ELearning course: `https://eclipseclp.org/ELearning/index.html`.
Support from Cisco Systems and the Silicon Valley Community Foundation is gratefully acknowledged.

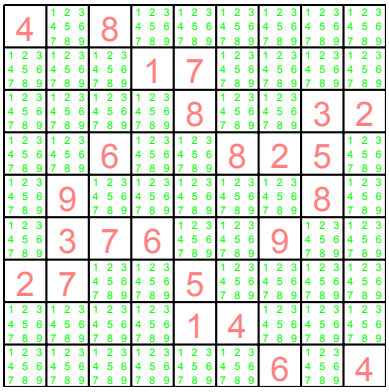
Example 2: Sudoku

- Global Constraints
  - Powerful modelling abstractions
  - Non-trivial propagation
  - Different consistency levels
- Example: Sudoku puzzle

Problem Definition

Sudoku

Fill in numbers from 1 to 9 so that each row, column and 3*x*3 block contain each number exactly once



4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

Model

- A variable for each cell, ranging from 1 to 9
- A 9x9 matrix of variables describing the problem
- Preassigned integers for the given hints
- `alldifferent` constraints for each row, column and 3x3 block

Sudoku Models

- ECLiPSe ▸ Show
- MiniZinc ▸ Show
- NumberJack ▸ Show
- CPMpy ▸ Show
- Choco-solver ▸ Show

ECLiPSe Sudoku Model (from https://eclipseclp.org/)

```
:- lib(ic).
:- import alldifferent/1 from ic_global.

top :-
    problem(Board),
    print_board(Board),
    sudoku(Board),
    labeling(Board),
    print_board(Board).

sudoku(Board) :-
    dim(Board, [N,N]),
    Board :: 1..N,
    ( for(I,1,N), param(Board) do
        alldifferent(Board[I,*]),
        alldifferent(Board[*,I])
    ),
    NN is integer(sqrt(N)),
    ( multifor([I,J],1,N,NN), param(Board,NN) do
        alldifferent(concat(Board[I..I+NN-1, J..J+NN-1]))
    ).

print_board(Board) :-
    dim(Board, [N,N]),
    ( for(I,1,N), param(Board,N) do
        ( for(J,1,N), param(Board,I) do
        X is Board[I,J],
        ( var(X) -> write("  _") ; printf(" %2d", [X]) )
        ), nl
    ), nl.
```

ECLiPSe Data Definition

```
problem([](
    [](4, _, 8, _, _, _, _, _, _),
    [](_, _, _, 1, 7, _, _, _, _),
    [](_, _, _, _, 8, _, _, 3, 2),
    [](_, _, 6, _, _, 8, 2, 5, _),
    [](_, 9, _, _, _, _, _, 8, _),
    [](_, 3, 7, 6, _, _, 9, _, _),
    [](2, 7, _, _, 5, _, _, _, _),
    [](_, _, _, _, 1, 4, _, _, _),
    [](_, _, _, _, _, _, 6, _, 4))).
```

Continue

MiniZinc Sudoku Model

```
int: s;
int: n=s*s;
array[1..n,1..n] of var 1..n: puzzle;

include "sudoku.dzn";

predicate alldifferent(array[int] of var int: x) =
  forall(i,j in index_set(x) where i < j)
    (x[i] != x[j]);

constraint forall(i in 1..n)
    (alldifferent([puzzle[i,j]| j in 1..n]));
constraint forall(j in 1..n)
    (alldifferent([ puzzle[i,j]| i in 1..n]));
constraint forall(i,j in 1..s)
    (alldifferent([puzzle[s*(i-1)+p, s*(j-1)+q]|
                   p,q in 1..s]));
solve satisfy;
```

MiniZinc Output

```
output [ "sudoku:\\n" ] ++
  [ show(puzzle[i,j]) ++
    if j = n then
       if i mod s = 0 /\ i < n then "\n\n"
       else "\n"
       endif
     else
       if j mod s = 0 then "  "
       else " "
       endif
     endif
    | i,j in 1..n ];
```


MiniZinc Data File (sudoku.dzn)

```
s=3;
puzzle=[|
    4, _, 8, _, _, _, _, _, _|
    _, _, _, 1, 7, _, _, _, _|
    _, _, _, _, 8, _, _, 3, 2|
    _, _, 6, _, _, 8, 2, 5, _|
    _, 9, _, _, _, _, _, 8, _|
    _, 3, 7, 6, _, _, 9, _, _|
    2, 7, _, _, 5, _, _, _, _|
    _, _, _, _, 1, 4, _, _, _|
    _, _, _, _, _, _, 6, _, 4|
|];
```

▸ Continue

NumberJack Sudoku Model

```
from Numberjack import *

def get_model(N, clues):
    grid = Matrix(N*N, N*N, 1, N*N)

    sudoku = Model([AllDiff(row) for row in grid.row],
                   [AllDiff(col) for col in grid.col],
                   [AllDiff(grid[x:x + N, y:y + N]) for x in range(0, N*N, N)
                                                    for y in range(0, N * N, N)],
                   [(x == int(v)) for x, v in
                       zip(grid.flat, "".join(open(clues)).split()) if v != '*']
                   )
    return grid, sudoku

def solve(param):
    N = param['N']
    clues = param['file']

    grid, sudoku = get_model(N, clues)

    solver = sudoku.load(param['solver'])
    solver.setVerbosity(param['verbose'])
    solver.setTimeLimit(param['tcutoff'])

    solver.solve()
```


NumberJack Data File

```
4 * 8 * * * * * *
* * * 1 7 * * * *
* * * * 8 * * 3 2
* * 6 * * 8 2 5 *
* 9 * * * * * 8 *
* 3 7 6 * * 9 * *
2 7 * * 5 * * * *
* * * * 1 4 * * *
* * * * * * 6 * 4
```

▸ Continue

CPMpy Sudoku Model(from https://github.com/CPMpy/)

```
import numpy as np
from cpmpy import *

# Variables
puzzle = intvar(1,9, shape=given.shape, name="puzzle")

model = Model(
    # Constraints on values (cells that are not empty)
    puzzle[given!=e] == given[given!=e], # numpy's indexing, vectorized equality
    # Constraints on rows and columns
    [AllDifferent(row) for row in puzzle],
    [AllDifferent(col) for col in puzzle.T], # numpy's Transpose
)

# Constraints on blocks
for i in range(0,9, 3):
    for j in range(0,9, 3):
        model += AllDifferent(puzzle[i:i+3, j:j+3]) # python's indexing

model.solve()
```


CPMpy Data Definition

```
e = 0 # value for empty cells
given = np.array([
    [4, e, 8, e, e, e, e, e, e],
    [e, e, e, 1, 7, e, e, e, e],
    [e, e, e, e, 8, e, e, 3, 2],
    [e, e, 6, e, e, 8, 2, 5, e],
    [e, 9, e, e, e, e, e, 8, e],
    [e, 3, 7, 6, e, e, 9, e, e],
    [2, 7, e, e, 5, e, e, e, e],
    [e, e, e, e, 1, 4, e, e, e],
    [e, e, e, e, e, e, 6, e, 4]
])
```

▸ Continue

Choco-solver Sudoku Model

```
Model model = new Model("Sudoku");
int blockSize = 3;
int m = blockSize*blockSize;

IntVar[][] vars = new IntVar[m][m];
for(int i=0;i<m;i++){
    for(int j=0;j<m;j++){
        vars[i][j] =  model.intVar("X"+i+""+j, 1, m);
        if (data[i][j]>0) {
            model.arithm(vars[i][j],"=",data[i][j]).post();
        }
    }
}
for(int i=0;i<m;i++){
    model.allDifferent(row(i,m,vars)).post();
    model.allDifferent(column(i,m,vars)).post();
}
for(int i=0;i<m;i+=blockSize){
    for(int j=0;j<m;j+=blockSize){
        model.allDifferent(block(i,j,blockSize,vars)).post();
    }
}
Solver solver = model.getSolver();
solver.solve();
```


Choco-solver Data

```
int[][] data = new int[m][m]{
    {4, 0, 8, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 1, 7, 0, 0, 0, 0},
    {0, 0, 0, 0, 8, 0, 0, 3, 2},
    {0, 0, 6, 0, 0, 8, 2, 5, 0},
    {0, 9, 0, 0, 0, 0, 0, 8, 0},
    {0, 3, 7, 6, 0, 0, 9, 0, 0},
    {2, 7, 0, 0, 5, 0, 0, 0, 0},
    {0, 0, 0, 0, 1, 4, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 6, 0, 4}
};
```

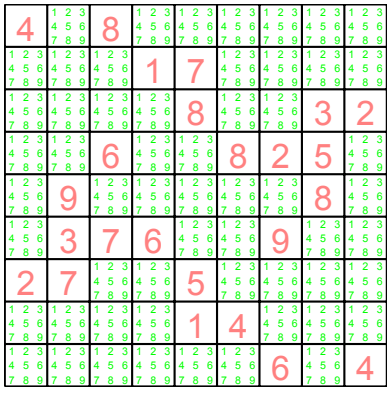
Choco-solver Utilities

```
IntVar[] row(int row, int size, IntVar[][] array){
    return array[row];
}

IntVar[] column(int col,int size,IntVar[][] array){
    IntVar[] column = new IntVar[size];
    for(int i=0; i<size; i++){
        column[i] = array[i][col];
    }
    return column;
}

IntVar[] block(int x,int y,int blockSize,IntVar[][] array){
    IntVar[] block = new IntVar[blockSize*blockSize];
    int k=0;
    for(int i=0;i<blockSize;i++){
        for(int j=0;j<blockSize;j++){
            block[k++] = array[x+i][y+j];
        }
    }
    return block;
}
```

Continue

Domain Visualizer

- Problem shown as matrix
- Each cell corresponds to a variable
- Instantiated: Shows integer value (large)
- Uninstantiated: Shows values in domain



Initial State (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9	1 5 7	1 6 7 9	1 5 6 7 9
3 6 9	2 5 6	3 5 9	1	7	2 3 5 6 9	4 5 8	4 6 9	5 6 8 9
6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 9	1 4 5 7	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	3 4 7	3 4 6	1 3 6 7	1 3 4 7	8	1 3 6 7
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 6 7 9	2 5 6 8	3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	3 5 6 7 8 9
3 7 9	1 2 5 8	1 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

After Setup (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 7 9	5 6 7 9
3 6 9	2 5 6	3 5 9	1	7	2 3 6 9	4 5 8	6 9	5 6 8 9
6 7 9	1 5 6	1 5 9	4 5 9	8	6 9	1 4 5 7	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 8 9	5	3 6 9	1 3 8	1 9	3 8 9
3 6 9	5 6 8	3 5 9	2 3 7 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 3 5 9	2 3 7 8 9	3 9	2 3 7 9	6	1 2 7 9	4

Can we do better?

- The alldifferent constraint is missing propagation
  - How can we do more propagation?
  - Do we know when we derive all possible information from the constraint?
- Constraints only interact by changing domains of variables

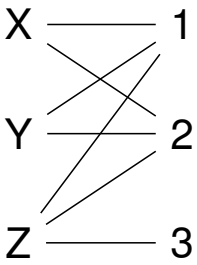
A Simpler Example

```
include "alldifferent.mzn";

var 1..2:X;
var 1..2:Y;
var 1..3:Z;

constraint alldifferent([X,Y,Z]);

solve satisfy;
```


Using Forward Checking

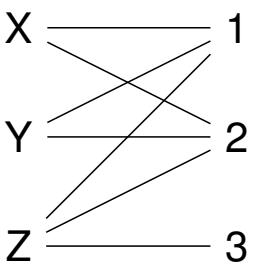
- No variable is assigned
- No reduction of domains
- But, values 1 and 2 can be removed from Z
- This means that Z is assigned to 3

Visualization of alldifferent as Graph



- Show problem as graph with two types of nodes
  - Variables on the left
  - Values on the right
- If value is in domain of variable, show link between them
- This is called a *bipartite* graph

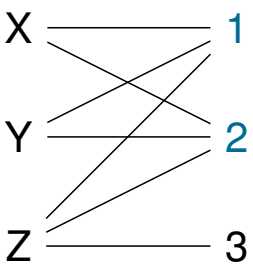
A Simpler Example



Value Graph for

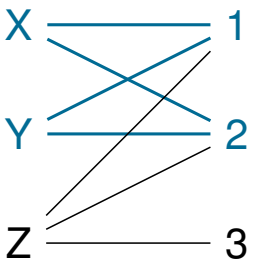
```
var 1..2:X;

var 1..2:Y;

var 1..3:Z;
```

A Simpler Example



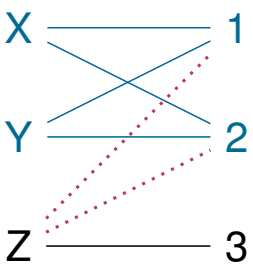
Check interval [1,2]

A Simpler Example



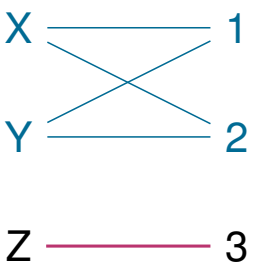
- Find variables completely contained in interval
- There are two: X and Y
- This uses up the capacity of the interval

A Simpler Example



No other variable can use that interval

A Simpler Example



Only one value left in domain of Z, this can be assigned

Idea (Hall Intervals)

- Take each interval of possible values, say size N
- Find all K variables whose domain is completely contained in interval
- If $K > N$ then the constraint is infeasible
- If $K = N$ then no other variable can use that interval
- Remove values from such variables if their bounds change
- If $K < N$ do nothing
- Re-check whenever domain bounds change

Implementation

- Problem: Too many intervals ($O(n^2)$) to consider
- Solution:
  - Check only those intervals which update bounds
  - Enumerate intervals incrementally
  - Starting from lowest(highest) value
  - Using sorted list of variables
- Complexity: $O(n \log(n))$ in standard implementations
- Important: Only looks at min/max bounds of variables

Bounds Consistency

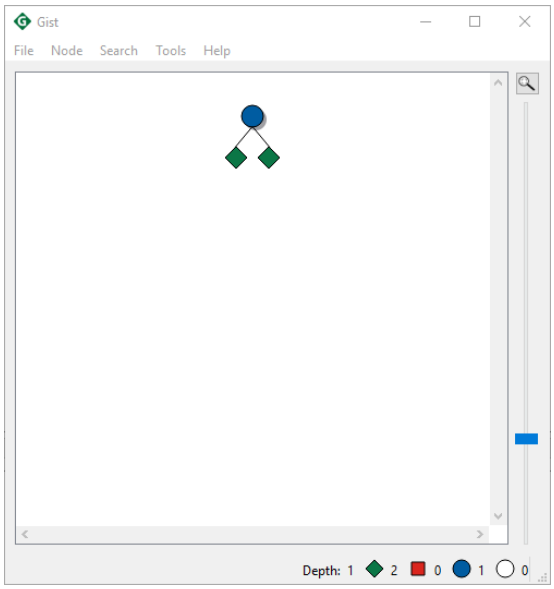
Definition

A constraint achieves *bounds consistency*, if for the lower and upper bound of every variable, it is possible to find values for all other variables between their lower and upper bounds which satisfy the constraint.

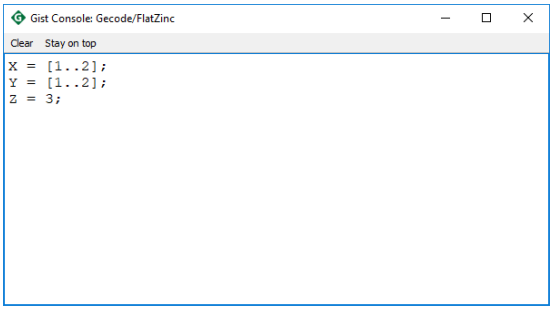
Annotation: :: bounds

```
include "alldifferent.mzn";

var 1..2:X;
var 1..2:Y;
var 1..3:Z;

constraint alldifferent([X,Y,Z]) :: bounds;
solve satisfy;
```

Running with Gecode Gist



All Solutions



Node Inspector (Root)

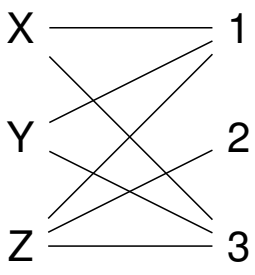
Can we do even better?

- Bounds consistency only considers min/max bounds
- Ignores "holes" in domain
- Sometimes we can improve propagation looking at those holes

Another Simple Example

```
include "alldifferent.mzn";

var {1,3}:X; % note enumerated domain
var {1,3}:Y;
var 1..3:Z; % note domain as interval

% annotated constraint
constraint alldifferent([X,Y,Z]) :: bounds;
solve satisfy;
```

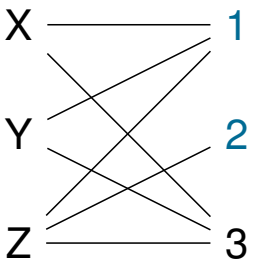

Another Simple Example



Value Graph for

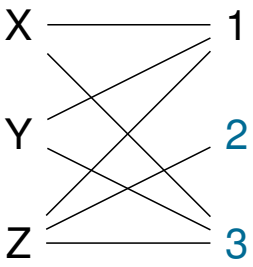
```
var {1,3}:X;

var {1,3}:Y;

var 1..3:Z;
```

Another Simple Example



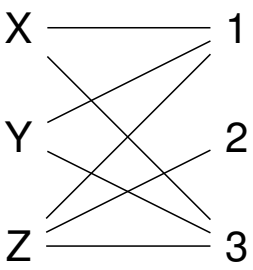
- Check interval [1,2]
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



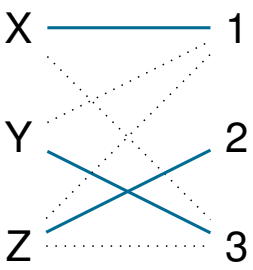
- Check interval [2,3]
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



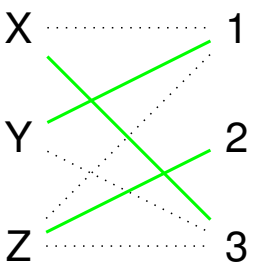
But, more propagation is possible, there are only two solutions

Another Simple Example



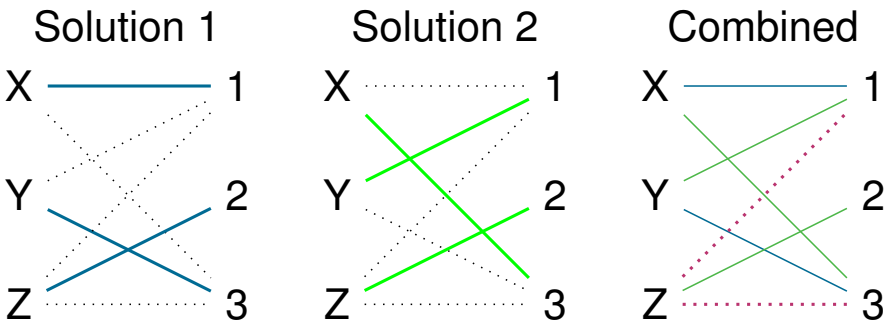
Solution 1: assignment in blue

Another Simple Example



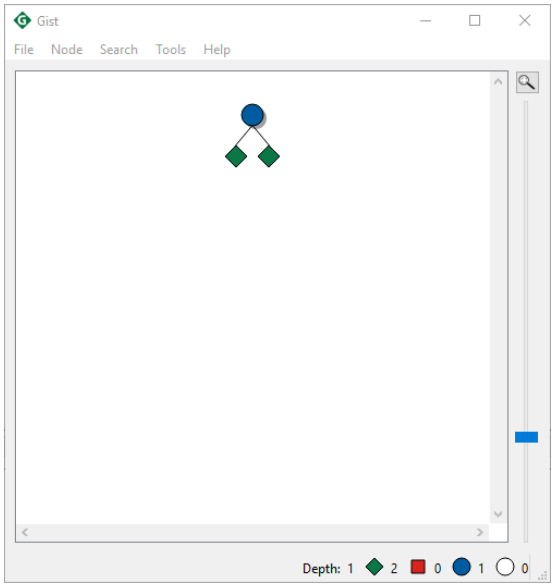
Solution 2: assignment in green

Another Simple Example

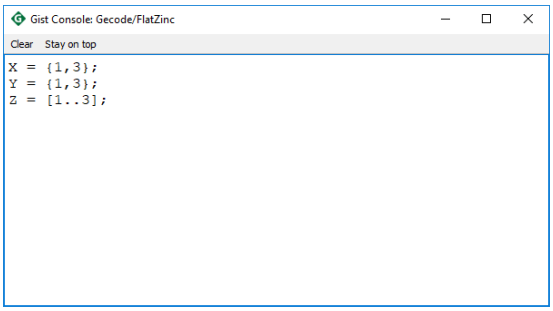


Combining solutions shows that Z=1 and Z=3 are not possible. Can we deduce this without enumerating solutions?

Bounds Consistency with Gecode Gist: No Propagation



All Solutions



Node Inspector (Root)

Solutions and Maximal Matchings

- A *Matching* is subset of edges which do not coincide in any node
- No matching can have more edges than number of variables
- Every solution corresponds to a *maximal matching* and vice versa
- If a link does not belong to some maximal matching, then it can be removed

Implementation

- Possible to compute all links which belong to some matching
  - Without enumerating all of them!
- Enough to compute **one** maximal matching
- Requires algorithm for *strongly connected components*
- Extra work required if more values than variables
- All links (values in domains) which are not supported can be removed
- Complexity: $O(n^{1.5}d)$

Domain Consistency

Definition

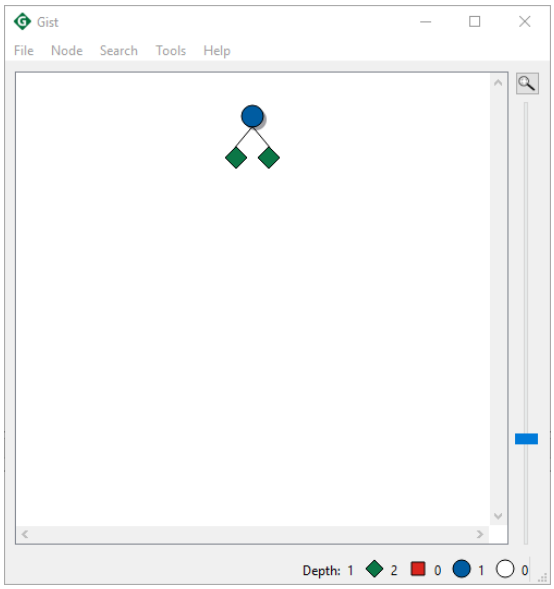
A constraint achieves *domain consistency*, if for every variable and for every value in its domain, it is possible to find values in the domains of all other variables which satisfy the constraint.

- Also called *generalized arc consistency (GAC)*
- or *hyper arc consistency*

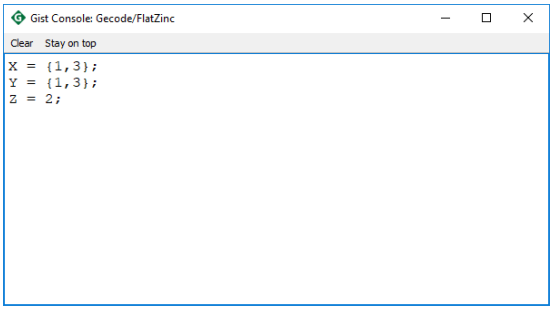
Simple Example Revisited

```
include "alldifferent.mzn";

var {1,3}:X; % note enumerated domain
var {1,3}:Y;
var 1..3:Z; % note domain as interval

% note different annotation
constraint alldifferent([X,Y,Z]) :: domain;
solve satisfy;
```


Domain Consistency with Gecode Gist: Propagation



All Solutions



Node Inspector (Root)

Can we still do better?

- NO! This extracts all information from this one constraint
- We could perhaps improve speed, but not propagation
- But possible to use different model
- Or model interaction of multiple constraints

Should all constraints achieve domain consistency?

- Domain consistency is usually more expensive than bounds consistency
  - Overkill for simple problems
  - Nice to have choices
- For some constraints achieving domain consistency is NP-hard
  - We have to live with more restricted propagation

Modified MiniZinc Program

```
int: s;
int: n=s*s;
array[1..n,1..n] of var 1..n: puzzle;

include "sudoku.dzn";

include "alldifferent.mzn";

constraint forall(i in 1..n)
    (alldifferent([puzzle[i,j]| j in 1..n])::domain);
constraint forall(j in 1..n)
    (alldifferent([ puzzle[i,j]| i in 1..n])::domain);
constraint forall(i,j in 1..s)
    (alldifferent([puzzle[s*(i-1)+p, s*(j-1)+q]|
                   p,q in 1..s])::domain);

solve satisfy;
```


Modified Choco-solver Sudoku Model

```
    Model model = new Model("Sudoku");
    int blockSize = 3;
    int m = blockSize*blockSize;

    IntVar[][] vars = new IntVar[m][m];
    for(int i=0;i<m;i++){
        for(int j=0;j<m;j++){
            vars[i][j] =  model.intVar("X"+i+""+j, 1, m);
            if (data[i][j]>0) {
                model.arithm(vars[i][j],"=",data[i][j]).post();
            }
        }
    }
// Consistency level AC: domain consistency, BC: bounds consistency, default: mix
    for(int i=0;i<m;i++){
        model.allDifferent(row(i,m,vars),AC).post();
        model.allDifferent(column(i,m,vars),AC).post();
    }
    for(int i=0;i<m;i+=blockSize){
        for(int j=0;j<m;j+=blockSize){
            model.allDifferent(block(i,j,blockSize,vars),AC).post();
        }
    }
    Solver solver = model.getSolver();
    solver.solve();
```

Initial State (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Propagation Steps (Domain Consistency)

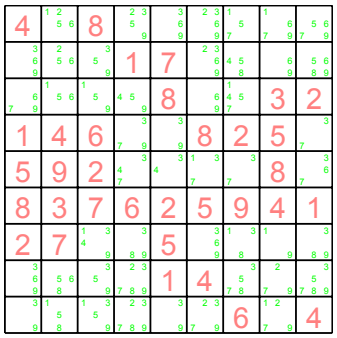
4	2	8	5	6	3	1	7 6 9	1 5 6 7 9
3 6 9	5	5 3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	7 3 9	3 9	8	2	5	7 3 9
5	9	2	7 3	4	1	1 4 7 3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	8 9 3	5	6	8	1	1 3 6 8 9
6	8	5 3 9	2	1	4	5 3 7 8	2 6 7 9	5
3 9	1	5	8	2 3 9	7	6	2	4

After Setup (Domain Consistency)

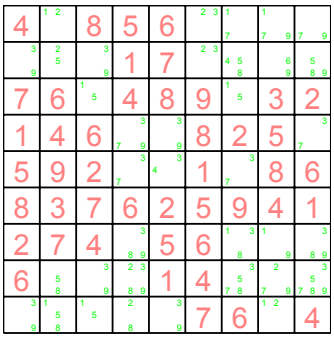
4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

Comparison

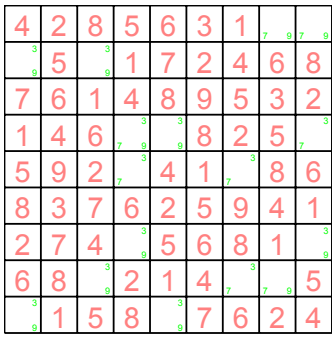
Forward Checking



Bounds Consistency



Domain Consistency



Typical?

- This does not always happen
- Sometimes, two methods produce same amount of propagation
- Possible to predict in certain special cases
- In general, tradeoff between speed and propagation
- Not always fastest to remove inconsistent values early
- But often required to find a solution at all

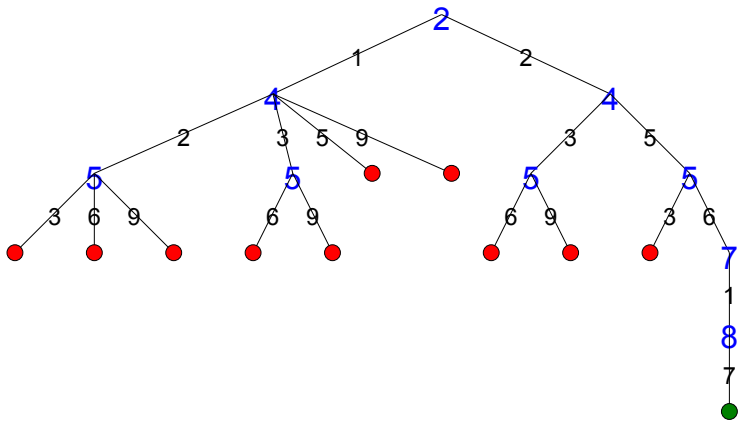
Simple search routine

- Enumerate variables in given order
- Try values starting from smallest one in domain
- Complete, chronological backtracking
- Advantage: Results can be compared with each other
- Disadvantage: Usually not a very good strategy

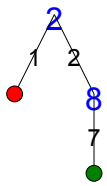
Asking for Naive Search in MiniZinc

```
solve :: int_search(
  puzzle,
  input_order,
  indomain_min)
satisfy;
```


Search Tree (Forward Checking)



Search Tree (Bounds Consistency)



Search Tree (Domain Consistency)

Trading Propagation Against Search

- If we perform more propagation, search is more constrained
- Fewer values left, fewer alternatives to explore in search
- Best compromise is not obvious
- But can be learned from examples or during search
- Annotations are optional
  - Some MiniZinc back-end solvers do the search they want, not the one you specify
  - Some solvers simply do not work in a way that these search annotations apply

Are there other Global Constraints?

- alldifferent is the most commonly used constraint
- Propagation methods can be explained
- But there are many more

Global Constraint Catalog

- `https://sofdem.github.io/gccat/`
- Description of 354 global constraints, 2800 pages
- Not all of them are widely used
- Detailed, meta-data description of constraints in Prolog

Families of Global Constraints

- Value Counting
  - alldifferent, global cardinality
- Scheduling
  - cumulative
- Properties of Sequences
  - sequence, no_valley
- Graph Properties
  - circuit, tree

Common Algorithmic Techniques

- Bi-Partite Matchning
- Flow Based Algorithms
- Automata
- Task Intervals
- Reduced Cost Filtering
- Decomposition