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# Visualization for Constraint Programming



Helmut Simonis

CRT-AI CP Week 2025

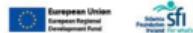
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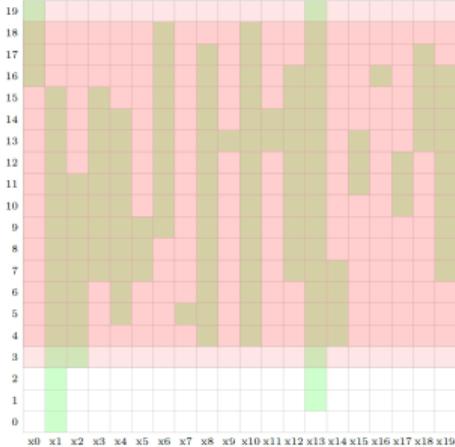
# Origins

- Derived from Tutorial at CP 2021  
(<https://cp2021.a4cp.org/tutorials.html>)
- By Helmut Simonis, Insight/UCC and Guido Tack, Monash University
- Video at <https://www.youtube.com/watch?v=AI-ZfQtMLAU>

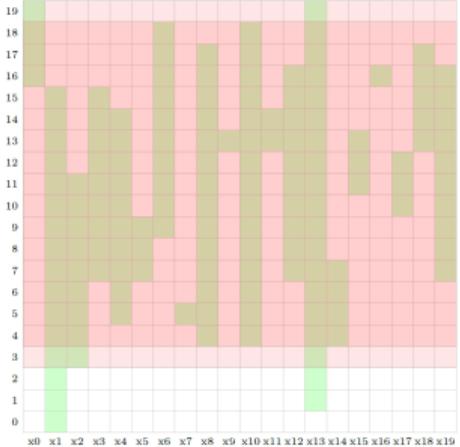
## Take-Away Message

- Visualization can help at different stages of the development process
- Discover problems early
- Understand what is happening without drowning in details
- Easily communicate with stakeholders without CP experience
- Decide how much integration with solver you need
- Complex use cases require visualization to be practical
- Generic vs. application specific visualization

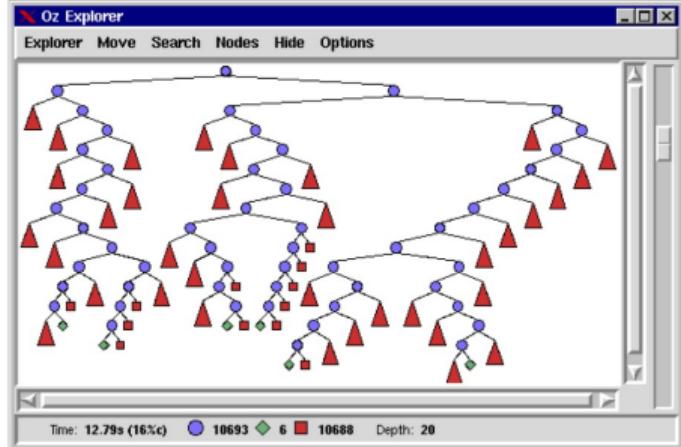
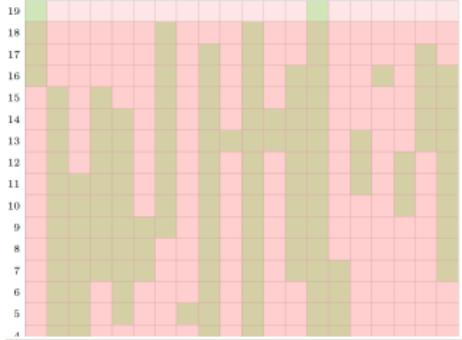
# Lots of pretty pictures, too!



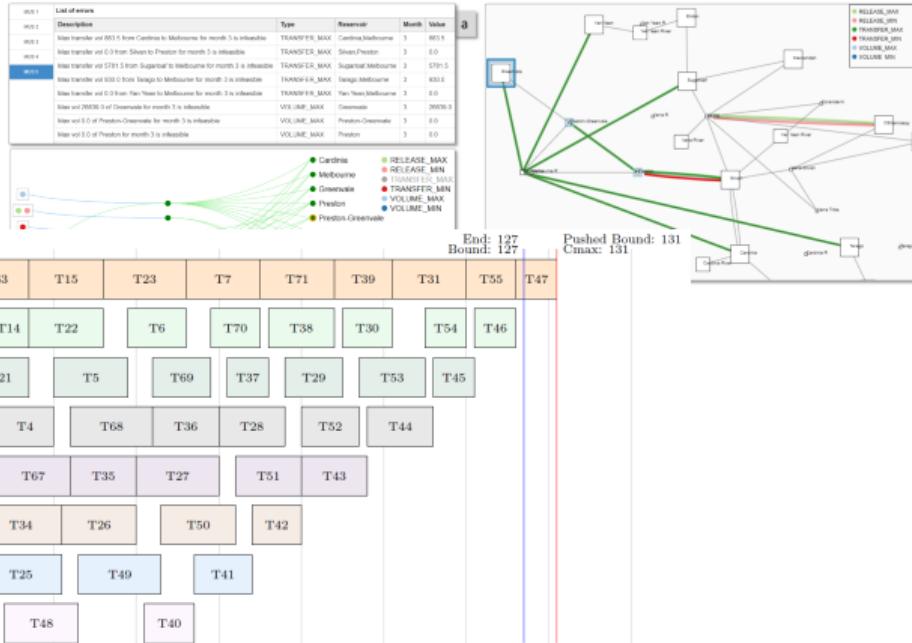
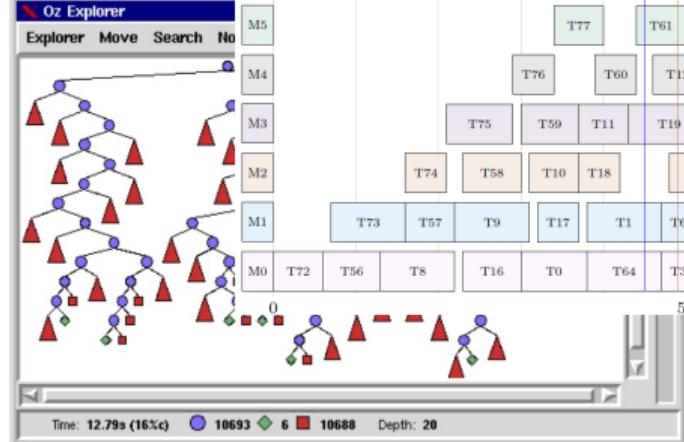
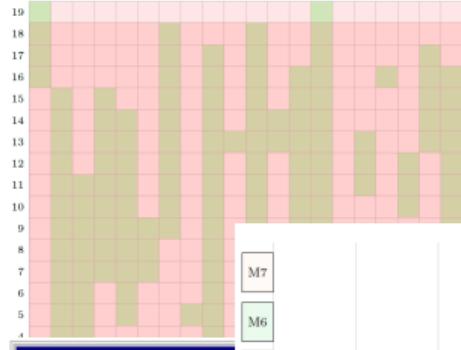
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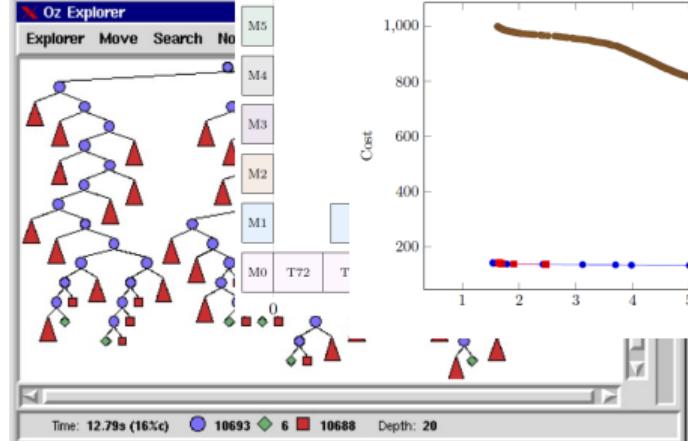
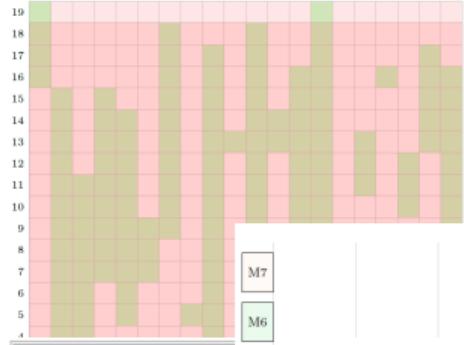
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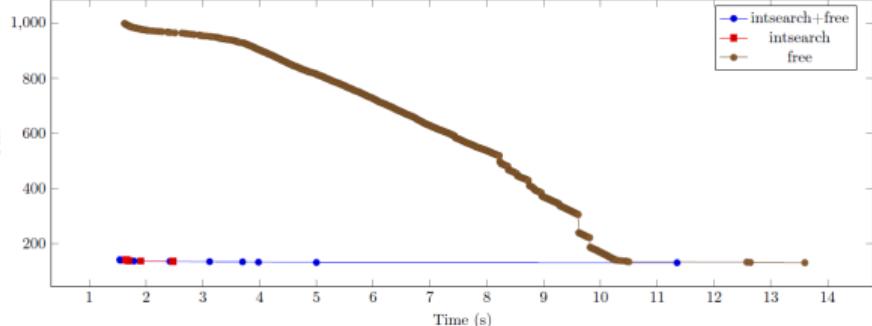
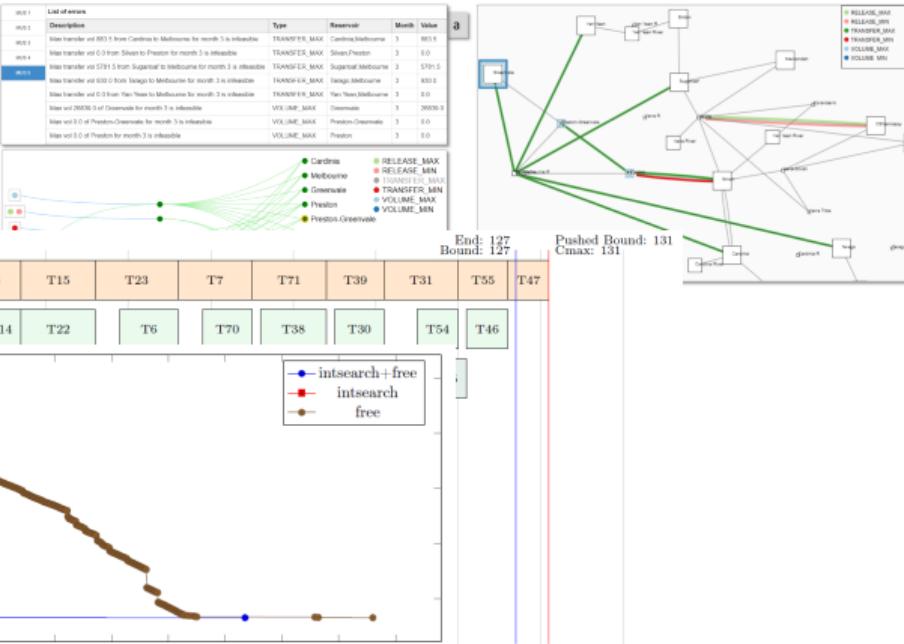
# Lots of pretty pictures, too!



# Lots of pretty pictures, too!



Earliest Start: 45      Actual Start: 49



**Lots of pretty pictures, too!**

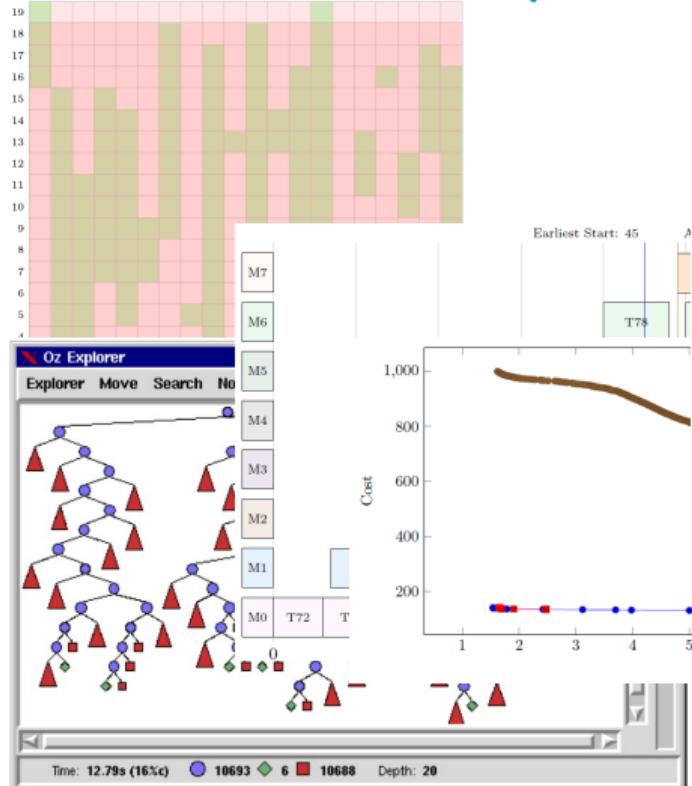
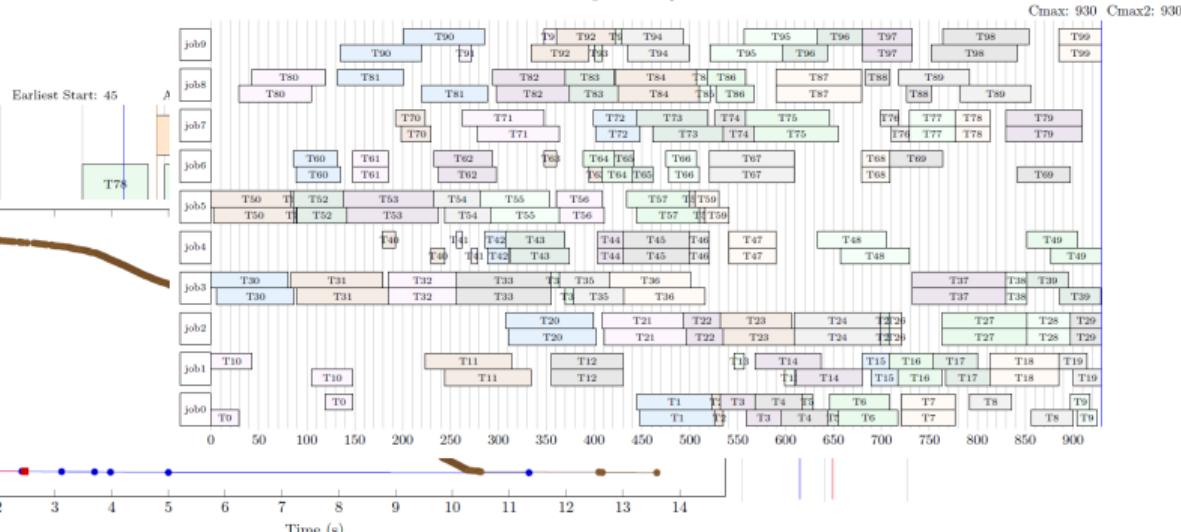


Figure 8: Comparison Job View



# Lots of pretty pictures, too!

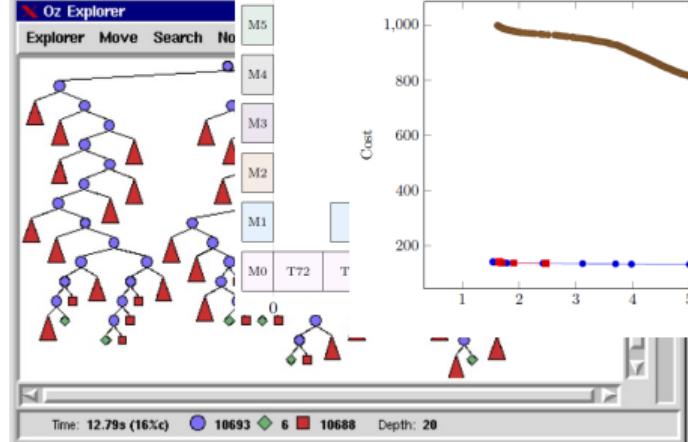
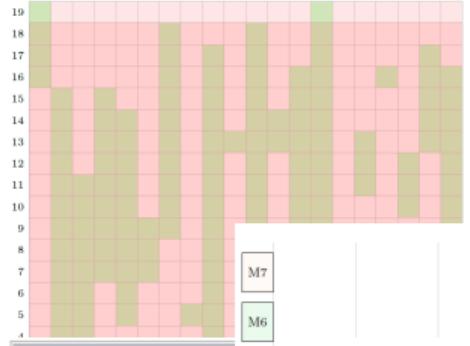
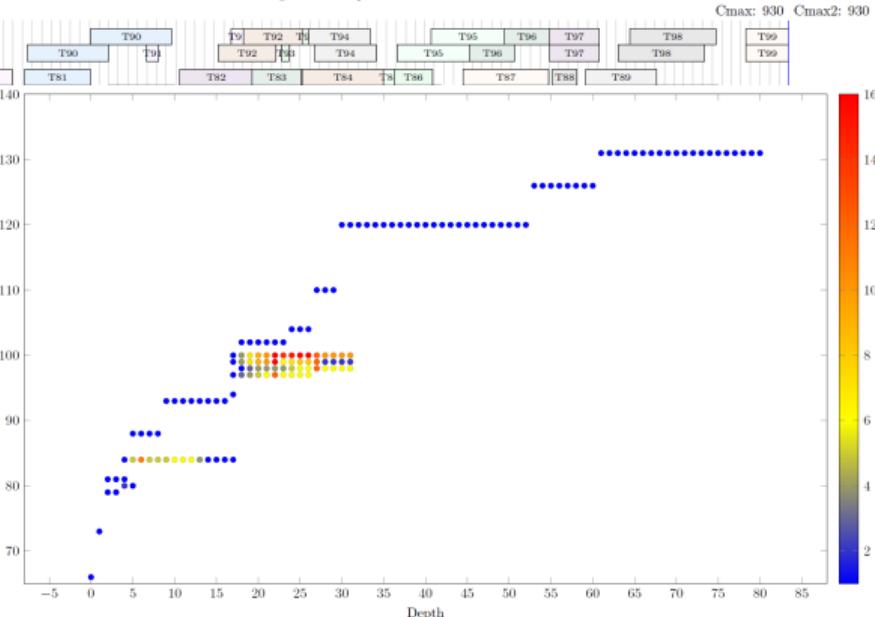
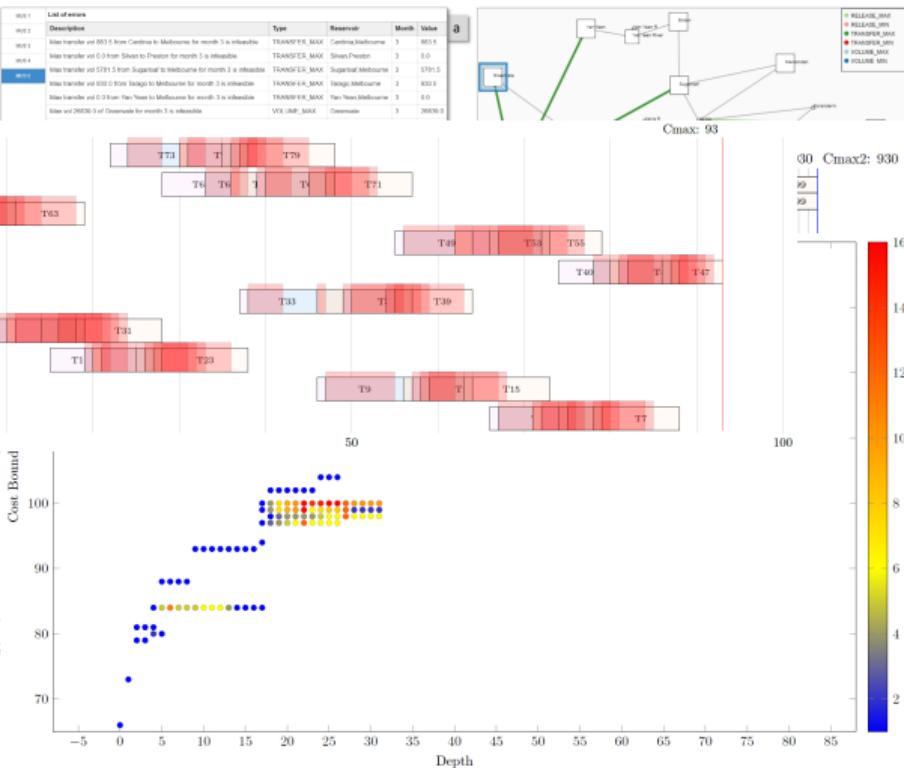
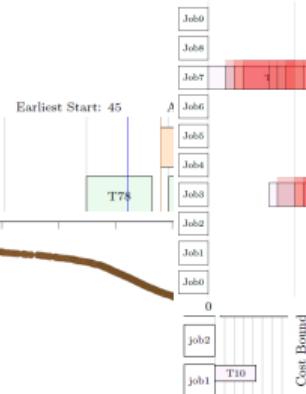
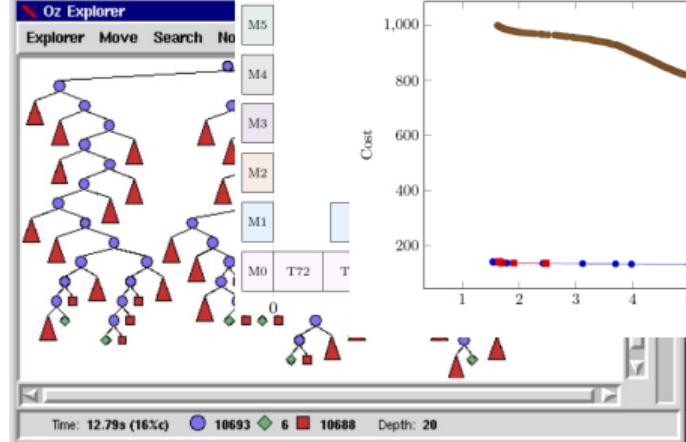
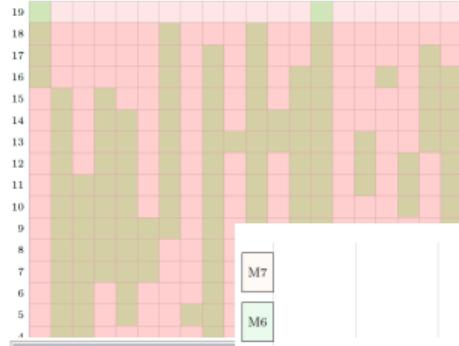


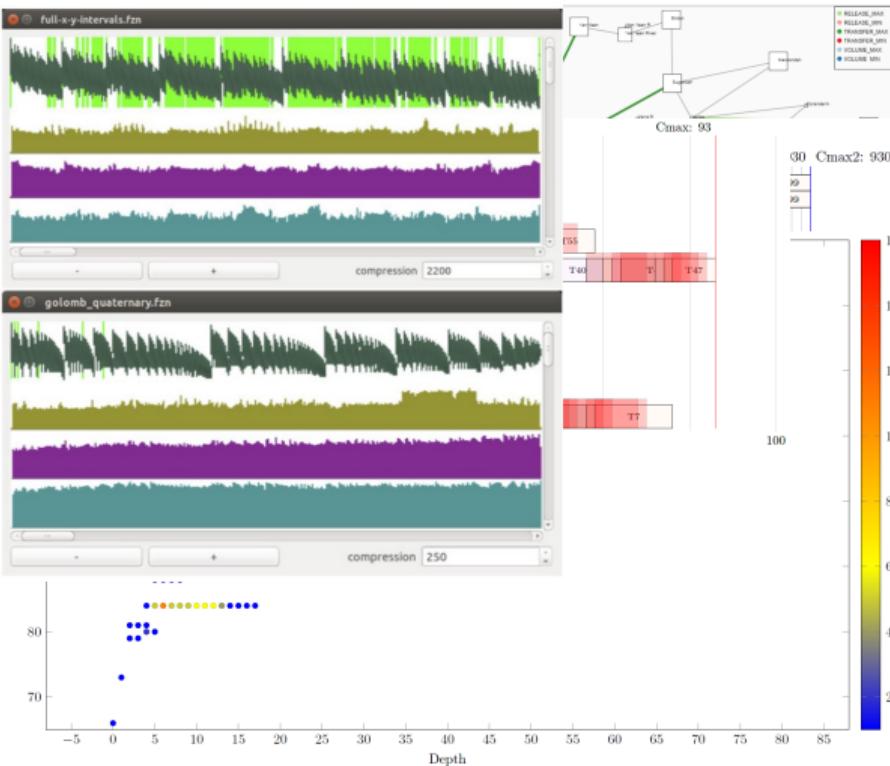
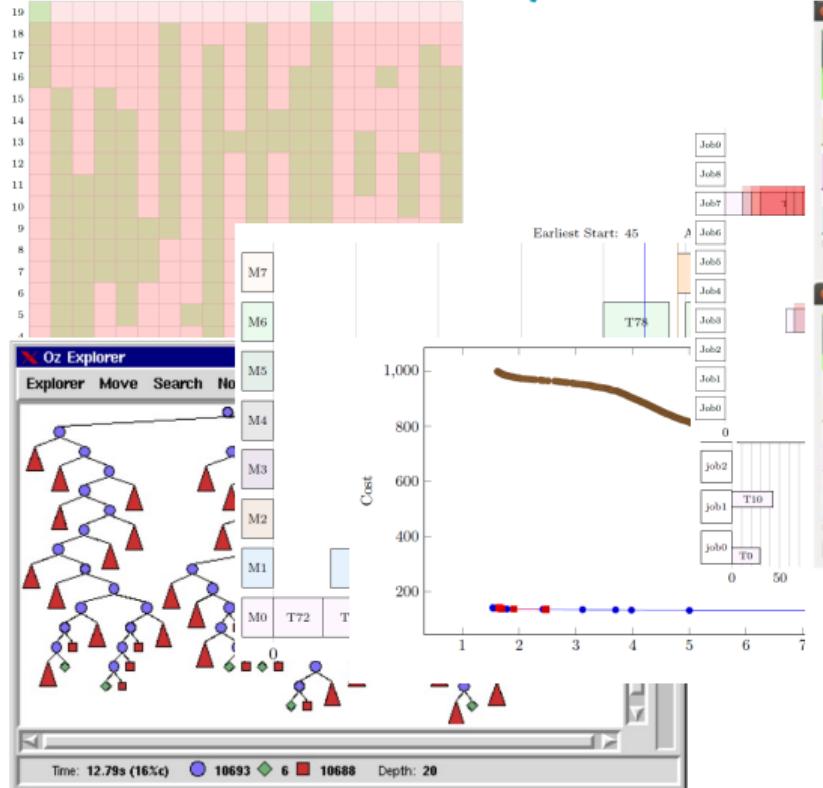
Figure 8: Comparison Job View



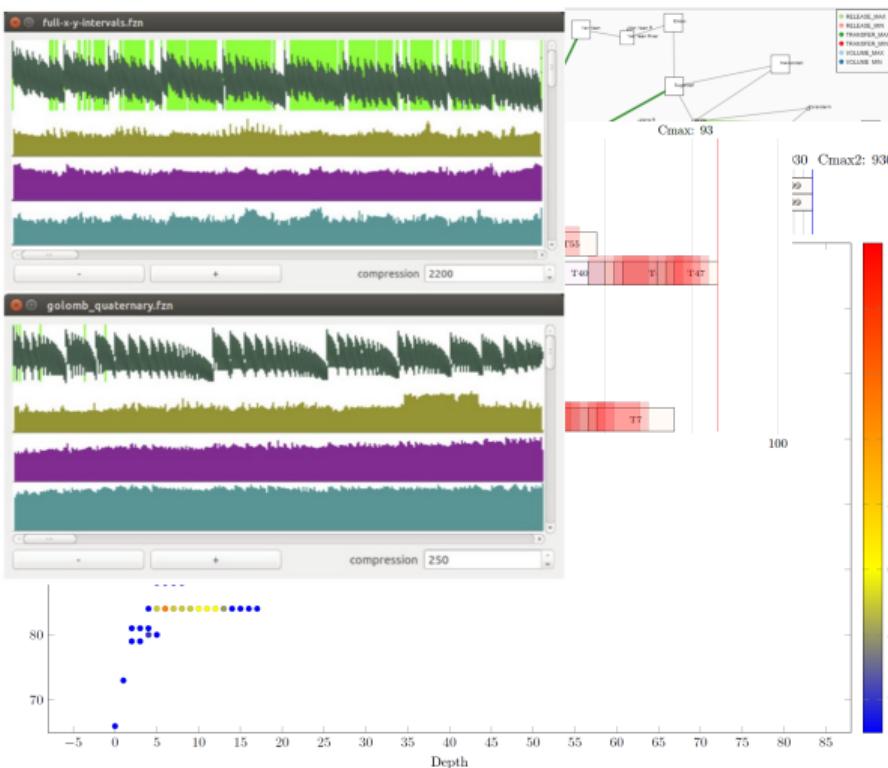
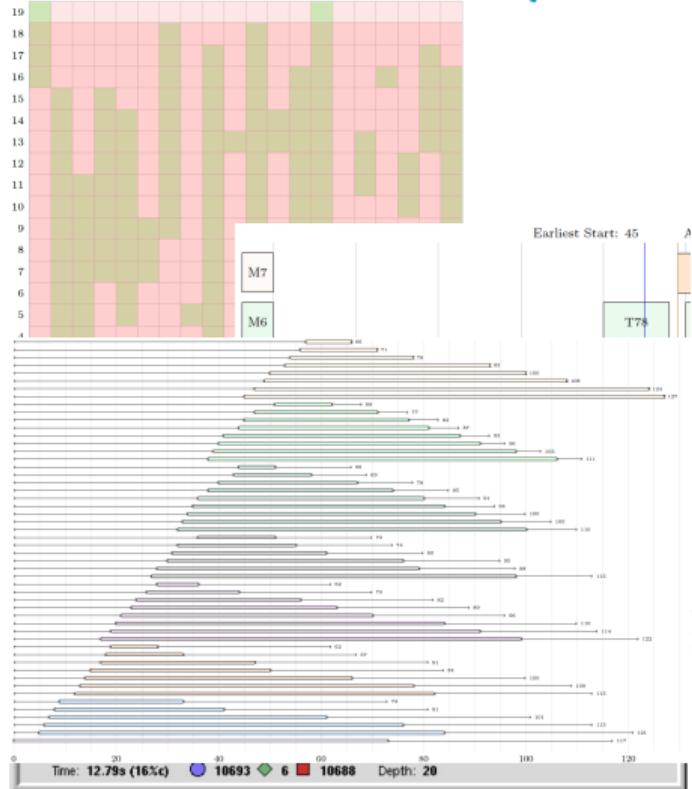
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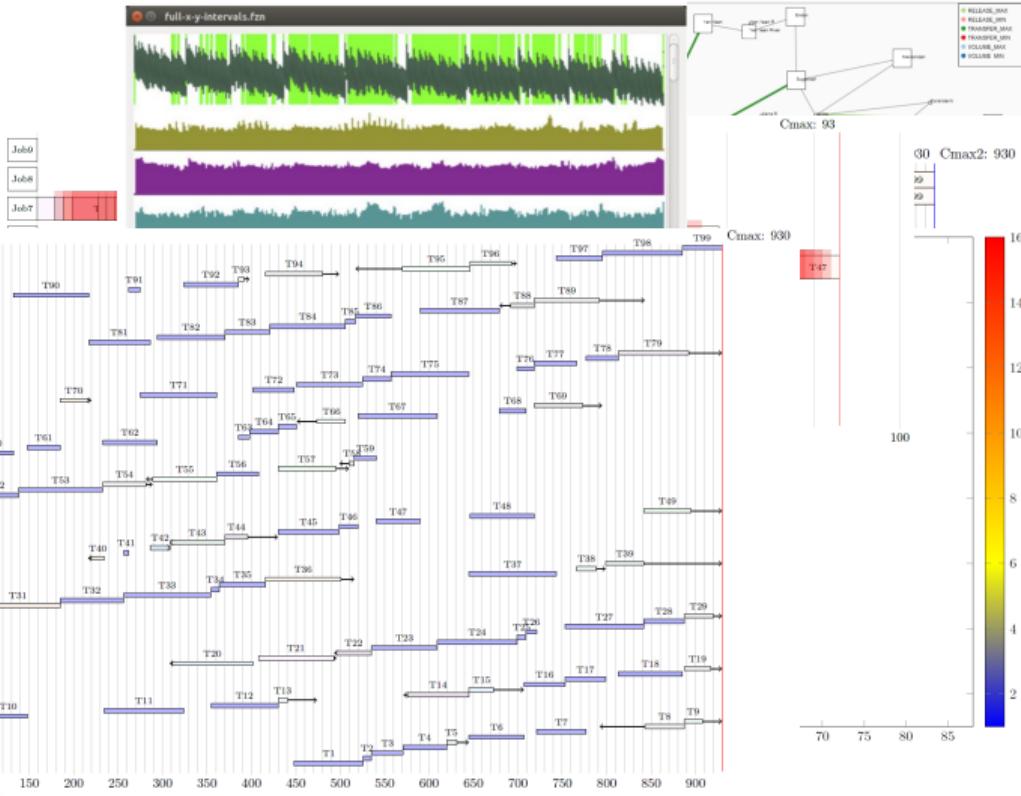
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# Outline

Introduction

Classification of Modelling Problems

Using Visualization to Understand Modelling Issues

- Model is inconsistent

- Solver starts, but does not return result

- No Further Improvement After Initial Solution

- User rejects valid solution

Summary

Bibliography

# Outline

Introduction

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# Background (Simonis)



- Partner in the ASSISTANT (<https://assistant-project.eu/>) H2020 ICT-38 project
- Visualization and Constraint Acquisition are part of WP4 (Scheduling and Production Planning)
- Focused on industrial case studies from Siemens Energy and Atlas Copco (flow-shop variants)

# Background (Tack)

- Monash University, Melbourne, Australia
- MiniZinc (<https://www.minizinc.org>) and Gecode (<https://www.gecode.org>)
- Implemented Gist (search tree visualisation), MiniZinc IDE (includes visualisation API), industry projects

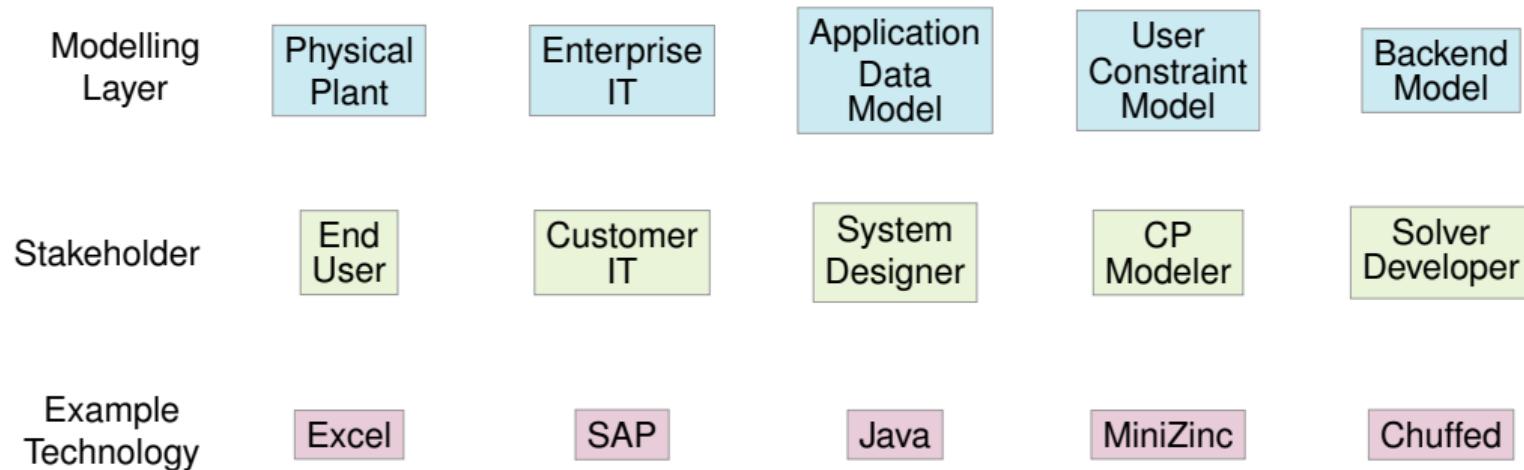
# Full Scale Development Process

- Considering the full process of building a CP based application
- Not just solving a given benchmark problem
- Involves many people, a lot of different tools

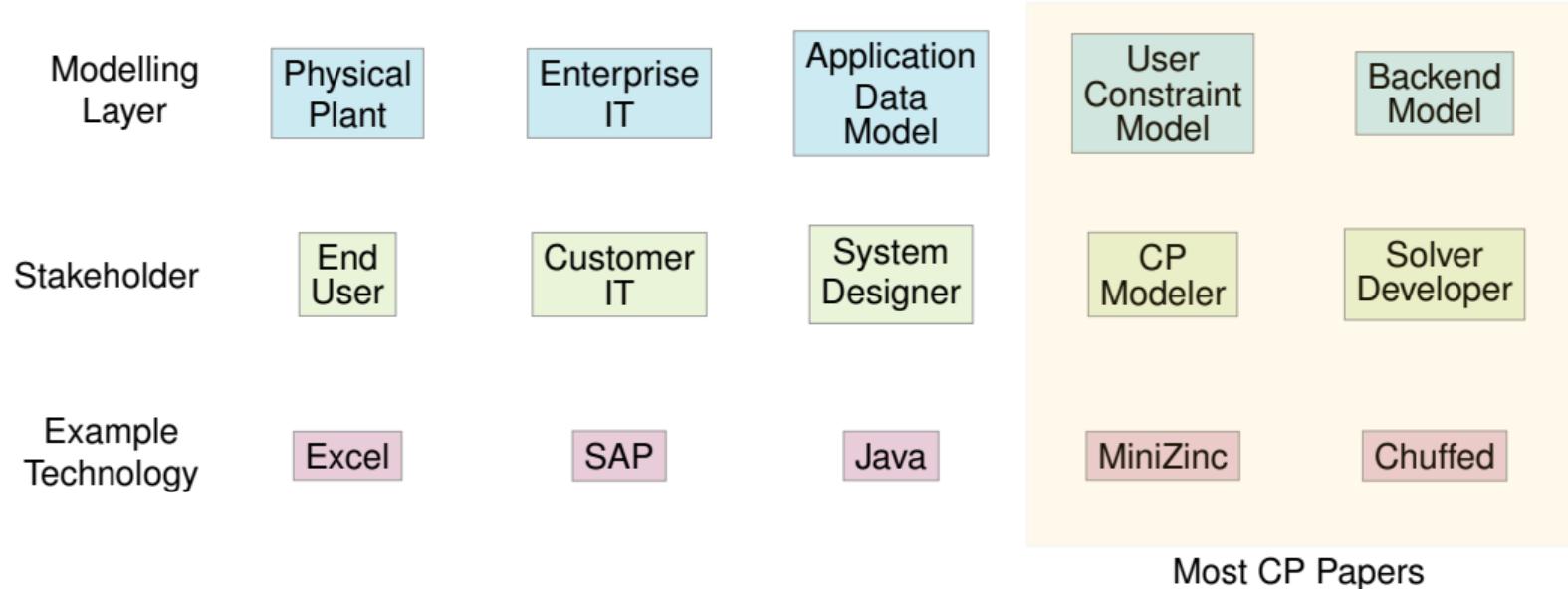
## Stakeholders (One person may have multiple roles)

- Application end user
- Domain expert
- Management
- Customer IT department
- System designer
- CP model developer
- Integration/front-end developer
- Solver developer

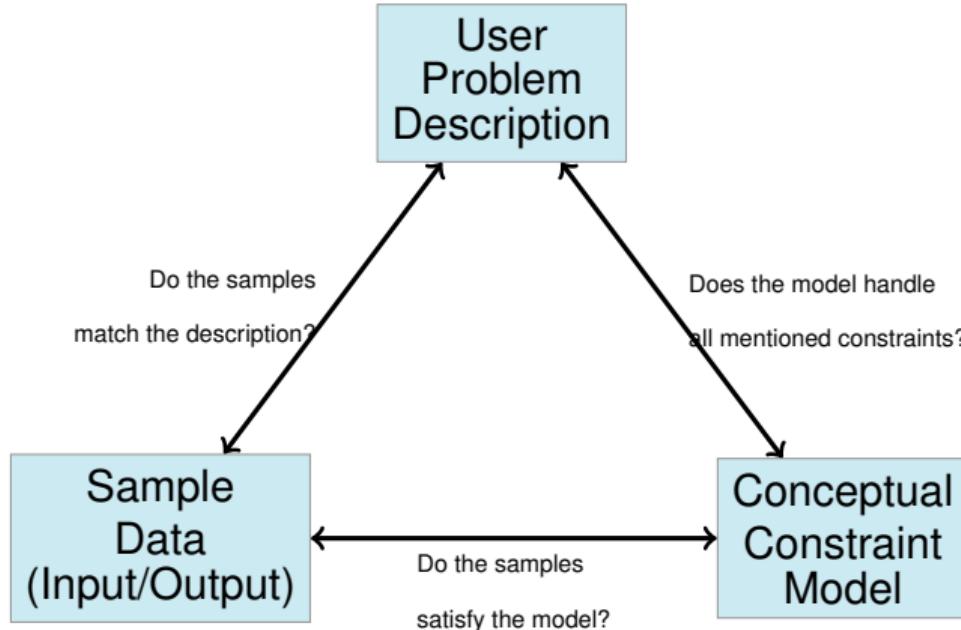
# Layers of Description (Scheduling Example)



# Layers of Description (Academic Focus)



# Triangle of Information



## Roles of Visualization Considered Here

- Help to build model
- Explain results to different stakeholders
- Build confidence in system
- Allow communication between stakeholders
- Present results to management/funding agency/general public

## Focus: Building and Maintaining the Model

- We will focus on using visualization to help develop the model
- For this we consider a problem classification of typical issues arising during development
- Visualization helps with resolving the issues, but does not solve them itself
- We are concentrating on using visualization as a tool for the developer
- Also used when maintaining a working system
  - You may not remember all the details of the model!
  - You should still understand the information in the visualization

## Other Roles of Visualization (not covered in detail)

- Improve the solver itself
- Understand what the solver is doing
- Teaching aid
- Outreach

# Important Distinction

- A generic visualization toolkit (used for multiple problems)
  - May need adaptation/ does not handle full range of problems encountered
  - Available at start of project
  - Reuse of components/improvement across multiple projects
- A problem specific visualization
  - Can be customized to use/handle specific problem features
  - Not available at start of project
  - Development cost can be prohibitive for single project

# Problem Specific: Sudoku Tool [Howell et al., 2018]

The screenshot shows a user interface for a Sudoku tool. At the top, there are buttons for LOAD, UPLOAD, SOLVE, and SUBMIT, followed by a gear icon. Below these are dropdown menus for Columns, Puzzles (set to 473), Const (set to SSGAC), #Clues (set to 22), and #Sols (set to 1). The main area displays a 9x9 Sudoku grid with some pre-filled numbers (e.g., A1=9, B2=3, C3=5, D4=2, E5=7, F6=2, G7=8, H8=7, I9=4) and empty cells. To the left of the grid, there are sections labeled A through I, each containing a 3x3 subgrid. Below the grid, there is a decorative footer with a repeating pattern of small icons.

Board

Domains     Assign Singletons

Puzzles Name	Const	#Clues	#Sols	I
HardestSudokuThread-00000	SSGAC	22	1	
HardestSudokuThread-00028	SSGAC	22	1	
HardestSudokuThread-00075	SSGAC	21	1	
HardestSudokuThread-00226	SSGAC	23	1	
HardestSudokuThread-00361	SSAC	23	1	
HardestSudokuThread-00027	SSGAC	22	1	
HardestSudokuThread-00029	SSGAC	22	1	
HardestSudokuThread-00031	SSGAC	22	1	
HardestSudokuThread-00007	SSGAC	22	1	
HardestSudokuThread-00044	SSGAC	23	1	
HardestSudokuThread-00050	SSGAC	21	1	
HardestSudokuThread-00680	SSAC	23	1	
HardestSudokuThread-00014	SSGAC	22	1	
HardestSudokuThread-00086	BSGAC + SSAC	21	1	
HardestSudokuThread-00063	SSGAC	21	1	
HardestSudokuThread-00010	SSGAC	22	1	
HardestSudokuThread-00055	SSGAC	20	1	
HardestSudokuThread-00020	SSGAC	22	1	
HardestSudokuThread-00026	SSGAC	22	1	
HardestSudokuThread-00216	SSGAC	23	1	
HardestSudokuThread-00219	SSGAC	23	1	
HardestSudokuThread-00225	SSGAC	22	1	
HardestSudokuThread-00358	SSAC	23	1	
HardestSudokuThread-00222	SSGAC	23	1	
HardestSudokuThread-00013	SSGAC	22	1	
HardestSudokuThread-00041	SSGAC	21	1	
HardestSudokuThread-00060	SSGAC	21	1	
HardestSudokuThread-00025	SSGAC	22	1	
HardestSudokuThread-00068	BSGAC + SSAC	21	1	
HardestSudokuThread-00044	SSGAC	21	1	
HardestSudokuThread-00245	SSGAC	22	1	
HardestSudokuThread-00072	SSAC	21	1	
HardestSudokuThread-00073	SSGAC	21	1	
HardestSudokuThread-00249	SSGAC	22	1	

- User control of solving process
- Understand the impact of consistency techniques
- Very much based on specific problem structure

## Generic Tool: CP-Viz [Simonis et al., 2010]

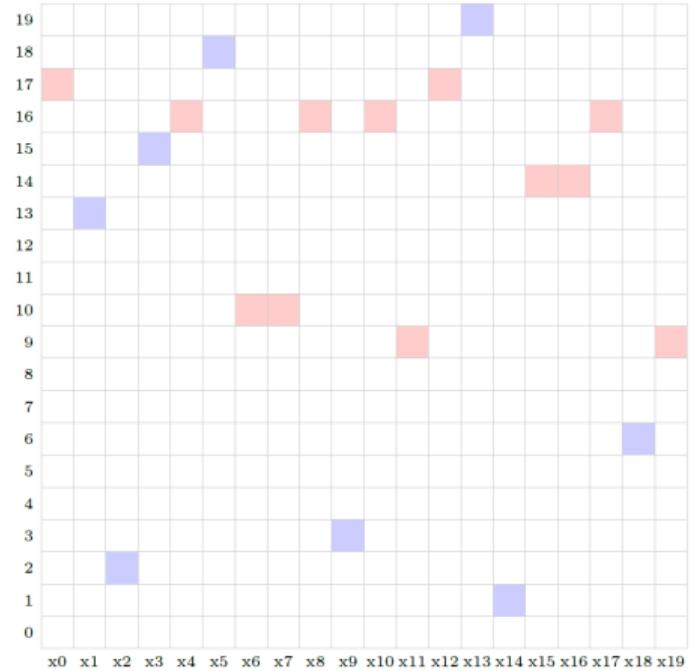
- Less interaction/user control
  - Available from start of development
  - Integrated into report/slides generation

## Common to Both Examples

- Emphasis on solution process, not on solution
- Understand how CP solves the problem
- Ways of configuring solver to change process
- Not focussed on
  - Modelling the problem
  - Checking that the solution is correct

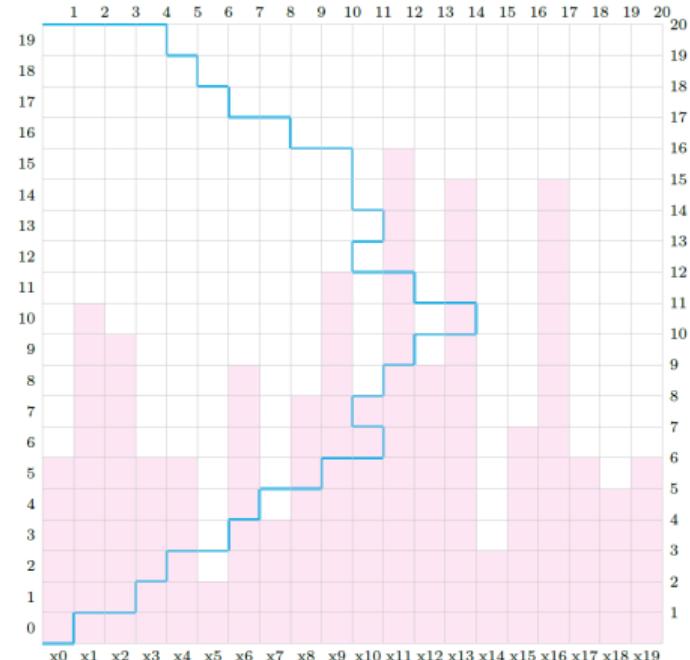
# Fundamental Types of Visualization for CP

- Check an assignment
  - May or may not satisfy constraint



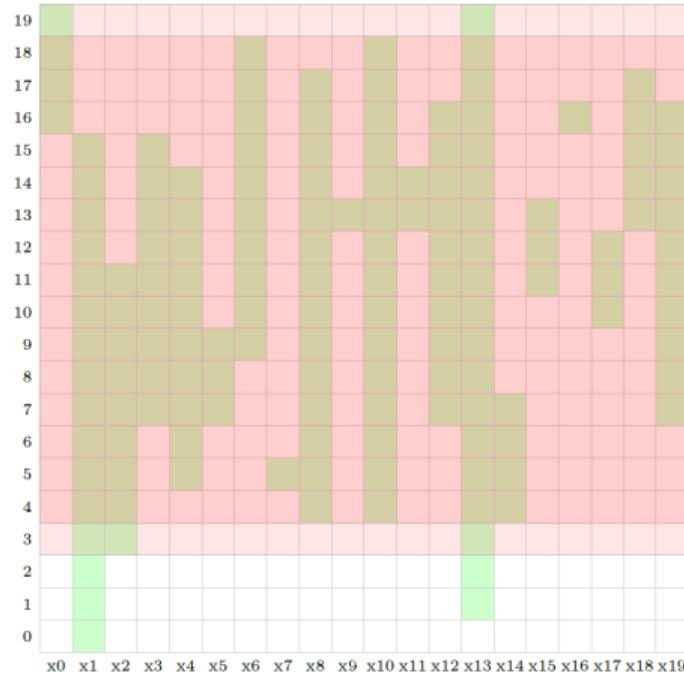
# Fundamental Types of Visualization for CP

- Check an assignment
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- Capacity view
  - How tight is the constraint, no assignment needed



# Fundamental Types of Visualization for CP

- Check an assignment
    - May or may not satisfy constraint
  - Capacity view
    - How tight is the constraint,  
no assignment needed
  - Explain failure
    - No assignment,  
constraint cannot be satisfied



# Fundamental Types of Visualization for CP

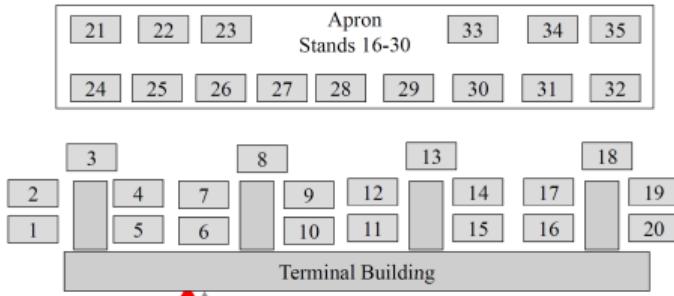
- Check an assignment
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- Capacity view
  - How tight is the constraint,  
no assignment needed
- Explain failure
  - No assignment,  
constraint cannot be satisfied
- Explain progress during search
  - Partial assignment, show propagation

	0	1	2	3	4	5	6	7	8	9
S										
E										
N										
D										
M			*							
O										
R										
Y										

from: HS. ECLiPSe ELearning Course

# Fundamental Types of Visualization for CP

- Check an assignment
  - May or may not satisfy constraint
- Capacity view
  - How tight is the constraint,  
no assignment needed
- Explain failure
  - No assignment,  
constraint cannot be satisfied
- Explain progress during search
  - Partial assignment, show propagation
- Show solution
  - Often application specific



from: HS. Standard Models for Finite Domain Constraint Modelling, PACT 97

## Used at Different Stages

**Assignment Checker** check manual or external solutions

**Capacity View** check for input data consistency

**Failure Explanation** model setup failed

**Propagation View** during search, detailed understanding

- Often too much information

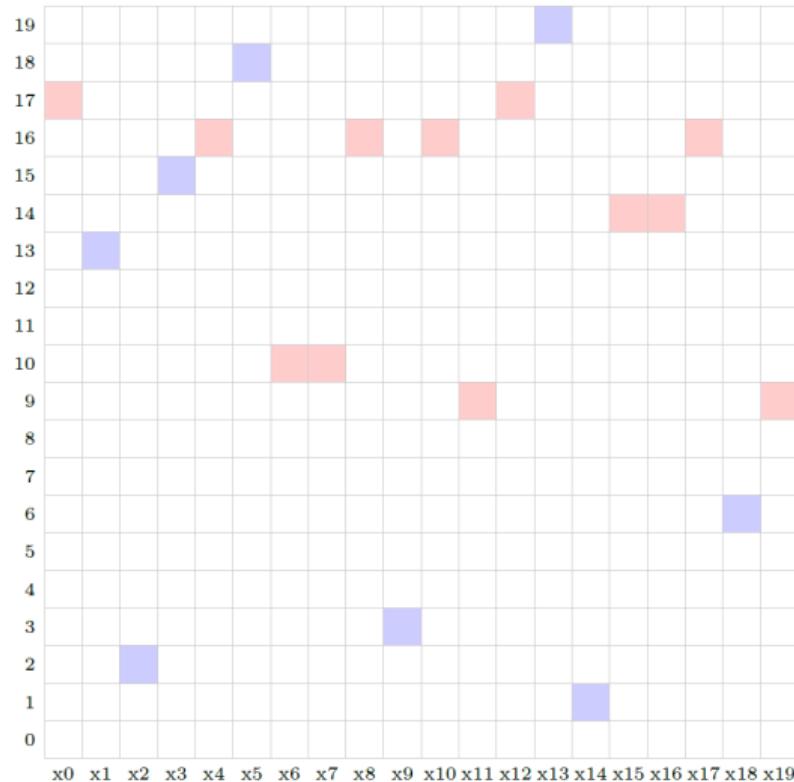
**Solution View** displaying results for end-user

- Often useful to translate into end-user concepts

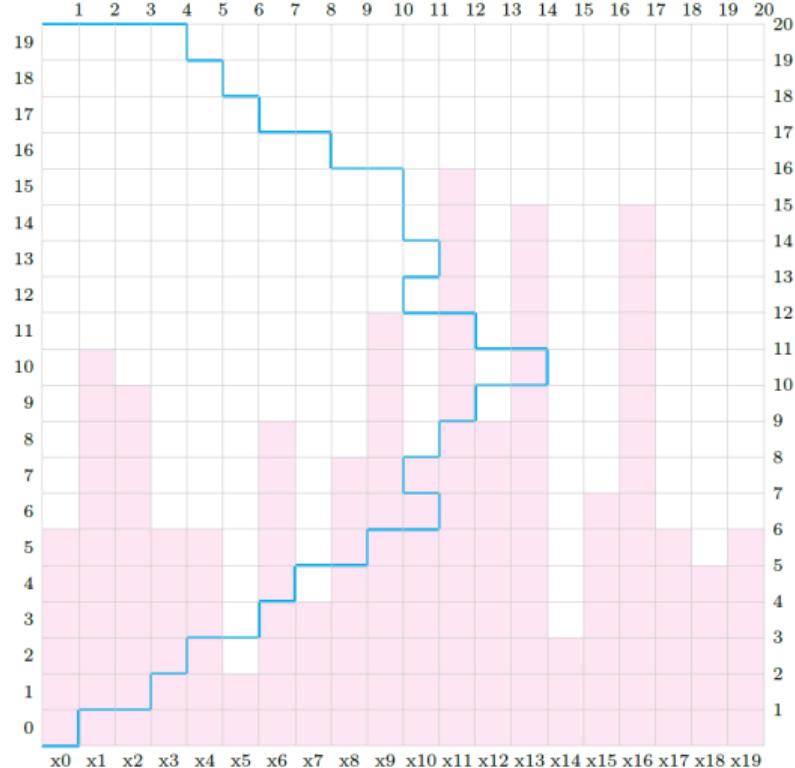
## Example: Alldifferent Visualizations

- We use the same representation as variable/value matrix
  - Columns: variables
  - Rows: values
- Other visualization forms possible (vector, bi-partite graph)
- Different visualizations use different APIs
  - Values only
  - Domain bounds/explicit domains
  - Requires information about methods used in solver (domain/bound consistency)

# AllDifferent: Inconsistent Assignment

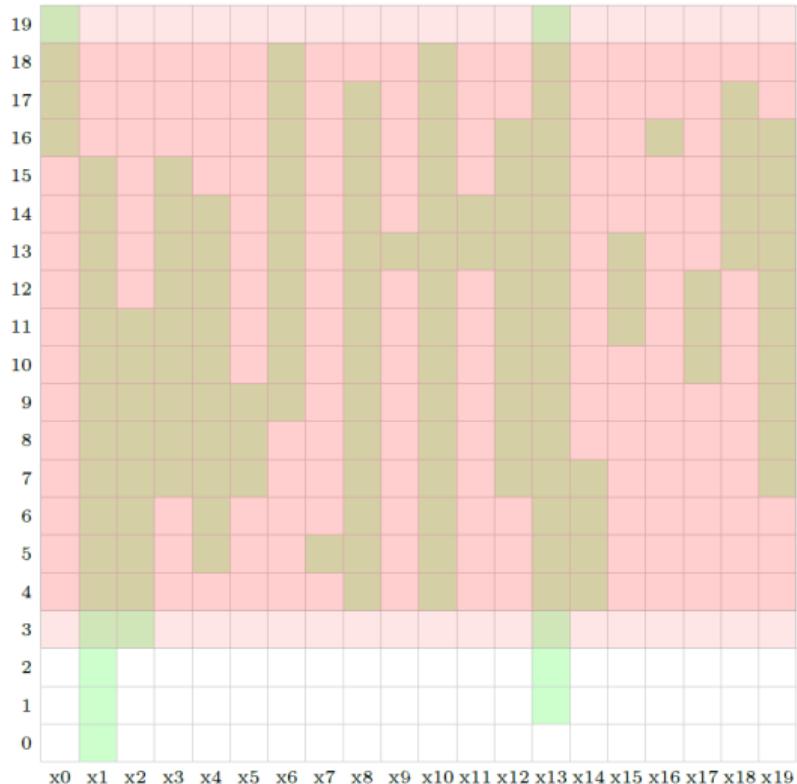


# AllDifferent: Capacity View



- Show many values are in the domain of each variable
- How many variables contain a value in their domain
- Highlights potential bottlenecks

# AllDifferent: Explaining Infeasibility



- Show value range that contains too many competing variables (bound consistency)
- All or only one explanation?
- Largest or smallest infeasible set?
- Dual explanation possible

# Outline

Introduction

**Classification of Modelling Problems**

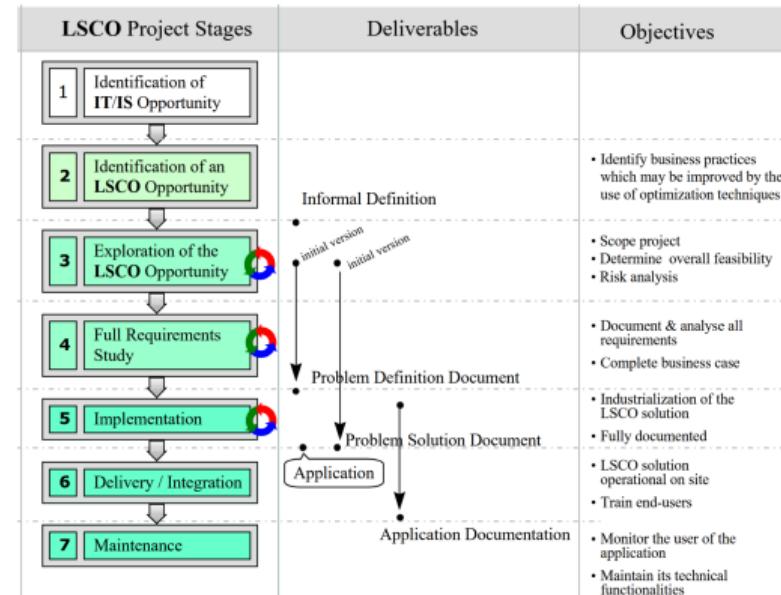
Using Visualization to Understand Modelling Issues

Summary

Bibliography

# Development Process

- Surprisingly little work describing how to develop and maintain models
- Foundation: European CHIC-2 project [Gervet, 1998]
- Not aware of papers in the last 10 years
- Most training material focused on
  - How to use a specific system
  - Explaining the principles behind CP
- Problems occur/re-occur at different points in project timeline



# Classifications of Problems with Models

1. It does not compile
2. A known solution is not accepted by the model (covered in slides)
3. *The model is inconsistent and rejected at startup* (covered in presentation)
4. *The model is inconsistent and fails after some search*
5. *The solver starts, but does not finish, returning neither yes or no*
6. A solution is found, but the user rejects it, because it violates some constraint that was not discussed before
7. *An initial solution is found, but no further improvements are found in a reasonable time*

## Classifications of Problems with Models (II)

8. The model has been working for some time,  
but suddenly it stops finding a solution, or violates a constraint
9. When the best solution is shown to the users,  
they immediately find a change that improves the quality
10. The solver says it is optimal, but there is a better solution
11. *The solver finds a solution, which satisfies all constraints, the user accepts it as correct, but doesn't like it, and wants a different one*

## Comments

- For some solvers, certain problems are not differentiated
  - MiniZinc: (3) and (4) look the same, as there is no separate consistency check at root node
  - Choco: You can ask to propagate constraints after setup, to separate (3) and (4)
  - Prolog based: Individual constraint may fail at posting, allowing fine grain analysis of failure (3)
- Access to internal state during search may not be possible/is difficult
- Reason for failure of constraint rarely provided

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## Using Visualization to Understand Modelling Issues

Model is inconsistent

Solver starts, but does not return result

No Further Improvement After Initial Solution

User rejects valid solution

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# A known solution is not accepted by the current model

- This can have different reasons
  - The data models do not match
  - A constraint stated was misunderstood
  - A constraint is soft, not hard
  - The known solution is not the plan, but the actual operation
- Role of visualization

## Running Example: Scheduling Problem

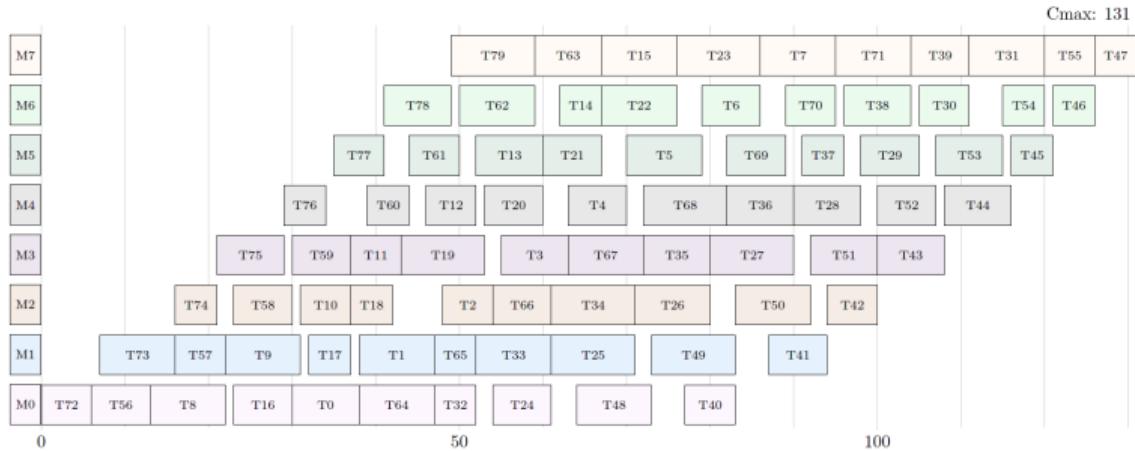
- Flowshop type problem
- Jobs consist of series of tasks processed on machines in same order
- Precedence between tasks of same job
- Disjunctive machines
- Tasks require operator during first part of run
  - Overall cumulative manpower limit

# Conceptual Model (MinZinc)

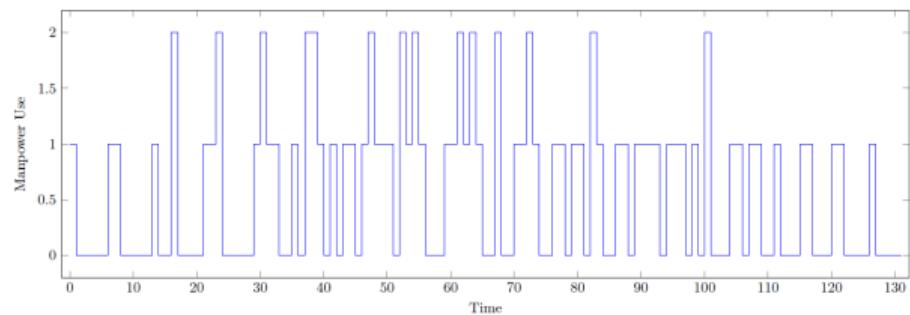
```
1 % variables
2 array[T] of var 0..ub:start;
3 var lb..ub:cmax;
4
5 % constraints
6 constraint forall (i in T)
7     (cmax >= start[i]+duration[i]);
8 constraint forall (p in P)
9     (start[prec[p,2]] >= start[prec[p,1]]+
10      duration[prec[p,1]]));
11 constraint forall(s in S)
12     (cumulative([start[i]|i in T where stage[i]=s],
13                 [duration[i]|i in T where stage[i] = s],
14                 [1|i in T where stage[i] = s],1));
15 constraint
16     cumulative([start[i]|i in T],
17                 [manpowerDuration[i]|i in T],
18                 [1|i in T],manpower);
19
20 solve minimize cmax;
```

- Data definition not shown for brevity

# Example Solution (10 Jobs/8 Machines)



- Typical visualization: Gantt Chart
- Machine view
- Other views available



- Typical visualization: Resource Profile
- Show operator use over time

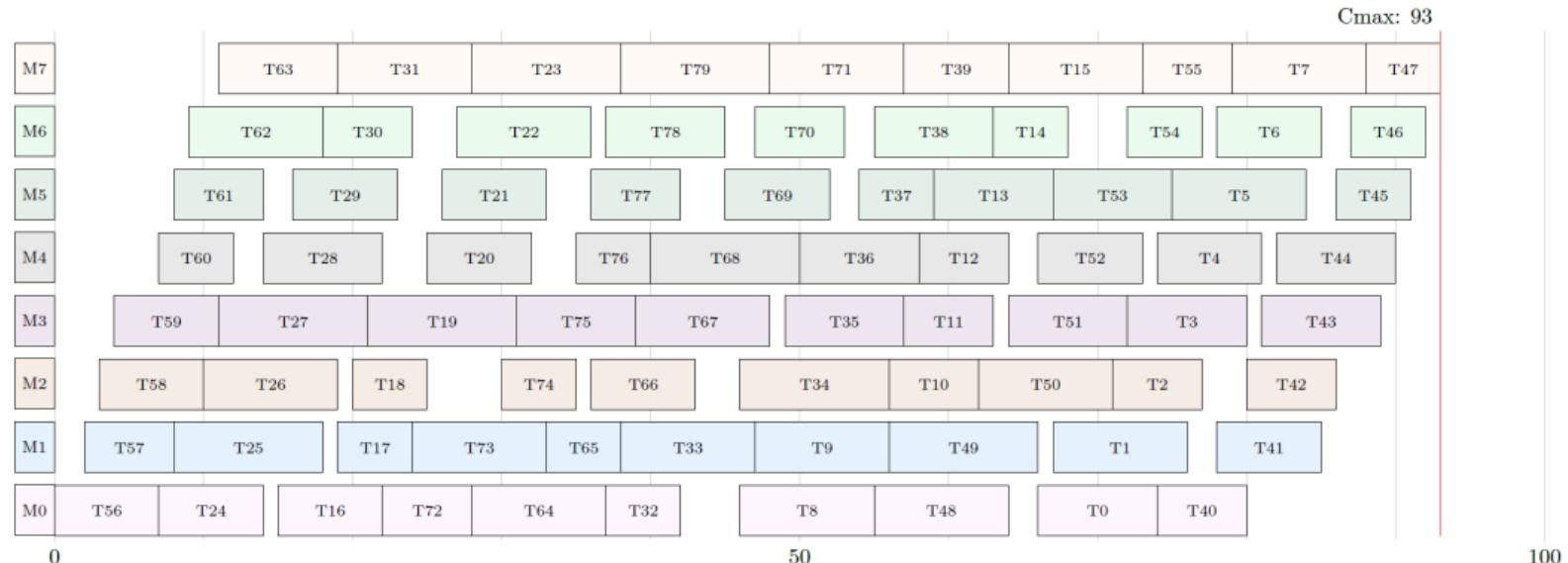
## Conceptual Models do not Match

- Different concept of time
  - Calendar time
  - Working time
- Rework tasks (added tasks)
- Skipped tasks for some jobs
- Preemption allowed
- Task stretching over downtime

# A Constraint was Misunderstood

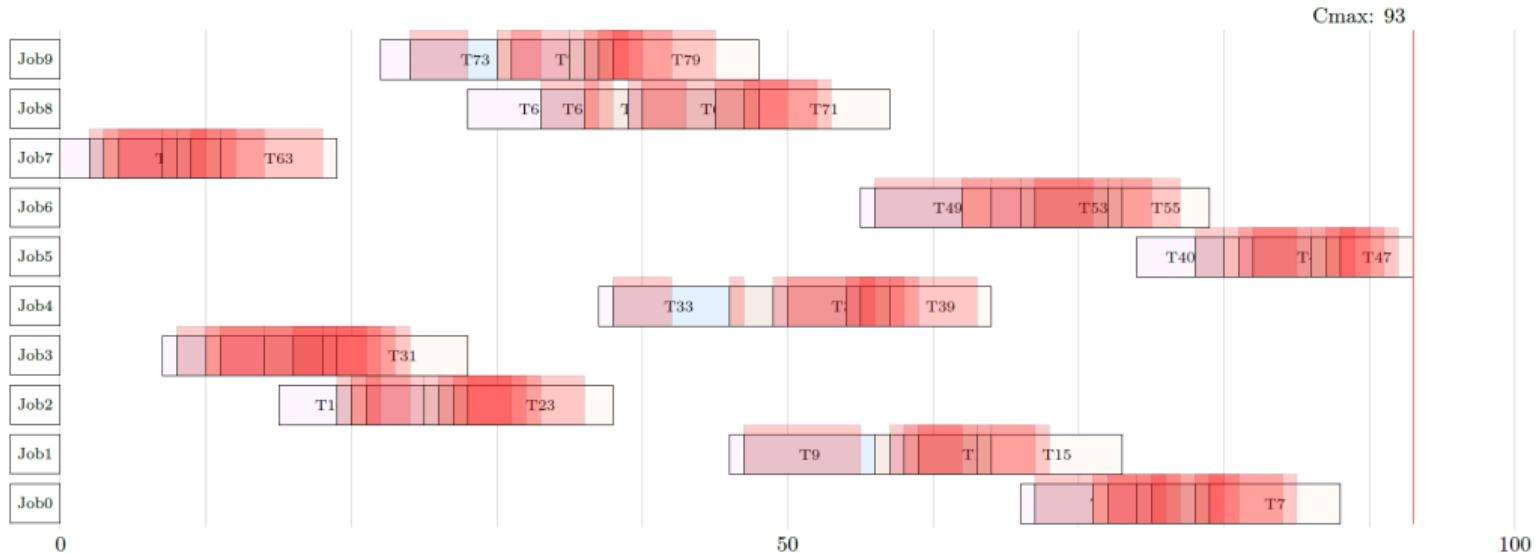
- Very rare that a formal description of constraints is given
- In most cases, problem derived from interviews with domain experts/end users
- Language barrier:
  - CP Modeller does not (yet) speak language of application domain
  - End users are not familiar with CP/optimization
- Example in Manufacturing:
  - Pipelined production, not strict end-to-start precedence

# Modified Problem: Gantt Chart with Pipelining (Machine View)



- Nothing to see here, we need other views to identify problems

# Modified Problem: Gantt Chart with Pipelining (Job View)



- Display in job view highlights numerous task overlaps
- Shades of red show how many tasks are active at the same time
- Uses semi-opaque overlay to indicate problems

# Modified Problem: Gantt Chart with Pipelining (Task View)



- Task view separates tasks, shows pipelining of production steps
- Next task of job can start as soon as some items of previous task are completed
- Must run until last items of previous tasks are finished

# Role of Visualization

- Check that conceptual model matches sample data
- Quickly highlight conflicts in sample data
- Visual Checker
  - Does not replace automatic checks on data or solutions
  - If nobody checks the visualization, no alarm is raised
- Understand problems with units
  - Time resolution
  - Stock levels/consumption

# The model is inconsistent and rejected at startup

- Two main concepts:
- Find minimal correction sets (MCS) / minimal unsatisfiable set (MUS)
  - Each MUS (there can be many) explains why the model does not provide a solution
  - Each MCS (there can be many) explains how the model can be made satisfiable
  - Overview on explanations in CP during PTHG21 workshop  
<http://www.cs.ucc.ie/~bg6/data/pthg2021.pdf> [Gupta et al., 2021]
- Explain unsatisfiable global constraints
  - Even if you know that a single global constraint is unsatisfiable, you need to understand why.
  - This is not trivial

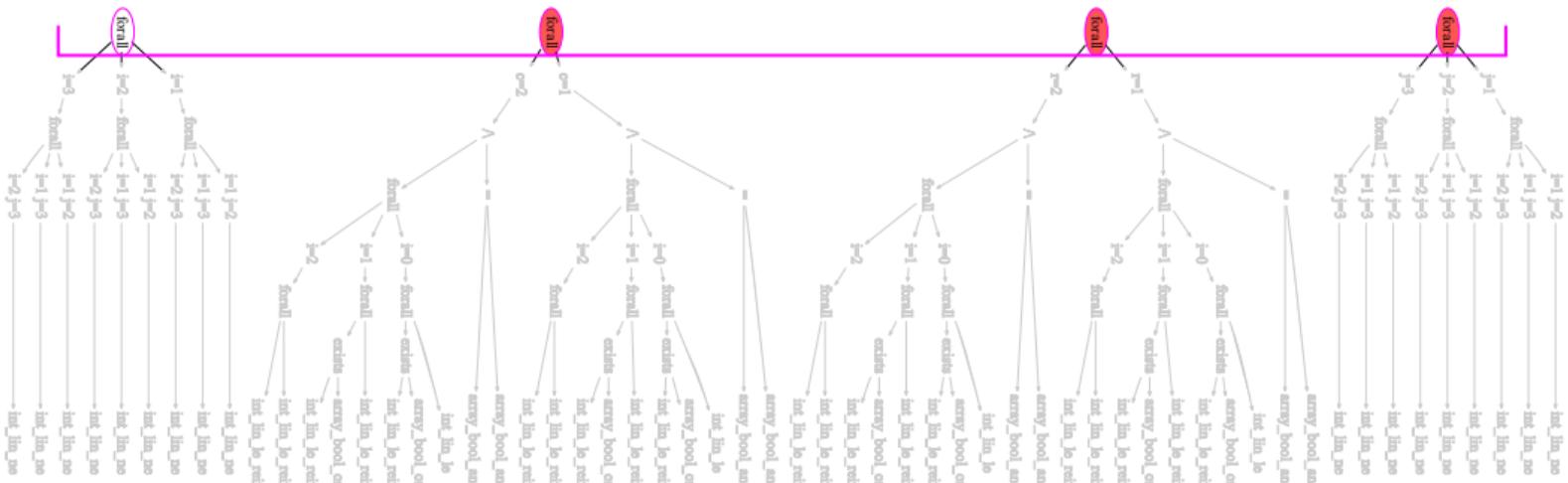
# Finding MCS/MUS

- Algorithms typically use *search*
- Enable/disable constraints systematically
- Problem: how to interpret results?
- Need to understand MCS/MUS at the level of the conceptual model, not the solver-level model

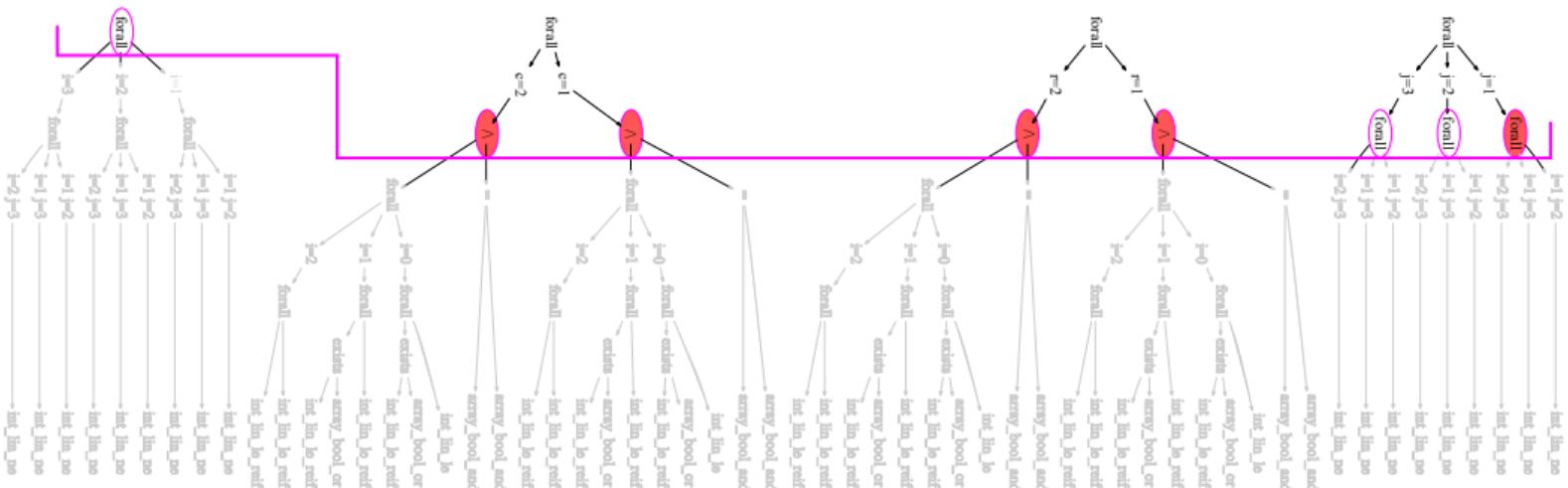
## Example: Latin Squares MUS

(This slide is a placeholder for a live demo in the presentation)

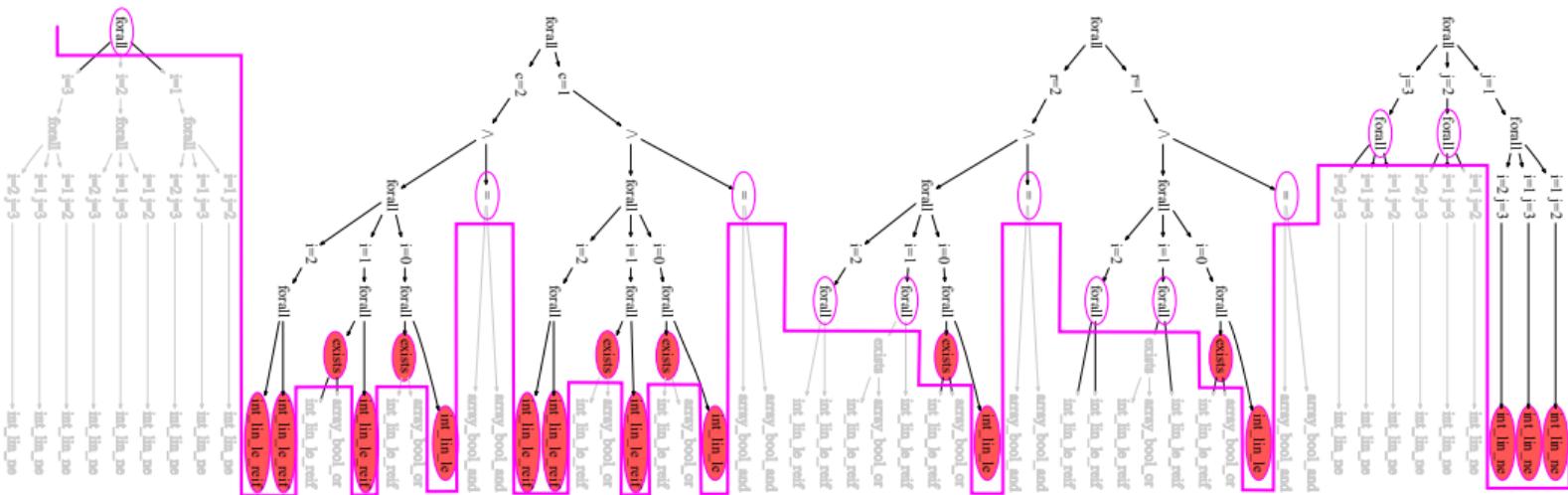
# Visualising MUS in terms of the model



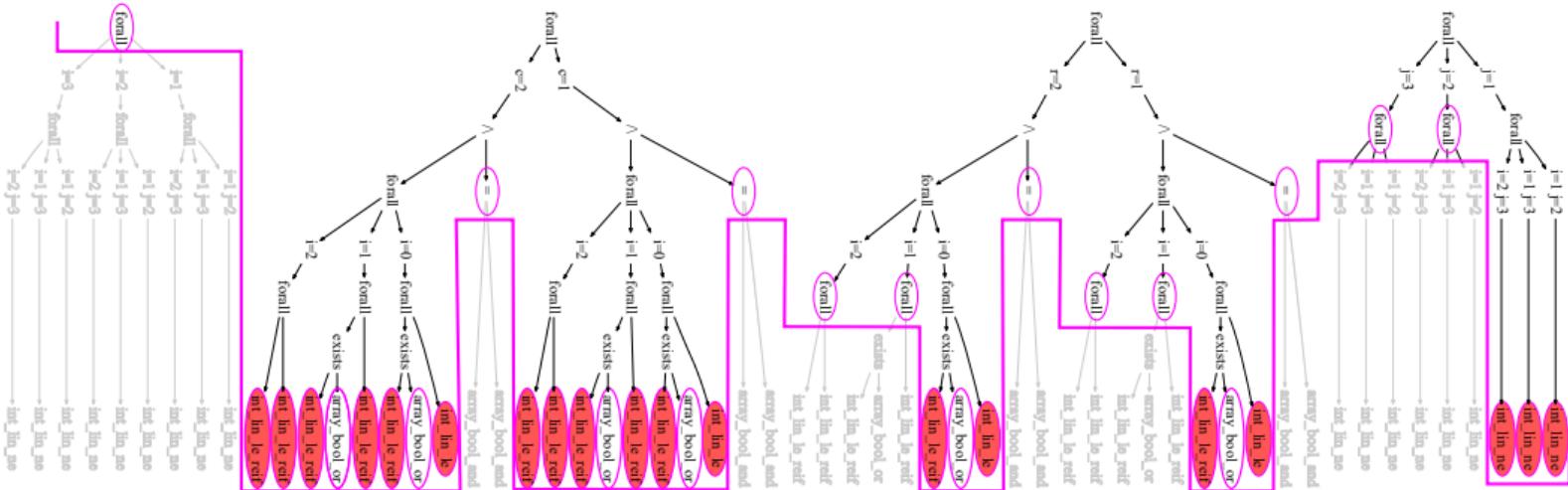
## Visualising MUS in terms of the model



## Visualising MUS in terms of the model

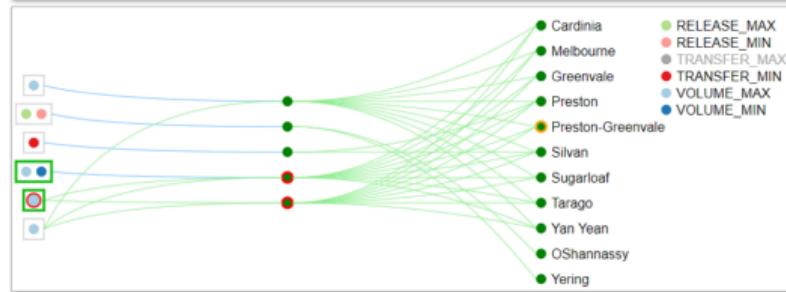


# Visualising MUS in terms of the model



# Example: Visual MUS exploration I [Senthooran et al., 2021]

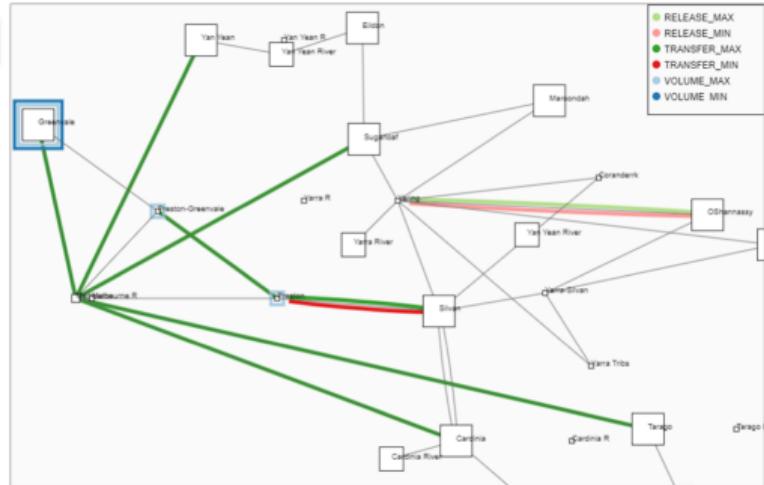
MUS 1	List of errors				
MUS 2	Description	Type	Reservoir	Month	Value
MUS 3	Max transfer vol 883.5 from Cardinia to Melbourne for month 3 is infeasible	TRANSFER_MAX	Cardinia,Melbourne	3	883.5
MUS 4	Max transfer vol 0.0 from Silvan to Preston for month 3 is infeasible	TRANSFER_MAX	Silvan,Preston	3	0.0
MUS 5	Max transfer vol 5781.5 from Sugarloaf to Melbourne for month 3 is infeasible	TRANSFER_MAX	Sugarloaf,Melbourne	3	5781.5
	Max transfer vol 930.0 from Tarago to Melbourne for month 3 is infeasible	TRANSFER_MAX	Tarago,Melbourne	3	930.0
	Max transfer vol 0.0 from Yan Yean to Melbourne for month 3 is infeasible	TRANSFER_MAX	Yan Yean,Melbourne	3	0.0
	Max vol 26839.0 of Greenvale for month 3 is infeasible	VOLUME_MAX	Greenvale	3	26839.0
	Max vol 0.0 of Preston-Greenvale for month 3 is infeasible	VOLUME_MAX	Preston-Greenvale	3	0.0
	Max vol 0.0 of Preston for month 3 is infeasible	VOLUME_MAX	Preston	3	0.0



a

b

c

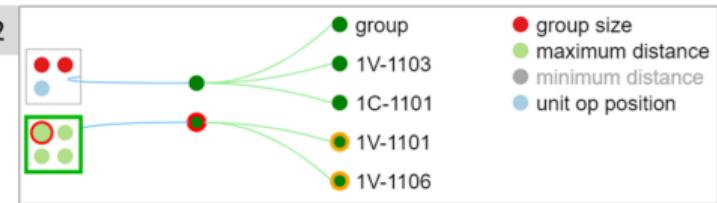


# Example: Visual MUS exploration II [Senthooran et al., 2021]

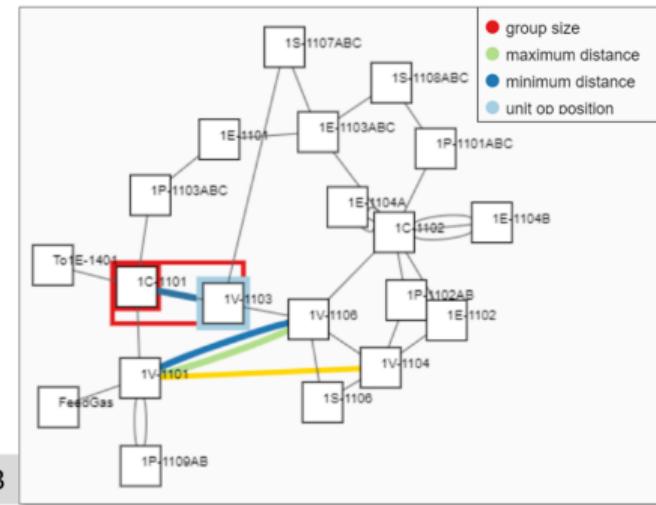
1

Set 1		List of errors				
Set 2	Description	Type	Unit Operations	Axis	Value	Fix Manually
	size of group group in X direction is insufficient	group size	group	X	9000	Select Group
	size of group group in Y direction is insufficient	group size	group	Y	9000	Select Group
	minimum distance 1500 between unit ops 1V-1103 and 1C-1101 in X+ direction is	minimum distance	1V-1103, 1C-1101	X+	1500	Distance Editor

2



3



# Explaining Failure of Single Global Constraints

## [Simonis et al., 2000]

- Approach 1: Integrate (parts of) constraint filtering into visualization
  - Example: Detect Hall sets for alldifferent, as shown above
- Approach 2: Provide debugging information about failure
  - Example: CHIP propagation events, oaDymPac trace format
- Approach 3: Use generic (SAT-based) methods to describe failures
  - Research challenge: How to present internal problem representation and learned clauses back to the user

## The model is inconsistent and fails after some search

- Constraints are not strong enough to detect infeasibility
- Good chance of using stronger lower bounds
- MUS based methods work, but can be slow

## The solver starts, but does not return yes or no

- The model may well be correct, but the search does not work
- The model is too tight, but the search space is too large to detect infeasibility
- Relaxing some resource limits should allow to find solutions
- Understanding what is happening requires some access to solver internals

# Why does the search not find a solution?

- Two aims of analysis
  - Understand what is happening
  - Suggest change to improve behaviour

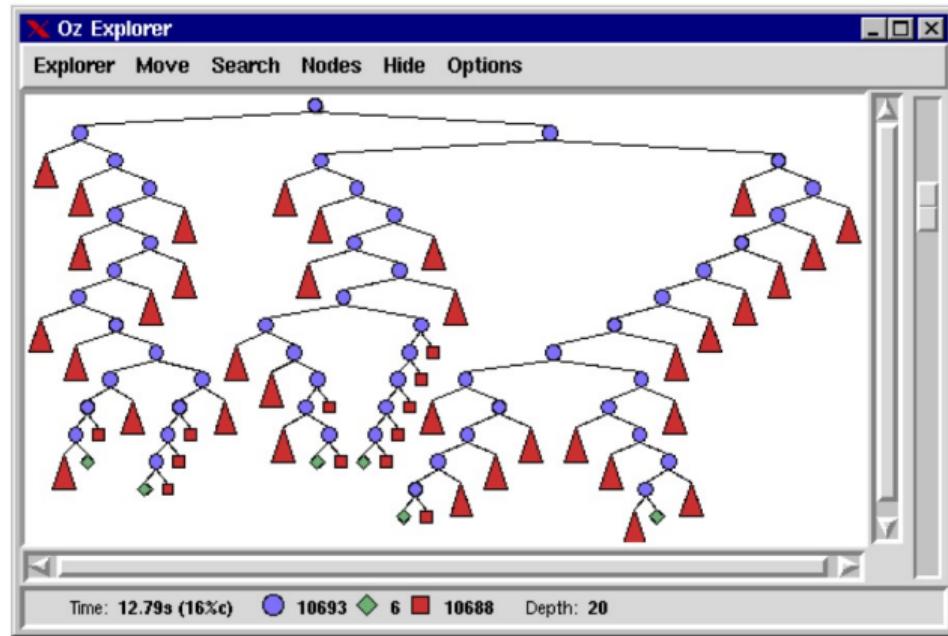
# Visualizations

- Search tree display
- Search depth display
- Heatmap

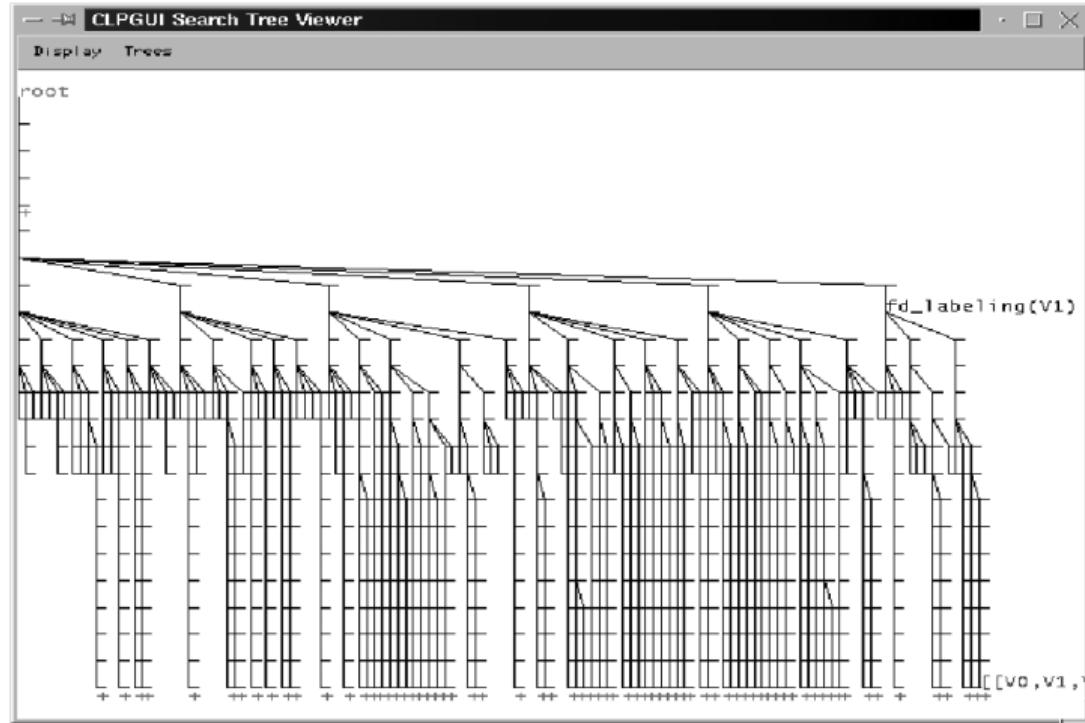
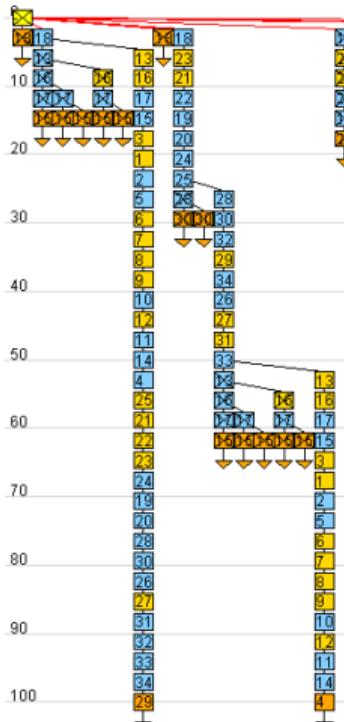
## What can we learn?

- How far do we progress in the search?
- How far do we backtrack?
- Is the value selection method working?
- Do we get stuck in an infeasible branch?
- Can we cut off infeasible branches by propagation?

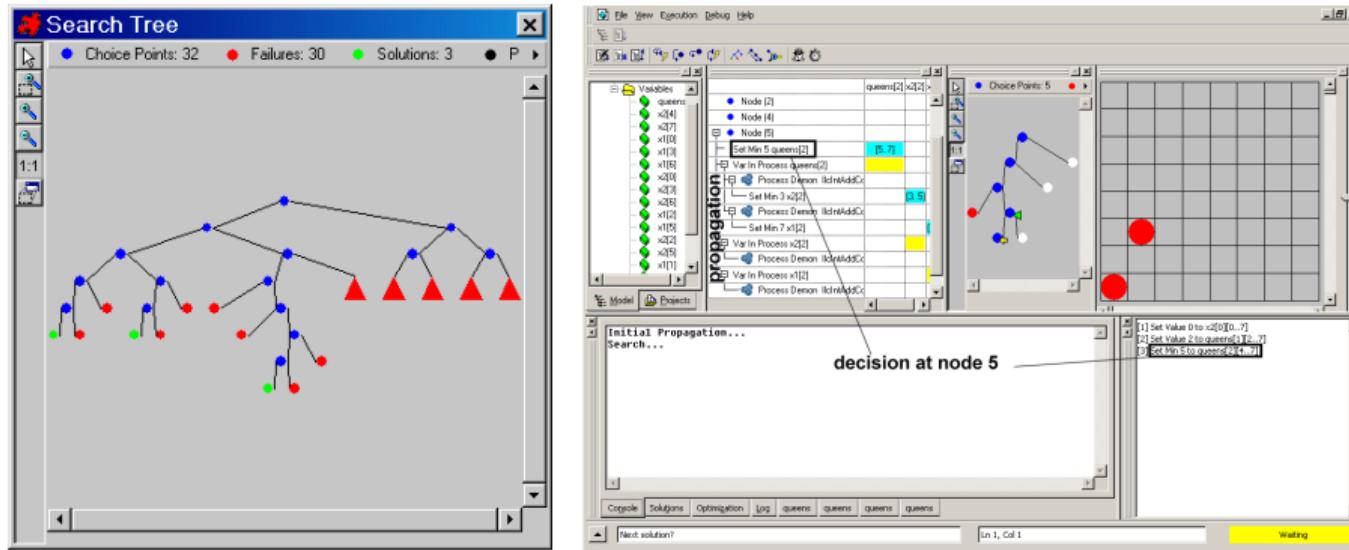
# Search Tree Display [Schulte, 1997]



# Search Tree Display [Simonis and Aggoun, 2000] [Fages et al., 2004]

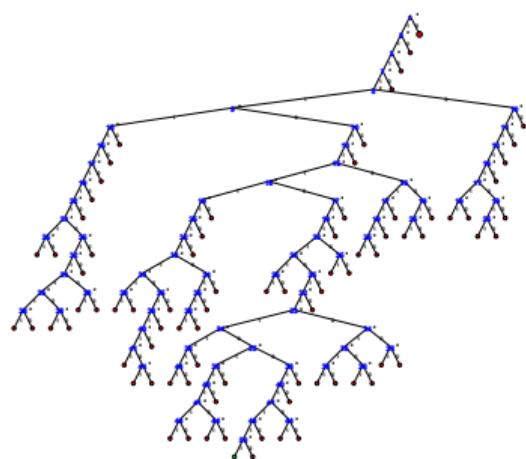


# Search Tree Display: ILOG Solver Debugger (2009)

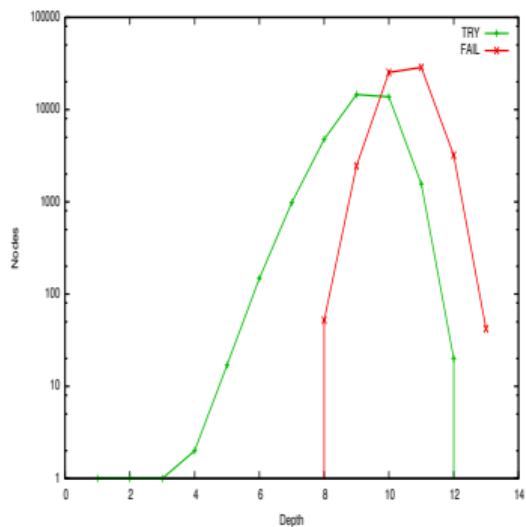


# Search Tree Display [Simonis et al., 2010]

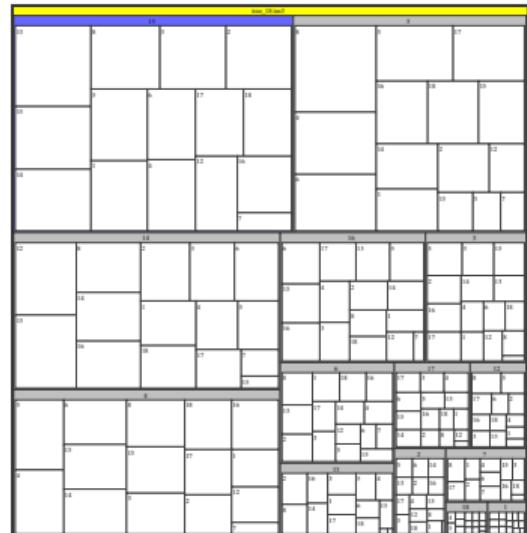
Tree Display



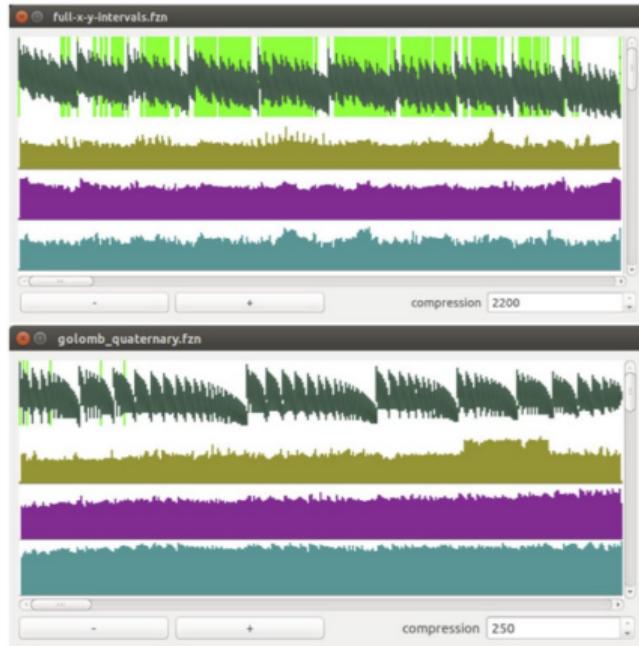
Failure Level



Failure Causes



# Pixel Tree [Shishmarev et al., 2016]



- See large-scale structure in very large trees
- Plot histograms on same horizontal scale (e.g. avg. failure depth, avg. domain reduction)

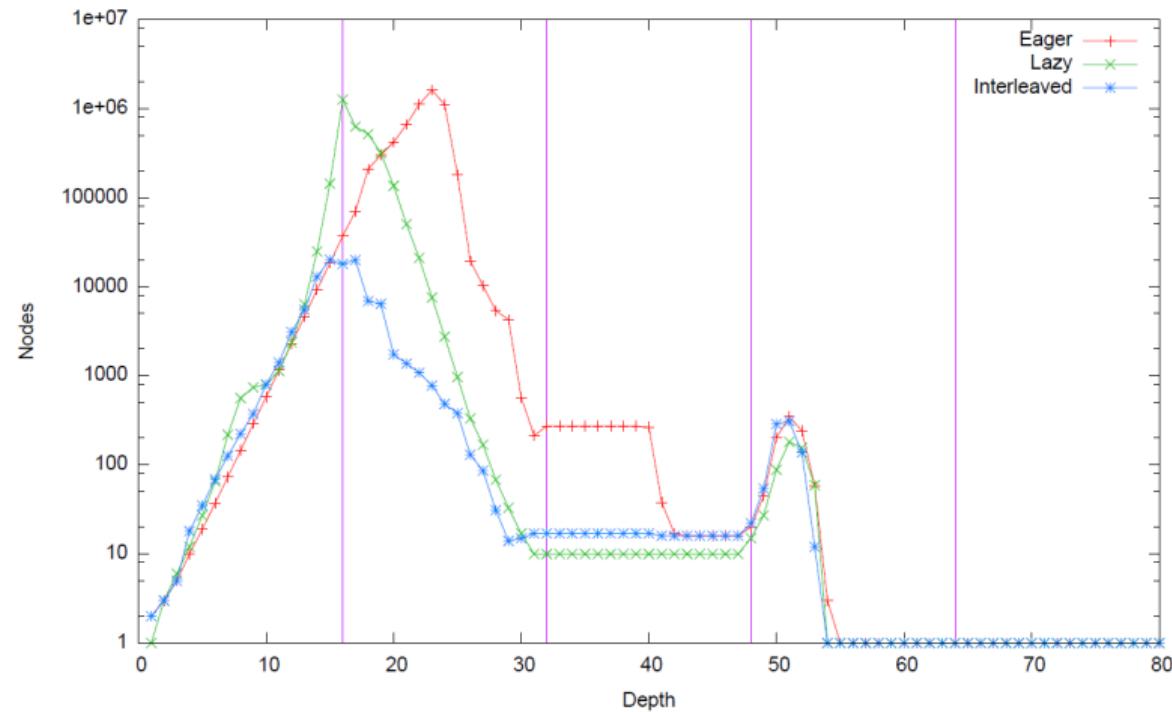
## Example: Similar Subtree Analysis

(This slide is a placeholder for a live demo in the presentation)

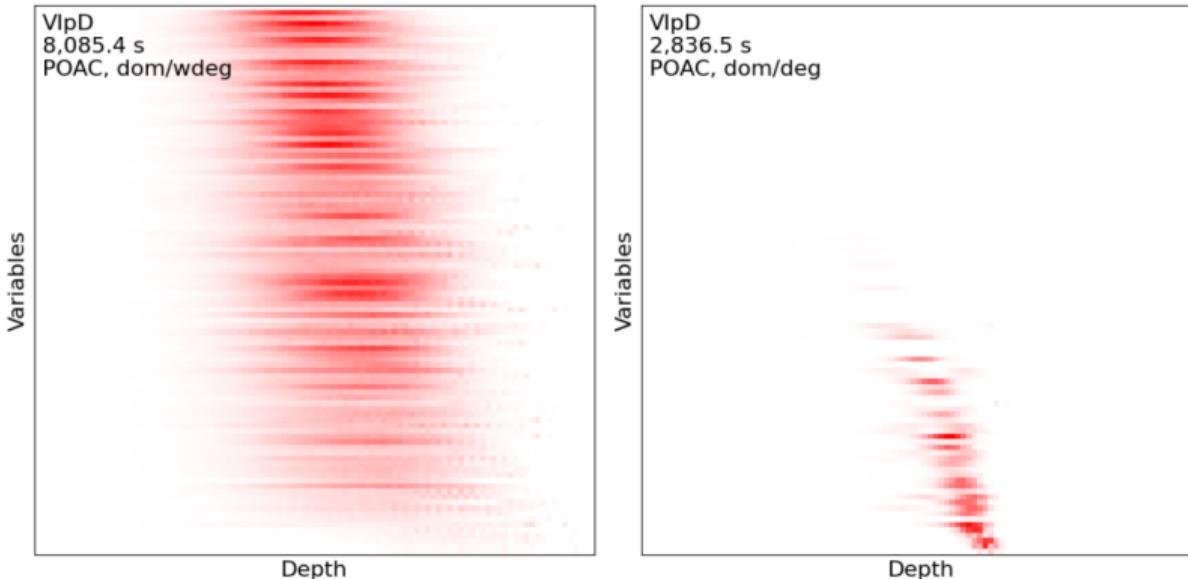
# Search Tree Visualization: Challenges

- How to visualize learning, diving, restarts, backjumps, non-chronological search, best-first search, ... ?
- What can we learn from very large trees?

# Search Depth Display [Simonis and O'Sullivan, 2011]



# Heatmap [Howell et al., 2020]



- Which variables are assigned at which depth of the search
- Where does backtracking occur

## A solution is found, but the user rejects it

- There may be a constraint that is missing in the description
- Visualization is key for discussion with users
- Users are very good at spotting problems
- Not so good at describing all constraints of problem

## Examples

- Task stretching over downtime not allowed
- Task starting just before shift-change not allowed

## An initial solution is found, but no further improvements are made

- Only for optimization problems
- Two basic scenarios
  - We are not finding better solutions due to limitations of search method
  - We have found the optimal solution, but have difficulty proving optimality
- Different approaches for both cases

# Limit of Search Routine

- Some search methods work for finding good initial solutions, but are poor exploring the search space
- Other methods are weak getting initial good solutions, but are effective to explore search space
- Reasons
  - Bad update of cost function
  - Making choices that are dominated
  - Causes deep backtracking in search tree
- Adding lower bound is not helping
- Solutions:
  - Adding constraints to update cost earlier in tree
  - Explore search tree only partially

## Running Example: Scheduling Problem

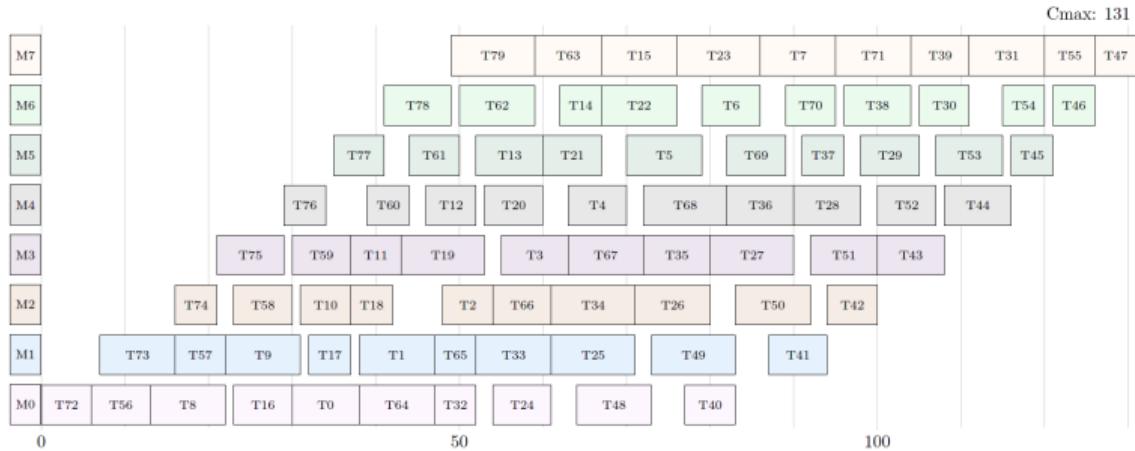
- Flowshop type problem
- Jobs consist of series of tasks processed on machines in same order
- Precedence between tasks of same job
- Disjunctive machines
- Tasks require operator during first part of run
  - Overall cumulative manpower limit

# Conceptual Model (MinZinc)

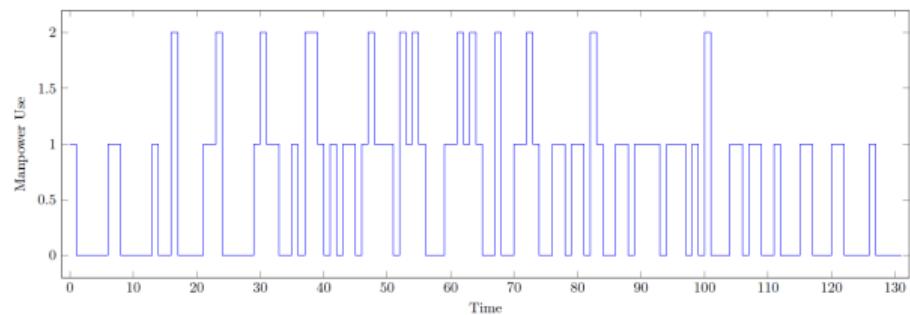
```
1 % variables
2 array[T] of var 0..ub:start;
3 var lb..ub:cmax;
4
5 % constraints
6 constraint forall (i in T)
7     (cmax >= start[i]+duration[i]);
8 constraint forall (p in P)
9     (start[prec[p,2]] >= start[prec[p,1]]+
10      duration[prec[p,1]]));
11 constraint forall(s in S)
12     (cumulative([start[i]|i in T where stage[i]=s],
13                 [duration[i]|i in T where stage[i] = s],
14                 [1|i in T where stage[i] = s],1));
15 constraint
16     cumulative([start[i]|i in T],
17                 [manpowerDuration[i]|i in T],
18                 [1|i in T],manpower);
19
20 solve minimize cmax;
```

- Data definition not shown for brevity

# Example Solution (10 Jobs/8 Machines)

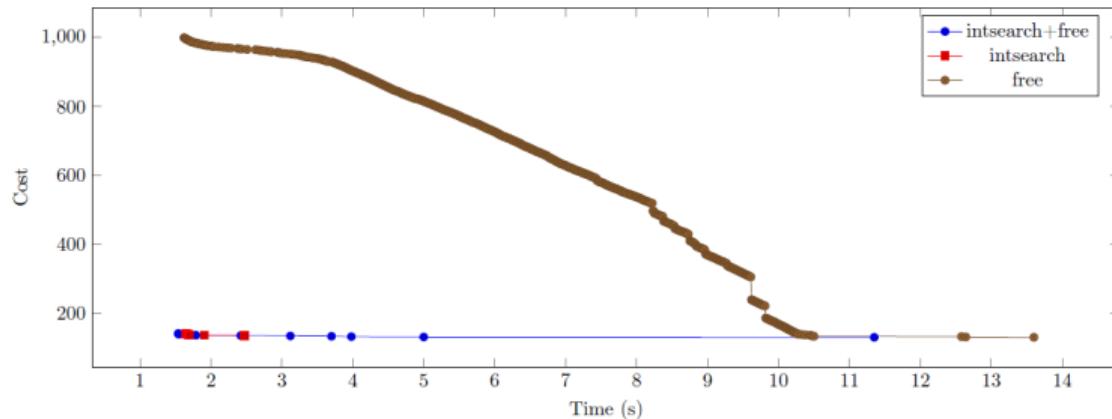


- Typical visualization: Gantt Chart
- Machine view
- Other views available



- Typical visualization: Resource Profile
- Show operator use over time

# Comparison of Three Search Methods

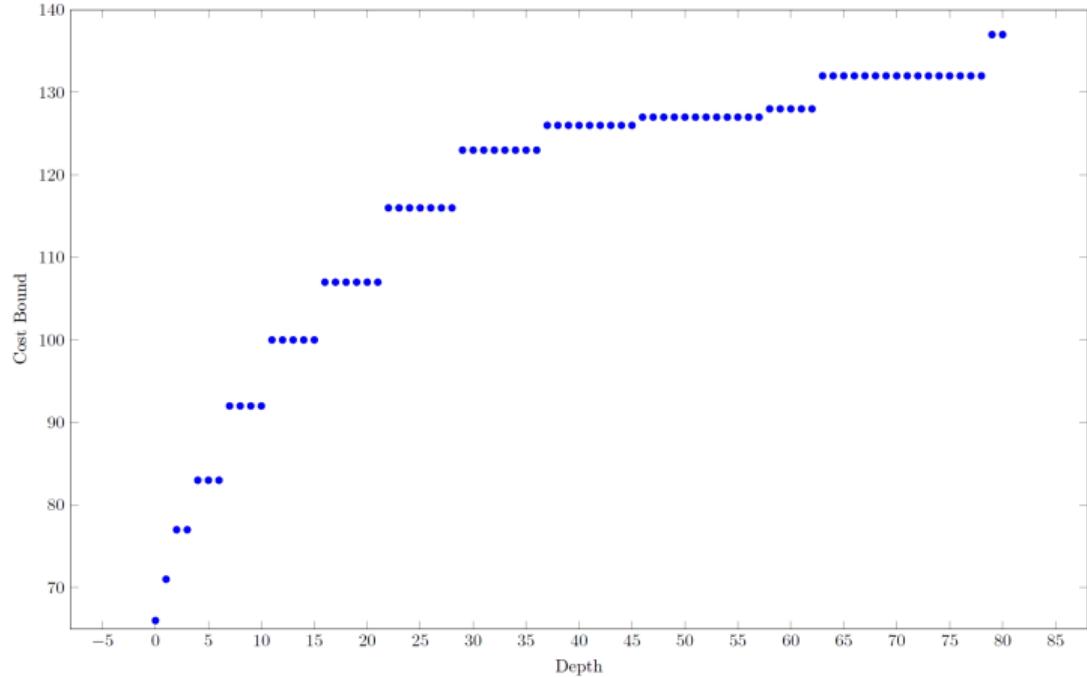


- *intsearch* finds goods solution initially, but gets stuck improving further
- *free* search starts with very poor cost, but continues to improve, reaches and proves optimum
- *free* search with `::intsearch` annotation starts with good solution, but also finds and proves optimum (alternates search steps between free and intsearch)

# Cost Estimate Visualization

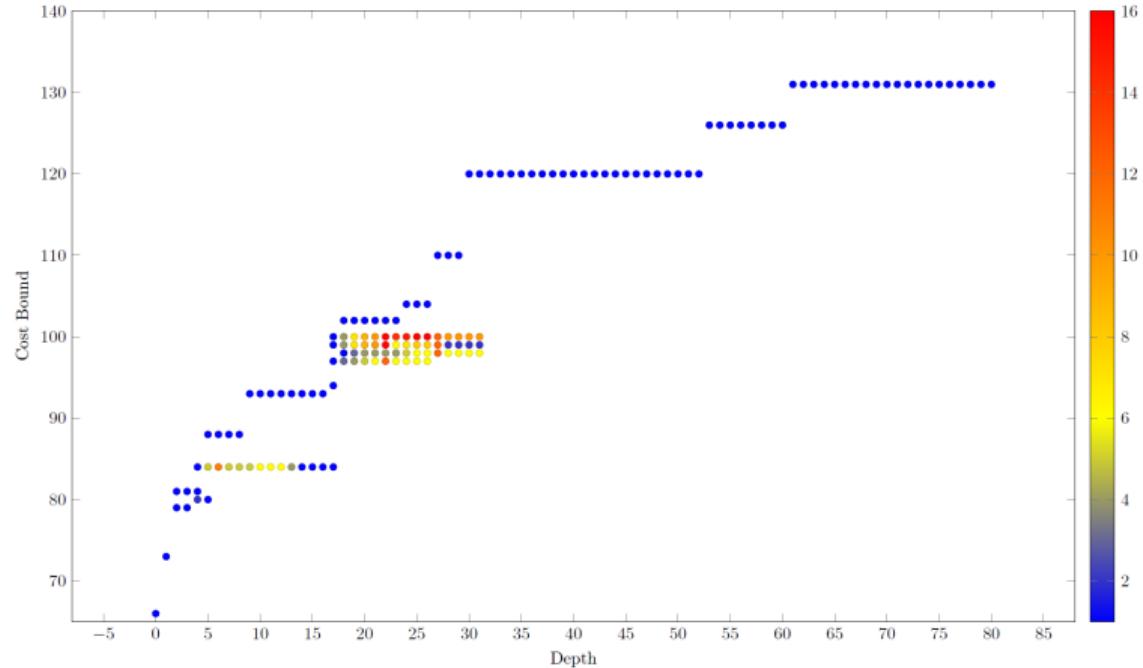
- See the lower bound of cost function over the depth of search
- On the left, initial bound due to propagation
- On the right, actual cost found
- How quickly does the lower bound rise?

# Cost Estimates for Finding First Solution (SICStus)



- Initial bound of 66 due to longest job
- No backtracking for finding first solution

# Finding Optimal Solution (Upper Bound 132)



- Color indicates number of nodes explored at this level
- Backtracking mainly between depth 20 and 30

# Difficult Proof of Optimality

- Do we really care, if we have the optimal solution?
- We can use lower bound to stop search without further exploration
- Requires really good lower bound, or smallish problem size
- Often enough if we are "close enough" to lower bound

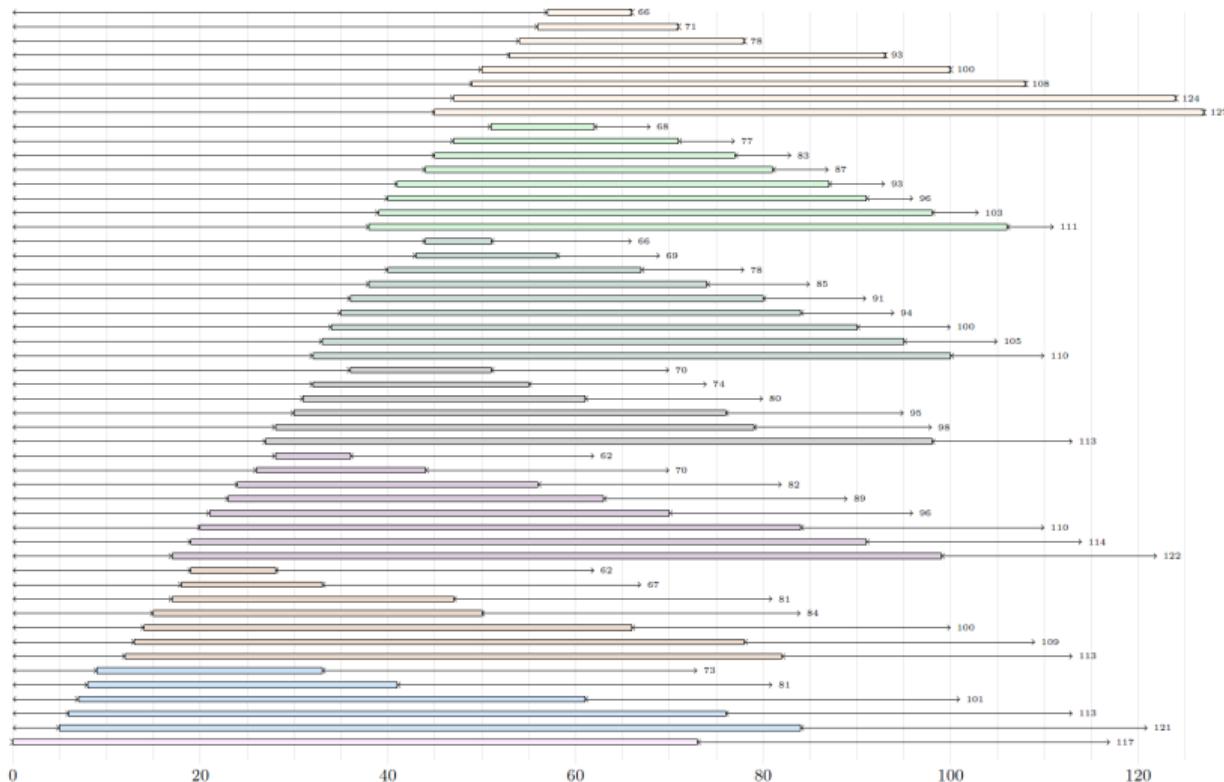
# Understanding Lower Bounds

- The lower bound on the objective obtained by propagation often is very weak
- Many specialized, problem specific bounds in literature
- Used to stop search if lower bound is reached (no search for optimality proof required)
- Interest of converting lower bounds into constraints
- Example: Flowshop subset bound

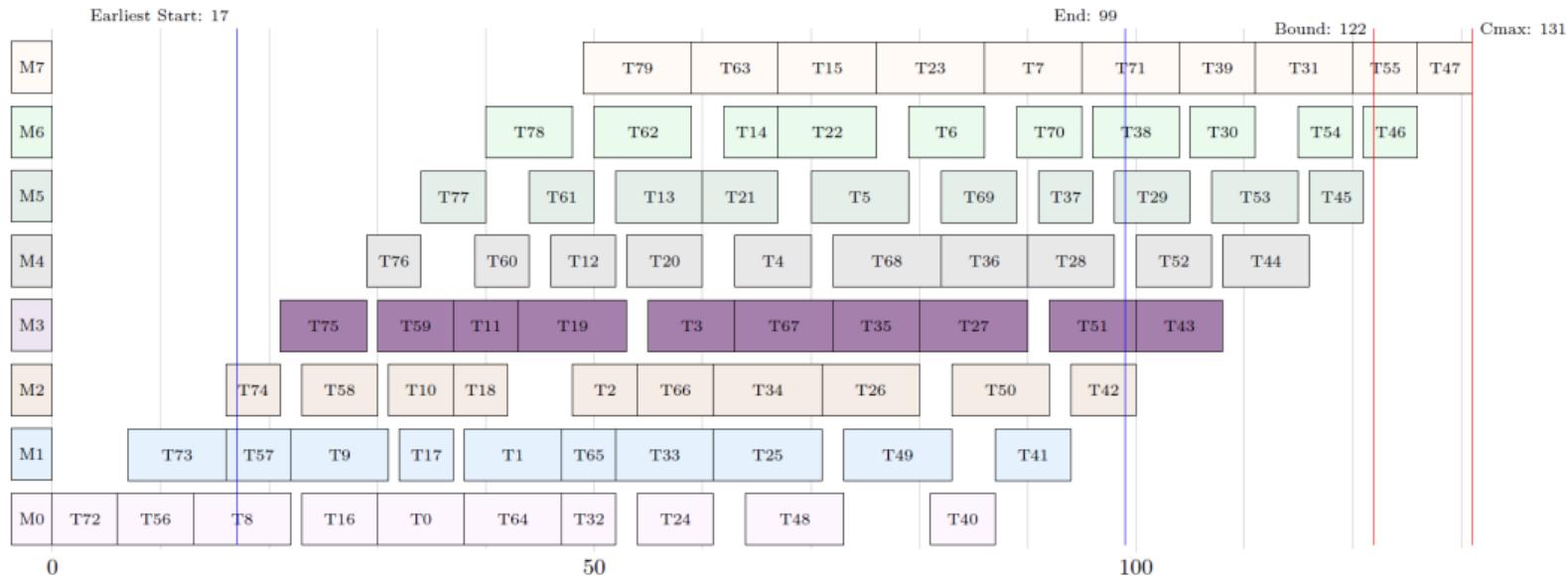
# Flow Shop Stage/Subset Bound

- Initial bound on cost of flowshop is sum of durations in longest job
  - This works well if there are few jobs
  - It is meaningless if there are many jobs
- Stage bound
  - Consider time to process one stage
  - Waiting time before first task of stage begins operating
  - Workload of all tasks of that stage
  - Time to finish later stages after last task of this stage ends
  - Bound is sum of these three elements
- Subset bound
  - Also works for any subset of tasks of the same stage

# Subset Bounds for Example Problem



# Lower Bound and Actual Solution (Stage 3)

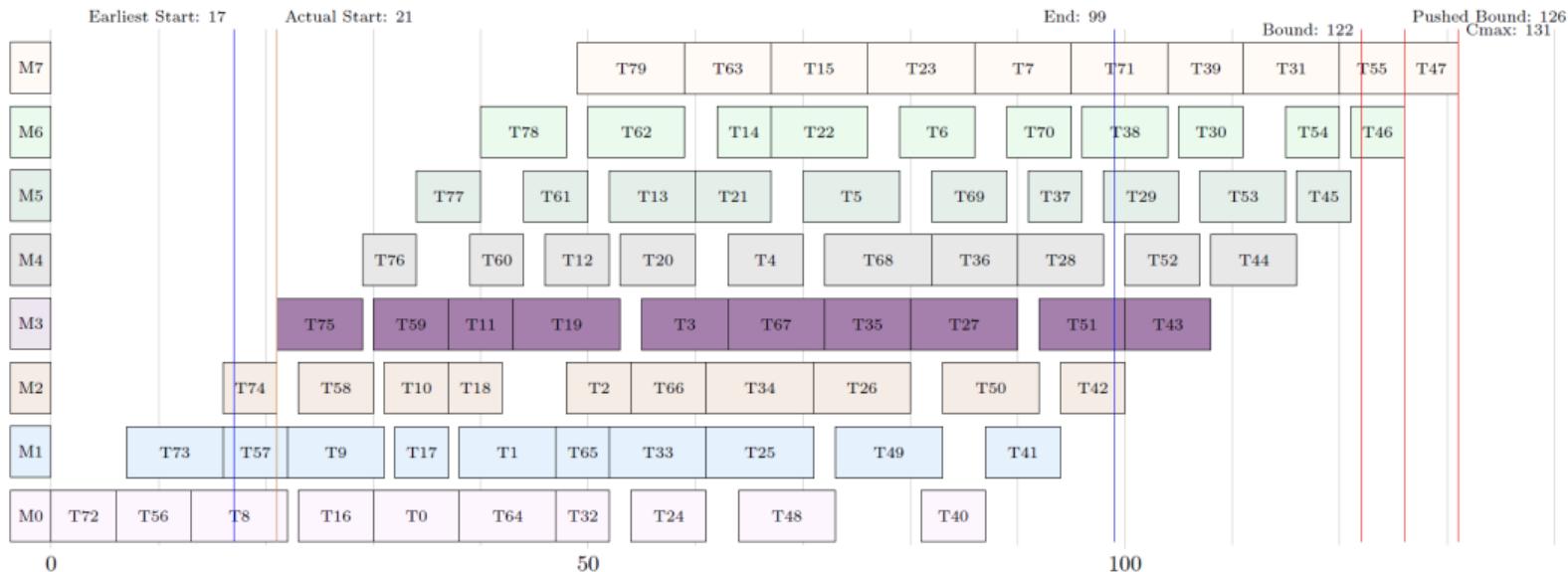


- Stage 3 does not begin at earliest start time, also has gaps in utilization

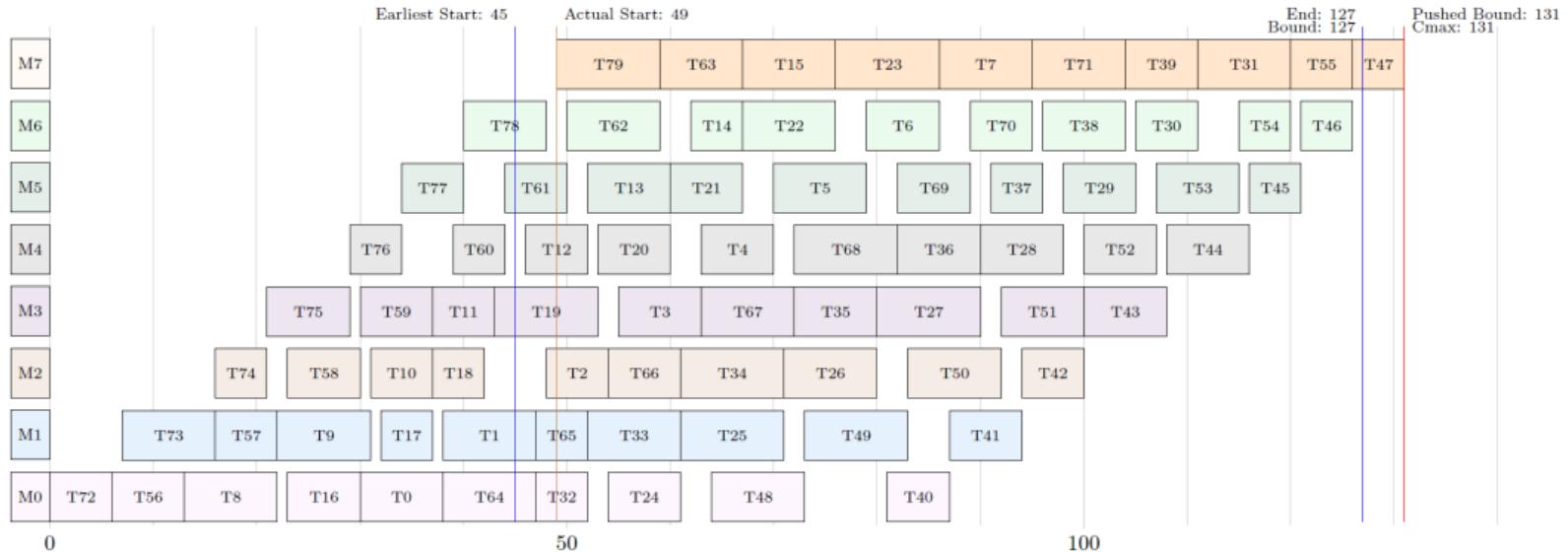
## Converting Bound to Constraint

- Bound assumes that tasks start at earliest possible time
- We can see in solution that this is not the case
- At start of first task of this stage, the workload and the time to end are still correct
- Replace earliest start possible with minimum start time of the tasks
- We know that the makespan must be greater than this
- This is a constraint we can add to model
- (We can keep track of this as an invariant)

# Pushing the Bound Example (Stage 3)

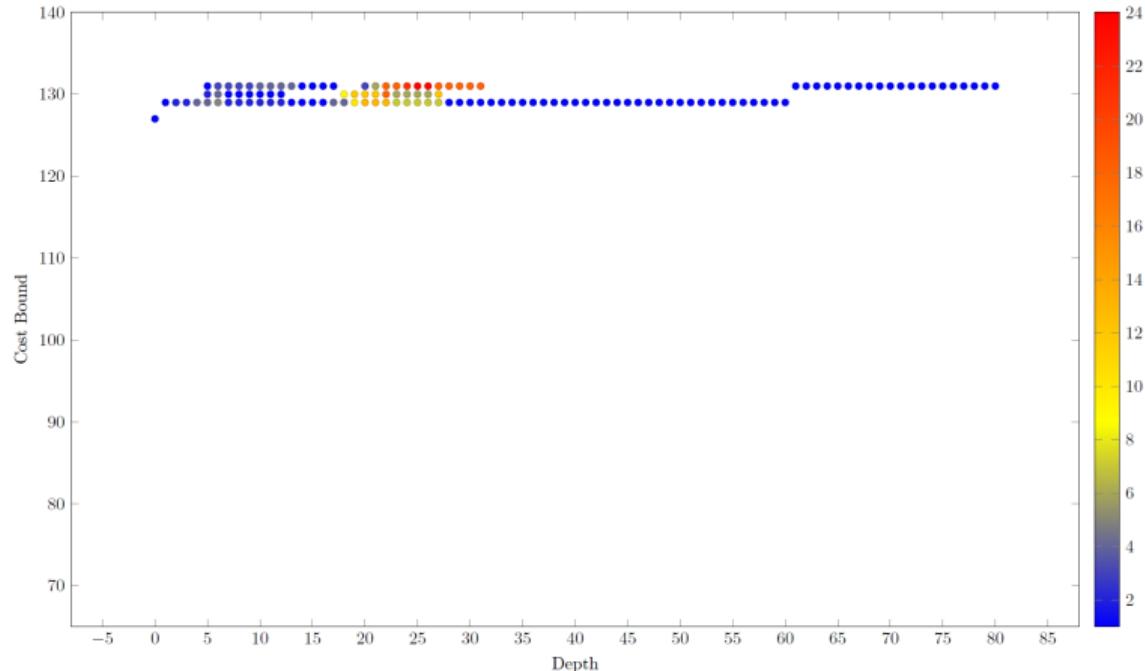


# Strongest Constraint for Stage 7



- When assigning T79 as first task of stage 7, we know that cost must be at least 131
- To improve, we must start stage 7 before time 49

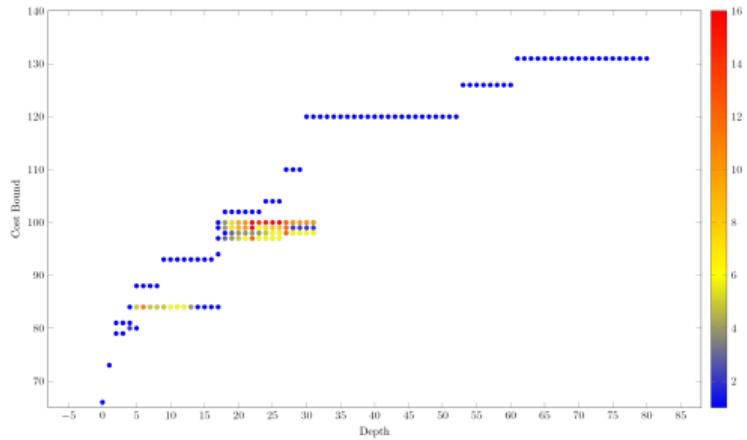
# Updated Cost Estimate in Search for Optimal Solution



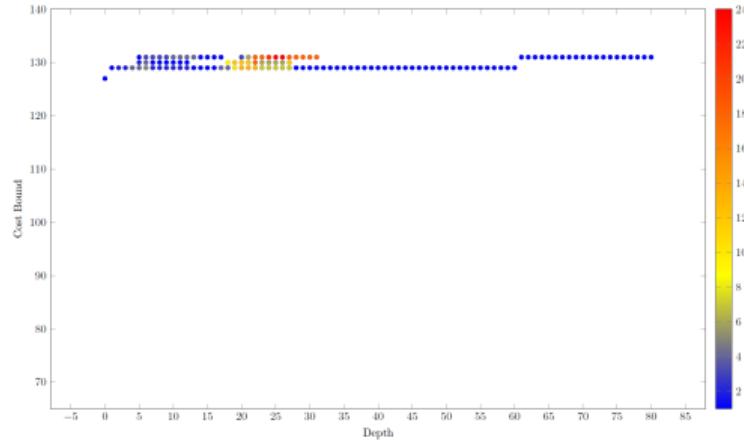
- Cost bound jumped from lower bound 127 to 129 after first task fixed

# Comparison of Cost Estimates

Without Subset Bound Constraint



With Subset Bound Constraint



- Does not reduce amount of search as much as expected
- This type of view is also useful to compare heuristic variable orderings

# Research Question

- How do we plot search depth in a clause learning solver?
- Actual depth of search tree varies
- Variables in search not directly linked to user defined variables

## The model has been working for some time, but suddenly it stops working, or violates a constraint

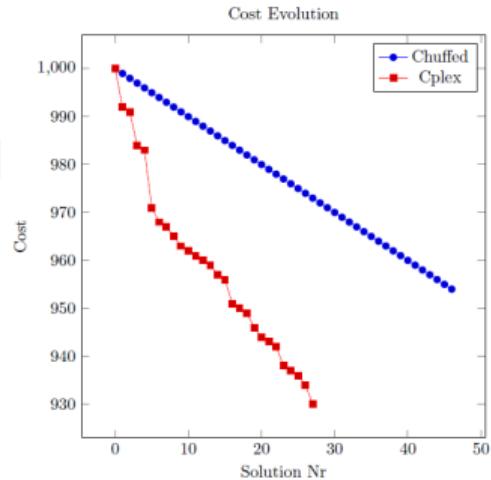
- Often due to changes in base data (resources/process)
- Data entry mistake (duration too long, resource use too high)
- Missing data (null values replaced by defaults)
- Missing preferences (no default process path given for new product)
- Change in real world not reflected in input data
  - New resources
  - Upgraded machines
  - Process changes

## The user can easily improve upon the best solution found

- This often happens with vehicle routing problems
- Unable to explore the search space completely
- Visualization gives an immediate picture of solution quality
- Manual changes correspond to local search moves
- One solution is to do a local search post-solving
- Careful: sometimes it only looks like an improvement
  - Especially if distance is based on actual road travel time
  - ...and visualization uses straight line links
- Having a nice visualization can become a constraint

# The solver says it is optimal, but there is a better solution

- Example MT10 with Chuffed
  - Classical scheduling benchmark [Carlier, Pinson 1989]
  - We know that optimal solution is 930
  - Compare Cplex/Chuffed solutions
  - Chuffed claims its best solution is optimal
  - But Cplex finds a better optimal solution
- This is really tough to detect for a new problem
  - Solution satisfies the constraints
  - Requires a priori knowledge of optimal cost
  - Or, two solvers that both find an optimal solution, and disagree on cost
  - (Optimal solutions with same cost can be quite different)

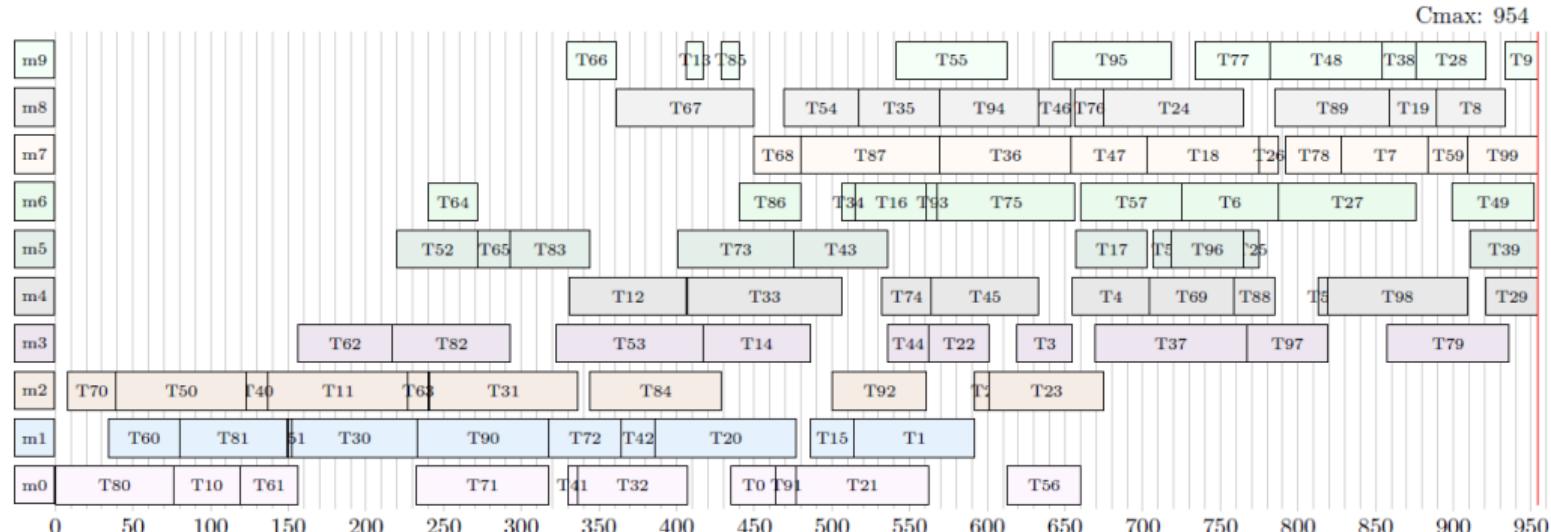


# Job-Shop Scheduling (Minizinc Program)

```
1 include "globals.mzn";
2 int:nrJobs;
3 int:nrRes;
4 set of int: J=1..nrJobs;
5 set of int: R=1..nrRes;
6 array[J,R] of int:taskUse;
7 array[J,R] of int:taskDuration;
8 include "mt10.dzn";
9 int:ub =1000;
10
11 array[J,R] of var 0..ub:start;
12 var 0..ub:objective;
13 constraint forall(j in J)
14   (objective >= start[j, nrRes]+taskDuration[j, nrRes]);
15 constraint forall(j in J, r in 1..nrRes-1)
16   (start[j, r+1] >= start[j, r]+taskDuration[j, r]);
17 constraint forall (r in R)
18   (cumulative([start[j,k]|j in J, k in R where taskUse[j,k]+1=r],
19               [taskDuration[j,k]|j in J, k in R where taskUse[j,k]+1=r],
20               [1|j in J, k in R where taskUse[j,k]+1=r],1)
21 );
22
23 solve minimize objective;
```

# "Optimal" Solution found with Chuffed (Cost 954)

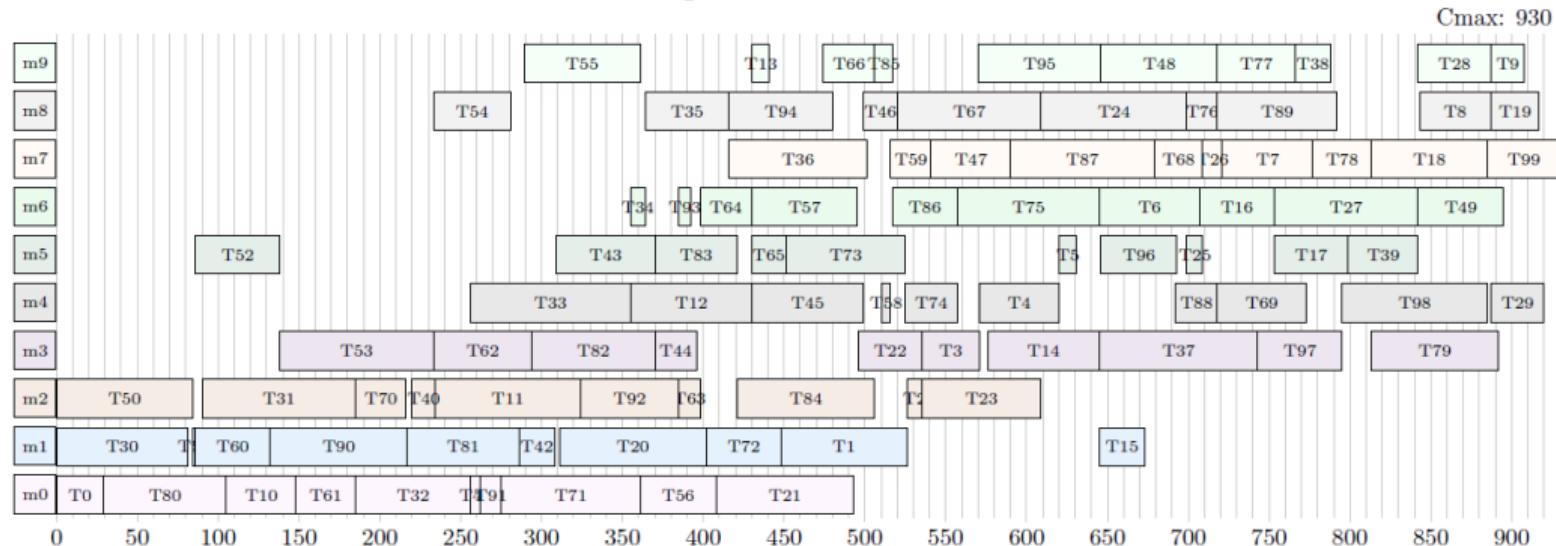
Figure 5: Machine View (Chuffed Solution)



- No reason to suspect that something is wrong
- Different upper bounds lead to different optimal values?

# Real Optimum found with Cplex (Cost 930)

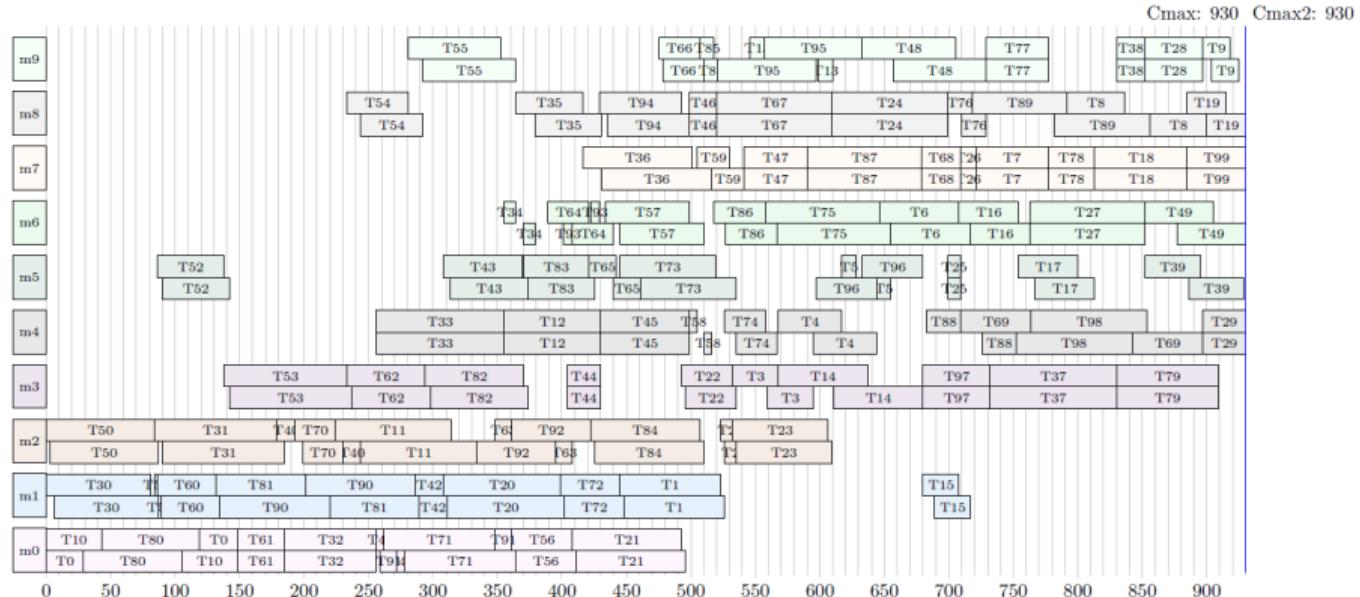
Figure 1: Machine View



- Normally, we would suspect the user program
- Identical MiniZinc program and data, only backend solver changed

# Comparing Two Optimal Solutions (Chuffed/Cplex)

Figure 7: Comparison Machine View

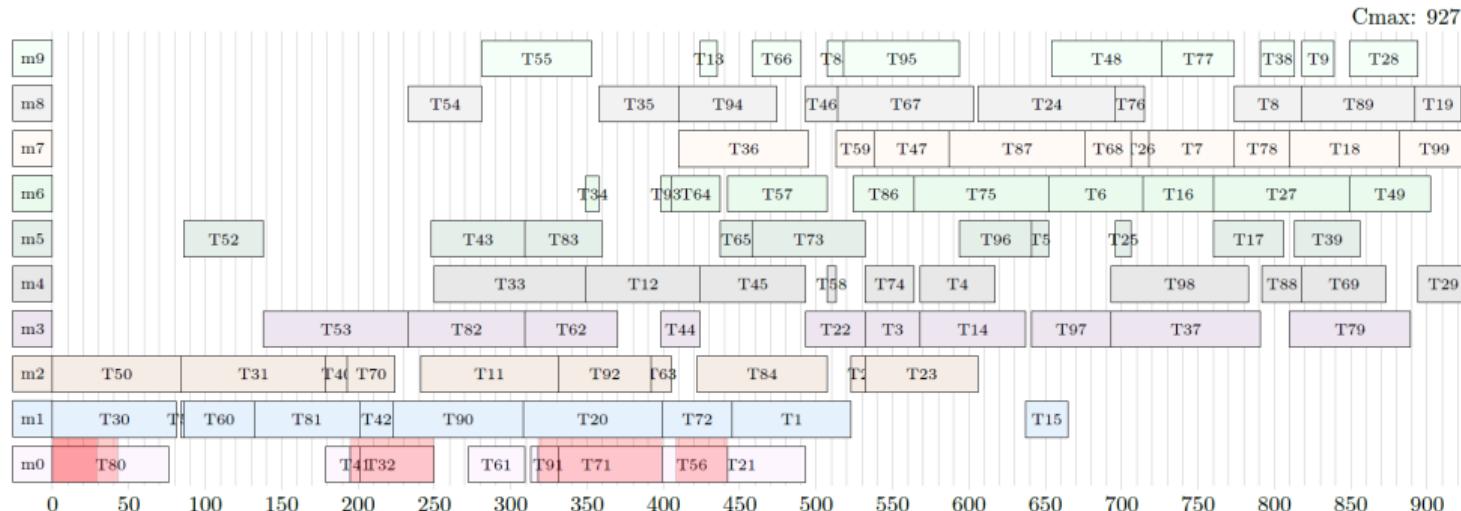


- Many tasks have different start times in the two solutions
- Order of some jobs on machines changed

## The solver says it has found a solution better than the known optimum

- Known benchmark results are not just useful to check performance
- Lower bounds can be used if optimal value not known
  - If you find solution better than lower bound, then either solution or bound (or both) are wrong
  - Only works if lower bound not used to prune search
- Double check before announcing sensational result

# Improved Solution for MT10?



- Highlights overlap of tasks on Machine m0
- Caused by mismatch of resource ids in data (0-9) and in program (1-10)
- No resource constraint for Machine m0

# Minizinc Program with Problem

```
1 include "globals.mzn";
2
3 int:nrJobs;
4 int:nrRes;
5
6 set of int: J=1..nrJobs;
7 set of int: R=1..nrRes;
8
9 array[J,R] of int:taskUse;
10 array[J,R] of int:taskDuration;
11 int:ub =1000;
12
13 array[J,R] of var 0..ub:start;
14 var 0..ub:objective;
15
16 constraint forall(j in J)
17   (objective >= start[j,nrRes]+taskDuration[j ,nrRes]);
18 constraint forall(j in J, r in 1..nrRes-1)
19   (start[j ,r+1] >= start[j ,r]+taskDuration[j ,r]);
20 constraint forall (r in R)
21   (cumulative([start[j,k]|j in J, k in R where taskUse[j ,k]=r],
22               [taskDuration[j,k]|j in J, k in R where taskUse[j ,k]=r],
23               [1|j in J, k in R where taskUse[j ,k]=r],1)
24
25 solve minimize objective;
```

# User Rejects Valid Solution

- Hardest cases, as the model is working as intended
- You may have the wrong objective
- You may only be exploring a small part of the search space
- The user is wrong

# Counterfactual Explanations

- The argument by the user is:
  - Instead of your current solution you should be doing *this and that*
  - These changes would produce a better solution
- Step 1: Can you really do *this and that*? Check if there are solutions which do these changes.
- Step 2: Does this really make an improvement with your current objective?
- Step 3: Can you modify the objective function to accept this as a better solution?
- Step 4: Can you modify the search so that you find this modified solution?
- Step 5: Can you modify the model so that this becomes a feasible solution?

# Counterfactual Explanations in AI

- Hot topic in "Explainable AI" [Stepin et al., 2021, Verma et al., 2020]
- First papers applying this to CP [Korikov and Beck, 2021]  
[Korikov et al., 2021]
  - Restricted to changing the weights of a cost function, not the constraints themselves
- Still a question of "What is a satisfactory explanation?"
  - Differentiate model-centric and user-centric view
  - A small change for the user may be a big change in the model and vice versa

## Show Flexibility in Solution

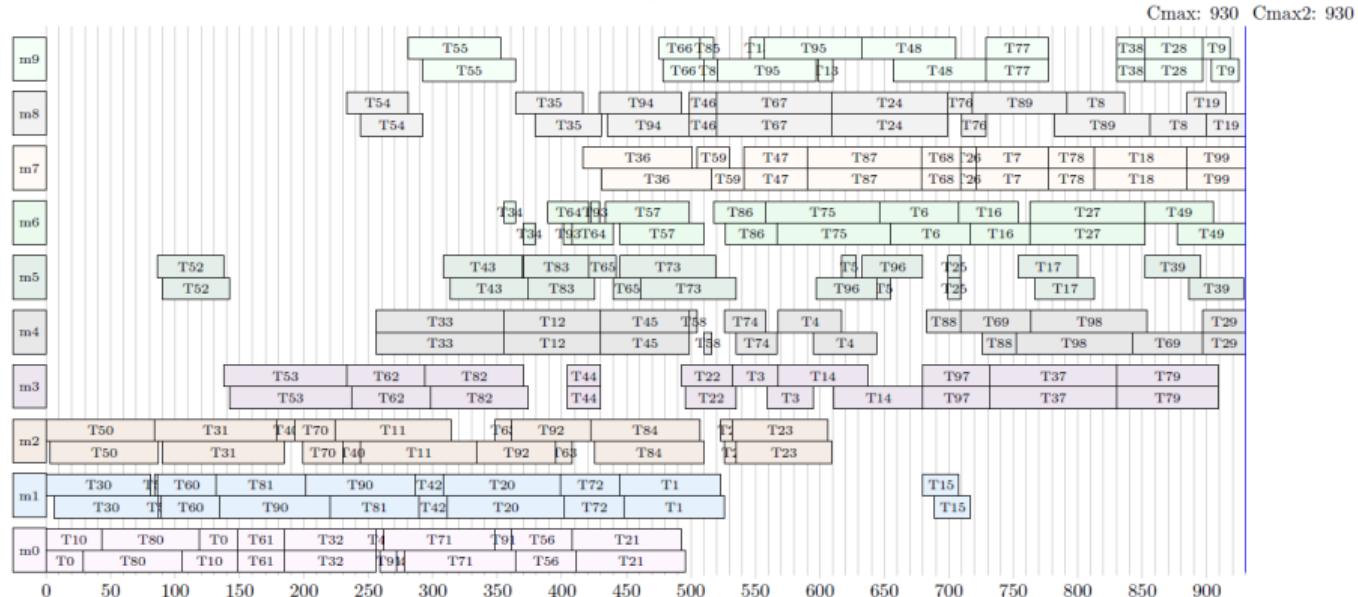
- Instead of showing single solution, show what is possible
- Compare two solutions on one visualization
- In a given solution, some variables can be assigned differently without affecting other variables
  - Easy to allow user to shift tasks within that range
- More flexibility by replacing resource constraints by inequalities
  - Keeps relative order of tasks
  - Can be pre-computed in polynomial time (interactive use may be possible)
- To find overall earliest start/latest end requires multiple solver runs (expensive, not interactive)

# Job-Shop Scheduling (Minizinc Program)

```
1 include "globals.mzn";
2 int:nrJobs;
3 int:nrRes;
4 set of int: J=1..nrJobs;
5 set of int: R=1..nrRes;
6 array[J,R] of int:taskUse;
7 array[J,R] of int:taskDuration;
8 include "mt10.dzn";
9 int:ub =1000;
10
11 array[J,R] of var 0..ub:start;
12 var 0..ub:objective;
13 constraint forall(j in J)
14   (objective >= start[j, nrRes]+taskDuration[j, nrRes]);
15 constraint forall(j in J, r in 1..nrRes-1)
16   (start[j, r+1] >= start[j, r]+taskDuration[j, r]);
17 constraint forall (r in R)
18   (cumulative([start[j,k]|j in J, k in R where taskUse[j,k]+1=r],
19               [taskDuration[j,k]|j in J, k in R where taskUse[j,k]+1=r],
20               [1|j in J, k in R where taskUse[j,k]+1=r],1)
21 );
22
23 solve minimize objective;
```

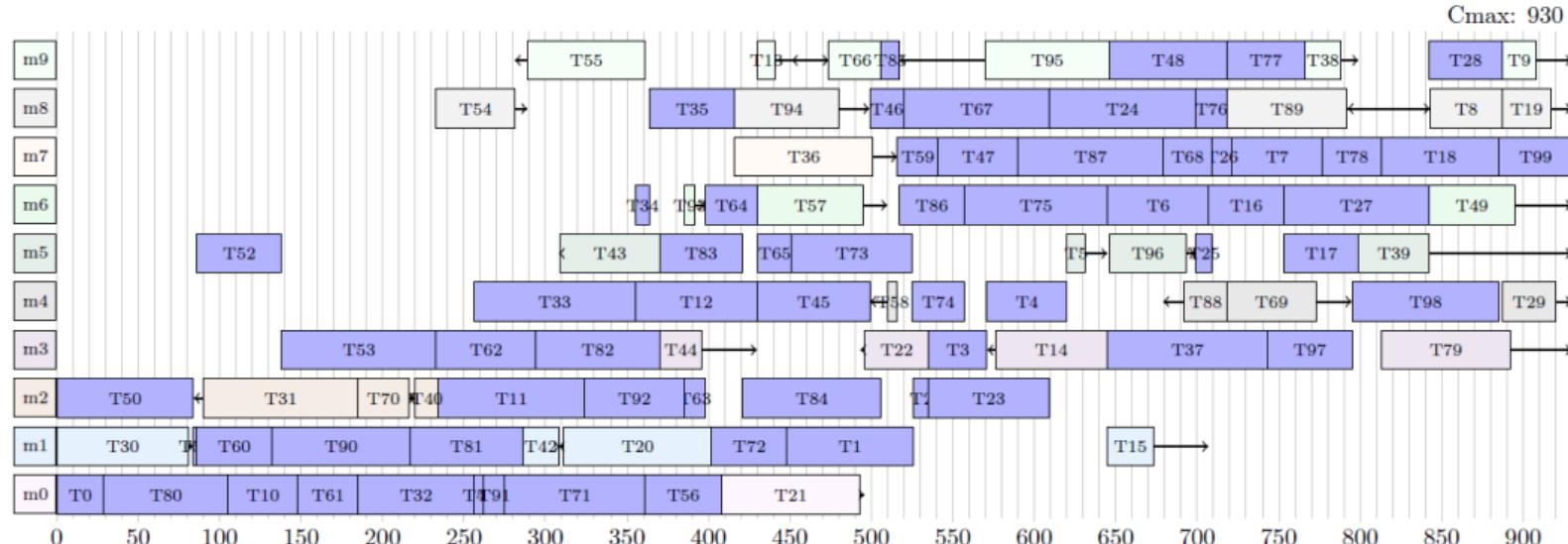
# Comparing Two Optimal Solutions (Chuffed/Cplex)

Figure 7: Comparison Machine View



- Many tasks have different start times in the two solutions
- Order of some jobs on machines changed

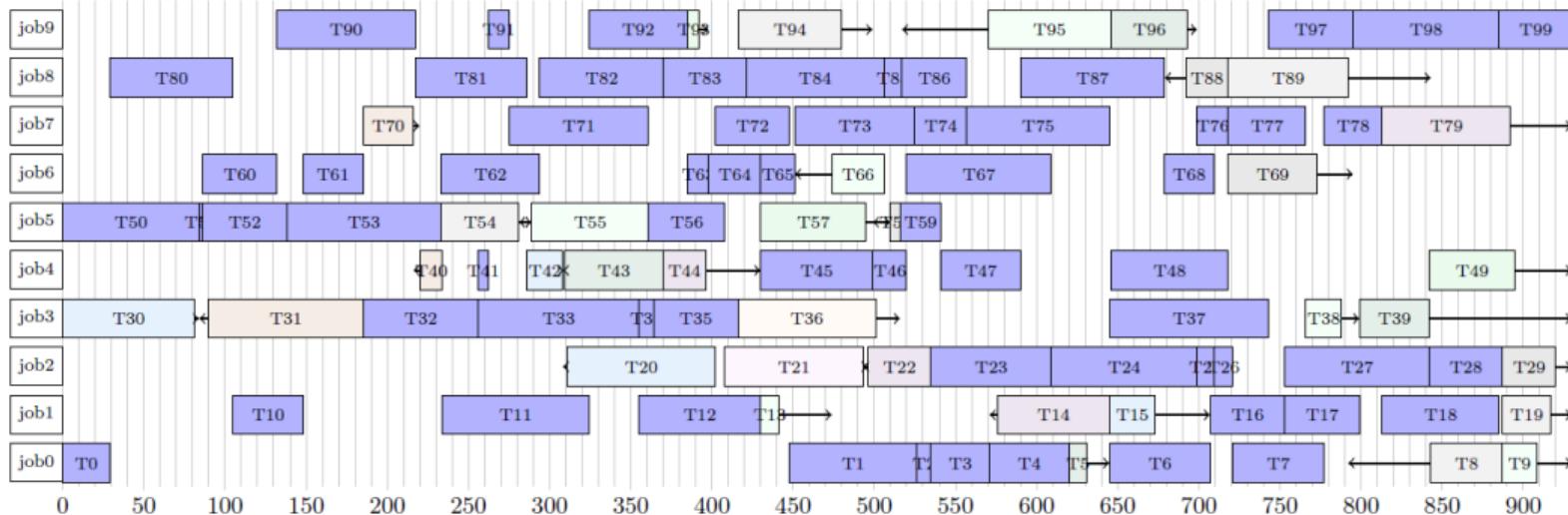
# Single Task Flexibility (Machine View)



- Show how much a task can move without changing any other task
- Blue tasks are not movable in current solution
- More complex when also maintaining other constraints

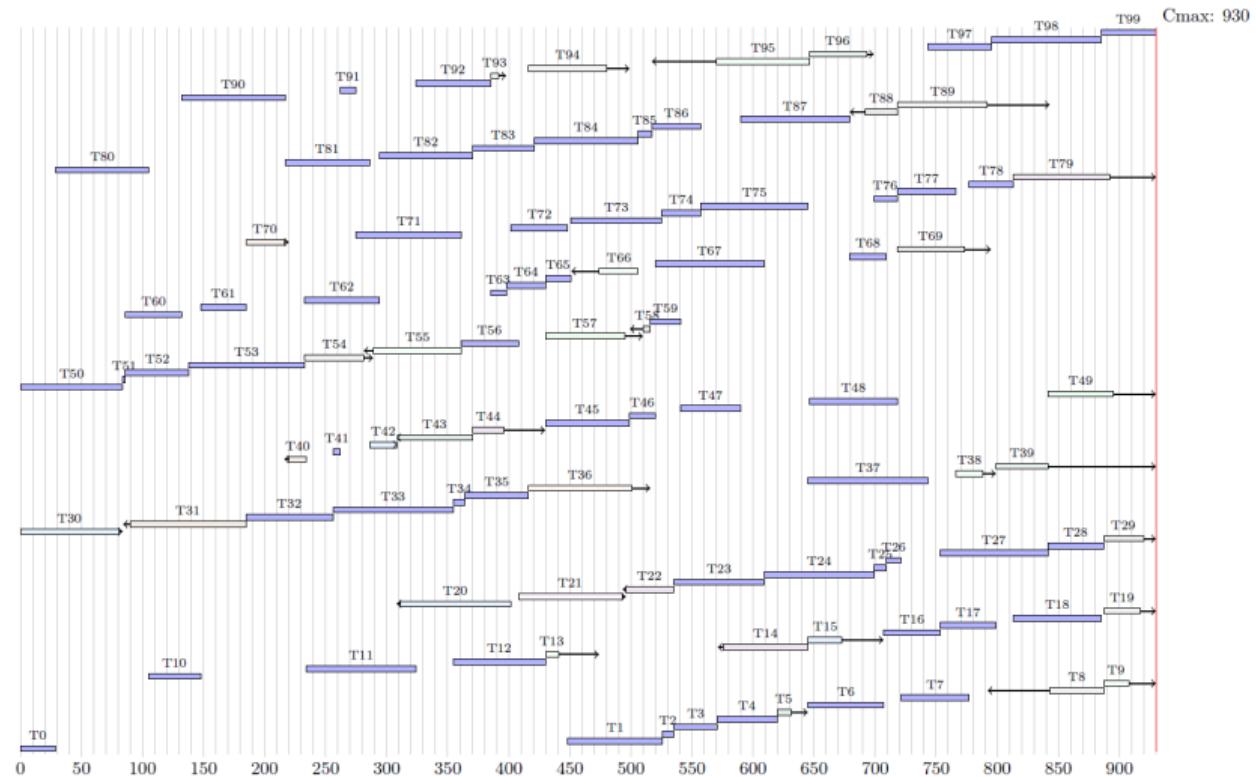
# Single Task Flexibility (Job View)

Cmax: 930



- Some tasks could move further by pushing other tasks (indicated in red)
- This requires running a constraint program to find bounds
- Keep task order on machines, keep objective value

# Single Task Flexibility (Task View)



# Outline

Introduction

Classification of Modelling Problems

Using Visualization to Understand Modelling Issues

Summary

Bibliography

# What have we discussed?

- How to use visualization in modelling problems
- How this integrates into development process
- What can be done with different tools
- Overview of literature in this area

# What did we leave out?

- Visualizations to understand and improve a solver

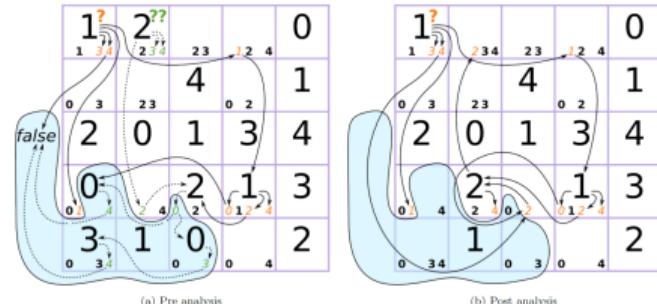
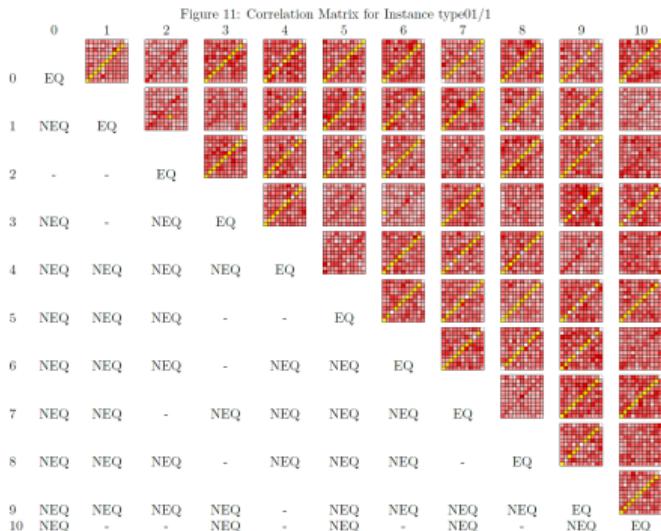


Figure 1: Conflict analysis on the implication graph of a  $5 \times 5$  QC\_COMPLETION problem

from [Downing et al., 2012]

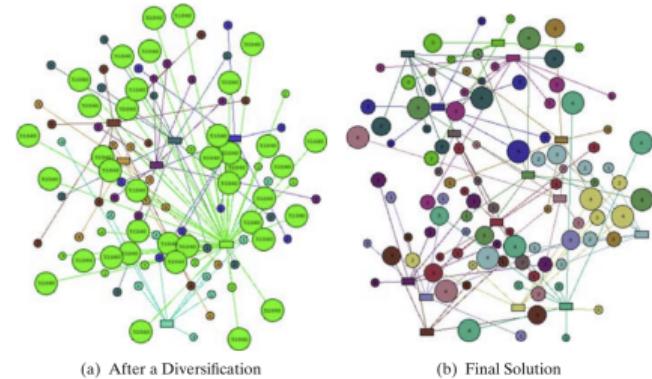
# What did we leave out?

- Visualizations to understand and improve a solver
- Tools for Constraint Acquisition



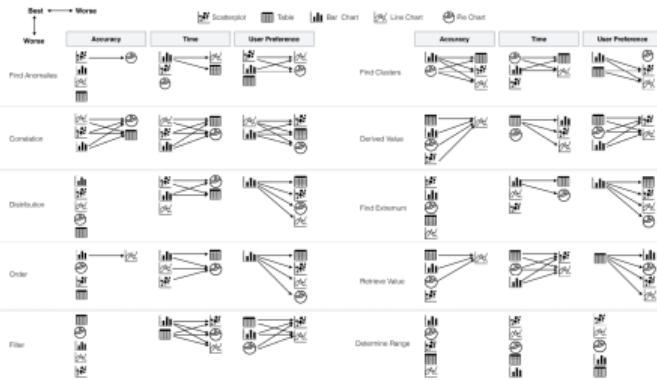
# What did we leave out?

- Visualizations to understand and improve a solver
- Tools for Constraint Acquisition
- **Visualization for Local Search and Constraint Based Local Search [Dooms et al., 2009]**



# What did we leave out?

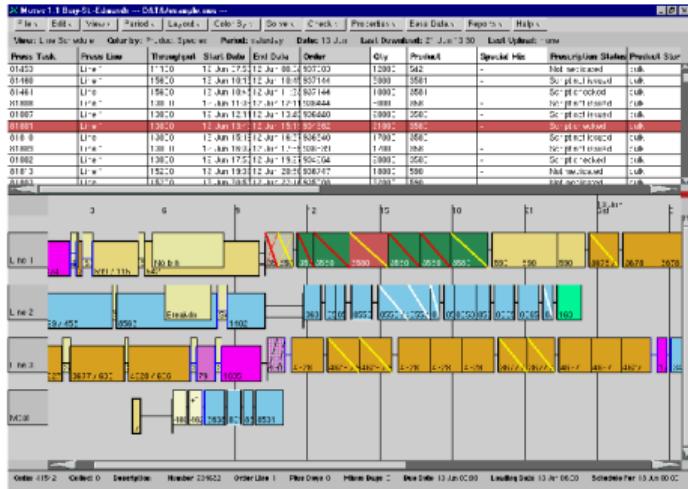
- Visualizations to understand and improve a solver
- Tools for Constraint Acquisition
- Visualization for Local Search and Constraint Based Local Search  
[Dooms et al., 2009]
- How to test effectiveness of visualization



from: [Saket et al., 2019]

# What did we leave out?

- Visualizations to understand and improve a solver
- Tools for Constraint Acquisition
- Visualization for Local Search and Constraint Based Local Search  
[Dooms et al., 2009]
- How to test effectiveness of visualization
- **Visualizations for specific applications**



from: COSYTEC. Moses Feedmill Scheduling System

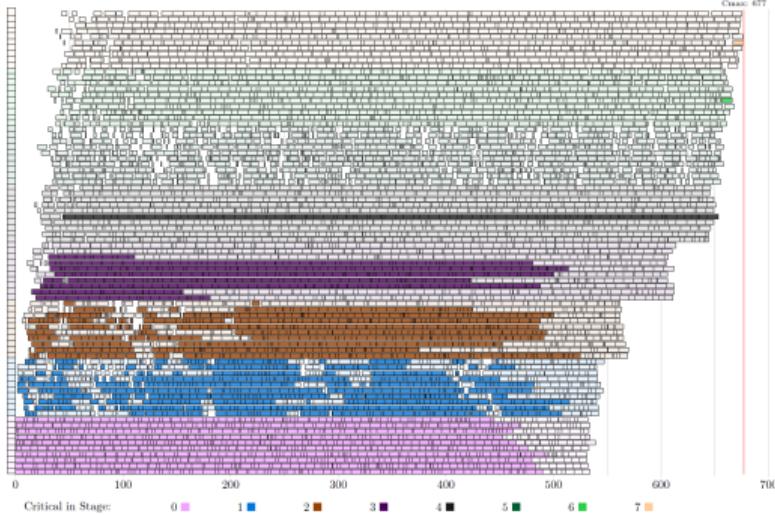
# What did we leave out?

- Visualizations to understand and improve a solver
- Tools for Constraint Acquisition
- Visualization for Local Search and Constraint Based Local Search  
[Dooms et al., 2009]
- How to test effectiveness of visualization
- Visualizations for specific applications
- **Visualization as opt-art**  
[Bosch, 2006, Cambazard et al., 2011]



# What did we leave out?

- Visualizations to understand and improve a solver
- Tools for Constraint Acquisition
- Visualization for Local Search and Constraint Based Local Search  
[Dooms et al., 2009]
- How to test effectiveness of visualization
- Visualizations for specific applications
- Visualization as opt-art  
[Bosch, 2006, Cambazard et al., 2011]
- Scalability issues



## Are the Examples Available?

- Much is available in the MiniZinc IDE
- Assistant based material not yet released
- Will become available as open-source
- Slide set available at

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