

Centre for
Data Analytics



Basic Modelling

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<https://eclipseclp.org/ELearning/index.html>.

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Example 2: Sudoku

- Global Constraints
 - Powerful modelling abstractions
 - Non-trivial propagation
 - Different consistency levels
- Example: Sudoku puzzle

Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

Problem Definition

Sudoku

Fill in numbers from 1 to 9 so that each row, column and 3x3 block contain each number exactly once

[illegible]

Problem Definition

Sudoku

Fill in numbers from 1 to 9 so that each row, column and 3x3 block contain each number exactly once

[illegible]

4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

Model

- A variable for each cell, ranging from 1 to 9
- A 9x9 matrix of variables describing the problem
- Preassigned integers for the given hints
- `alldifferent` constraints for each row, column and 3x3 block

Sudoku Models

- ECLiPSe [▶ Show](#)
- MiniZinc [▶ Show](#)
- NumberJack [▶ Show](#)
- CPMpy [▶ Show](#)
- Choco-solver [▶ Show](#)

ECLiPSe Sudoku Model (from <https://eclipseclp.org/>)

```
:- lib(ic).
:- import alldifferent/1 from ic_global.

top :-
    problem(Board),
    print_board(Board),
    sudoku(Board),
    labeling(Board),
    print_board(Board).

sudoku(Board) :-
    dim(Board, [N,N]),
    Board :: 1..N,
    ( for(I,1,N), param(Board) do
        alldifferent(Board[I,*]),
        alldifferent(Board[*,I])
    ),
    NN is integer(sqrt(N)),
    ( multifer([I,J],1,N,NN), param(Board,NN) do
        alldifferent(concat(Board[I..I+NN-1, J..J+NN-1]))
    ).

print_board(Board) :-
    dim(Board, [N,N]),
    ( for(I,1,N), param(Board,N) do
        ( for(J,1,N), param(Board,I) do
            X is Board[I,J],
            ( var(X) -> write("  ") ; printf(" %2d", [X]) )
        ), nl
    ), nl.
```

ECLiPSe Data Definition

```
problem([ (
    [ (4, _, 8, _, _, _, _, _),
      [ (_, _, _, 1, 7, _, _, _),
        [ (_, _, _, 8, _, _, 3, 2),
          [ (_, _, 6, _, _, 8, 2, 5, _),
            [ (_, 9, _, _, _, _, 8, _),
              [ (_, 3, 7, 6, _, _, 9, _),
                [ (2, 7, _, _, 5, _, _, _),
                  [ (_, _, _, 1, 4, _, _, _),
                    [ (_, _, _, _, _, 6, _, 4) ) ) ) ) ) ) ) ) .
```

► Continue

MiniZinc Sudoku Model

```
int: s;  
int: n=s*s;  
array[1..n,1..n] of var 1..n: puzzle;  
  
include "sudoku.dzn";  
  
predicate alldifferent(array[int] of var int: x) =  
  forall(i,j in index_set(x) where i < j)  
    (x[i] != x[j]);  
  
constraint forall(i in 1..n)  
  (alldifferent([puzzle[i,j] | j in 1..n]));  
constraint forall(j in 1..n)  
  (alldifferent([ puzzle[i,j] | i in 1..n]));  
constraint forall(i,j in 1..s)  
  (alldifferent([puzzle[s*(i-1)+p, s*(j-1)+q] |  
    p,q in 1..s]));  
solve satisfy;
```

MiniZinc Output

```
output [ "sudoku:\n" ] ++  
  [ show(puzzle[i,j]) ++  
    if j = n then  
      if i mod s = 0 /\ i < n then "\n\n"  
      else "\n"  
      endif  
    else  
      if j mod s = 0 then " "  
      else " "  
      endif  
    endif  
  | i,j in 1..n ];
```

MiniZinc Data File (sudoku.dzn)

```
s=3;  
puzzle=[ |  
  4, _, 8, _, _, _, _, _ |  
  _, _, _, 1, 7, _, _, _ |  
  _, _, _, _, 8, _, _, 3 |  
  _, _, 6, _, _, 8, 2, 5 |  
  _, 9, _, _, _, _, _, 8 |  
  _, 3, 7, 6, _, _, 9, _ |  
  2, 7, _, _, 5, _, _, _ |  
  _, _, _, _, 1, 4, _, _ |  
  _, _, _, _, _, 6, _, 4 |  
  | ];
```

▶ Continue

NumberJack Sudoku Model

```
from Numberjack import *

def get_model(N, clues):
    grid = Matrix(N*N, N*N, 1, N*N)

    sudoku = Model([AllDiff(row) for row in grid.row],
                   [AllDiff(col) for col in grid.col],
                   [AllDiff(grid[x:x + N, y:y + N]) for x in range(0, N*N, N)
                                                            for y in range(0, N * N, N)],
                   [(x == int(v)) for x, v in
                    zip(grid.flat, "".join(open(clues)).split()) if v != '*']
                  ])
    return grid, sudoku

def solve(param):
    N = param['N']
    clues = param['file']

    grid, sudoku = get_model(N, clues)

    solver = sudoku.load(param['solver'])
    solver.setVerbosity(param['verbose'])
    solver.setTimeLimit(param['tcutoff'])

    solver.solve()
```

NumberJack Data File

```
4 * 8 * * * * * *
* * * 1 7 * * * *
* * * * 8 * * 3 2
* * 6 * * 8 2 5 *
* 9 * * * * * 8 *
* 3 7 6 * * 9 * *
2 7 * * 5 * * * *
* * * * 1 4 * * *
* * * * * 6 * 4
```

▶ Continue

CPMpy Sudoku Model (from <https://github.com/CPMpy/>)

```
import numpy as np
from cpmPy import *

# Variables
puzzle = intvar(1,9, shape=given.shape, name="puzzle")

model = Model(
    # Constraints on values (cells that are not empty)
    puzzle[given!=e] == given[given!=e], # numpy's indexing, vectorized equality
    # Constraints on rows and columns
    [AllDifferent(row) for row in puzzle],
    [AllDifferent(col) for col in puzzle.T], # numpy's Transpose
)

# Constraints on blocks
for i in range(0,9, 3):
    for j in range(0,9, 3):
        model += AllDifferent(puzzle[i:i+3, j:j+3]) # python's indexing

model.solve()
```

CPMpy Data Definition

```
e = 0 # value for empty cells
given = np.array([
    [4, e, 8, e, e, e, e, e, e],
    [e, e, e, 1, 7, e, e, e, e],
    [e, e, e, e, 8, e, e, 3, 2],
    [e, e, 6, e, e, 8, 2, 5, e],
    [e, 9, e, e, e, e, e, 8, e],
    [e, 3, 7, 6, e, e, 9, e, e],
    [2, 7, e, e, 5, e, e, e, e],
    [e, e, e, e, 1, 4, e, e, e],
    [e, e, e, e, e, e, 6, e, 4]
])
```

▶ Continue

Choco-solver Sudoku Model

```
Model model = new Model("Sudoku");
int blockSize = 3;
int m = blockSize*blockSize;

IntVar[][] vars = new IntVar[m][m];
for(int i=0;i<m;i++){
    for(int j=0;j<m;j++){
        vars[i][j] = model.intVar("X"+i+" "+j, 1, m);
        if (data[i][j]>0) {
            model.arithm(vars[i][j],"=",data[i][j]).post();
        }
    }
}
for(int i=0;i<m;i++){
    model.allDifferent(row(i,m,vars)).post();
    model.allDifferent(column(i,m,vars)).post();
}
for(int i=0;i<m;i+=blockSize){
    for(int j=0;j<m;j+=blockSize){
        model.allDifferent(block(i,j,blockSize,vars)).post();
    }
}
Solver solver = model.getSolver();
solver.solve();
```

Choco-solver Data

```
int[][] data = new int[m][m]{  
    {4, 0, 8, 0, 0, 0, 0, 0, 0},  
    {0, 0, 0, 1, 7, 0, 0, 0, 0},  
    {0, 0, 0, 0, 8, 0, 0, 3, 2},  
    {0, 0, 6, 0, 0, 8, 2, 5, 0},  
    {0, 9, 0, 0, 0, 0, 0, 8, 0},  
    {0, 3, 7, 6, 0, 0, 9, 0, 0},  
    {2, 7, 0, 0, 5, 0, 0, 0, 0},  
    {0, 0, 0, 0, 1, 4, 0, 0, 0},  
    {0, 0, 0, 0, 0, 0, 6, 0, 4}  
};
```

Choco-solver Utilities

```
IntVar[] row(int row, int size, IntVar[][] array){
    return array[row];
}

IntVar[] column(int col,int size,IntVar[][] array){
    IntVar[] column = new IntVar[size];
    for(int i=0; i<size; i++){
        column[i] = array[i][col];
    }
    return column;
}

IntVar[] block(int x,int y,int blockSize,IntVar[][] array){
    IntVar[] block = new IntVar[blockSize*blockSize];
    int k=0;
    for(int i=0;i<blockSize;i++){
        for(int j=0;j<blockSize;j++){
            block[k++] = array[x+i][y+j];
        }
    }
    return block;
}
```

► Continue

Domain Visualizer

- Problem shown as matrix
- Each cell corresponds to a variable
- Instantiated: Shows integer value (large)
- Uninstantiated: Shows values in domain

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	3	2	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

Initial State (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9	

Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

►► Skip Animation

Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1	3	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	3	2
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1	3	5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1	3	5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

[◀ Back to Start](#)

▶▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3	2 3 4 5 6 8 9	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1	3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1	3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1	3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 8 9	1 4 5 6 7 8 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 7 9	1 4 5 6 7 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	8 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9
1 5 6 7 8 9	3 4 5 6 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 5 6 7 8 9	1 2 4 5 6 8	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 4 5 6 7 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9
1 3 5 6 7 9	1 4 4	6 4 5 9	1 3 4 7 9	1 3 4 7 9	8 4 7 9	2 4 7 9	5 4 7 9	1 3 4 7 9
1 3 5 6 7 8 9	9 4 5 9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3 4 5 9	7 4 5 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 4 5 9	7 4 5 9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	
1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4	3 8	2	5	1 3 4 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 5 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 4 5 8	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 6 4 8 9	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 3 4 5 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9 7	1 2 3 5 6 9 7	1 3 5	1 2 3 5 6 7 9	1 2 3 4 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	4 5 8	3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	4 5 7	3	2	
1 3 7 9	1 4	6	4 3 7 9	4 9	3 8	2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7	
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8	
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	4 3 8 9	5	1 3 6 4 9	1 3 8	1 3 6 4 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 9	1 2 3 5 6 7 9	6	1 2 4 6 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	3 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	4	6	4 3 7 9	4 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 3 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	3 5 6 8 9
1 5 6 7 9	1 5 6	1 5	4 5 7 9	8	1 5 6 7 9	1 4 5	3	2
1 3 7 9	1 4	6	4 3 7 9	3 4	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 4 6 8 9	1 3 4 6 8 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9	1 3 5 7	1 2 6 9	1 3 5 6 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
1 5 6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5	1 7 9	1 5 6 9
5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
1 5 6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 4 5 9 7	3	2	
1	4	6	4 7 9	3 4 9	8	2	5	1 3 7 9
5	9	2	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 5 6 7	8	1 3 5 6 7
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5 7	1 6 9 7	1 5 6 9
3 6 9	2 5 6	3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 4 5 9 7	3 4 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	4 7	4 6 7	1 3 3 1 6 4 7	1 3 8	1 3 6 7	
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 6 7 9	2 5 6 8	3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
3 7 9	1 2 5 8	1 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5 7 9	1 6 7 9	1 5 6 9 9
3 6 9	2 5 6	3 5 9	1	7	2 3 5 6 9 8	4 5 8 9	4 6 9 8	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9 9	8	5 6 9 7	1 4 5 7 9	3	2
1	4	6	3 7 9	3 9 9	8	2	5	3 7 9
5	9	2	3 4 7	3 4 7	1 3 7 9	1 3 4 7	8	1 3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 6 4 9	1 3 6 8 9
3 6 7 9	2 5 6 8 9	3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
3 7 9	1 2 5 8	1 5 9	2 3 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 9	5 6 7 9
3 6 9	2 5 6	5 9	1	7	2 3 6 4 5 9 8		6 9	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9	8	1 6 4 5 9 7	3	2	
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4 7	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 6 9	3 6 8 9
3 6 7 9	2 5 6 8	3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
3 5 7 9	1 2 5 8	1 5 9	2 3 5 7 8 9	3	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 9	5 6 7 9
3 6 9	2 5 6	5 9	1	7	2 3 6 4 5 9 8		6 9	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9	8	1 6 4 5 9 7	3	2	
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 6 9	3 6 8 9
3 6 9	5 6 8	5 9	2 3 5 7 8 9	1	4	5 7 8	2 6 7 9	3 5 6 7 8 9
3 9	1 5 8	1 5 9	2 3 5 7 8 9	3 9	1 2 3 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 9	5 6 7 9
3 6 9	2 5 6	5 9	1	7	2 3 6 4 5 9 8		6 9	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9	8	1 6 4 5 9 7	3	2	
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 8 9	5	3 6 9	1 3 4 8	1 6 9	3 6 8 9
3 6 9	5 6 8	5 9	3 2 3 7 8 9	1	4	5 7 8	3 2 6 9	3 5 6 7 8 9
3 9	1 5 8	1 5 9	3 2 3 7 8 9	3	2 3 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	³ 6 9	^{2 3} 6 9	¹ 5 7	¹ 6 9	^{5 6} 7 9
³ 6 9	² 5 6	³ 5 9	1	7	^{2 3} 6 4 5 9 8	⁶ 4 5 8	⁶ 5 6 8 9	^{5 6} 7 9
¹ 6 7 9	¹ 5 6	¹ 5 9	^{4 5} 4 5 9	8	¹ 6 4 5 9 7	³ 2 5	³ 2 5	³ 2 5
1	4	6	³ 7 9	³ 8 9	¹ 2 5	¹ 3 8	¹ 4 9	¹ 5 9
5	9	2	⁴ 7	⁴ 8 9	¹ 2 5	¹ 3 8	¹ 4 9	¹ 5 9
8	3	7	6	2	5	9	4	1
2	7	¹ 4	³ 8 9	5	³ 6 9	¹ 2 5	¹ 3 8	¹ 4 9
³ 6 9	^{5 6} 8	⁵ 9 7 8 9	^{2 3} 1 4	² 3 8 9	² 3 8 9	⁵ 6 9	⁵ 6 9	⁵ 6 9
³ 1 9	¹ 5 8	¹ 3 9	^{2 3} 7 8 9	³ 6 9	^{2 3} 1 4	¹ 2 5	¹ 3 8	¹ 4 9

◀ Back to Start

After Setup (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	³ 6 9	^{2 3} 6 9	¹ 5 7	¹ 6 7	^{5 6} 7 9
³ 6 9	² 5 6	³ 5 9	1	7	^{2 3} 6 9	^{4 5} 8 9	⁶ 7 9	^{5 6} 8 9
⁶ 7 9	¹ 5 6	¹ 5 9	^{4 5} 7 9	8	¹ 6 9	^{4 5} 7 9	3	2
1	4	6	³ 7 9	³ 7 9	8	2	5	³ 7 9
5	9	2	⁴ 7 9	³ 7 9	^{1 3} 7 9	³ 7 9	8	³ 6 7
8	3	7	6	2	5	9	4	1
2	7	^{1 3} 4 9	³ 8 9	5	^{3 1} 6 9	^{3 1} 8 9	¹ 7 9	³ 8 9
³ 6 9	^{5 6} 7 8	³ 5 9	^{2 3} 7 8 9	1	4	⁵ 7 8	² 7 9	³ 5 7 8 9
^{3 1} 5 9	¹ 5 9	³ 7 8 9	^{2 3} 7 8 9	³ 7 9	^{2 3} 7 9	6	^{1 2} 7 9	4

Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

- Bounds Consistency

- Domain Consistency

- Comparison

Search

Other Global Constraints

Can we do better?

- The alldifferent constraint is missing propagation
 - How can we do more propagation?
 - Do we know when we derive all possible information from the constraint?
- Constraints only interact by changing domains of variables

A Simpler Example

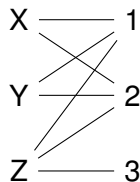
```
include "alldifferent.mzn";  
  
var 1..2:X;  
var 1..2:Y;  
var 1..3:Z;  
  
constraint alldifferent([X,Y,Z]);  
  
solve satisfy;
```

Using Forward Checking

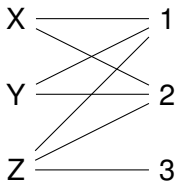
- No variable is assigned
- No reduction of domains
- But, values 1 and 2 can be removed from Z
- This means that Z is assigned to 3

Visualization of all different as Graph

- Show problem as graph with two types of nodes
 - Variables on the left
 - Values on the right
- If value is in domain of variable, show link between them
- This is called a *bipartite* graph



A Simpler Example



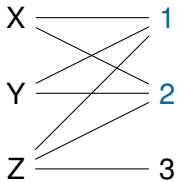
Value Graph for

```
var 1..2:X;
```

```
var 1..2:Y;
```

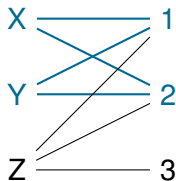
```
var 1..3:Z;
```

A Simpler Example



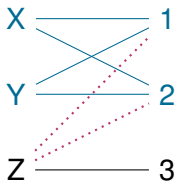
Check interval $[1,2]$

A Simpler Example



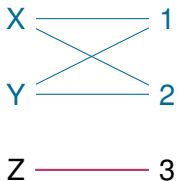
- Find variables completely contained in interval
- There are two: X and Y
- This uses up the capacity of the interval

A Simpler Example



No other variable can use that interval

A Simpler Example



Only one value left in domain of Z,
this can be assigned

Idea (Hall Intervals)

- Take each interval of possible values, say size N
- Find all K variables whose domain is completely contained in interval
- If $K > N$ then the constraint is infeasible
- If $K = N$ then no other variable can use that interval
- Remove values from such variables if their bounds change
- If $K < N$ do nothing
- Re-check whenever domain bounds change

Implementation

- Problem: Too many intervals ($O(n^2)$) to consider
- Solution:
 - Check only those intervals which update bounds
 - Enumerate intervals incrementally
 - Starting from lowest(highest) value
 - Using sorted list of variables
- Complexity: $O(n \log(n))$ in standard implementations
- Important: Only looks at min/max bounds of variables

Bounds Consistency

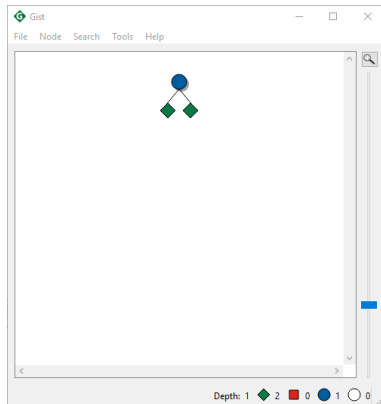
Definition

A constraint achieves *bounds consistency*, if for the lower and upper bound of every variable, it is possible to find values for all other variables between their lower and upper bounds which satisfy the constraint.

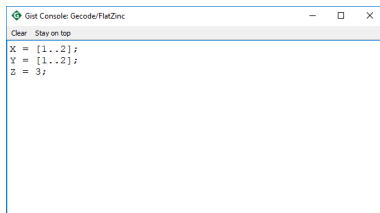
Annotation: :: bounds

```
include "alldifferent.mzn";  
  
var 1..2:X;  
var 1..2:Y;  
var 1..3:Z;  
  
constraint alldifferent([X,Y,Z]) :: bounds;  
solve satisfy;
```

Running with Gecode Gist



All Solutions



Node Inspector (Root)

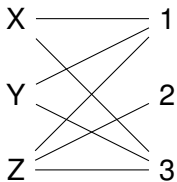
Can we do even better?

- Bounds consistency only considers min/max bounds
- Ignores “holes” in domain
- Sometimes we can improve propagation looking at those holes

Another Simple Example

```
include "alldifferent.mzn";  
  
var {1,3}:X; % note enumerated domain  
var {1,3}:Y;  
var 1..3:Z; % note domain as interval  
  
% annotated constraint  
constraint alldifferent([X,Y,Z]) :: bounds;  
solve satisfy;
```

Another Simple Example



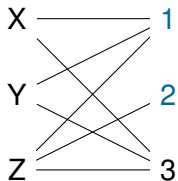
Value Graph for

```
var {1, 3}:X;
```

```
var {1, 3}:Y;
```

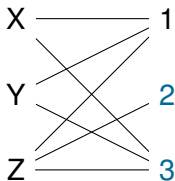
```
var 1..3:Z;
```


Another Simple Example



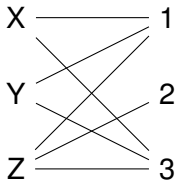
- Check interval $[1,2]$
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



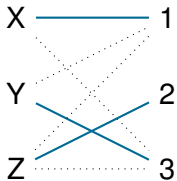
- Check interval $[2,3]$
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



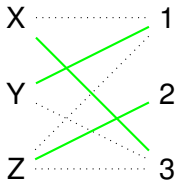
But, more propagation is possible,
there are only two solutions

Another Simple Example



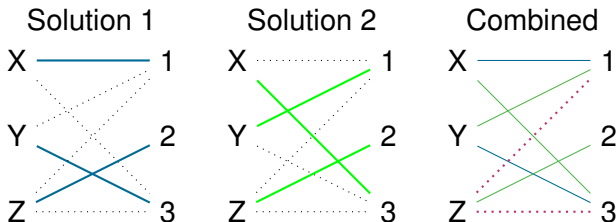
Solution 1: assignment in blue

Another Simple Example



Solution 2: assignment in green

Another Simple Example

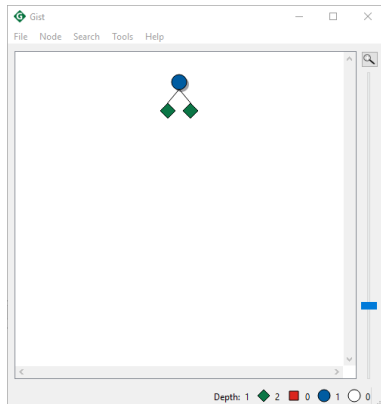


Combining solutions shows that $Z=1$ and $Z=3$ are not possible.

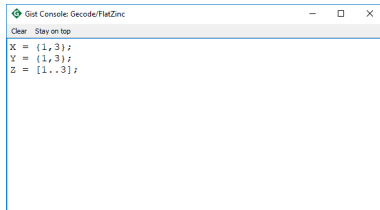
Another Simple Example

Can we deduce this without enumerating solutions?

Bounds Consistency with Gecode Gist: No Propagation



All Solutions



Node Inspector (Root)

Solutions and Maximal Matchings

- A *Matching* is subset of edges which do not coincide in any node
- No matching can have more edges than number of variables
- Every solution corresponds to a *maximal matching* and vice versa
- If a link does not belong to some maximal matching, then it can be removed

Implementation

- Possible to compute all links which belong to some matching
 - Without enumerating all of them!
- Enough to compute **one** maximal matching
- Requires algorithm for *strongly connected components*
- Extra work required if more values than variables
- All links (values in domains) which are not supported can be removed
- Complexity: $O(n^{1.5}d)$

Domain Consistency

Definition

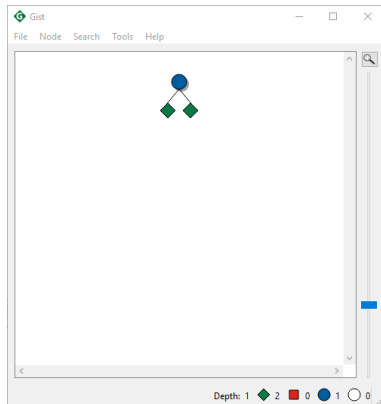
A constraint achieves *domain consistency*, if for every variable and for every value in its domain, it is possible to find values in the domains of all other variables which satisfy the constraint.

- Also called *generalized arc consistency (GAC)*
- or *hyper arc consistency*

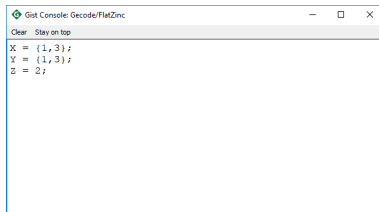
Simple Example Revisited

```
include "alldifferent.mzn";  
  
var {1,3}:X; % note enumerated domain  
var {1,3}:Y;  
var 1..3:Z; % note domain as interval  
  
% note different annotation  
constraint alldifferent([X,Y,Z]) :: domain;  
solve satisfy;
```

Domain Consistency with Gecode Gist: Propagation



All Solutions



Node Inspector (Root)

Can we still do better?

- NO! This extracts all information from this one constraint
- We could perhaps improve speed, but not propagation
- But possible to use different model
- Or model interaction of multiple constraints

Should all constraints achieve domain consistency?

- Domain consistency is usually more expensive than bounds consistency
 - Overkill for simple problems
 - Nice to have choices
- For some constraints achieving domain consistency is NP-hard
 - We have to live with more restricted propagation

Modified MiniZinc Program

```
int: s;  
int: n=s*s;  
array[1..n,1..n] of var 1..n: puzzle;  
  
include "sudoku.dzn";  
  
include "alldifferent.mzn";  
  
constraint forall(i in 1..n)  
    (alldifferent([puzzle[i,j] | j in 1..n])::domain);  
constraint forall(j in 1..n)  
    (alldifferent([ puzzle[i,j] | i in 1..n])::domain);  
constraint forall(i,j in 1..s)  
    (alldifferent([puzzle[s*(i-1)+p, s*(j-1)+q] |  
                    p,q in 1..s])::domain);  
  
solve satisfy;
```


Modified Choco-solver Sudoku Model

```
Model model = new Model("Sudoku");
int blockSize = 3;
int m = blockSize*blockSize;

IntVar[][] vars = new IntVar[m][m];
for(int i=0;i<m;i++){
    for(int j=0;j<m;j++){
        vars[i][j] = model.intVar("X"+i+" "+j, 1, m);
        if (data[i][j]>0) {
            model.arithm(vars[i][j],"=",data[i][j]).post();
        }
    }
}

// Consistency level AC: domain consistency, BC: bounds consistency, default: mix
for(int i=0;i<m;i++){
    model.allDifferent(row(i,m,vars),AC).post();
    model.allDifferent(column(i,m,vars),AC).post();
}
for(int i=0;i<m;i+=blockSize){
    for(int j=0;j<m;j+=blockSize){
        model.allDifferent(block(i,j,blockSize,vars),AC).post();
    }
}

Solver solver = model.getSolver();
solver.solve();
```

Initial State (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9	

Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
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» Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9 7	1 2 3 5 6 9 7	1 2 3 5 6 9 7	1 2 3 5 6 9 7	1 2 3 5 6 9 7
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◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 6 4 8 9	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9 7	1 2 3 5 6 9 7	1 3 5	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4	3 9	8	2	5
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 3 4 5 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9 7	1 2 3 5 6 9 7	1 3 5	1 2 3 5 6 7 9	1 2 3 4 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	4 5 8	3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3 2		
1 3 7 9	1 4	6	4 3 7 9	4 9	3 8	2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7	
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8	
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9	
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 5 6 7 8 9	2 3 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 9	1 2 3 5 6 7 9	6	1 2 4 6 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 7 9	2 3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	4	6	4 3 7 9	4 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 5 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 3 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4	8	1 5 6 9	4 5	3	2
1 3 9	4	6	3 7 9	3 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6 4	2 5	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9	1 3 5 7	1 2 6 9	1 3 5 6 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4	8	5 6 9	1 5	3	2
1 3 9	4	6	3 7 9	3 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5	2 3 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5 7	1 6 7 9	1 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
7	1 5 6	1 5 9	4	8	5 6 9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	1 3 7 9
5	9	2	2 3 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	4	3 8 9	5	1 3 6 4 9 8	1 3 4	1 6 4 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 7 9	3 5 6 7 8 9
1 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	¹ ₇	⁶ ₉	¹ ₇ ⁵ ₉ ⁶ ₉
³ ₆ ⁹	5	³ ₅ ⁹	1	7	2	4	6	8	
7	6	1	4	8	9	5	3	2	
1	4	6	³ ₇ ⁹	³ ₉	8	2	5	³ ₇ ⁹	
5	9	2	³ ₇	4	1	¹ ₄ ³ ₇	8	6	
8	3	7	6	2	5	9	4	1	
2	7	4	³ ₈ ⁹	5	6	8	1	¹ ₈ ³ ₉ ^{6₉}	
6	8	³ ₅ ⁹	2	1	4	³ ₅ ^{8₇ ²₇ ^{6₉}}	² ₇ ^{6₉}	5	
³ ₉	1	5	8	² ₉ ³	7	6	2	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7</small>	<small>9</small>	<small>7</small>	<small>9</small>
<small>3</small>	5	<small>3</small>	1	7	2	4	6	8		
7	6	1	4	8	9	5	3	2		
1	4	6	<small>7</small>	<small>3</small>	<small>9</small>	8	2	5	<small>3</small>	<small>7</small>
5	9	2	<small>7</small>	<small>3</small>	4	1	<small>7</small>	8	6	
8	3	7	6	2	5	9	4	1		
2	7	4	<small>3</small>	<small>9</small>	5	6	8	1	<small>3</small>	<small>9</small>
6	8	<small>3</small>	<small>9</small>	2	1	4	<small>7</small>	<small>3</small>	<small>7</small>	<small>9</small>
<small>3</small>	1	5	8	<small>3</small>	<small>9</small>	7	6	2	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7</small>	<small>9</small>	<small>7</small>	<small>9</small>
<small>3</small>	5	<small>3</small>	1	7	2	4	6	8		
<small>9</small>		<small>9</small>								
7	6	1	4	8	9	5	3	2		
1	4	6	<small>7</small>	<small>3</small>	<small>9</small>	8	2	5	<small>3</small>	<small>7</small>
5	9	2	<small>3</small>	<small>7</small>	4	1	<small>3</small>	8	6	
8	3	7	6	2	5	9	4	1		
2	7	4	<small>3</small>	<small>9</small>	5	6	8	1	<small>3</small>	<small>9</small>
6	8	<small>3</small>	<small>9</small>	2	1	4	<small>3</small>	<small>7</small>	<small>9</small>	5
<small>3</small>	<small>9</small>	1	5	8	<small>3</small>	7	6	2	4	

◀ Back to Start

▶ Skip Animation

Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

◀ Back to Start

After Setup (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9</small>	<small>7 9</small>
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3</small>	<small>7 9</small>	5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

Comparison

Forward Checking

4	1 2	8	2 3	3	2 3	1	1	6	5 6
3	2	5	3	1	7	2 3	4	5	6
6	5	6	9	4	5	8	6	5	6
7	5	1	5	4	5	8	6	5	6
1	4	6	7	9	9	8	2	5	7
5	9	2	4	7	4	7	7	8	7
8	3	7	6	2	5	9	4	1	3
2	7	4	9	8	6	5	3	1	3
3	6	5	3	2	3	1	4	5	3
6	8	5	9	7	8	9	7	8	9
3	1	1	3	2	3	2	3	7	8
9	8	5	9	7	8	9	7	8	9

Bounds Consistency

4	1 2	8	5	6	2 3	1	1		
3	2		3		2 3	4	5	6	5
7	6	1	5	4	8	9	1	5	3
1	4	6	7	9	9	8	2	5	7
5	9	2	3	4	1	7	8	6	
8	3	7	6	2	5	9	4	1	3
2	7	4		5	6		3	1	3
6	5		3	2	3	1	4	5	3
3	1	1	5	2	3	2	3	7	8
9	8	5	9	7	8	9	7	8	9
3	1	1	3	2	3	2	3	7	8
9	8	5	9	7	8	9	7	8	9

Domain Consistency

4	2	8	5	6	3	1			
3	5		1	7	2	4	6	8	
7	6	1	4	8	9	5	3	2	
1	4	6	7	9	9	8	2	5	7
5	9	2	7	4	1	7	8	6	
8	3	7	6	2	5	9	4	1	3
2	7	4		5	6	8	1		3
6	8		2	1	4	7	7	8	9
3	1	1	5	2	3	2	3	7	8
9	8	5	9	7	8	9	7	8	9
3	1	1	3	2	3	2	3	7	8
9	8	5	9	7	8	9	7	8	9

Typical?

- This does not always happen
- Sometimes, two methods produce same amount of propagation
- Possible to predict in certain special cases
- In general, tradeoff between speed and propagation
- Not always fastest to remove inconsistent values early
- But often required to find a solution at all

Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

Simple search routine

- Enumerate variables in given order
- Try values starting from smallest one in domain
- Complete, chronological backtracking
- Advantage: Results can be compared with each other
- Disadvantage: Usually not a very good strategy

Asking for Naive Search in MiniZinc

```
solve :: int_search(  
    puzzle,  
    input_order,  
    indomain_min)  
satisfy;
```

Search Tree (Forward Checking)

2

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 7 9	5 6 9
3 6 9	2 5 6 9	3 5 9	1 7	2 3 6 9	4 5 8	6 5 8 9	5 6 8 9	
6 7 9	1 5 6 9	1 5 9	4 5 8	8	1 6 7	3 4 5 7	2 3 6	
1 6 9	4 5 8	6 7 9	3 4 7	8 3 7	2 5 7	5 6 8	3 6 7	
5 6 9	9 2 7	2 4 7	3 4 7	3 1 7	3 3 7	8 9 7	3 6 7	
8 3 7	3 7 9	6 7 9	2 5 7	5 9 7	9 4 7	4 1 7	1 3 7	
2 7 9	7 3 9	1 4 9	3 8 9	5 6 9	3 1 9	3 1 9	3 3 9	
3 6 9	5 6 8	3 5 9	2 3 7	1 4 7	3 5 7	2 6 7	3 5 7	
3 1 9	1 5 8	1 3 9	2 3 7	3 2 7	6 1 7	2 2 7	4 4 7	

Search Tree (Forward Checking)

2
1
4

4	1	8	2 3 5 9	3 6 5 9	2 3 6 9	1 5 7 8	1 6 7 9	5 6 7 9
3 6 5 9	2 5 6 8	5 9	1 7	2 3 6 8	4 5 7 8	6 8 7 9	5 6 7 9	5 6 7 9
6 8 7 9	1 5 6 8	1 5 6 8	4 5 7 8	8	1 6 7 8	3 2		
1 4 6	7 8	9	8 2 5	3 6				
5 9 2	4 7	3 4	1 3 3	8	7			
8 3 7 6	2 5 9 4 1							
2 7	1 3 3 4 9 8 9	5	3 1 3 1 6 9 8					
3 6 5 8	5 9 7 8 9	1 4	3 2 7 8 7 9 7 8 9					
3 1 5 8	1 3 2 3 5 7 8 9	3 2 3 6 7 9	6	1 2 7 9	4			

Search Tree (Forward Checking)

2
1
4
2
5

4	1	8	2		3	2	3			6	5	6
	3	2	3		6	6	5		7	7	9	9
	6	5	5	1	7	3	4	5		6	5	6
	7	6	5	4	5	8		1		3	2	
1	4	6			3		8	2	5		7	3
5	9	2	4		3	1	3		8		7	6
8	3	7	6	2	5	9	4	1				
2	7				5							
	3	5	3		1	4						
	6	8	5		7	8	9	6	8	9		
	3	1	3		3	2	3		6	1	2	4

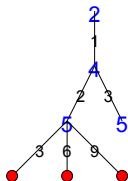
2
1
4
2
5
3
●

4	1	8	2	3	3		6	5		6	5	6
3	2			6	7		9	7	7	9	7	9
6	9	5	9	1	7		3	4	5	6	5	8
6	5	6	5	4	5	8	1	4	5	3	2	
7	9		9				9	7				3
1	4	6			8	2	5		7			
			7	9								
5	9	2		4	3	1	3		3		3	6
			7	4		7	7			7		
8	3	7	6	2	5	9	4	1				
			1	3	3		3	1	3	1		3
2	7		4			5		6				
			9	8	9			8		9	8	9
3	5	6	5			1	4		3	2		3
			9	7	8	9		7	8	7	9	8
9			1	3	3		2	3		1	2	
	5		9	7	8	9		9	7	9		
	8							6		7	9	4

[illegible]

4	1	8	2	9	3	3						
3	2			9	6	5			7	6	5	6
6	5	5	3	1	7	3	4	5		6	5	6
9		5	9			9	8		9	5	8	9
				4	5		1					
7	6	5	5	5	9	8	6	4	5	3	2	
1	4	6			3	8	2	5			3	
			7	9	9					7		
5	9	2	4	3	3	1	3		3		3	6
			7	4	7		7			8	7	
8	3	7	6	2	5	9	4	1				
2	7		1	3		3	1	3	1		3	
			4	9	8	9	6	9		8	9	8
3	6	5	6	5	3	3		3	2		3	5
9	8			7	8	9		5		9	7	8
								7	8	7	9	7
3		5	1	3	3	3	2	3		1	2	
			5									
9	8		9	7	8	9	9	7	6	7	9	4

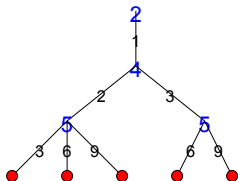
Search Tree (Forward Checking)



4	1	8	3	3	2	3			6	5	6
3	2	3	1	7	2	3	5		7	9	9
6	5	5	4	5	8	1	3	2		6	5
7	6	5	8		6	4	5		6	5	6
1	4	6	7	8	8	2	5		7		3
5	9	2	4	3	1	3	3		8	3	6
8	3	7	6	2	5	9	4	1			
2	7	1	3	5	3	1	3	1		3	
3	6	3	2	1	6	8			9	8	9
9	8	5	7	8	9	7	8	9	7	8	9
3	1	3	2	3	3	2	3	1	2		
9	5	5	9	7	8	9	9	7	9	4	

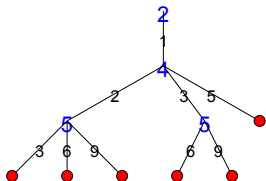
[illegible]

Search Tree (Forward Checking)



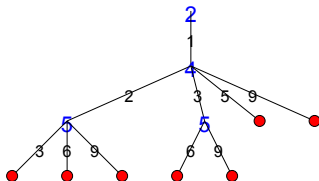
4	1	8	3	9	2				6	5	6
	3	2	3	1	7		2		6	4	5
	6	5	5	4	5	8		1	6	4	5
7	6	5	5	4	5	8		6	4	5	3
1	4	6				8	2	5			3
5	9	2	4	4	3	1	3	3	8		3
8	3	7	6	2	5	9	4	1			
2	7		1	3		3	1	3	1		3
	3	6	5	3	2	1	4		3	2	3
	6	5	5	7	8	9	1	4	7	5	6
3	1	5	1	3	2	3	2	3	6	1	2
9	8	9	7	8	9	9	7	9	7	9	4

Search Tree (Forward Checking)



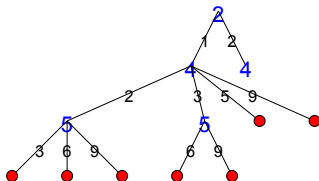
4	1	8	5	3	2	3	5	6	5	6
3	2	5	1	7	2	3	4	5	6	5
7	6	5	4	5	8	1	6	4	5	3
1	4	6	7	8	2	5	7	3	6	1
5	9	2	4	3	1	3	3	8	7	6
8	3	7	6	2	5	9	4	1	3	6
2	7	1	3	3	5	3	1	3	1	3
3	6	5	2	3	1	4	7	5	7	8
3	5	1	3	2	3	3	2	3	1	2
9	8	9	7	8	9	9	7	9	4	8

Search Tree (Forward Checking)



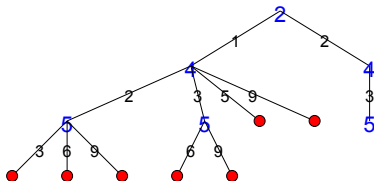
4	1	8	9	3	2	3		6	5	6
	3	2	5	3	1	7			7	9
7	6	5	5	4	5	8		1	3	2
1	4	6		3	3	8	2	5		3
5	9	2	4	3	1	3	3	8		3
8	3	7	6	2	5	9	4	1		
2	7		3	3	5		3	1	3	3
	3	5	3	2	3	1	4		2	3
3	6	5	9	7	8	9	1	4	7	8
3	1	3	2	3	3	2	3	6	1	2
9	5	8	9	7	8	9	9	7	9	4

Search Tree (Forward Checking)



4	2	8	2 3 5 9	3 3 6 9	2 3 5 9	1 5 6 7	1 6 7 9	5 6 8 9
3 6 9 9	2 5 6 9	5 5	1 7	2 3 6 9	4 5 8 9	6 6 7 9	5 6 8 9	
1 5 6 7	1 5 6 9	4 5 8 9	8	1 1 6 7	3 2 4 5 8 9	3 2 4 5 8 9		
1 4 5 9	6 6 7 9	7 8 9 9	8	2 5 3 6	8 2 9 7	5 7 6 9		
8 3 2 7	7 6 8 9	2 5 3 6	2 5 3 6	9 4 1 7	9 4 1 7	4 1 5 8		
2 7 3 6	1 3 4 9	3 3 8 9	5	3 1 6 8	3 1 4 9	3 1 4 9		
3 6 9 9	5 5 8 9	2 3 7 8	1 4	3 2 6 9	3 2 4 9	5 6 8 9		
3 1 9 8	1 5 6 9	2 3 7 8	3 2 6 9	1 2 4 9	6 7 8 9	4 1 5 8		

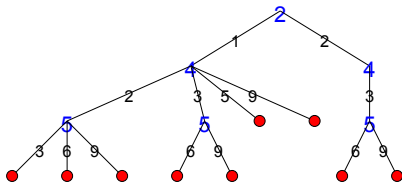
Search Tree (Forward Checking)



4	2	8	3	3	3	1	1		
3	6	5	6	6	6	5	7	6	5
9	9	9	9	9	9	9	9	9	9
1	4	6	7	8	2	5	7		
5	9	2	4	3	1	3	3	8	3
8	3	7	6	2	5	9	4	1	
2	7	1	3	5	3	1	3	1	3
3	6	5	3	6	9	8	9	8	9
9	9	9	9	9	9	9	9	9	9
3	1	5	1	3	2	3	3	2	3
9	9	9	9	9	9	9	9	9	9
6	5	7	8	9	7	8	9	7	8
9	9	9	9	9	9	9	9	9	9
3	1	5	1	3	2	3	3	2	3
9	9	9	9	9	9	9	9	9	9
6	5	7	8	9	7	8	9	7	8
9	9	9	9	9	9	9	9	9	9

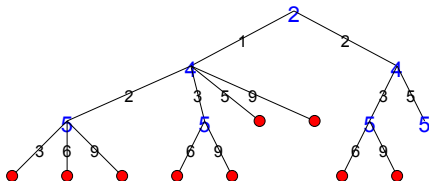
[illegible]

Search Tree (Forward Checking)



4	2	8	3	9	6	1	5	1	6	5	6
3	6	5	5	3	1	7	2	4	5	8	6
6	9	5	9	9			9	8	9	8	9
1	5	6	1	5	4	5	8	1	6	5	6
7	6	9	5	9	9	8	6	4	5	3	2
1	4	6			3	8	2	5		3	
			7	9	9					7	
5	9	2	4	7	4	1	3	3		3	6
						7	7			8	7
8	3	7	6	2	5	9	4	1			
2	7	1	3	4							
		9	8	9	5	3	6	9	1	3	1
3	6	5	6	5	3	2			3	2	3
9	8	9	8	9	7	8	9	5	7	8	5
3	1	5	9	1	3	2	3	2	3	1	2
9	8	5	9	7	8	9	9	7	9	6	4

Search Tree (Forward Checking)

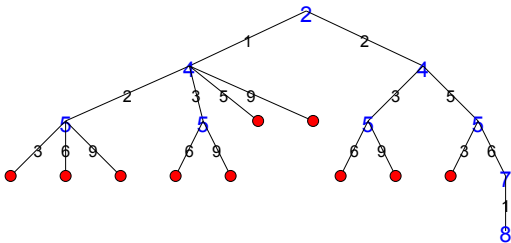


4	2	8	5	3	3	1	1		
3	6	5	1	7	2	4	5	6	5
6	9	5	4	8	6	4	5	6	5
7	9	5	4	8	6	4	5	6	5
1	4	6	8	2	5	7			
5	9	2	4	3	1	3	3	8	3
8	3	7	6	2	5	9	4	1	
2	7	4	9	5	3	1	3	1	3
3	6	5	1	4	5	6	7	9	7
3	1	5	1	3	2	3	3	2	3
9	8	5	9	7	8	9	7	8	9
9	8	5	9	7	8	9	7	8	9

[illegible]

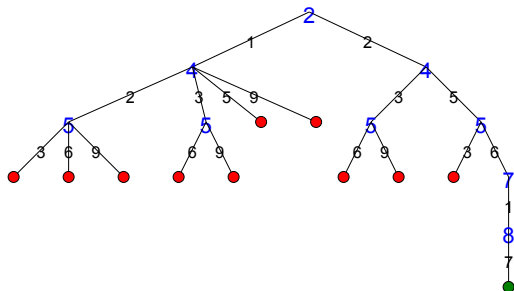
4	2	8	5	6	3	1	1	6	6
3 6 9	5 6	5 9	3 9	1	7	2	4 5 8	6 9 8	5 6 8
1 6 9	5 6	1 5	4	8	9	1 4 5	3	2	
1	4	6			8	2	5		
5	9	2			1		8		
8	3	7	6	2	5	9	4	1	
2	7			5	6				
3 6 9	5 6 8	5 9	3 2 3	1	4		3 2 3	6 8 9	
3 1 9	5 9	1 5	3 2 3		7	6	1 2 9		4

Search Tree (Forward Checking)



4	2	8	5	6	3	1		
				1	7	2		
				4	8	9		3
1	4	6			8	2	5	
5	9	2			1		8	
8	3	7	6	2	5	9	4	1
2	7			5	6			
				1	4			
				7	6			4

Search Tree (Forward Checking)



4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

Search Tree (Bounds Consistency)

2

4	1 2	8	5	6	2 3	1	1		
3	2 5	3	1	7	2 3	4 5 8	6 5 9	7 9	9
7	6	1 5	4	8	9	1 5	3	2	
1	4	6	7 9	3 9	8	2	5	7	3
5	9	2	7 3	4 3	1	7 3	8	6	
8	3	7	6	2	5	9	4	1	
2	7	4	3 8 9	5	6	1 3 1	8 9	8 9	3
6	5 8	3 2 3	1	4	7 8	5 3 2	7 8 9	5 8 9	3
3 1	5 8	1 5	2 8	3 9	7	6	1 2	4	

2
1
●

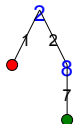
4	1	8	5	6					
				1	7				
7	6		4	8				3	2
1	4	6			8	2	5		
5	9	2			1			8	6
8	3	7	6	2	5	9	4	1	
2	7	4		5	6				
6				1	4				
					7	6			4

Search Tree (Bounds Consistency)



4	2	8	5	6	3	1		
3	5		3	1	7	2	4	6
7	6	1	4	8	9	5	3	2
1	4	6		8	2	5		
5	9	2		1		8	6	
8	3	7	6	2	5	9	4	1
2	7	4		5	6	8	1	
6	8		2	1	4			5
	1	5	8	7	6	2	4	

Search Tree (Bounds Consistency)



4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

Search Tree (Domain Consistency)

8

4	2	8	5	6	3	1		
³ ₉	5	³ ₉	1	7	2	4	⁷ ₉	⁷ ₉
7	6	1	4	8	9	5	3	2
1	4	6	³ ₇	³ ₉	8	2	5	³ ₇
5	9	2	³ ₇	4	1	³ ₇	8	6
8	3	7	6	2	5	9	4	1
2	7	4	³ ₉	5	6	8	1	³ ₉
6	8	³ ₉	2	1	4	³ ₇	⁷ ₉	5
³ ₉	1	5	8	³ ₉	7	6	2	4

Search Tree (Domain Consistency)

8
7
●

4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

Trading Propagation Against Search

- If we perform more propagation, search is more constrained
- Fewer values left, fewer alternatives to explore in search
- Best compromise is not obvious
- But can be learned from examples or during search
- Annotations are optional
 - Some MiniZinc back-end solvers do the search they want, not the one you specify
 - Some solvers simply do not work in a way that these search annotations apply

Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

Are there other Global Constraints?

- alldifferent is the most commonly used constraint
- Propagation methods can be explained
- But there are many more

Global Constraint Catalog

- <https://sofdem.github.io/gccat/>
- Description of 354 global constraints, 2800 pages
- Not all of them are widely used
- Detailed, meta-data description of constraints in Prolog

Families of Global Constraints

- Value Counting
 - alldifferent, global cardinality
- Scheduling
 - cumulative
- Properties of Sequences
 - sequence, no_valley
- Graph Properties
 - circuit, tree

Common Algorithmic Techniques

- Bi-Partite Matchning
- Flow Based Algorithms
- Automata
- Task Intervals
- Reduced Cost Filtering
- Decomposition