

Centre for  
Data Analytics



# Basic Modelling

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<https://eclipseclp.org/ELearning/index.html>.

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## Example 2: Sudoku

- Global Constraints
  - Powerful modelling abstractions
  - Non-trivial propagation
  - Different consistency levels
- Example: Sudoku puzzle

# Outline

## Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

# Problem Definition

## Sudoku

Fill in numbers from 1 to 9 so that each row, column and 3x3 block contain each number exactly once

4	1 2 3	8	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	1 7	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	8	4 5 6	4 5 6	3 2	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	6	4 5 6	4 5 6	8 2 5	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	9	4 5 6	4 5 6	8	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	3 7 6	4 5 6	4 5 6	9	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	5	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	1 4	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	6	4 5 6	4
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9

## Problem Definition

# Sudoku

Fill in numbers from 1 to 9 so that each row, column and 3x3 block contain each number exactly once

4	1	2		1	2	3	2	3	1	2	3	2	3				
4	5	6	8		4	5	6	4	5	6	4	5	6				
7	8	9		7	8	9	7	8	9	7	8	9	7	8	9		
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1		
4	5	6	4	5	6	1	7		4	5	6	4	5	6	4		
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
4	5	6	4	5	6	8		4	5	6	8		4	5	6	8	
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
4	5	6	4	5	6	1	7		4	5	6	4	5	6	4	5	6
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
4	5	6	4	5	6	8	2		4	5	6	8	2		4	5	6
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
4	5	6	4	5	6	1	7		4	5	6	4	5	6	4	5	6
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
4	5	6	4	5	6	1	7		4	5	6	4	5	6	4	5	6
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
4	5	6	4	5	6	1	7		4	5	6	4	5	6	4	5	6
7	8	9	7	8	9	7	8	9	7	8	9	7	8	9	7	8	9
1	2	3	1	2													

4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

# Model

- A variable for each cell, ranging from 1 to 9
- A 9x9 matrix of variables describing the problem
- Preassigned integers for the given hints
- `alldifferent` constraints for each row, column and 3x3 block



# Sudoku Models

- ECLiPSe [▶ Show](#)
- MiniZinc [▶ Show](#)
- NumberJack [▶ Show](#)
- CPMpy [▶ Show](#)
- Choco-solver [▶ Show](#)

# ECLiPSe Sudoku Model (from <https://eclipseclp.org/>)

```
:- lib(ic).
:- import alldifferent/1 from ic_global.

top :-
    problem(Board),
    print_board(Board),
    sudoku(Board),
    labeling(Board),
    print_board(Board).

sudoku(Board) :-
    dim(Board, [N,N]),
    Board :: 1..N,
    ( for(I,1,N), param(Board) do
        alldifferent(Board[I,*]),
        alldifferent(Board[*,I])
    ),
    NN is integer(sqrt(N)),
    ( multifer([I,J],1,N,NN), param(Board,NN) do
        alldifferent(concat(Board[I..I+NN-1, J..J+NN-1]))
    ).

print_board(Board) :-
    dim(Board, [N,N]),
    ( for(I,1,N), param(Board,N) do
        ( for(J,1,N), param(Board,I) do
            X is Board[I,J],
            ( var(X) -> write("  ") ; printf(" %2d", [X]) )
        ), nl
    ), nl.
```

# ECLiPSe Data Definition

```
problem([(  
  [(4, _, 8, _, _, _, _, _),  
   (_, _, _, 1, 7, _, _, _),  
   (_, _, _, _, 8, _, _, 3, 2),  
   (_, _, 6, _, _, 8, 2, 5, _),  
   (_, 9, _, _, _, _, 8, _),  
   (_, 3, 7, 6, _, _, 9, _, _),  
   (2, 7, _, _, 5, _, _, _),  
   (_, _, _, _, 1, 4, _, _, _),  
   (_, _, _, _, _, 6, _, 4)])]).
```

► Continue

# MiniZinc Sudoku Model

```
int: s;  
int: n=s*s;  
array[1..n,1..n] of var 1..n: puzzle;  
  
include "sudoku.dzn";  
  
predicate alldifferent(array[int] of var int: x) =  
  forall(i,j in index_set(x) where i < j)  
    (x[i] != x[j]);  
  
constraint forall(i in 1..n)  
  (alldifferent([puzzle[i,j] | j in 1..n]));  
constraint forall(j in 1..n)  
  (alldifferent([ puzzle[i,j] | i in 1..n]));  
constraint forall(i,j in 1..s)  
  (alldifferent([puzzle[s*(i-1)+p, s*(j-1)+q] |  
    p,q in 1..s]));  
solve satisfy;
```

# MiniZinc Output

```
output [ "sudoku:\\n" ] ++  
  [ show(puzzle[i,j]) ++  
    if j = n then  
      if i mod s = 0 /\ i < n then "\\n\\n"  
      else "\\n"  
      endif  
    else  
      if j mod s = 0 then " "  
      else " "  
      endif  
    endif  
  | i,j in 1..n ];
```

# MiniZinc Data File (sudoku.dzn)

```
s=3;
puzzle=[ |
  4, _, 8, _, _, _, _, _ |
  _, _, _, 1, 7, _, _, _ |
  _, _, _, _, 8, _, _, 3, 2 |
  _, _, 6, _, _, 8, 2, 5, _ |
  _, 9, _, _, _, _, _, 8, _ |
  _, 3, 7, 6, _, _, 9, _, _ |
  2, 7, _, _, 5, _, _, _, _ |
  _, _, _, _, 1, 4, _, _, _ |
  _, _, _, _, _, 6, _, 4 |
| ];
```

▶ Continue

# NumberJack Sudoku Model

```
from Numberjack import *

def get_model(N, clues):
    grid = Matrix(N*N, N*N, 1, N*N)

    sudoku = Model([AllDiff(row) for row in grid.row],
                   [AllDiff(col) for col in grid.col],
                   [AllDiff(grid[x:x + N, y:y + N]) for x in range(0, N*N, N)
                                                            for y in range(0, N * N, N)],
                   [(x == int(v)) for x, v in
                    zip(grid.flat, "".join(open(clues)).split()) if v != '*']
                  ])
    return grid, sudoku

def solve(param):
    N = param['N']
    clues = param['file']

    grid, sudoku = get_model(N, clues)

    solver = sudoku.load(param['solver'])
    solver.setVerbosity(param['verbose'])
    solver.setTimeLimit(param['tcutoff'])

    solver.solve()
```

# NumberJack Data File

```
4 * 8 * * * * * *
* * * 1 7 * * * *
* * * * 8 * * 3 2
* * 6 * * 8 2 5 *
* 9 * * * * * 8 *
* 3 7 6 * * 9 * *
2 7 * * 5 * * * *
* * * * 1 4 * * *
* * * * * 6 * 4
```

▶ Continue



# CPMpy Sudoku Model (from <https://github.com/CPMpy/>)

```
import numpy as np
from cpmPy import *

# Variables
puzzle = intvar(1,9, shape=given.shape, name="puzzle")

model = Model(
    # Constraints on values (cells that are not empty)
    puzzle[given!=e] == given[given!=e], # numpy's indexing, vectorized equality
    # Constraints on rows and columns
    [AllDifferent(row) for row in puzzle],
    [AllDifferent(col) for col in puzzle.T], # numpy's Transpose
)

# Constraints on blocks
for i in range(0,9, 3):
    for j in range(0,9, 3):
        model += AllDifferent(puzzle[i:i+3, j:j+3]) # python's indexing

model.solve()
```

# CPMpy Data Definition

```
e = 0 # value for empty cells
given = np.array([
    [4, e, 8, e, e, e, e, e, e],
    [e, e, e, 1, 7, e, e, e, e],
    [e, e, e, e, 8, e, e, 3, 2],
    [e, e, 6, e, e, 8, 2, 5, e],
    [e, 9, e, e, e, e, e, 8, e],
    [e, 3, 7, 6, e, e, 9, e, e],
    [2, 7, e, e, 5, e, e, e, e],
    [e, e, e, e, 1, 4, e, e, e],
    [e, e, e, e, e, e, 6, e, 4]
])
```

▶ Continue

# Choco-solver Sudoku Model

```
Model model = new Model("Sudoku");
int blockSize = 3;
int m = blockSize*blockSize;

IntVar[][] vars = new IntVar[m][m];
for(int i=0;i<m;i++){
    for(int j=0;j<m;j++){
        vars[i][j] = model.intVar("X"+i+" "+j, 1, m);
        if (data[i][j]>0) {
            model.arithm(vars[i][j],"=",data[i][j]).post();
        }
    }
}
for(int i=0;i<m;i++){
    model.allDifferent(row(i,m,vars)).post();
    model.allDifferent(column(i,m,vars)).post();
}
for(int i=0;i<m;i+=blockSize){
    for(int j=0;j<m;j+=blockSize){
        model.allDifferent(block(i,j,blockSize,vars)).post();
    }
}
Solver solver = model.getSolver();
solver.solve();
```

# Choco-solver Data

```
int[][] data = new int[m][m]{  
    {4, 0, 8, 0, 0, 0, 0, 0, 0},  
    {0, 0, 0, 1, 7, 0, 0, 0, 0},  
    {0, 0, 0, 0, 8, 0, 0, 3, 2},  
    {0, 0, 6, 0, 0, 8, 2, 5, 0},  
    {0, 9, 0, 0, 0, 0, 0, 8, 0},  
    {0, 3, 7, 6, 0, 0, 9, 0, 0},  
    {2, 7, 0, 0, 5, 0, 0, 0, 0},  
    {0, 0, 0, 0, 1, 4, 0, 0, 0},  
    {0, 0, 0, 0, 0, 0, 6, 0, 4}  
};
```

# Choco-solver Utilities

```
IntVar[] row(int row, int size, IntVar[][] array){
    return array[row];
}

IntVar[] column(int col,int size,IntVar[][] array){
    IntVar[] column = new IntVar[size];
    for(int i=0; i<size; i++){
        column[i] = array[i][col];
    }
    return column;
}

IntVar[] block(int x,int y,int blockSize,IntVar[][] array){
    IntVar[] block = new IntVar[blockSize*blockSize];
    int k=0;
    for(int i=0;i<blockSize;i++){
        for(int j=0;j<blockSize;j++){
            block[k++] = array[x+i][y+j];
        }
    }
    return block;
}
```

► Continue

# Domain Visualizer

- Problem shown as matrix
- Each cell corresponds to a variable
- Instantiated: Shows integer value (large)
- Uninstantiated: Shows values in domain

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	3	2	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	1 2 3 4 5 6 7 8 9

# Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

# Initial State (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9	



# Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

►► Skip Animation

# Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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## Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1	3	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	3	2
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1	3	1 2 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1	3	5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 7 9	1 4 5 6 7 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	8 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9
1 5 6 7 8 9	3 4 5 6 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 4 5 6 7 9	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	1 3 4 7 9	8	2	5	1 3 4 7 9	
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 4 5 9	1 4 5 4 5 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9	
1 3 7 9	1 4	6	4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 9	9	1 2 3 4 5 7	2 3 4 5 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4	3 8	2	5	1 3 4 7 9
1 3 5 6 7 9	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	2 3 4 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 3 4 5 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9 7	1 2 3 5 6 9 7	1 3 5	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9 8	3 4 5 8	2 3 4 5 8 9	2 3 4 5 8 9	2 3 4 5 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2	
1 3 7 9	1 4	6	4 3 7 9	4 9	3 8	2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 7	
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8	
2	7	1 3 4	3 4 9 8 9	5	1 3 6 4 9 8	1 3 6 4 9 8	1 3 4 6 8 9	1 3 4 6 8 9	
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	2 3 5 6 7 8 9	2 3 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9 7	1 2 3 5 6 9 7	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 9	1 2 3 5 6 7 9	6	1 2 4 6 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 6 9	1 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 3 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	3 5 6 8 9
1 5 6 7 9	1 5 6	1 5	4 5 7 9	8	1 5 6 7 9	1 4 5	3	2
1 3 7 9	1 4	6	4 3 7 9	3 4	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 4 6 8 9	1 3 4 6 8 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9	1 3 5 7	1 2 6 7	1 3 5 6 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
1 5 6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5	1 7 9	1 5 6 9
5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
1 5 6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 4 5 9 7	3	2	
1	4	6	4 7 9	3 4 9	8	2	5	1 3 7 9
5	9	2	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 5 6 7	8	1 3 5 6 7
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5 7	1 6 9 7	1 5 6 9
3 6 9	2 5 6	3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 4 5 9 7	3 4 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	3 4 7	3 4 6 7	1 3 3 1 6 4 7	1 3 8	1 3 6 7	
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9 8	1 3 4 6 9 8	1 3 6 8 9	
3 6 7 9	2 5 6 8	3 5 9 7 8 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9 7 8 9	
3 7 9	1 2 5 8	1 5 9	3 7 8 9	2 3 5	2 3 1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5 7	1 6 7 9	1 5 6 9
3 6 9	2 5 6	3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9	8	5 6 4 5 9 7	3 4 5 7	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	3 4 7	3 4 7	1 3 7	1 3 4 7	8	1 3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 6 7 9	2 5 6 8	3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 7 9	3 5 6 7 8 9
3 7 9	1 2 5 8	1 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 9	5 6 7 9
3 6 9	2 5 6	5 9	1	7	2 3 6 4 5 9 8		6 9	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9	8	1 6 4 5 9 7	3	2	
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 6 9	3 6 8 9
3 6 7 9	2 5 6 8	3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
3 5 7 9	1 2 5 8	1 5 9	2 3 5 7 8 9	3	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 9	5 6 7 9
	3 6 9	2 5 6	3 5 9	1 7	2 3 6 9	4 5 8	6 9	5 6 8 9
	1 6 9	1 5 6	1 5 9	4 5 9	8	1 6 9	4 5 7	3 2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 8	1 6 9	3 6 8 9
3 6 9	5 6 8	5 9	2 3 5 7 8 9	1 4	3 5 7 8	2 7	3 6 7 9	3 5 6 7 8 9
3 9	1 5 8	1 5 9	2 3 5 7 8 9	3 9	1 2 3 7	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 6 7	1 6 7	5 6 9
3 6 9	2 5 6	5 9	1	7	2 3 6 4 5 9 8		6 9	5 6 8 9
1 6 7 9	1 5 6	1 5 9	4 5 9	8	1 6 4 5 9 7	3	2	
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4 7	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 8 9	5	3 6 9	1 3 4 8	1 6 9	3 6 8 9
3 6 9	5 6 8	5 9	3 2 3 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 8 9
3 9	1 5 8	1 5 9	3 2 3 7 8 9	3 9	2 3 7 9	6	1 2 7 9	4

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# Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 9	5 6 7 9
3 6 9	2 5 6	5 9	1	7	2 3 6 4 5 9 8		6 9	5 6 8 9
1 6 7	1 5 6	1 5 9	4 5 9	8	1 6 4 5 9 7	3	2	
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4	3 8 9	5	3 1 6 8 9	3 1 9	1 9	3 8 9
3 6 9	5 6 8	5 9	2 3 7 8 9	1	4	5 3 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 5 9	2 3 7 8 9	3 9	2 3 7 9	6	1 2 7 9	4

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# After Setup (Forward Checking)

4	<sup>1 2</sup> 5 6	8	<sup>2 3</sup> 5 9	<sup>3</sup> 6 9	<sup>2 3</sup> 6 9	<sup>1</sup> 5 7	<sup>1</sup> 6 7	<sup>5 6</sup> 7 9
<sup>3</sup> 6 9	<sup>2</sup> 5 6	<sup>3</sup> 5 9	1	7	<sup>2 3</sup> 6 9	<sup>4 5</sup> 8 9	<sup>6</sup> 7 9	<sup>5 6</sup> 8 9
<sup>6</sup> 7 9	<sup>1</sup> 5 6	<sup>1</sup> 5 9	<sup>4 5</sup> 7 9	8	<sup>1</sup> 6 9	<sup>4 5</sup> 7 9	3	2
1	4	6	<sup>3</sup> 7 9	<sup>3</sup> 7 9	8	2	5	<sup>3</sup> 7 9
5	9	2	<sup>4</sup> 7 9	<sup>3</sup> 7 9	<sup>1 3</sup> 7 9	<sup>3</sup> 7 9	8	<sup>3</sup> 6 7
8	3	7	6	2	5	9	4	1
2	7	<sup>1 3</sup> 4 9	<sup>3</sup> 7 9	5	<sup>3 1</sup> 6 9	<sup>3 1</sup> 7 9	<sup>1</sup> 8 9	<sup>3</sup> 7 9
<sup>3</sup> 6 9	<sup>5 6</sup> 7 9	<sup>3</sup> 5 9	<sup>2 3</sup> 7 9	1	4	<sup>5</sup> 7 9	<sup>2</sup> 7 9	<sup>3</sup> 5 7
<sup>3 1</sup> 5 9	<sup>1</sup> 5 9	<sup>3</sup> 7 9	<sup>2 3</sup> 7 9	<sup>3</sup> 7 9	<sup>2 3</sup> 7 9	6	<sup>1 2</sup> 7 9	4

# Outline

Problem

Initial Propagation (Forward Checking)

**Improved Reasoning**

- Bounds Consistency

- Domain Consistency

- Comparison

Search

Other Global Constraints

# Can we do better?

- The alldifferent constraint is missing propagation
  - How can we do more propagation?
  - Do we know when we derive all possible information from the constraint?
- Constraints only interact by changing domains of variables



# A Simpler Example

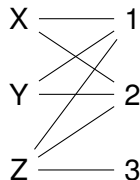
```
include "alldifferent.mzn";  
  
var 1..2:X;  
var 1..2:Y;  
var 1..3:Z;  
  
constraint alldifferent([X,Y,Z]);  
  
solve satisfy;
```

# Using Forward Checking

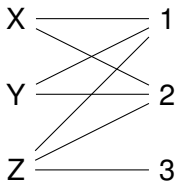
- No variable is assigned
- No reduction of domains
- But, values 1 and 2 can be removed from Z
- This means that Z is assigned to 3

# Visualization of alldifferent as Graph

- Show problem as graph with two types of nodes
  - Variables on the left
  - Values on the right
- If value is in domain of variable, show link between them
- This is called a *bipartite* graph



# A Simpler Example



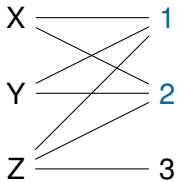
Value Graph for

```
var 1..2:X;
```

```
var 1..2:Y;
```

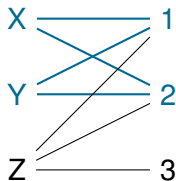
```
var 1..3:Z;
```

# A Simpler Example



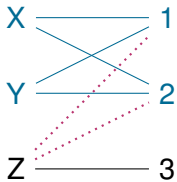
Check interval  $[1,2]$

# A Simpler Example



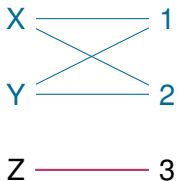
- Find variables completely contained in interval
- There are two: X and Y
- This uses up the capacity of the interval

# A Simpler Example



No other variable can use that interval

# A Simpler Example



Only one value left in domain of Z,  
this can be assigned



# Idea (Hall Intervals)

- Take each interval of possible values, say size  $N$
- Find all  $K$  variables whose domain is completely contained in interval
- If  $K > N$  then the constraint is infeasible
- If  $K = N$  then no other variable can use that interval
- Remove values from such variables if their bounds change
- If  $K < N$  do nothing
- Re-check whenever domain bounds change

# Implementation

- Problem: Too many intervals ( $O(n^2)$ ) to consider
- Solution:
  - Check only those intervals which update bounds
  - Enumerate intervals incrementally
  - Starting from lowest(highest) value
  - Using sorted list of variables
- Complexity:  $O(n \log(n))$  in standard implementations
- Important: Only looks at min/max bounds of variables

# Bounds Consistency

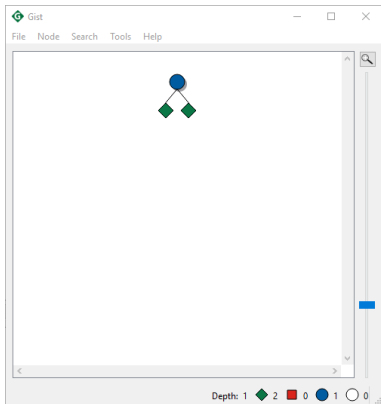
## Definition

A constraint achieves *bounds consistency*, if for the lower and upper bound of every variable, it is possible to find values for all other variables between their lower and upper bounds which satisfy the constraint.

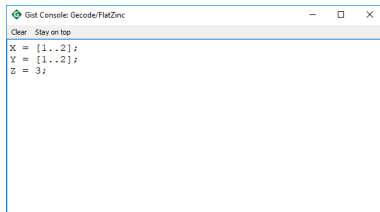
## Annotation: :: bounds

```
include "alldifferent.mzn";  
  
var 1..2:X;  
var 1..2:Y;  
var 1..3:Z;  
  
constraint alldifferent([X,Y,Z]) :: bounds;  
solve satisfy;
```

# Running with Gecode Gist



All Solutions



Node Inspector (Root)

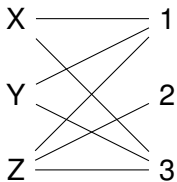
# Can we do even better?

- Bounds consistency only considers min/max bounds
- Ignores “holes” in domain
- Sometimes we can improve propagation looking at those holes

## Another Simple Example

```
include "alldifferent.mzn";  
  
var {1,3}:X; % note enumerated domain  
var {1,3}:Y;  
var 1..3:Z; % note domain as interval  
  
% annotated constraint  
constraint alldifferent([X,Y,Z]) :: bounds;  
solve satisfy;
```

# Another Simple Example



Value Graph for

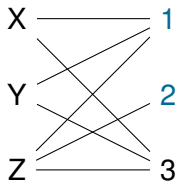
```
var {1, 3}:X;
```

```
var {1, 3}:Y;
```

```
var 1..3:Z;
```

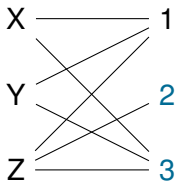


# Another Simple Example



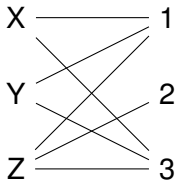
- Check interval  $[1,2]$
- No domain of a variable completely contained in interval
- No propagation

## Another Simple Example



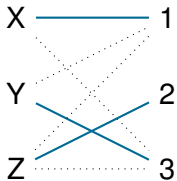
- Check interval  $[2,3]$
- No domain of a variable completely contained in interval
- No propagation

## Another Simple Example



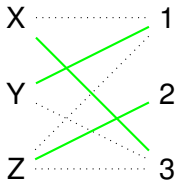
But, more propagation is possible,  
there are only two solutions

# Another Simple Example



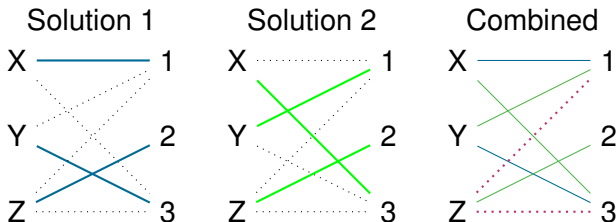
Solution 1: assignment in blue

# Another Simple Example



Solution 2: assignment in green

## Another Simple Example

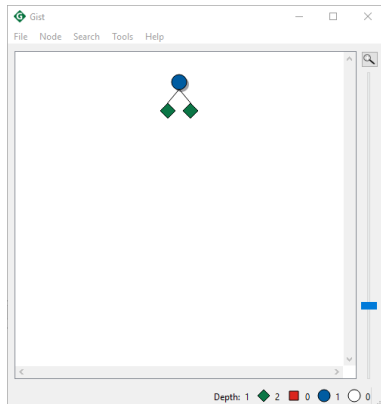


Combining solutions shows that  $Z=1$  and  $Z=3$  are not possible.

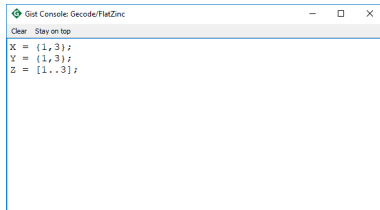
# Another Simple Example

Can we deduce this without enumerating solutions?

# Bounds Consistency with Gecode Gist: No Propagation



All Solutions



Node Inspector (Root)



# Solutions and Maximal Matchings

- A *Matching* is subset of edges which do not coincide in any node
- No matching can have more edges than number of variables
- Every solution corresponds to a *maximal matching* and vice versa
- If a link does not belong to some maximal matching, then it can be removed

# Implementation

- Possible to compute all links which belong to some matching
  - Without enumerating all of them!
- Enough to compute **one** maximal matching
- Requires algorithm for *strongly connected components*
- Extra work required if more values than variables
- All links (values in domains) which are not supported can be removed
- Complexity:  $O(n^{1.5}d)$

# Domain Consistency

## Definition

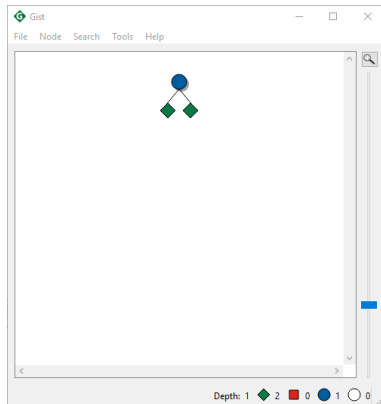
A constraint achieves *domain consistency*, if for every variable and for every value in its domain, it is possible to find values in the domains of all other variables which satisfy the constraint.

- Also called *generalized arc consistency (GAC)*
- or *hyper arc consistency*

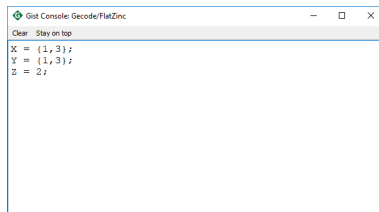
# Simple Example Revisited

```
include "alldifferent.mzn";  
  
var {1,3}:X; % note enumerated domain  
var {1,3}:Y;  
var 1..3:Z; % note domain as interval  
  
% note different annotation  
constraint alldifferent([X,Y,Z]) :: domain;  
solve satisfy;
```

# Domain Consistency with Gecode Gist: Propagation



All Solutions



Node Inspector (Root)

# Can we still do better?

- NO! This extracts all information from this one constraint
- We could perhaps improve speed, but not propagation
- But possible to use different model
- Or model interaction of multiple constraints

# Should all constraints achieve domain consistency?

- Domain consistency is usually more expensive than bounds consistency
  - Overkill for simple problems
  - Nice to have choices
- For some constraints achieving domain consistency is NP-hard
  - We have to live with more restricted propagation

# Modified MiniZinc Program

```
int: s;  
int: n=s*s;  
array[1..n,1..n] of var 1..n: puzzle;  
  
include "sudoku.dzn";  
  
include "alldifferent.mzn";  
  
constraint forall(i in 1..n)  
    (alldifferent([puzzle[i,j] | j in 1..n])::domain);  
constraint forall(j in 1..n)  
    (alldifferent([ puzzle[i,j] | i in 1..n])::domain);  
constraint forall(i,j in 1..s)  
    (alldifferent([puzzle[s*(i-1)+p, s*(j-1)+q] |  
                    p,q in 1..s])::domain);  
  
solve satisfy;
```



# Modified Choco-solver Sudoku Model

```
Model model = new Model("Sudoku");
int blockSize = 3;
int m = blockSize*blockSize;

IntVar[][] vars = new IntVar[m][m];
for(int i=0;i<m;i++){
    for(int j=0;j<m;j++){
        vars[i][j] = model.intVar("X"+i+" "+j, 1, m);
        if (data[i][j]>0) {
            model.arithm(vars[i][j],"=",data[i][j]).post();
        }
    }
}

// Consistency level AC: domain consistency, BC: bounds consistency, default: mix
for(int i=0;i<m;i++){
    model.allDifferent(row(i,m,vars),AC).post();
    model.allDifferent(column(i,m,vars),AC).post();
}
for(int i=0;i<m;i+=blockSize){
    for(int j=0;j<m;j+=blockSize){
        model.allDifferent(block(i,j,blockSize,vars),AC).post();
    }
}

Solver solver = model.getSolver();
solver.solve();
```

# Initial State (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9	

# Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9	

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# Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1	3	1 2 3	1 2 3	1	7	1 2 3	1 2 3	1 2 3	1 2 3
5	6	4 5 6	4 5 6			4 5 6	4 5 6	4 5 6	4 5 6
7	8 9	7 8 9	7 8 9			7 8 9	7 8 9	7 8 9	7 8 9
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3		
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	3	2
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9		
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3	5	1 2 3
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	4	5 6
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9	7	8 9
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3	1 2 3	1 2 3
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	4	5 6
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9	7	8 9
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3	1 2 3	1 2 3
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	4	5 6
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9	7	8 9
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3	1 2 3	1 2 3
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	4	5 6
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9	7	8 9
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3	1 2 3	1 2 3
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	4	5 6
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9	7	8 9
1	3	1 2 3	1 2 3	1	2 3	1 2 3	1 2 3	1 2 3	1 2 3
5	6	4 5 6	4 5 6	4	5 6	4 5 6	4 5 6	4	5 6
7	8 9	7 8 9	7 8 9	7	8 9	7 8 9	7 8 9	7	8 9

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# Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3	2 3 4 5 6 8 9	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	3	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	3	2
1	3	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1	3	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1	3	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 7 9	1 4 5 6 7 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	8 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9
1 5 6 7 8 9	1 2 4 5 6 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 5 6 7 8 9	1 2 4 5 6 8	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 4 5 6 7 9	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	1 3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 7 9	3 4 7 9	1 3 4 7 9	8	2	5
1 3 5 6 7 8 9	9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	
1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 9	9	1 2 3 4 5 7	2 3 4 5 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4	3 8	2	5	1 3 4 7 9
1 3 5 6 7 9	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	2 3 4 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 4	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 4 5 8	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4	3 8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 5 6 7 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 9	3	2	
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7	
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8	
2	7	1 3 4	3 4 8 9	5	1 3 6 9	1 3 4 6 8 9	1 3 4 6 8 9	1 3 4 6 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 3 4 5 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9 7	1 2 3 5 6 9 7	1 3 5 7	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9 8	3 4 5 8	2 3 4 5 8 9	2 3 4 5 8 9	2 3 4 5 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2	
1 3 7 9	1 4	6	4 3 7 9	4 9	3 8	2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 7	
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8	
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9 8	1 3 6 4 9 8	1 3 4 6 8 9	1 3 4 6 8 9	
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	2 3 5 6 7 8 9	2 3 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9 7	1 2 3 5 6 9 7	6	1 2 3 4 5 6 7 8 9	4	

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 9	1 3 5	1 2 6 7 9	1 2 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 9	1 2 3 5 6 7 9	6	1 2 4 6 7 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 6 9	1 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	4	6	4 3 7 9	4 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 9	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5	1 2 6 7 9	1 3 5 6 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4	8	1 5 6 9	4 5	3	2
1 3 9	4	6	3 7 9	3 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6 4	2 5	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4	3 4 8 9	5	1 3 6 4 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 3 5 7	1 2 6 9	1 3 5 6 9 7
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9 7	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4	8	5 6 9	1 5	3	2
1 3 9	4	6	3 7 9	3 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5 7	2 3 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 7	1 5 7	1 6 7 9	1 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9 8	4 5 8	4 6 9	5 6 8 9
7	1 5 6	1 5 9	4	8	5 6 9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	1 3 7 9
5	9	2	2 3 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	4	3 8 9	5	1 3 6 9	1 3 4 8	1 6 4 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 7 9	6	1 2 7 9	4

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# Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<sup>1</sup> <sub>7</sub>	<sup>1</sup> <sub>6</sub> <sup>5</sup> <sub>9</sub>
<sup>3</sup> <sub>6</sub> <sup>9</sup>	5	<sup>3</sup> <sub>5</sub> <sup>9</sup>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<sup>3</sup> <sub>7</sub> <sup>9</sup>	<sup>3</sup> <sub>9</sub>	8	2	5	<sup>3</sup> <sub>7</sub> <sup>9</sup>
5	9	2	<sup>3</sup> <sub>7</sub>	4	<sup>1</sup> <sub>4</sub> <sup>3</sup> <sub>7</sub>	1	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<sup>3</sup> <sub>8</sub> <sup>9</sup>	5	6	8	1	<sup>1</sup> <sub>8</sub> <sup>3</sup> <sub>6</sub> <sup>9</sup>
6	8	<sup>3</sup> <sub>5</sub> <sup>9</sup>	2	1	4	<sup>3</sup> <sub>5</sub> <sup>8<sup>9</sup></sup>	<sup>2</sup> <sub>7</sub> <sup>6<sup>9</sup></sup>	5
<sup>3</sup> <sub>9</sub>	1	5	<sup>2</sup> <sub>3</sub> <sup>9</sup>	8	7	6	2	4

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# Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	8	2	5	<small>7 9 3</small>	
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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# Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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# Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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# Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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# Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9</small>	<small>7 9</small>
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 9</small>	4	1	<small>7 9</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 9</small>	<small>7 9</small>	5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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# After Setup (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9</small>	<small>7 9</small>
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3</small>	<small>7 9</small>	5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4



# Comparison

## Forward Checking

4	1 2	8	2 3	3	2 3	1	1	6	5 6
3	2	5	3	1 7	2 3	4 5	6	5 6	
6	5 6	9		8	4 5	6	5 6		
7	5	1	9	4 5	8	9	7	8 9	
1	4	6	7	9	8	2	5	7	3
5	9	2	4	7	8				
8	3	7	6	2	5	9	4	1	
2	7	4	9	8	6	5	3	1	3
3	6	5	3	2	3	1	4	5	8 9
6	8	9	7	8 9	1	4	5	3	2
3	1	1	3	2	3	2	3	7	8 9
9	8	5	9	7	8 9	9	7	8	4

## Bounds Consistency

4	1 2	8	5	6	2 3	1	1		
3	2		3	1 7	2 3	4 5	6	5	
7	6	1	5	4	8	9	1	5	3
1	4	6	7	9	8	2	5	7	
5	9	2	3	4	1		8	6	
8	3	7	6	2	5	9	4	1	
2	7	4		5	6				
6	5		3	2	3	1	4	5	8 9
3	1	1	5	2	3	2	3	7	8 9
9	8	5	9	7	8 9	9	7	8	4

## Domain Consistency

4	2	8	5	6	3	1			
3	5		1	7	2	4	6	8	
7	6	1	4	8	9	5	3	2	
1	4	6	7	9	8	2	5	7	
5	9	2	7	4	1		8	6	
8	3	7	6	2	5	9	4	1	
2	7	4		5	6	8	1		
6	8		2	1	4				
3	1	1	5	2	3	2	3	7	8 9
9	8	5	9	7	8 9	9	7	8	4

# Typical?

- This does not always happen
- Sometimes, two methods produce same amount of propagation
- Possible to predict in certain special cases
- In general, tradeoff between speed and propagation
- Not always fastest to remove inconsistent values early
- But often required to find a solution at all

# Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

# Simple search routine

- Enumerate variables in given order
- Try values starting from smallest one in domain
- Complete, chronological backtracking
- Advantage: Results can be compared with each other
- Disadvantage: Usually not a very good strategy

# Asking for Naive Search in MiniZinc

```
solve :: int_search(  
    puzzle,  
    input_order,  
    indomain_min)  
satisfy;
```

# Search Tree (Forward Checking)

2

4	1 2 3 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 7 9	5 6 9
3 6 9	2 5 6 9	3 5 9	1 7	2 3 6 9	4 5 8	6 5 8 9	5 6 8 9	
6 7 9	1 5 6 9	1 5 9	4 5 8	8	1 6 7	3 4 5 7	2	
1	4	6	7 8 9	3 8	2 5 7	5	3	
5	9	2	4 7 9	3 4 7	1 3 7	3 8	3 6 7	
8	3	7	6	2 5 7	9	4	1	
2	7	1 3 4 9	3 8 9	5	3 6 9	1 8 9	1 9 8 9	3
3 6 9	5 8 9	3 5 7 8 9	2 3 7 8 9	1 4	3 5 7 8	2 7 9	5 8 9	3
3 1 9	1 5 8	1 3 7 8 9	2 3 7 8 9	3 2 7 9	6	1 2 7 9	4	

# Search Tree (Forward Checking)

2  
1  
4

4	1	8	2 3 5 9	3 6 5 9	2 3 6 9	1 5 7 8	1 6 7 9	5 6 7 9
3 6 9 9	2 5 6 8	5 9	1 7	2 3 6 8	4 5 7 8	6 9	5 6 8 9	
6 7 9 9	1 5 6 8	1 5 6 8	4 5 7 8	8 9	1 6 7 8	3 2		
1 4	6 7	8 9	3 8	2 5	7 9			
5 9	2 4	7 8	3 4	1 3 7 8	3 8	7 9		
8 3	7 6	2 5	9 4	1 9				
2 7	1 4 9 8	3 9 8 9	3 5	1 3 6 8	3 1 9 8	1 9 8 9		
3 6 9 8	5 8 9 8	5 7 8 9	1 4	2 3 7 8	3 7 8 9	2 9 7 8	5 6 8 9	
3 1 9 8	1 5 6 8	1 3 7 8	2 3 9 8	3 2 7 8	6 9	1 2 7 8	4 9	

# Search Tree (Forward Checking)

2  
1  
4  
2  
5

4	1	8	2		3	2	3			6	5	6
	3	2	3		6	6	5		7	7	9	9
	6	5	6	1	7	2	3	4	5	6	5	6
	7	6	5	4	5	8	1	6	4	5	3	2
1	4	6			3	8	2	5			7	3
5	9	2	4		3	1	3	3		8	3	6
8	3	7	6	2	5	9	4	1				
2	7		3	5		3	1	3	1		3	
	3	5	3	2	3	1	4	3	2		5	3
	6	8	5	1	4	7	8	7	9	7	8	9
	3	1	3	2	3	3	2	3	1	2		4



# Search Tree (Forward Checking)

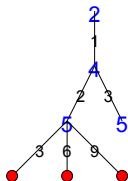
2  
1  
4  
2  
5  
3  
●

4	1	8	2	3	3	5		6	5	6
	2	5	1	7	3	4	5	6	5	6
	5	5	5	4	8	6	4	3	2	
1	4	6	7	8	2	5	7			3
5	9	2	4	3	1	3	3	8	7	3
8	3	7	6	2	5	9	4	1		
2	7	1	3	5	3	1	3	1	3	
	5	5	1	4	5	7	8	7	9	7
	5	1	3	3	3	2	3	1	2	4



4	1	8	2	9	3	3					
3	2			9	6	5		7	6	5	6
6	5	6	5	9	3	6	4	5	6	5	6
9						9	8		9	8	9
	6	5	6	5	4	5	1				
7	9			9	9	8	6	4	5	3	2
1	4	6			3	8	2	5			3
					7	9				7	
5	9	2			3	1	3				3
			4	4	7	7	7			8	6
8	3	7	6	2	5	9	4	1			
2	7		1	3		3	1	3	1		3
			4	9	8	9	6				
							8		9	8	9
3	6	5	6	5	3	3		3	2		3
9	8				7	8	9	5		5	3
								7	8	7	8
3	5		1	3	3	3	2	3	1	2	
	9	8	5	9	7	8	9		9		
								6	7	9	4

# Search Tree (Forward Checking)

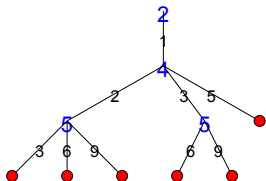


4	1	8	3	3	2	3			6	5	6
	3	2	3	6	6	5			7	9	9
	6	5	5	9	9	7			6	5	6
7	6	5	5	4	5	8		1	3	2	
				9	8	9		6	4	5	6
1	4	6	1	7				9	8		3
5	9	2	4	3	1	3		3	8		3
8	3	7	6	2	5	9		4	1		6
2	7	1	3	5		3	1	3	1		3
	3	6	5	9	8	9		8	9	8	9
	6	8	5	7	8	9		1	4		3
3	1	3	2	3	3	2		3	2		3
	5	1	3	2	3	3		6	1	2	
9	8	9	7	8	9	9		7	9	4	

4	1	8	3	6	2						
3 6 9	2 5 6	5 9	3 9	1 7	2 6 9	4 5 8		6 9	5 7	7 9	5 6 9
	5 6	5 9	4 5	8	1 6 9	4 5		3	2		
7 9					3 9						3 9
1	4	6			8	2	5				
			7 9		1 3		3				3 6
5	2		4 7	4	7		8				
8	3	7	6	2	5	9	4	1			
2	7	1 3 4		5		1 3 9					
	3 6 9	5 6	5 9	3 2	1 4		3 2 3				3 9
3 9	5 8	1 5	7 8 9			6					4

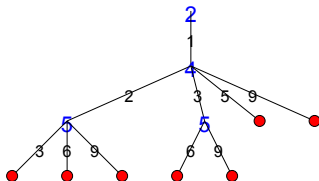
4	1	8	3	9	2		6	5		6	5	6
3	2			1	7	2	6	4	5	6	5	6
6	5	5	9				9	8		9	8	9
7	6	5	5	4	5	8	6	4	5	3	2	
1	4	6			3	8	2	5			3	
5	9	2	4		3	1	3		3		3	6
8	3	7	6	2	5	9	4	1				
2	7		1	3		3	1	3	1		3	
			4	9	8	9	5	6	9	8	9	8
3	6	5	6	5	9	7	8	9			3	5
9	8				1	4		5	6	7	8	9
3	5		1	3	2		3	2	3	1	2	
9	8		5	9	7	8	9		6	7	9	4

# Search Tree (Forward Checking)



4	1	8	5	3	2	3	5	6	5	6
3	2	5	1	7	2	3	4	5	6	5
7	6	5	4	5	8	1	6	4	5	3
1	4	6	7	8	2	5	7	3	6	1
5	9	2	4	3	1	3	3	8	7	3
8	3	7	6	2	5	9	4	1	3	3
2	7	1	3	3	5	3	1	3	1	3
3	6	5	3	2	3	1	4	7	8	9
3	5	1	3	2	3	3	2	3	1	2
9	8	9	7	8	9	9	7	9	4	4

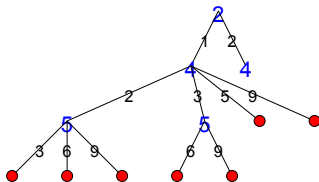
# Search Tree (Forward Checking)



4	1	8	9	3	2	3	3	6	5	6	5	6
3	2	5	3	1	7	2	3	6	4	5	6	5
7	6	5	5	4	5	8	1	6	4	5	6	5
1	4	6	7	8	2	5	7	3	8	7	3	6
5	9	2	4	3	1	3	3	7	8	7	3	6
8	3	7	6	2	5	9	4	1	3	1	3	3
2	7	1	3	3	5	3	1	3	1	3	1	3
3	6	5	3	2	3	1	4	7	8	7	9	7
3	1	3	2	3	3	2	3	6	1	2	4	4

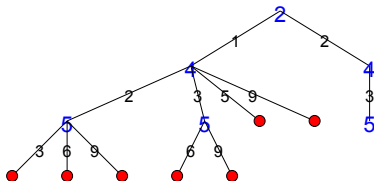


# Search Tree (Forward Checking)



4	2	8	2 3 5 9	3 3 6 9	2 3 6 9	1 5 7 9	1 6 7 9	5 6 8 9
3 6 9 9	2 5 6 9	5 9	1 7	2 3 6 9	4 5 8 9	6 5 7 9	5 6 8 9	
1 5 6 9	1 5 6 9	4 5 8 9	8	1 1 6 9	3 4 7 9	3 2 7 9		
1 4 5 9	6 7 9 9	3 8 7 9	8	2 3 6 9	5 7 9 9			
8 3 2 7	7 6 9 9	2 5 6 9	9 4 9 9	1 3 6 9	3 1 7 9	3 1 7 9	3 3 7 9	
2 7 9 9	1 3 6 9	3 2 7 9	5	3 1 6 9	3 1 7 9	3 1 7 9	3 3 7 9	
3 6 9 9	5 8 9 9	5 9 7 9	1 4 7 9	1 4 7 9	3 2 7 9	3 2 7 9	3 3 7 9	
3 1 9 9	1 5 6 9	1 3 6 9	3 2 7 9	3 2 6 9	3 2 7 9	3 2 7 9	3 3 7 9	

# Search Tree (Forward Checking)

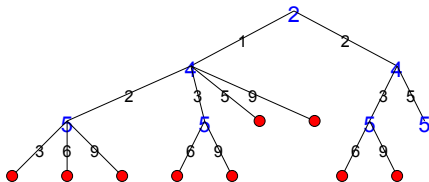


4	2	8	3	3	3	1	1		
3	6	5	6	6	6	5	7	6	5
9	9	9	9	9	9	7	7	9	9
1	4	6	7	8	8	2	5	7	
5	9	2	4	3	3	3	8	3	6
8	3	7	6	2	5	9	4	1	
2	7	1	3	3	5	3	1	3	3
3	6	5	9	9	9	8	9	8	9
9	9	9	9	9	9	9	9	9	9
3	1	5	1	3	2	3	3	2	3
9	9	9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9	9	9

[illegible]

[illegible]

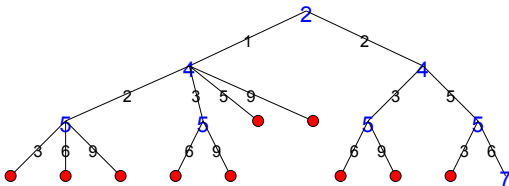
# Search Tree (Forward Checking)



4	2	8	5	3	3	1	1		
	3	5	3	1	2	4	6	5	6
	6	5	9	8	6	4	5	6	6
7	6	1	5	4	8	6	3	2	
	9	5	9	8	9	7	8		
1	4	6		3	8	2	5		3
			7	8					
5	9	2	4	3	1	3	3	8	6
			7	8	4	7	7		
8	3	7	6	2	5	9	4	1	
2	7	1	3	3	5	3	1	3	3
		4	9	8	9	8	9	8	9
3	6	5	2	3	1	4	5	2	3
	8	5	7	8	9	7	8	7	8
3	1	1	3	2	3	3	2	3	
		5	5	9	7	9	6		
9	8	9	7	8	9	9	7	9	4

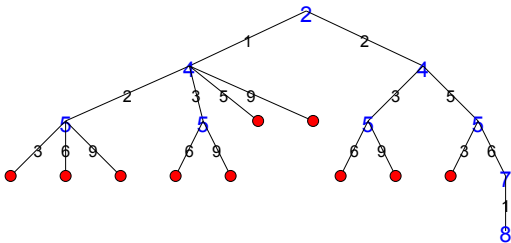
[illegible]

# Search Tree (Forward Checking)



4	2	8	5	6	3	1	1	6	6
3	5	5	1	7	2	4	5	6	5
6	5	5	4	8	9	4	5	3	2
1	4	6	7	8	8	2	5	7	3
5	9	2	4	3	1	7	8	3	6
8	3	7	6	2	5	9	4	1	3
2	7	4	9	8	5	6	1	3	1
3	5	5	7	8	9	1	4	7	5
3	1	5	1	3	2	3	7	6	4

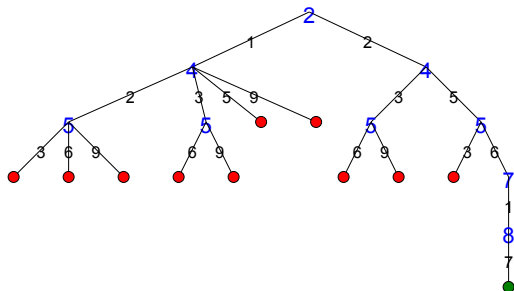
# Search Tree (Forward Checking)



4	2	8	5	6	3	1		
				1	7	2		
				4	8	9		3
1	4	6			8	2	5	
5	9	2			1		8	
8	3	7	6	2	5	9	4	1
2	7			5	6			
				1	4			
					7	6		4



# Search Tree (Forward Checking)



4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

# Search Tree (Bounds Consistency)

2

4	1 2	8	5	6	2 3	1	1		
3	2 5	3	1	7	2 3	4 5 8	6 5 9	7 9	7 9
7	6	1 5	4	8	9	1 5	3	2	
1	4	6	7 9	3 9	8	2	5	7	3
5	9	2	7 3	4 3	1	7 3	8	6	
8	3	7	6	2	5	9	4	1	
2	7	4	3 8 9	5	6	1 3 1	8 9	8 9	3
6	5 8	3 2 3	1	4	5 3	2	5 3	7 8 9	8 9
3 1	5 8	1 5	2 8	3 9	7 6	1 2	4		

2  
1  
●

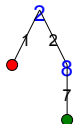
4	1	8	5	6					
				1	7				
7	6		4	8				3	2
1	4	6			8	2	5		
5	9	2			1			8	6
8	3	7	6	2	5	9	4	1	
2	7	4		5	6				
6				1	4				
					7	6			4

# Search Tree (Bounds Consistency)



4	2	8	5	6	3	1		
3	5		3	1	7	2	4	6
7	6	1	4	8	9	5	3	2
1	4	6		8	2	5		
5	9	2		1		8	6	
8	3	7	6	2	5	9	4	1
2	7	4		5	6	8	1	
6	8		2	1	4			5
	1	5	8	7	6	2	4	

# Search Tree (Bounds Consistency)



4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

# Search Tree (Domain Consistency)

8

4	2	8	5	6	3	1		
<sup>3</sup> <sub>9</sub>	5	<sup>3</sup> <sub>9</sub>	1	7	2	4	<sup>7</sup> <sub>9</sub>	<sup>7</sup> <sub>9</sub>
7	6	1	4	8	9	5	3	2
1	4	6	<sup>3</sup> <sub>7</sub>	<sup>3</sup> <sub>9</sub>	8	2	5	<sup>3</sup> <sub>7</sub>
5	9	2	<sup>3</sup> <sub>7</sub>	4	1	<sup>3</sup> <sub>7</sub>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<sup>3</sup> <sub>9</sub>	5	6	8	1	<sup>3</sup> <sub>9</sub>
6	8	<sup>3</sup> <sub>9</sub>	2	1	4	<sup>3</sup> <sub>7</sub>	<sup>7</sup> <sub>9</sub>	5
<sup>3</sup> <sub>9</sub>	1	5	8	<sup>3</sup> <sub>9</sub>	7	6	2	4

# Search Tree (Domain Consistency)

8  
7  
●

4	2	8	5	6	3	1	7	9
3	5	9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3	9	8	2	5	7
5	9	2	7	4	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	9	5	6	8	1	3
6	8	3	2	1	4	7	9	5
9	1	5	8	3	7	6	2	4

# Trading Propagation Against Search

- If we perform more propagation, search is more constrained
- Fewer values left, fewer alternatives to explore in search
- Best compromise is not obvious
- But can be learned from examples or during search
- Annotations are optional
  - Some MiniZinc back-end solvers do the search they want, not the one you specify
  - Some solvers simply do not work in a way that these search annotations apply



# Outline

Problem

Initial Propagation (Forward Checking)

Improved Reasoning

Search

Other Global Constraints

# Are there other Global Constraints?

- alldifferent is the most commonly used constraint
- Propagation methods can be explained
- But there are many more

# Global Constraint Catalog

- <https://sofdem.github.io/gccat/>
- Description of 354 global constraints, 2800 pages
- Not all of them are widely used
- Detailed, meta-data description of constraints in Prolog

# Families of Global Constraints

- Value Counting
  - alldifferent, global cardinality
- Scheduling
  - cumulative
- Properties of Sequences
  - sequence, no\_valley
- Graph Properties
  - circuit, tree

# Common Algorithmic Techniques

- Bi-Partite Matchning
- Flow Based Algorithms
- Automata
- Task Intervals
- Reduced Cost Filtering
- Decomposition