A Model for Designing the Electoral Constuencies in Ireland

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1 Introduction

2 Problem Description and Related Work

Different from many combinatorial optimization problems encountered in industry, the defining constraints for the election districting problems are well defined by law.

We quote from Article 16 of the *Irish Constitution* [4]:

> 2°The number of members shall from time to time be fixed by law, but the total number of members of Dáil Éireann shall not be fixed at less than one member for each thirty thousand of the population, or at more than one member for each twenty thousand of the population.
> 3°The ratio between the number of members to be elected at any time for each constituency and the population of each constituency, as ascertained at the last preceding census, shall, so far as it is practicable, be the same throughout the country.
> 6°No law shall be enacted whereby the number of members to be returned for any constituency shall be less than three.

More details about the creation of the constituencies is given in the *Electoral Reform Act* of 2022 [5], which in particular defines the following constraints in Section 57 (2):

> (a) the total number of members of Dáil Éireann, subject to Article 16.2.2 of the Constitution, shall be not less than 171 and not more than 181;
> (b) each constituency shall return 3, 4 or 5 members;
> (c) the breaching of county boundaries shall be avoided as far as practicable;

(d) each constituency shall be composed of contiguous areas;
(e) there shall be regard to geographic considerations including significant physical features and the extent of and the density of population in each constituency;
(f) subject to this section, the Commission shall endeavour to maintain continuity in relation to the arrangement of constituencies.

The *Electoral Commission* was tasked with producing a proposal for the Constituency Review, the current draft proposal is described in [7]. It describes the proposed revision of the constituencies for elections for Dáil Éireann, which we use for a baseline comparison, and for the simpler Irish constituency layout for the European Parliament, which we will not discuss in this paper.

In their terms of reference (page 18 of [7]), the Commission extended (c) above as

the breaching of county boundaries shall be avoided as far as practicable (shall be deemed not to include a reference to the boundary of a city or any boundary between any two of the counties of Dún Laoghaire-Rathdown, Fingal and South Dublin);

This means that the constituencies inside the Dublin area are not affected by the country borders, we follow this decision in our model, by treating Dublin as a single "county".

While the solution given in [7] is largely based on the existing electoral districts, it is important to note that in the solution given changes to a number of electoral districts are required (for example on page 127 for Cork North-Central). To achieve comparable results our solution may require similar changes.

As the Irish election system is quite different from many other countries, results on election districting [1] are not directly transferable to the Irish situation. Only [6] directly addresses the Irish scenario. Their model tries to allocate electoral districts to constituencies, satisfying the size limits imposed by law. To deal with the contiguity constraint, they use a constraint model based on the global *tree* constraint [2] which ensures that all districts allocated to a constituency form one contiguous area. The model suffers from scalability issues, and only a partial solution for Galway is presented in the paper, with an execution time of 100 hours quoted on a multi-core computer.

3   Model

In this section we describe our top-level model of deciding how many constituencies of which size should be created in each county, and which transfer of areas between neighbouring counties should be considered to achieve the best compromise between the total number of seats considered, balancing the size of the constituencies, respecting the county boundaries, and creating constituencies with a mix of different sizes. Different stakeholders will want to weight the elements of this objective in different ways, we show that by stating additional constraints different parts of the overall solution space can be explored.

3.1 Constants

We use the set I of all counties in the country, and use i as an index. We consider the set J of different constituency types, and use j as an index. In the current Irish case there are three constituency types, with resp. three, four and five seats per constituency. Additional types may be considered for evaluation.

We use the following constant values in the model, some are defined as data, while others can be chosen as parameters or cost factors for a specific run.

n total population
lb lower bound on number of persons per seat, defined as 20,000 in constitution
ub upper bound on number of persons per seat, defined as 30,000 in constitution
k total number of seats, can be selected between n/lb and n/ub
a average size of one seat, e.g. n/k
Δ maximum allowed variation of the actual size of a seat, i.e. the number of persons represented by each seat must fall in the interval $[a - \Delta, a + \Delta]$
p_i population of county i
s_j number of seats in a constituency of type j
$c_{i_1 i_2}$ Boolean value, true iff counties i_1 and i_2 are neighbours
α cost per cross-border transfer
β cost per person assigned to a constituency outside their home county
γ_j cost of a constituency of type j

3.2 Variables

The main decision variables of the model are used for two purposes. The x_{ij} variables indicate how many constituencies of type j we create in county i. The variables $y_{i_1 i_2}$ and $z_{i_1 i_2}$ consider the reallocation of persons living in county i_1 to a constituency in county i_2. The y variables are zero/one decision variables, indicating if a transfer is required at all, while the z continuous variables count the number of persons re-allocated. We use auxiliary integer variables u_j to constrain or count the number of constituencies of a type j at a national level.

x_{ij} non-negative integer variable stating how many constituencies of size j we create in county i
$y_{i_1 i_2}$ zero-one integer variable stating whether we assign population in county i_1 to a constituency defined in county i_2. The transfer only exists if counties i_1 and i_2 are neighbours.
$z_{i_1 i_2}$ non negative variable stating how many people we transfer from county i_1 to county i_2. p_{i_1} is an upper bound for this variable. In a first version of the model we assume that any specific number of persons can be transferred, possibly requiring modifications of electoral districts.
u_j the total number of constituencies of size j

3.3 Constraints

The total number of seats allocated to constituencies must be equal to the total number of seats given.

$$\sum_{i \in I} \sum_{j \in J} x_{ij} s_j = k \tag{1}$$

The number of constituencies of each type j is equal to the sum of constituencies of that type in all counties.

$$\forall_{j \in J} : \quad \sum_{i \in I} x_{ij} = u_j \tag{2}$$

The effective population of a county i (the population itself plus all transfers into the county minus all transfers out of the county) must be covered by the number of assigned seats of the county. A seat must at least represent $a - \Delta$ and at most $a + \Delta$ persons.

$$\forall_{i \in I} : \quad (a - \Delta) \sum_{j \in J} x_{ij} s_j \leq p_i + \sum_{i_1 \in I} z_{i_1 i} - \sum_{i_2 \in I} z_{i i_2} \leq (a + \Delta) \sum_{j \in J} x_{ij} s_j \tag{3}$$

Note that this form of the constraint assumes that we can split the total population in a county between the different constituencies exactly proportional to the number of seats allocated. This may require some changes to the electoral districts to balance the numbers.

The decision variable for a inter-country transfer is linked to the counting variable for the same transfer by a *bigM* constraint. If the decision variable is zero, then the counting variable must also be zero. Inversely, if the count is greater than zero, then the decision variable must be one.

$$\forall_{i_1, i_2 \in I} : \quad z_{i_1 i_2} \leq p_{i_1} y_{i_1 i_2} \tag{4}$$

The total number of persons assigned to any constituency outside their county i is limited by the population of the county. This value is reached if no constituency is allocated to the county at all.

$$\forall_{i \in I} : \quad \sum_{i_1 \in I} z_{i i_1} \leq p_i \tag{5}$$

For any pair of neighbouring counties, we can only transfer population in one direction,

$$\forall_{i_1, i_2 \in I} : \quad y_{i_1 i_2} + y_{i_2 i_1} \leq 1 \tag{6}$$

We can only have a transfer between neighbouring counties, e.g. transfers between counties that are not neighbours are not allowed.

$$\forall_{i_1, i_2 \in I} : \quad \neg c_{i_1 i_2} \Rightarrow (y_{i_1 i_2} = 0) \tag{7}$$

If we want to ensure that no seat represents more than ub persons, we can enforce an additional inequality stating that the effective number of persons in a county must not exceed the ub value per seat allocated to the county.

$$\forall_{i \in I} : \quad p_i + \sum_{i_1 \in I} z_{i_1 i} - \sum_{i_2 \in I} z_{i i_2} \leq ub \sum_{j \in J} x_{ij} s_j \tag{8}$$

3.4 Objective

We consider the following elements of the objective of the problem. Different stakeholder may consider only some of the objective terms, or assign different weights to the cost terms.

– The actual variance of the number of population per assigned seat in each constituency.
– The number of county border breaches, where part of a county is assigned to a constituency place in a neighbouring county
– The total size of the population assigned to a constituency spanning multiple counties
– The number of constituencies of different sizes, which can vary between three and five seats per constituency. Different stakeholders may prefer more smaller constituencies, or more of the larger ones.

The objective function for the problem is then defined as

$$\min \alpha \sum_{i_1,i_2 \in I} y_{i_1 i_2} + \beta \sum_{i_1,i_2 \in I} z_{i_1 i_2} + \sum_{j \in J} \gamma_j u_j \tag{9}$$

3.5 Performance Indicators

In order to compare and evaluate solutions, we compute different key performance indicators (KPIs) from the solutions found. We use them to compare solutions obtained for different input parameter values and constraints selected.

3.6 Experiments

3.7 Data

We use the population data for Ireland from the last Census in 2022 [3], Table F1004A on the CSO website provides county level counts, we aggregate the values for Dublin from the counties involved. Note that we treat Cork city and Galway city as individual entries separate from the corresponding counties.

3.8 Baseline Results

We use the solution proposed in [7] as a baseline. Figure 1 shows the proposed constituency structure, based on a total population of 5,149,139 and 174 seats for the next Dáil, an increase of 14 over the current parliament. The solution creates 43 constituencies (13 of size three, 15 each of sizes four and five). Six inter-county transfers are suggested, affecting xxx persons. The percentage variance in sizes varies from -8.13% to 8.08%, for a total range of 16.21%. Compared to the current constituency structure, 82% of all constituencies will be changed.

Table 2 shows the inter-county transfers extracted manually from the descriptions in [7]. Note that we find more transfers than stated in the document.

Table 1. Key Performance Indicator Definition

Symbol	Definition	Explanation
k	parameter	total number of seats allocated
Δ	parameter	maximum $\pm$ range of persons per seat
x	$\sum_{i \in I, j \in J} x_{ij}$	total number of constituencies allocated
t	k/x	average number of seats in the constituencies
a	n/k	average number of persons represented by seat
y	$\sum_{i_1, i_2 \in I} y_{i_1 i_2}$	total number of inter-county allocations of non-zero size
z	$\sum_{i_1, i_2 \in I} z_{i_1 i_2}$	total number of persons allocated to constituencies outside their home county
q_i	$p_i + \sum_{i_1 \in I} z_{i_1 i} - \sum_{i_2 \in I} z_{i i_2}$	effective population for county i
f_i	$\sum_{j \in J} x_{ij} s_j$	number of seats allocated to county i
e_i	q_i / f_i	actual number of persons per seat for county i
$\bar{e}$	$\max_{i \in I} e_i$	largest number of persons represented by any seat
$\underline{e}$	$\min_{i \in I} e_i$	smallest number of persons represented by any seat
r	$100.0(\bar{e} - \underline{e})/a$	representation range percentage
w_j	$100.0 u_j / x$	percentage of constituencies of type j
		variance of voting power per person

Table 2. Inter-County Transfers in Baseline Solution

Constituency	From	To	Volume	Percentage of Total From
Sligo-Leitrim	Leitrim	Sligo	35,199	100.00%
Roscommon-Galway	Galway	Roscommon	14,468	
Cavan-Monaghan	Monaghan	Cavan	65,288	100.00%
Louth	Meath	Louth	16,403	
Tipperary-North	Kilkenny	Tipperary	6,431	
Longford-Westmeath	Longford	Westmeath	46,751	100.00%
Carlow-Kilkenny	Carlow	Kilkenny	61,968	100.00%
Wicklow-Wexford	Wicklow	Wexford	35,708	
Total				n/a

Fig. 1. Baseline Solution from [7]

4   Results

5   Conclusion

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A   County Population Based on 2022 Census

Table 3 shows the population data used in [7] for the counties considered. Our program uses identical data obtained directly from the CSO website [3] in machine readable form.

Table 3. County Population

County	2022 population	Growth since 2016 %	Growth since 2016 population
Carlow	61,968	8.8	5,036
Cavan	81,704	7.3	5,528
Clare	127,938	7.7	9,121
Cork	584,156	7.6	41,288
Donegal	167,084	5	7,892
Dublin	1,458,154	7.6	110,795
Galway	277,737	7.8	19,679
Kerry	156,458	5.9	8,751
Kildare	247,774	11.4	25,270
Kilkenny	104,160	5	4,928
Laois	91,877	8.5	7,180
Leitrim	35,199	9.8	3,155
Limerick	209,536	7.5	14,637
Longford	46,751	14.4	5,878
Louth	139,703	8.4	10,819
Mayo	137,970	5.7	7,463
Meath	220,826	13.2	25,782
Monaghan	65,288	6.4	3,902
Offaly	83,150	6.7	5,189
Roscommon	70,259	8.9	5,715
Sligo	70,198	7.1	4,663
Tipperary	167,895	5.2	8,342
Waterford	127,363	9.6	11,187
Westmeath	96,221	8.4	7,451
Wexford	163,919	9.5	14,197
Wicklow	155,851	9.4	13,426

B   Baseline Constituencies

Table 4 shows a table of the 43 constituencies proposed in [7] for a total of 174 seats, with their individual number of seats, allocated population, persons per

TD, and variance against the national average of 29,593. The largest negative variance of -8.13% occurs in Kildare South, the largest positive variance of 8.08% occurs for the Clare constituency. Note that 17 of the proposed constituencies are assigned more than 30,000 persons per TD.

Table 4. Baseline Constituency Structure

Recommended Constituency	Number of TDs	Population 2022	Population per TD	% variance from National Average Population per TD
Carlow-Kilkenny	5	159,697	31,939	7.93
Cavan-Monaghan	5	146,992	29,398	-0.66
Clare	4	127,938	31,985	8.08
Cork East	4	111,458	27,865	-5.84
Cork North-Central	5	142,378	28,476	-3.78
Cork North-West	3	84,312	28,104	-5.03
Cork South-Central	5	153,441	30,688	3.70
Cork South-West	3	92,567	30,856	4.27
Donegal	5	157,700	31,540	6.58
Dublin Bay North	5	149,062	29,812	0.74
Dublin Bay South	4	124,257	31,064	4.97
Dublin Central	4	127,302	31,826	7.54
Dublin Fingal East	3	92,366	30,789	4.04
Dublin Fingal West	3	88,167	29,389	-0.69
Dublin Mid-West	5	142,140	28,428	-3.94
Dublin North-West	3	85,322	28,441	-3.89
Dublin Rathdown	4	113,625	28,406	-4.01
Dublin South-Central	4	119,383	29,846	0.85
Dublin South-West	5	158,935	31,787	7.41
Dublin West	5	137,360	27,472	-7.17
Dún Laoghaire	4	120,235	30,059	1.57
Galway East	4	113,254	28,314	-4.32
Galway West	5	150,015	30,003	1.39
Kerry	5	156,458	31,292	5.74
Kildare North	5	139,031	27,806	-6.04
Kildare South	4	108,743	27,186	-8.13
Laois	3	91,877	30,626	3.49
Limerick City	4	119,041	29,760	0.57
Limerick County	3	90,495	30,165	1.93
Longford-Westmeath	5	142,972	28,594	-3.37
Louth	5	156,106	31,221	5.50
Mayo	5	137,970	27,594	-6.75
Meath East	4	114,631	28,658	-3.16
Meath West	3	89,792	29,931	1.14
Offaly	3	83,150	27,717	-6.34
Roscommon-Galway	3	84,727	28,242	-4.56
Sligo-Leitrim	4	114,781	28,695	-3.03
Tipperary North	3	87,799	29,266	-1.10
Tipperary South	3	86,527	28,842	-2.54
Waterford	4	127,363	31,841	7.60
Wexford	4	114,176	28,544	-3.54
Wicklow	4	120,143	30,036	1.50
Wicklow-Wexford	3	85,451	28,484	-3.75
Total	**174**	**5,149,139**	**29,593**	