

# Oven Scheduling Case Study

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Constraint Based Production Scheduling

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## Key Points

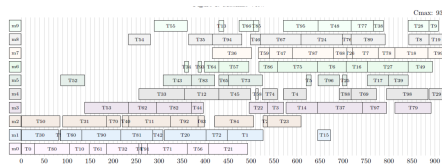
- Discusses two topics:
  - Solve a very specific industrial scheduling problem from the ASSISTANT EU project
  - Discuss the general issue of short-term scheduling vs. long-term objectives

## Research Challenge

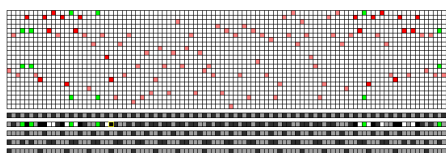
- Often the long-term business objectives are not visible in the operational decision problem
- We optimize a short-term objective without understanding the impact in the long term
- What choices should we make in short-term to improve overall result?
- Especially important when future data not yet visible
- Surprisingly, this problem is rarely discussed in literature

## Examples

- Production Scheduling
- Nearly all scheduling benchmarks use  $c_{max}$  (makespan) as objective
- Why?
- Do we want to close factory as rapidly as possible?



- Car Sequencing
- The best heuristics push difficult cars to the edge of schedule
- Because they are easier to schedule this way
- But: It makes it hard to schedule next day



## Examples

- Personnel Rostering
- Satisfy working rules and demands for period
- But: rules apply on a rolling horizon
- Easy to over-constrain problem for next period



- Transportation Planning
- Build daily delivery tours, optimizing cost
- Where are your trucks at 10PM?
- Also, avoid cherry-picking at start of week



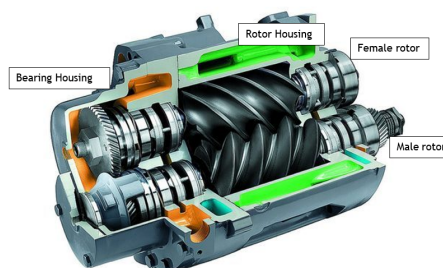
### Problem Studied Here

- Example from the ASSISTANT EU project (ended last year)
- Oven schedule for one of the industrial partners
- Schedule tasks on a set of ovens
- Tasks can share oven only if they are compatible
- Conflicting objectives
  - Energy use of ovens very significant, reduce when ovens are used
  - Waiting for an oven affects quality of product
- Jobs only visible when previous process step starts
- Currently scheduled by hand, industry partner expressed strong need for change

### What does this look like in the real world?



Industrial Oven



Rotors in Compressor

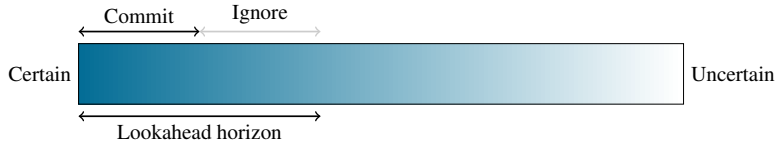
### Solution Approach: Constraint Programming

- Declarative modelling approach for combinatorial problems
  - Problem expressed in terms of variables and constraints
- Global constraints
  - Combines expressive modelling abstractions and powerful reasoning
  - Examples: disjunctive, cumulative, global\_cardinality
- Compositional: Add constraints as required
- Main application areas
  - Scheduling, rostering, transportation
  - Also: test generation, verification, configuration



### Overall Decomposition (Standard)

- We can only see that far into future
- We do not want to take decisions now that we might regret later
- We have to make some decisions now otherwise we never do anything
- *Rolling horizon* decomposition
  - We schedule up to *lookahead horizon* units into the future
  - We commit to implement resulting schedule only to up *commitHorizon*
  - We reschedule when we receive new information, or we reach the end of commitment
  - We solve each short-term sub problem based on short-term objectives



### Short-Term Schedule Modelling

- Challenge: There is no global constraint to express the oven resource constraint
- We are not able to invest a lot of time/resources to develop such a constraint
- Two choices:
  - Two traditional models with variables linking them (Lackner et al, Constraints 2023)
  - Direct model expressing conditions as disjunctions of basic constraints

### The Standard Pieces

- Jobs  $N$  consisting of multiple stages  $Q$ , tasks for each stage of each job, running on machines  $M$
- Release dates  $r_i$  of jobs given by up-stream schedule
- WiP  $w_k$  on certain machines resulting from earlier schedule
- Machine  $m_{ij}$  and start variables  $s_{ij}$  for each task
- Precedence constraints between tasks of each jobs, with total waiting time  $c_i$  when waiting for resource
- Total number of ovens used in schedule  $nrOvens$  by  $nvalue$  constraint

$$nvalue(nrOvens, [m_{ij}|i \in N, j \in Q] ++ [k|k \in M \text{ s.t. } w_k > 0])$$

### Resource Constraints

We start from the basic decomposition of the disjunctive machine choice constraint

$$\begin{aligned} \forall_{i_1, i_2 \in N} \forall_{j_1, j_2 \in Q} \text{ s.t. } \langle i_1, j_1 \rangle \neq \langle i_2, j_2 \rangle : \quad & m_{i_1 j_1} \neq m_{i_2 j_2} \vee \\ & s_{i_1 j_1} \geq s_{i_2 j_2} + d_{i_2 j_2} \vee \\ & s_{i_2 j_2} \geq s_{i_1 j_1} + d_{i_1 j_1} \end{aligned}$$

Express case where tasks share an oven (only when types and stages are the same)

$$\begin{aligned} \forall_{i_1, i_2 \in N} \text{ s.t. } i_1 \neq i_2 \forall_{j \in Q} : \quad & m_{i_1 j} \neq m_{i_2 j} \vee \\ & s_{i_1 j} \geq s_{i_2 j} + d_{i_2 j} \vee \\ & s_{i_2 j} \geq s_{i_1 j} + d_{i_1 j} \vee \\ & (t_{i_1 j_1} = t_{i_2 j_2} \wedge m_{i_1 j} = m_{i_2 j} \wedge s_{i_1 j} = s_{i_2 j}) \end{aligned}$$

### Limit stacking

Need binary variables  $b_{i_1 i_2 j}$  to state that two jobs  $i_1$  and  $i_2$  share oven in stage  $j$

$$\begin{aligned} \forall_{i_1, i_2 \in N} \text{ s.t. } i_1 < i_2 \forall_{j \in Q} : \quad & (b_{i_1 i_2 j} = 0 \wedge (m_{i_1 j} \neq m_{i_2 j} \vee \\ & s_{i_1 j} \geq s_{i_2 j} + d_{i_2 j} \vee \\ & s_{i_2 j} \geq s_{i_1 j} + d_{i_1 j})) \vee \\ & (b_{i_1 i_2 j} = 1 \wedge t_{i_1 j_1} = t_{i_2 j_2} \wedge m_{i_1 j} = m_{i_2 j} \wedge s_{i_1 j} = s_{i_2 j}) \end{aligned}$$

Count how many jobs share stage  $j$  with job  $i$

$$\forall_{i \in N} \forall_{j \in Q} : z_{ij} = \sum_{i_1=1}^{i-1} b_{i_1 i j} + \sum_{i_2=i+1}^n b_{i i_2 j}$$

Limit how many tasks can be stacked together

$$\forall_{i \in N} \forall_{j \in Q} : z_{ij} < \text{maxStacked}$$

### This should not work!

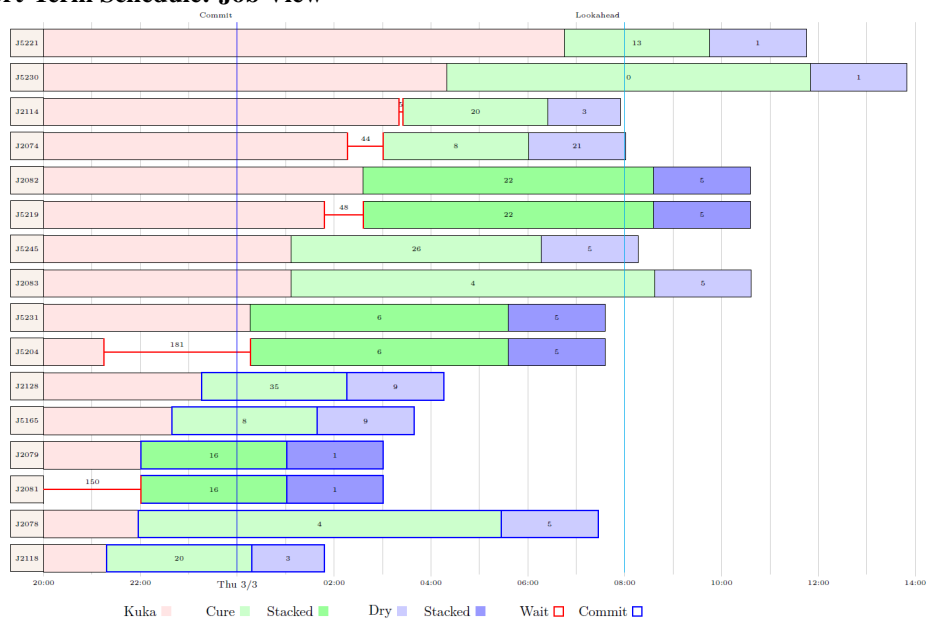
- Weakness of basic decomposition model was the reason to develop the scheduling constraints in the first place
- Does not scale well to thousands of tasks
- But model is well suited to some solvers
  - SAT based solvers, Chuffed, CP-SAT (OR-Tools)
  - MIP solvers
- This works (only) as long as problem size stays manageable

### Compound Objective

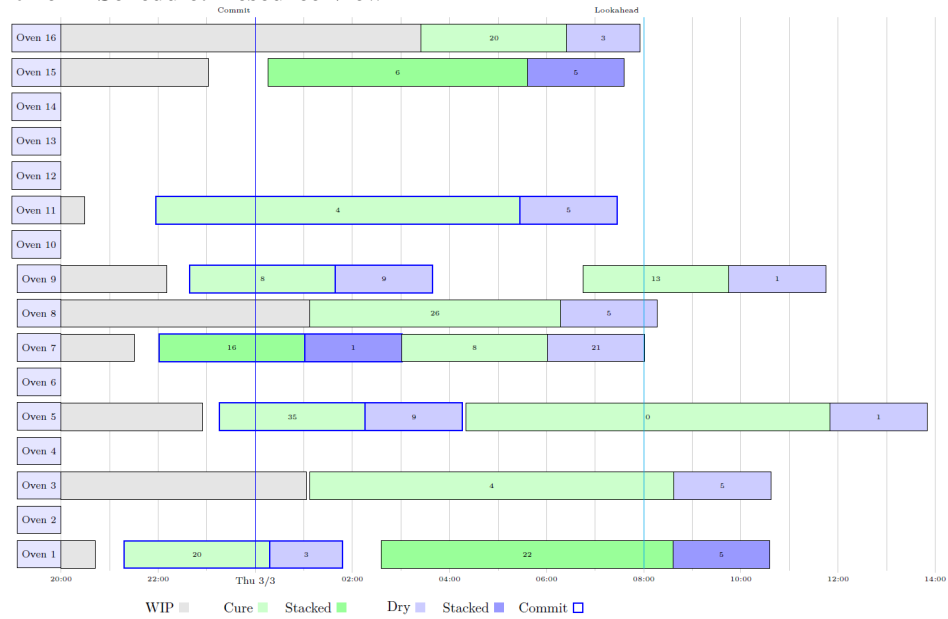
$$\min \alpha_1 \sum_{i \in N} c_i + \alpha_2 \text{nrOvens} + \alpha_3 \sum_{i \in N, j \in Q} z_{ij}$$

- Three conflicting elements
  - Total waiting time for jobs
  - Number of ovens used
  - Number of tasks stacked (negative coefficient)
- Reducing waiting time requires using more ovens
- Improved stacking will require for one job to wait until second is ready

### Short-Term Schedule: Job View



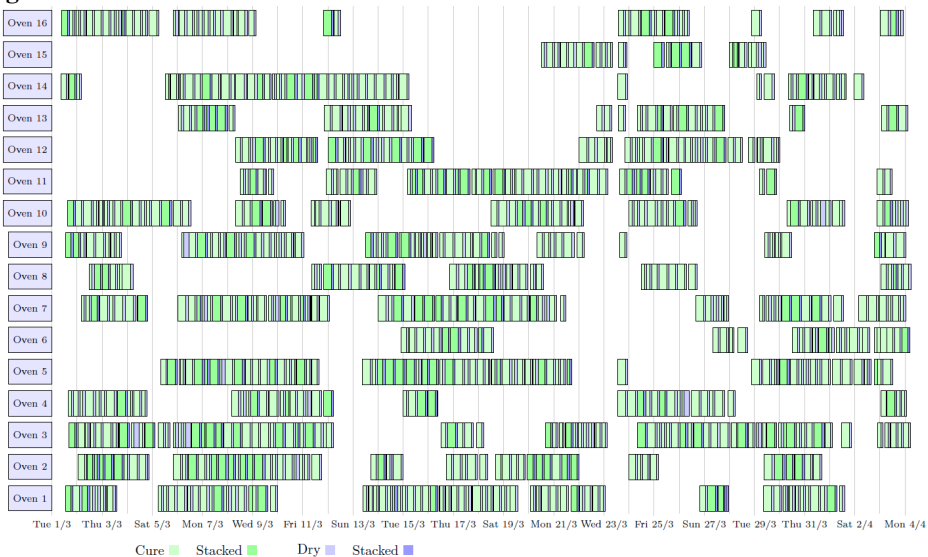
## Short Term Schedule: Resource View



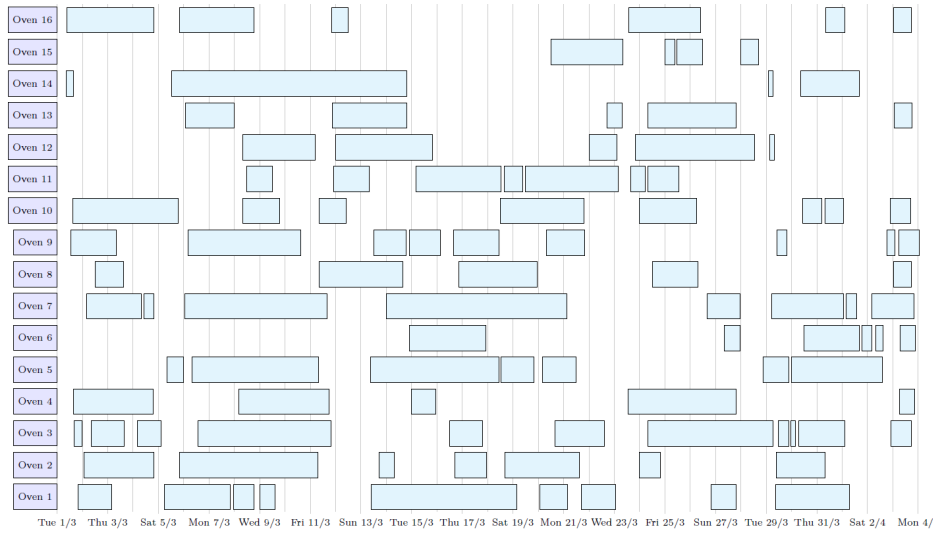
## Are the short-term solutions good?

- We solve many problems to optimality, depending on solver
- Optimality gap is small, increasing search time helps a bit
- But are we optimizing the best possible objective?

## Long Term Schedule: Detailed Schedule



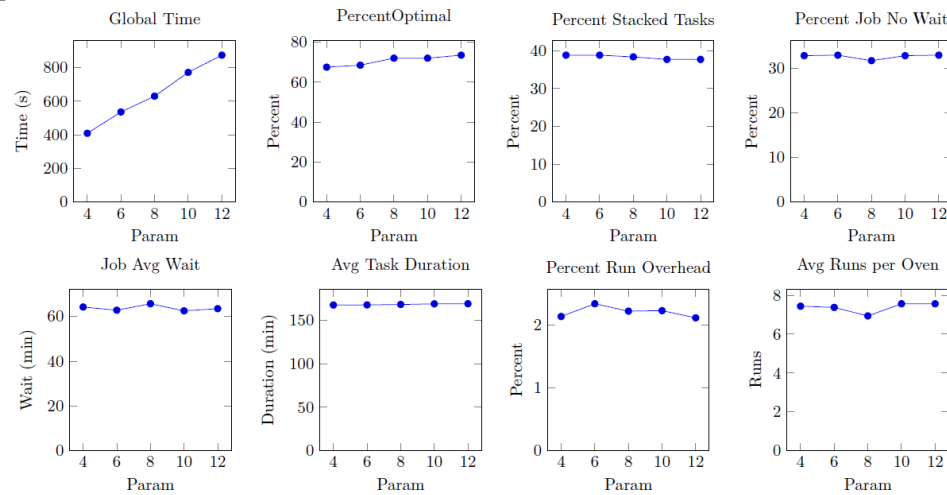
## Long Term Schedule: Abstracted Oven Runs



## Is that a good global schedule? KPIs

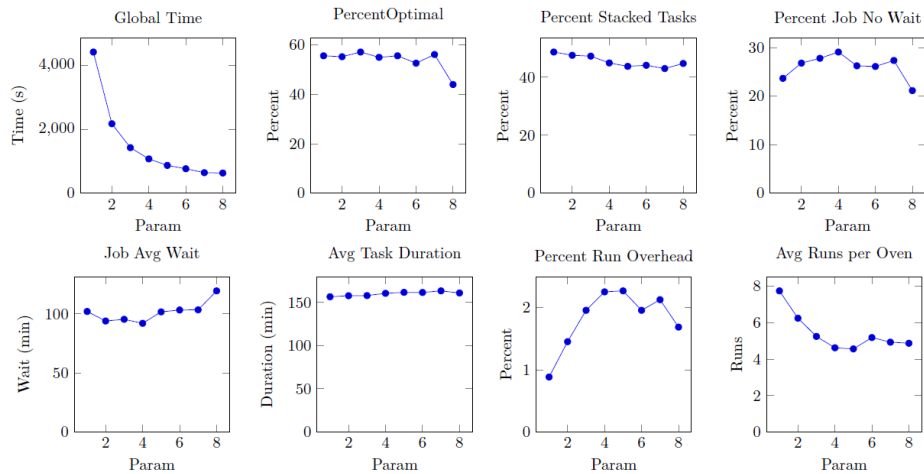
| Name                   | Unit               | Explanation                                                     |
|------------------------|--------------------|-----------------------------------------------------------------|
| Global Time            | Seconds            | Total time for solving all sub problems                         |
| Nr Jobs                | -                  | Total number of jobs scheduled                                  |
| Nr Tasks               | -                  | Total number of tasks scheduled                                 |
| Percent Optimal        | Percentage (0-100) | How many sub problems were solved to optimality                 |
| Percent Stacked Tasks  | Percentage (0-100) | Percentage of all tasks scheduled that were stacked             |
| Percent Jobs No Wait   | Percentage (0-100) | Percentage of jobs that were scheduled without any waiting time |
| Job Average Wait       | Minutes            | Average wait time over all jobs                                 |
| Job Maximal Wait       | Minutes            | Largest waiting time for any job scheduled                      |
| Ovens Used             | -                  | Total number of ovens used during period                        |
| Avg Task Duration      | Minutes            | Average tasks duration (influenced by stacking)                 |
| Oven Runs              | -                  | Number of oven runs over total horizon                          |
| Run Overhead Percent   | Percentage (0-100) | Overhead during oven runs when machine is idle                  |
| Avg Runs per Oven Used | -                  | Average number of oven runs per oven used                       |

## Impact of Lookahead Parameter

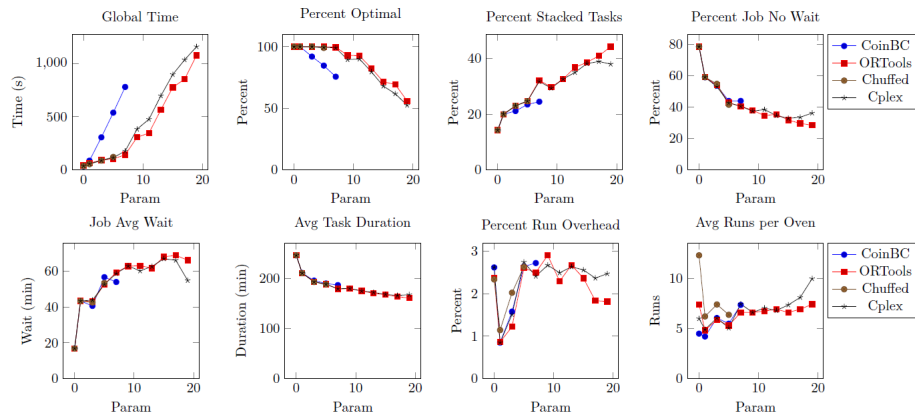


## Impact of CommitHorizon Parameter





## Comparing Different Solvers



## Is the global solution really good?

- We schedule with limited information
- Hindsight is 20/20, we cannot expect best possible solution from partial information
- Process Challenge: Can we improve data visibility?
- Demand is variable over time, no steady-state solution
- Modelling Challenge: Can we define a short-term objective that produces better long-term solutions?
- Algorithm Challenge: Can we solve the global problem to optimality?
  - Assumes "a priori" visibility of data
  - This would provide a lower bound
  - But we need optimality to use as bound

## Summary

- Discussed a non-standard oven scheduling problem from industry
- Models with decomposition of resource constraints
- Good/very good short-term solutions

- But is the overall schedule close to the global optimum?
- In any case, industry partner was happy with solution and analysis