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# Concepts

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## Constraint Based Production Scheduling



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# Key Points



- We introduce the core concepts used in scheduling
- Different layers of description
  - What we are doing (jobs, tasks, resources)
  - Why we are scheduling (orders, products, processes)
- Temporal Relations
- Process description
- Problem classification
- Visualization

# Outline



## Core Concepts

- Jobs, Tasks and Resources
- Orders, Products, Processes

Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

Summary

# Most basic description of scheduling problem



- *Job*
  - Collection of activities required to manufacture one object/lot/order
  - Overall start/end determined by starts and ends of its tasks
- *Task*
  - Individual activities required for manufacture
  - Have defined start, end (typical: variables) and duration (sometimes fixed)
  - Often performed on one specific resource (more on that later)
- *Resources*
  - Resources are needed to perform the tasks
- Very compact representation of scheduling problem
- But, where does that information come from?

# Scheduling orders



- An *order* specifies a need for a certain *product* at a given time in a specific quantity
- There may be multiple ways of making the *product* (multiple *processes*)
- We assume that the process to use is decided when placing the order
- Each order corresponds to a job, with its constituent tasks
- There may be limited visibility of future orders



- Each *process* consists of one or more *process steps*
- A process step contains a duration formula to describe how long it lasts
- The order of *process steps* is defined by *process sequences*
- The resources needed are defined by *resource needs* (described later on)
- Tasks are created for each process step, their duration is based on the duration formula and order quantity

# Where do the orders come from?



- Made to order
  - Each order is caused by a customer request
  - Defines due date, release date often implied
- Made to stock
  - Orders are satisfied from stock
  - Inventory control strategy decides when to make product
  - Often called stock orders
  - More complex variant integrates production planning and detailed scheduling
  - Example later in course

# Outline



## Core Concepts

### Temporal Relations

- Relations between Tasks

- Relation between Tasks and Jobs

- Jobs: Release and Due Date

- Relations between jobs

## Alternative Processes, Bill of Materials

## Problem Classification

## Key Visualization Methods

## Summary

# Temporal Relations



- Temporal constraints between tasks and/or jobs
- Defined by the manufacturing process
- In simple cases
  - A single sequence of process steps performed in that order
  - Each task must finish before the next one can start



# Annotations on Features

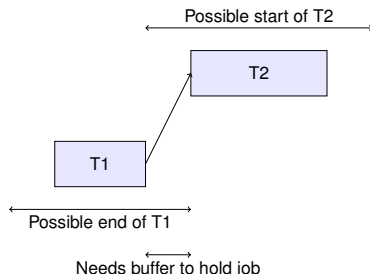


- ✓ Currently available in scheduling tool
- (✓) Will be available shortly
- ✗ Currently not available, may be added in future version

# The Most Common Relation: EndBeforeStart ✓



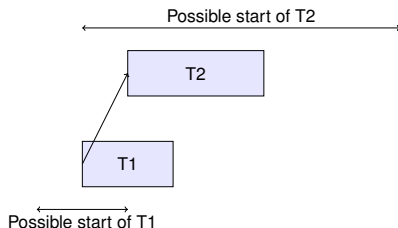
- States that one task (T1) must end before the next one (T2) can start
- Typical for manufacturing process based on the same item
- Addition: offset
  - Wait at least offset units between end and start
  - For example cooling, drying time outside a machine



## Less Common: StartBeforeStart ✓

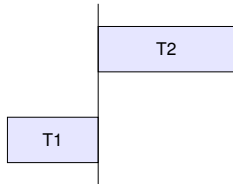


- States that one task (T2) can start any time after the start of another task (T1)
- Uncommon in manufacturing, occurs in project management
- Example later on on assembly line balancing

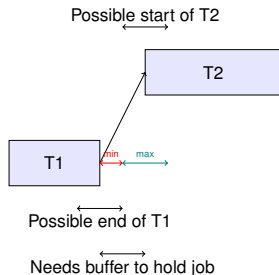


- Sometimes, two steps must follow each other immediately
- The item made would spoil
  - Product specific
- There is no space to hold item
  - Machine specific, buffers
- End of one task (T1) must be equal to start of next task (T2)
- May mean delay of start of task T1

Start of T2 is equal to End of T1



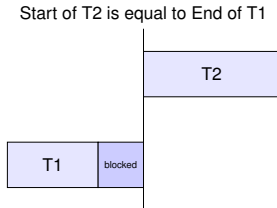
- Limit how long we can wait between tasks
  - Cooling enough, but not too much
  - Baking: rise time
- Impose both lower and upper waiting time limit
- Makes it more difficult to find solutions



# Blocking ✓



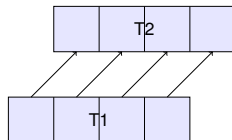
- Sometimes, two steps must follow each other immediately
- There is no space to store item between machines
- Keep item on previous machine until needed
- That machine is now *blocked*
- Duration of task T1 is extended until start of T2
- *Use with caution! Easy to deadlock*



## Special Case: Pipelining X



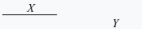
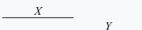

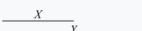
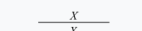

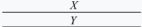
- Sometimes, we can start on the next task while the first is still running
- Possible if one jobs produces multiple items (lots,...)
- As soon as the first item is finished, take it to the next machine to process it there
- Overlaps T1 and T2 as much as possible
- Details can get complex



# More General: Relations between Intervals



- First introduced by Allen (1983)
- 13 relations between intervals
- Allows composition of relations
- Constraint reasoning on sets of relations

| Relation            | Illustration   | Interpretation  |
|---------------------|--|---|
| $X < Y$<br>$Y > X$  |  | X precedes Y<br>Y is preceded by X                                |
| $X m Y$<br>$Y mi X$ |  | X meets Y<br>Y is met by X ( <i>i</i> stands for <i>inverse</i> ) |
| $X o Y$<br>$Y oi X$ |  | X overlaps with Y<br>Y is overlapped by X                         |
| $X s Y$<br>$Y si X$ |  | X starts Y<br>Y is started by X                                   |
| $X d Y$<br>$Y di X$ |  | X during Y<br>Y contains X  |
| $X f Y$<br>$Y fi X$ |  | X finishes Y<br>Y is finished by X                                |
| $X = Y$             |  | X is equal to Y   |

from Wikipedia: [https://en.wikipedia.org/wiki/](https://en.wikipedia.org/wiki/Allen%27s_interval_algebra)

[Allen%27s\\_interval\\_algebra](https://en.wikipedia.org/wiki/Allen%27s_interval_algebra)

# Start and End of Jobs ✓



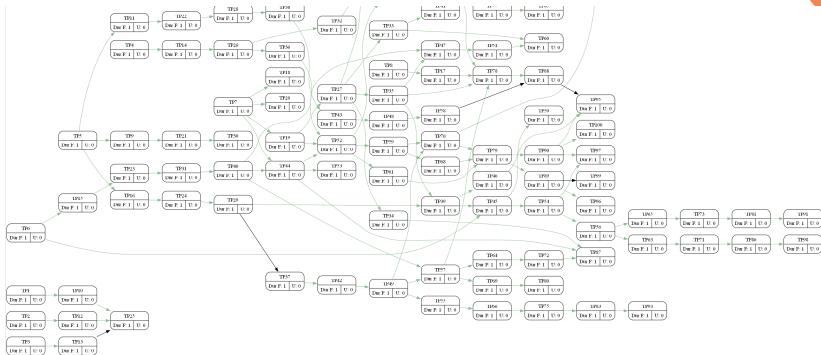
- The start of a job is equal to the start of the earliest task of the job
- The end of a job is equal to the latest end of any of its tasks
- Also called: the job *spans* its tasks
- Sometimes very simple
  - Start of job is start of first process step
  - End of job is end of last process step
  - But, do we know which steps will be first or last?

# An Example of a Simple Process



- The steps form a precedence chain
- Easy to identify first and last step

# An Example of a More Complex Process



- There is no clear first or last process step

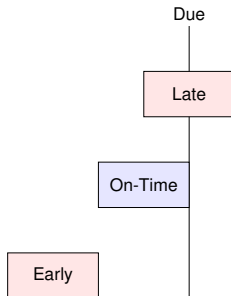
# Jobs: Release and Due Dates ✓



- The execution of a job may be constrained in time
- *Release dates* states earliest time a job can start
- *Due dates* states latest time a job can end
- These may or may not be hard constraints!
- A job will be *late* if it ends after the due date
- A job will be *early* if it ends before the due date
- A job will be *on-time* if it ends at the due date

Release

Due



# Relations between Jobs X



- There may be relations between jobs as well
- For example, jobs for the same product may be arranged by due date
- Do not allow to run job for a later due date before any job with an earlier due date
- Orders for the same customer, but different products, may be constrained
- Most common:
  - Jobs for intermediate products must finish in time for their use later on

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

Summary

# More Complexity



- We have ignored a lot of potential complications
  - Alternative processes
  - Alternative process paths
  - Alternative resources
- Intermediate products
- Impact of raw material availability

# Intermediate products x



- Some production operations are assembly steps
- Combine multiple intermediate products together
- These intermediate products need to be made as well
- There are processes for those products

# Raw materials X



- Sometimes, a process step needs certain raw materials
- These are not made within the scheduled part of the plant
- They come from stock, inventory control problem
- Do we schedule production and then order raw materials?
- Do we schedule based on the available raw materials?

# Bill of Materials (BoM), Bill of Processes X



- Enterprise systems will describe which items are needed to make a product
- Tree like structure, indicates the intermediate product/raw material needed and its quantity
- *BoM explosion* derive all required input materials for a given set of orders
- We may want to know at which step of process we need which materials (Bill of processes)
- This is where you use SAP, big database, trivial calculation
- Becomes hard if processes not fixed

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

## Problem Classification

Job-Shop

Flow-Shop

Open-Shop

RCPSP

$\alpha/\beta/\gamma$  Notation

Key Visualization Methods

# Problem Classification

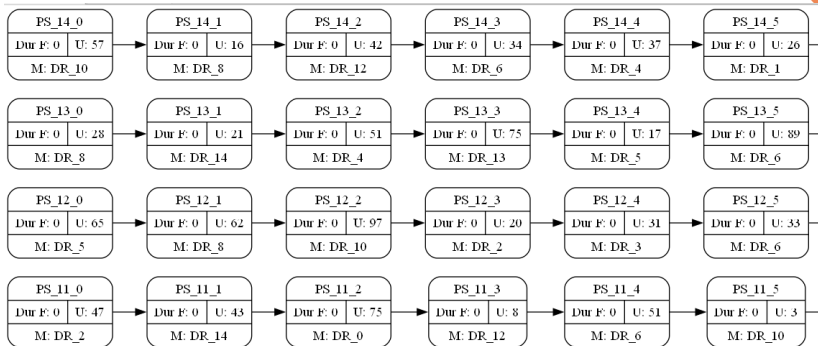


- Most real-world problems are messy, with many special conditions and exceptions
- Academic research prefers well-structured problems
- Scheduling research often focuses on well-structured problem types
  - Easier to understand
  - Possible to exploit structure
  - Easier to compare results
- A small number of problem types are very common in research



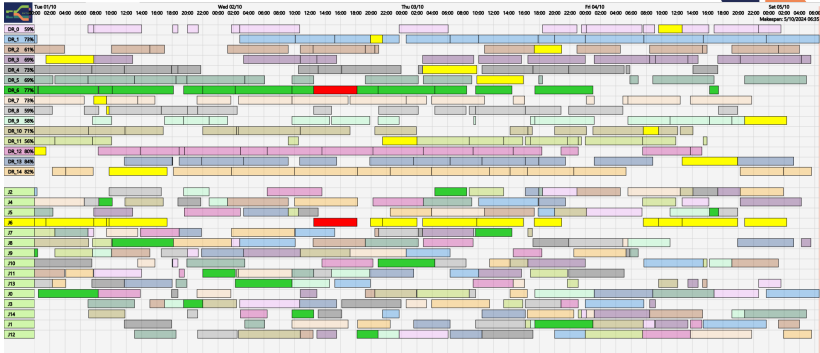
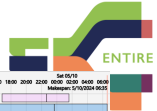
- Consists of a number of jobs and a number of machines
- Each job visits each machine, but possibly in a different order, depending on process
- Tasks of a job are linked as a precedence chain
- Objective is to minimize overall end, the *makespan*

# Example Job-Shop Process



- Note that the order of machines visited is different for each process

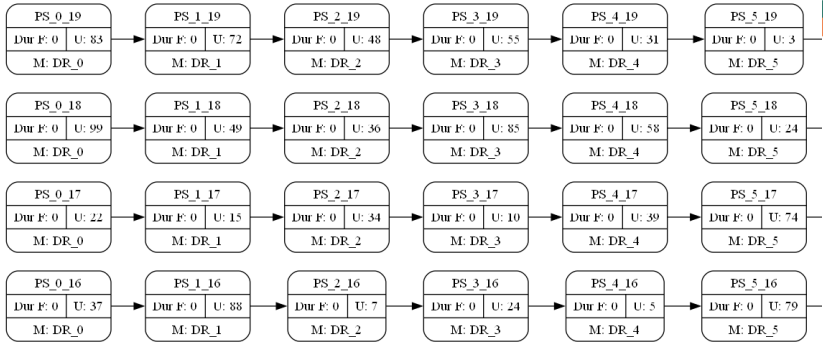
# Example Job-Shop Solution



- One task is selected (in red), in both Machine and Job Gantt Chart
- Tasks are colored by machine, note coloring in jobs is different for each job

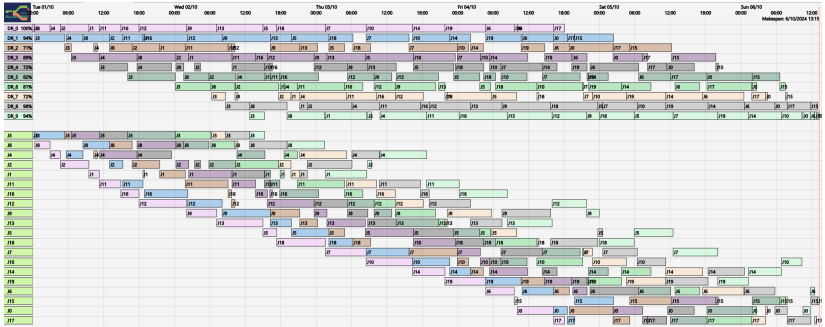


- Consists of a number of jobs and a number of machines
- Each jobs visits each machine, all jobs in the same order
- Tasks of a job a linked in a precedence chain
- Objective is to minimize overall end, the *makespan*



- Note that each process visits the machines in order DR\_0, DR\_1, ...

# Example Flow-Shop Solution



- Tasks are colored by machine, note the regular pattern in the Job Gantt Chart



- Consists of a number of jobs and a number of machines
- Each jobs visits each machine, we have to choose the sequence individually for each order
- There are no temporal constraints between tasks, but tasks of the same job cannot overlap
- Objective is to minimize overall end, the *makespan*

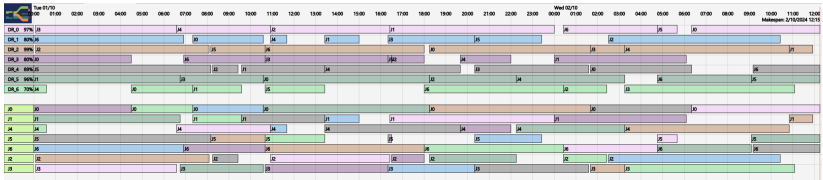
# Open Shop Example Process



- Only showing details of one process
- No prescribed sequence between process steps
- Easier to find a task to run next
- Much larger search space

|             |                |
|-------------|----------------|
| ▼ Processes |                |
| process_0   | PS_0_6         |
| process_1   | Dur F: 0 U: 56 |
| process_2   | M: DR_4        |
| process_3   |                |
| process_4   | PS_0_5         |
| process_5   | Dur F: 0 U: 92 |
| process_6   | M: DR_5        |
|             |                |
|             | PS_0_4         |
|             | Dur F: 0 U: 71 |
|             | M: DR_0        |
|             |                |
|             | PS_0_3         |
|             | Dur F: 0 U: 34 |
|             | M: DR_6        |
|             |                |
|             | PS_0_2         |
|             | Dur F: 0 U: 54 |
|             | M: DR_3        |
|             |                |
|             | PS_0_1         |
|             | Dur F: 0 U: 39 |
|             | M: DR_1        |
|             |                |
|             | PS_0_0         |
|             | Dur F: 0 U: 89 |
|             | M: DR_2        |

# Open-Shop Example Solution



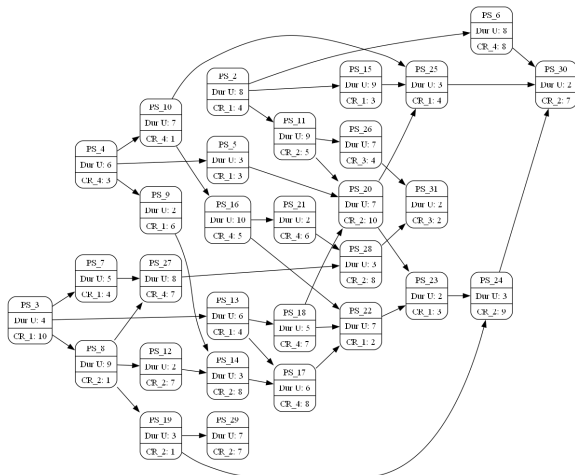
- Example solution for 7x7 open shop example
- Order of tasks within jobs not constrained
- Note that machines are still idle in optimal solution

# Resource Constrained Project Scheduling Problem (RCPSP) ✓



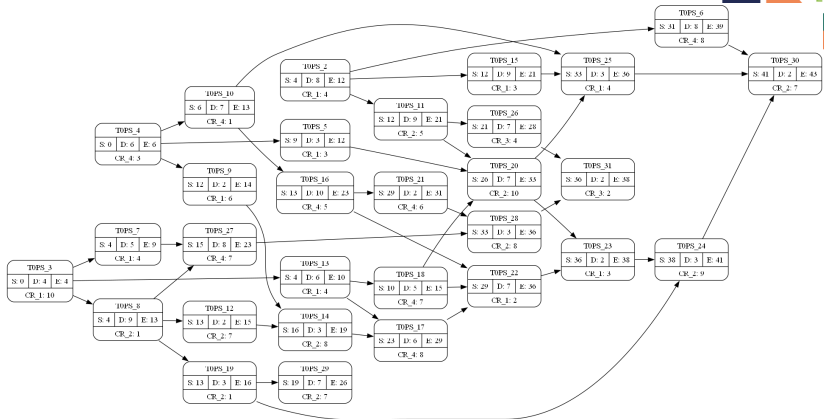
- Problem class from project management
- One project (one job), many tasks
- Precedence graph is arbitrary DAG
- Cumulative resources
- Variants with alternative modes, process alternatives ✗

# RCPSP Process Diagram



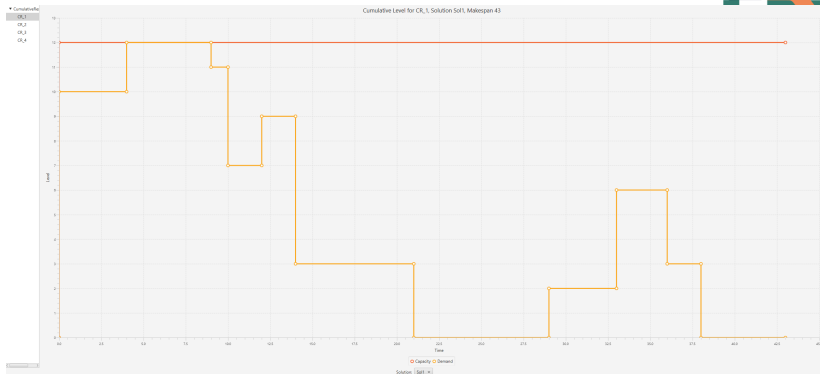
- Multiple sources, sinks
- Multiple cumulative resources

# RCPSP PERT Diagram



- Resource constraints influence schedule, no single critical path

# RCPSP Cumulative Chart



- Multiple cumulative resource, not busy all the time
- Not all instances are very hard to solve



- The previous classes are good for algorithm research, but not very practical
- General scheme to describe problem type introduced in 1979
- Based on three parameters
  - $\alpha$  resource structure, stages
  - $\beta$  temporal relations
  - $\gamma$  objective
- $P2/r_j, \bar{d}_j/C_{\max}$  : One stage, two identical parallel machines, hard release and due dates, objective makespan
- More detailed description at <https://encyclopedia.pub/entry/30497>

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

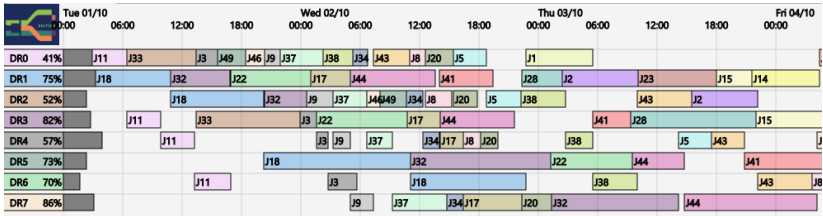
Key Visualization Methods

Summary



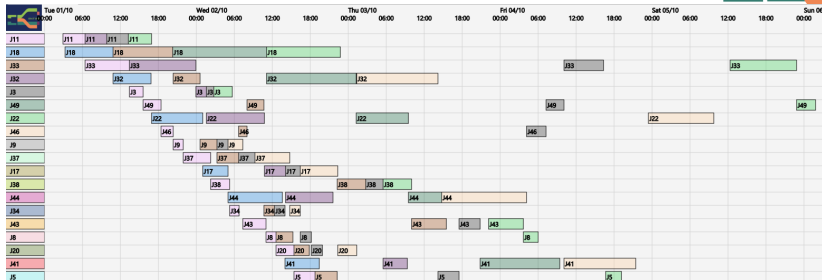
- Visualization is key to present and to understand results
- Many different ways to give an overview of schedule, and highlight problems
- Some diagrams types are used a lot, and are provided in our generic scheduling tool
- Customization is key

# Machine Gantt Chart ✓



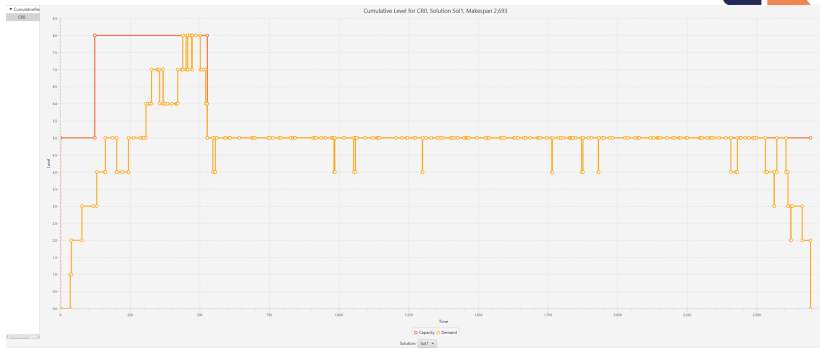
- Shows all tasks that are assigned to each machine
- Tasks should not overlap
- Also shows work in progress (WiP), down-times
- Optional display of setup and idle times

# Job Gantt Chart ✓



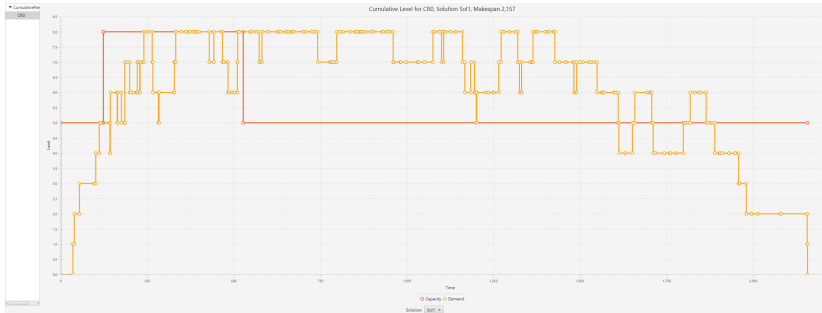
- Shows all tasks of a job in one line
- Only works for single chain of process steps
- Possible display of earliness, lateness
- Optional display of waiting and transport times

# Cumulative Resource Chart ✓

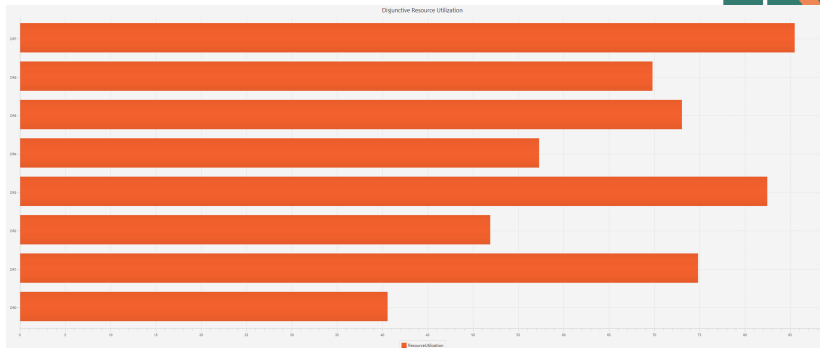


- Shows resource utilization of cumulative resource over time
- Utilization should be below capacity profile
- Unless we relax the cumulative resource constraint

# Cumulative Resource Constraint Relaxed

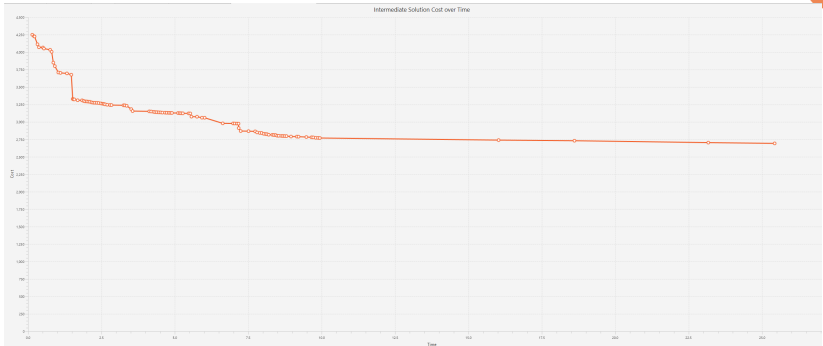


# Resource Utilization ✓



- Shows utilization of machines as percentage of active time
- Helpful to identify bottleneck machines
- Information also shown in Machine Gantt

# Intermediate Solutions ✓

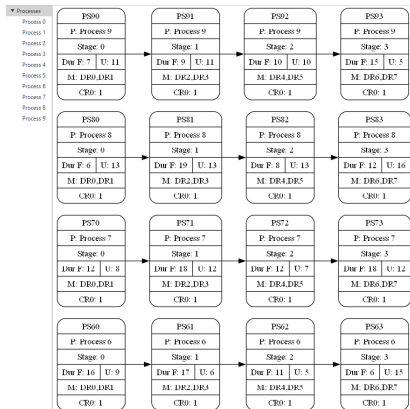


- Shows intermediate solutions found over time
- Useful to see if enough/too much time is allocated

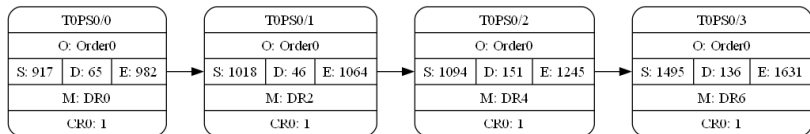
# Process Diagram ✓



- See all details of one process in one image
- Can also look at all processes in one diagram
- Options to show/hide different fields



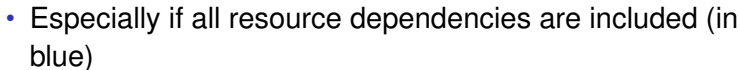
# PERT Chart(Program Evaluation Review Technique) ✓



- Show details of job as a graph
- Useful if task graph is not a chain
- Often used in project management



| TS2P50-3 |        |        |
|----------|--------|--------|
| 352      | E: 174 | E: 480 |



The logo for Entire Business, featuring a stylized 'E' composed of green, blue, and orange geometric shapes, followed by the word 'ENTIRE' in green capital letters.

- Shows weekly structure for one or more years
- Indicates public holidays, shut-downs, etc
- Indicating working days, KPI for each day

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- We introduced the key concepts for scheduling problems
- Orders, products, processes
- Jobs and tasks
- Existing problem classifications
  - Academic
  - Limited practical usefulness
  - Used for benchmarking
- Key visualization ideas