



Enterprise  
Ireland



Funded by the  
European Union  
NextGenerationEU

**EDIH** | European  
Digital Innovation  
Hubs Network



# Concepts

**Helmut Simonis**

## Constraint Based Production Scheduling



# Licence



This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. To view a copy of this license, visit <https://creativecommons.org/licenses/by-nc-sa/4.0/>.

This license requires that reusers give credit to the creator. It allows reusers to distribute, remix, adapt, and build upon the material in any medium or format, for noncommercial purposes only. If others modify or adapt the material, they must license the modified material under identical terms.



# Acknowledgments



This publication was developed as part of the ENTIRE EDIH project, which received funding from Enterprise Ireland and the European Commission.

Part of this work is based on research conducted with the financial support of Science Foundation Ireland under Grant number 12/RC/2289-P2 at Insight the SFI Research Centre for Data Analytics at UCC, which is co-funded under the European Regional Development Fund.

Part of this work is based on research conducted within the ASSISTANT European project, under the framework program Horizon 2020, ICT-38-2020, Artificial intelligence for manufacturing, grant agreement number 101000165.

# Key Points



- We introduce the core concepts used in scheduling
- Different layers of description
  - What we are doing (jobs, tasks, resources)
  - Why we are scheduling (orders, products, processes)
- Temporal Relations
- Process description
- Problem classification
- Visualization

# Outline



## Core Concepts

Jobs, Tasks and Resources

Orders, Products, Processes

## Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

Summary

# Most basic description of scheduling problem



- *Job*
  - Collection of activities required to manufacture one object/lot/order
  - Overall start/end determined by starts and ends of its tasks
- *Task*
  - Individual activities required for manufacture
  - Have defined start, end (typical: variables) and duration (sometimes fixed)
  - Often performed on one specific resource (more on that later)
- *Resources*
  - Resources are needed to perform the tasks
  - Very compact representation of scheduling problem
  - But, where does that information come from?

# Scheduling orders



- An *order* specifies a need for a certain *product* at a given time in a specific quantity
- There may be multiple ways of making the *product* (multiple *processes*)
- We assume that the process to use is decided when placing the order
- Each order corresponds to a job, with its constituent tasks
- There may be limited visibility of future orders

# Process Description



- Each *process* consists of one or more *process steps*
- A process step contains a duration formula to describe how long it lasts
- The order of *process steps* is defined by *process sequences*
- The resources needed are defined by *resource needs* (described later on)
- Tasks are created for each process step, their duration is based on the duration formula and order quantity

# Where do the orders come from?



- Made to order
  - Each order is caused by a customer request
  - Defines due date, release date often implied
- Made to stock
  - Orders are satisfied from stock
  - Inventory control strategy decides when to make product
  - Often called stock orders
  - More complex variant integrates production planning and detailed scheduling
  - Example later in course

# Outline



Core Concepts

## Temporal Relations

Relations between Tasks

Relation between Tasks and Jobs

Jobs: Release and Due Date

Relations between jobs

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

Summary

# Temporal Relations



- Temporal constraints between tasks and/or jobs
- Defined by the manufacturing process
- In simple cases
  - A single sequence of process steps performed in that order
  - Each task must finish before the next one can start



# Annotations on Features

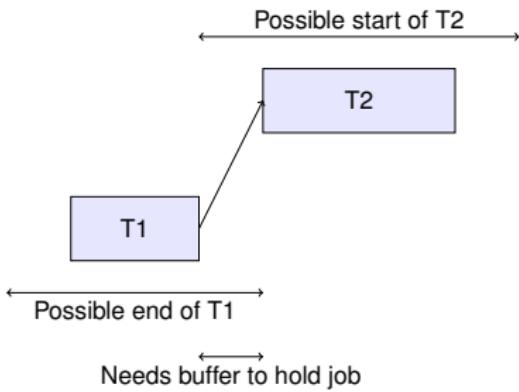


- ✓ Currently available in scheduling tool
- (✓) Will be available shortly
- ✗ Currently not available, may be added in future version

# The Most Common Relation: EndBeforeStart ✓



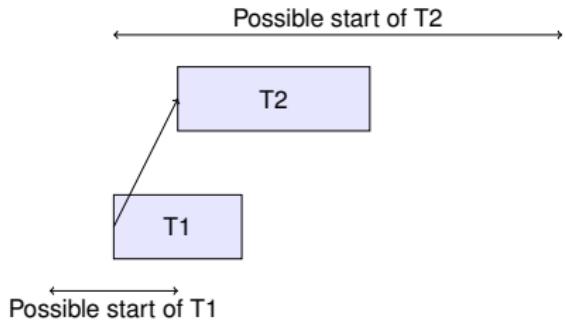
- States that one task (T1) must end before the next one (T2) can start
- Typical for manufacturing process based on the same item
- Addition: offset
  - Wait at least offset units between end and start
  - For example cooling, drying time outside a machine



# Less Common: StartBeforeStart ✓



- States that one task (T2) can start any time after the start of another task (T1)
- Uncommon in manufacturing, occurs in project management
- Example later on on assembly line balancing

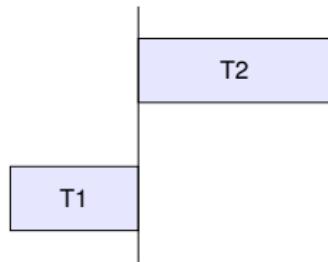


# NoWait ✓



- Sometimes, two steps must follow each other immediately
- The item made would spoil
  - Product specific
- There is no space to hold item
  - Machine specific, buffers
- End of one task (T1) must be equal to start of next task (T2)
- May mean delay of start of task T1

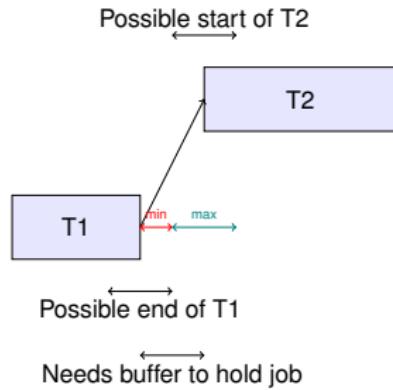
Start of T2 is equal to End of T1



# MaxWait (✓)



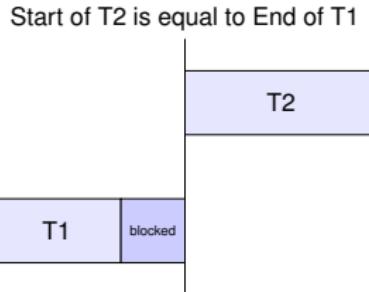
- Limit how long we can wait between tasks
  - Cooling enough, but not too much
  - Baking: rise time
- Impose both lower and upper waiting time limit
- Makes it more difficult to find solutions



# Blocking ✓



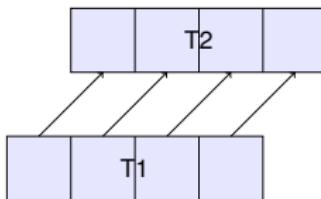
- Sometimes, two steps must follow each other immediately
- There is no space to store item between machines
- Keep item on previous machine until needed
- That machine is now *blocked*
- Duration of task T1 is extended until start of T2
- *Use with caution! Easy to deadlock*



# Special Case: Pipelining X



- Sometimes, we can start on the next task while the first is still running
- Possible if one job produces multiple items (lots,...)
- As soon as the first item is finished, take it to the next machine to process it there
- Overlaps T1 and T2 as much as possible
- Details can get complex



# More General: Relations between Intervals X



- First introduced by Allen (1983)
- 13 relations between intervals
- Allows composition of relations
- Constraint reasoning on sets of relations

Relation	Illustration	Interpretation
$X < Y$	$\underline{\hspace{1cm} X \hspace{1cm}}$	X precedes Y
$Y > X$	$\underline{\hspace{1cm} \hspace{1cm} Y}$	Y is preceded by X
$X \text{ m } Y$	$\underline{\hspace{1cm} X \hspace{1cm}} \underline{\hspace{1cm} \hspace{1cm} Y}$	X meets Y
$Y \text{ mi } X$	$\underline{\hspace{1cm} \hspace{1cm} Y} \underline{\hspace{1cm} \hspace{1cm}}$	Y is met by X ( <i>i</i> stands for <i>inverse</i> )
$X \text{ o } Y$	$\underline{\hspace{1cm} X \hspace{1cm}} \underline{\hspace{1cm} \hspace{1cm} Y}$	X overlaps with Y
$Y \text{ oi } X$	$\underline{\hspace{1cm} \hspace{1cm} Y} \underline{\hspace{1cm} \hspace{1cm}}$	Y is overlapped by X
$X \text{ s } Y$	$\underline{\hspace{1cm} X \hspace{1cm}} \underline{\hspace{1cm} \hspace{1cm} Y}$	X starts Y
$Y \text{ si } X$	$\underline{\hspace{1cm} \hspace{1cm} Y} \underline{\hspace{1cm} \hspace{1cm}}$	Y is started by X
$X \text{ d } Y$	$\underline{\hspace{1cm} X \hspace{1cm}} \underline{\hspace{1cm} \hspace{1cm} Y}$	X during Y
$Y \text{ di } X$	$\underline{\hspace{1cm} \hspace{1cm} Y} \underline{\hspace{1cm} \hspace{1cm}}$	Y contains X
$X \text{ f } Y$	$\underline{\hspace{1cm} \hspace{1cm} Y} \underline{\hspace{1cm} X \hspace{1cm}}$	X finishes Y
$Y \text{ fi } X$	$\underline{\hspace{1cm} \hspace{1cm} Y} \underline{\hspace{1cm} \hspace{1cm}}$	Y is finished by X
$X = Y$	$\underline{\hspace{1cm} X \hspace{1cm}} \underline{\hspace{1cm} \hspace{1cm} Y}$	X is equal to Y

from Wikipedia: <https://en.wikipedia.org/wiki/>

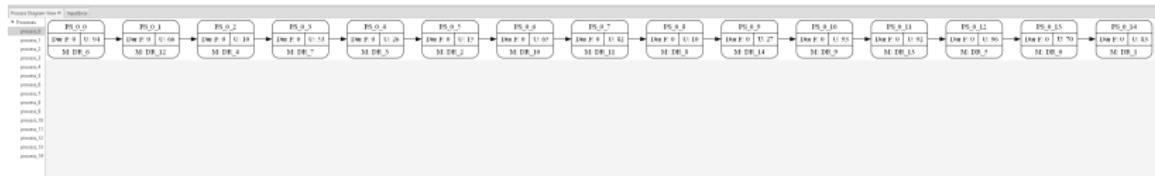
Allen%27s\_interval\_algebra

# Start and End of Jobs ✓

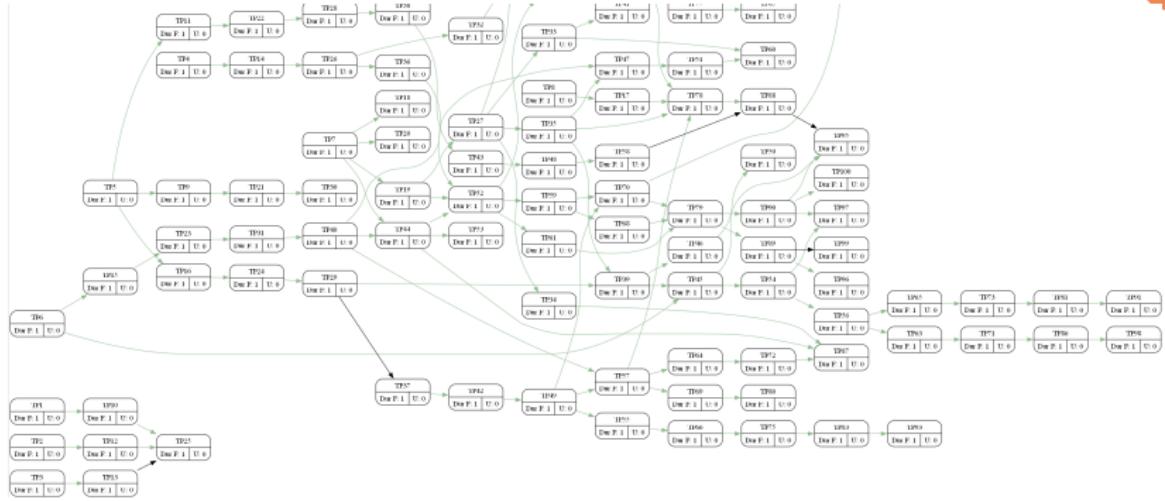


- The start of a job is equal to the start of the earliest task of the job
- The end of a job is equal to the latest end of any of its tasks
- Also called: the job *spans* its tasks
- Sometimes very simple
  - Start of job is start of first process step
  - End of job is end of last process step
  - But, do we know which steps will be first or last?

# An Example of a Simple Process



# An Example of a More Complex Process

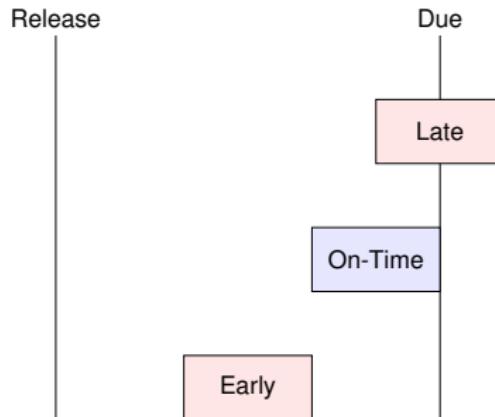


- There is no clear first or last process step

# Jobs: Release and Due Dates ✓



- The execution of a job may be constrained in time
- *Release dates* states earliest time a job can start
- *Due dates* states latest time a job can end
- These may or may not be hard constraints!
- A job will be *late* if it ends after the due date
- A job will be *early* if it ends before the due date
- A job will be *on-time* if it ends at the due date



# Relations between Jobs X



- There may be relations between jobs as well
- For example, jobs for the same product may be arranged by due date
- Do not allow to run job for a later due date before any job with an earlier due date
- Orders for the same customer, but different products, may be constrained
- Most common:
  - Jobs for intermediate products must finish in time for their use later on

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

Summary

# More Complexity



- We have ignored a lot of potential complications
  - Alternative processes
  - Alternative process paths
  - Alternative resources
- Intermediate products
- Impact of raw material availability

# Intermediate products X



- Some production operations are assembly steps
- Combine multiple intermediate products together
- These intermediate products need to be made as well
- There are processes for those products

# Raw materials X



- Sometimes, a process step needs certain raw materials
- These are not made within the scheduled part of the plant
- They come from stock, inventory control problem
- Do we schedule production and then order raw materials?
- Do we schedule based on the available raw materials?

# Bill of Materials (BoM), Bill of Processes X



- Enterprise systems will describe which items are needed to make a product
- Tree like structure, indicates the intermediate product/raw material needed and its quantity
- *BoM explosion* derive all required input materials for a given set of orders
- We may want to know at which step of process we need which materials (Bill of processes)
- This is where you use SAP, big database, trivial calculation
- Becomes hard if processes not fixed

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

## Problem Classification

Job-Shop

Flow-Shop

Open-Shop

RCPS

$\alpha/\beta/\gamma$  Notation

## Key Visualization Methods

# Problem Classification



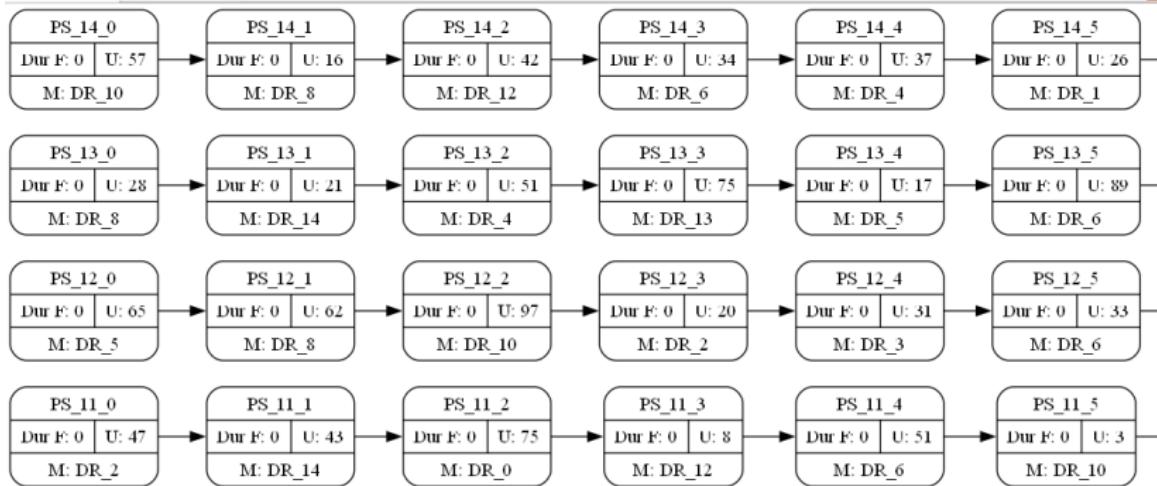
- Most real-world problems are messy, with many special conditions and exceptions
- Academic research prefers well-structured problems
- Scheduling research often focuses on well-structured problem types
  - Easier to understand
  - Possible to exploit structure
  - Easier to compare results
- A small number of problem types are very common in research

# Job-Shop ✓



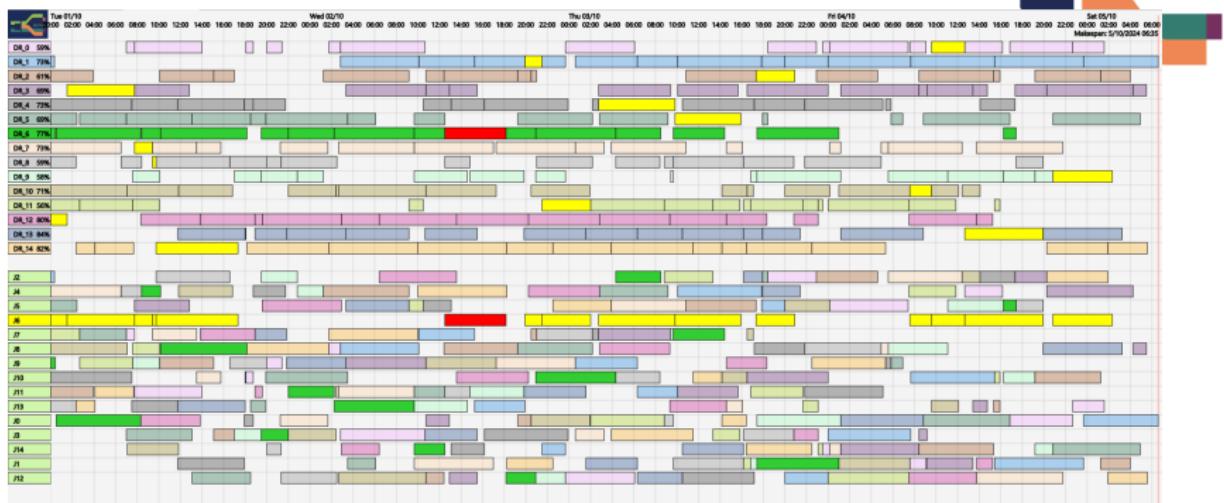
- Consists of a number of jobs and a number of machines
- Each job visits each machine, but possibly in a different order, depending on process
- Tasks of a job are linked as a precedence chain
- Objective is to minimize overall end, the *makespan*

# Example Job-Shop Process



- Note that the order of machines visited is different for each process

# Example Job-Shop Solution

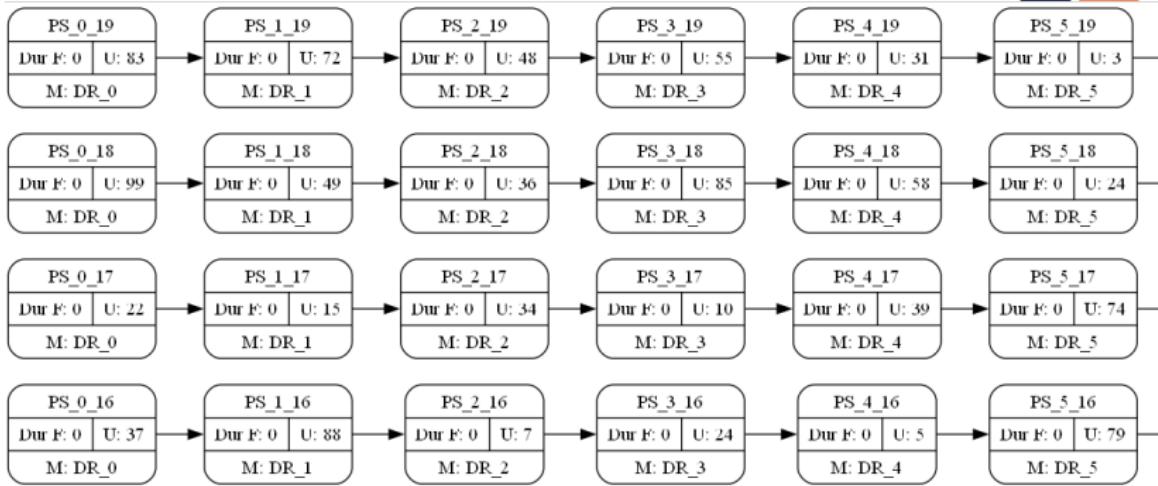


- One task is selected (in red), in both Machine and Job Gantt Chart
- Tasks are colored by machine, note coloring in jobs is different for each job

# Flow-Shop ✓

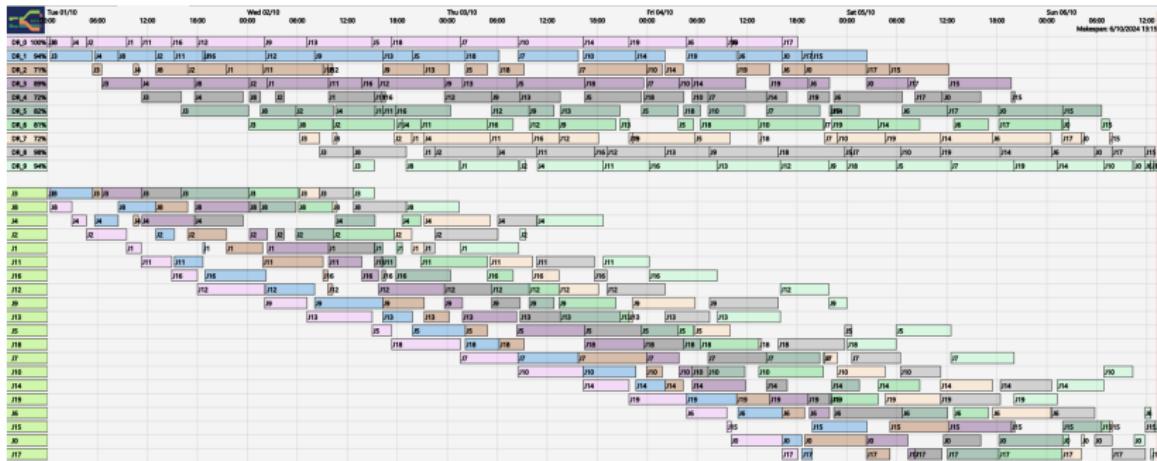


- Consists of a number of jobs and a number of machines
- Each job visits each machine, all jobs in the same order
- Tasks of a job are linked in a precedence chain
- Objective is to minimize overall end, the *makespan*



- Note that each process visits the machines in order DR\_0, DR\_1, ...

# Example Flow-Shop Solution



- Tasks are colored by machine, note the regular pattern in the Job Gantt Chart

# Open-Shop ✓



- Consists of a number of jobs and a number of machines
- Each job visits each machine, we have to choose the sequence individually for each order
- There are no temporal constraints between tasks, but tasks of the same job cannot overlap
- Objective is to minimize overall end, the *makespan*

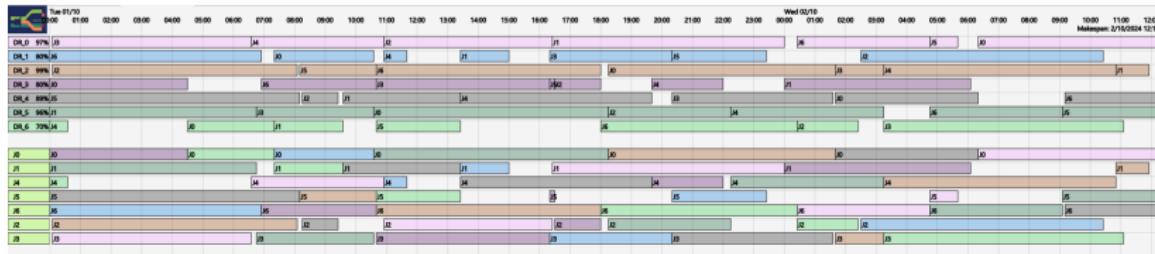
# Open Shop Example Process



- Only showing details of one process
- No prescribed sequence between process steps
- Easier to find a task to run next
- Much larger search space

▼ Processes		
process_0	PS_0_6	
process_1	Dur F: 0	U: 56
process_2	M: DR_4	
process_3		
process_4		
process_5		
process_6	PS_0_5	
	Dur F: 0	U: 92
	M: DR_5	
	PS_0_4	
	Dur F: 0	U: 71
	M: DR_0	
	PS_0_3	
	Dur F: 0	U: 34
	M: DR_6	
	PS_0_2	
	Dur F: 0	U: 54
	M: DR_3	
	PS_0_1	
	Dur F: 0	U: 39
	M: DR_1	
	PS_0_0	
	Dur F: 0	U: 89
	M: DR_2	

# Open-Shop Example Solution



- Example solution for 7x7 open shop example
- Order of tasks within jobs not constrained
- Note that machines are still idle in optimal solution

# Resource Constrained Project Scheduling Problem (RCPSP) (✓)



- Problem class from project management
- One project (one job), many tasks
- Precedence graph is arbitrary DAG
- Cumulative as well as disjunctive resources
- Variants with process alternatives

# $\alpha/\beta/\gamma$ Notation



- The previous classes are good for algorithm research, but not very practical
- General scheme to describe problem type introduced in 1979
- Based on three parameters
  - $\alpha$  resource structure, stages
  - $\beta$  temporal relations
  - $\gamma$  objective
- $P2/r_j, \bar{d}_j/C_{\max}$  : One stage, two identical parallel machines, hard release and due dates, objective makespan
- More detailed description at  
<https://encyclopedia.pub/entry/30497>

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

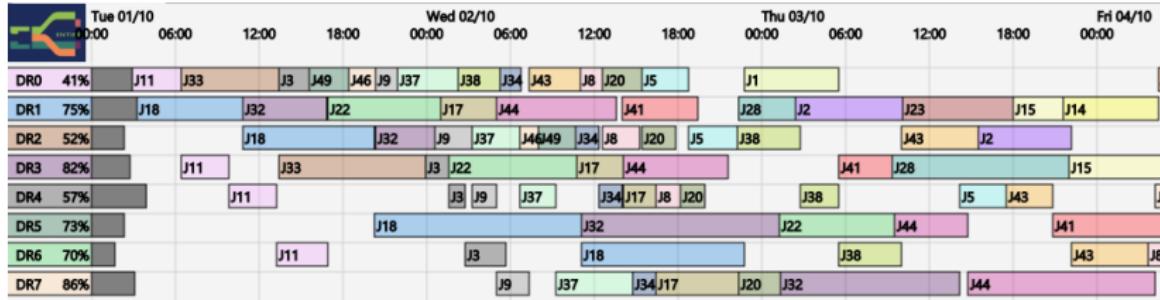
Summary

# Visualization



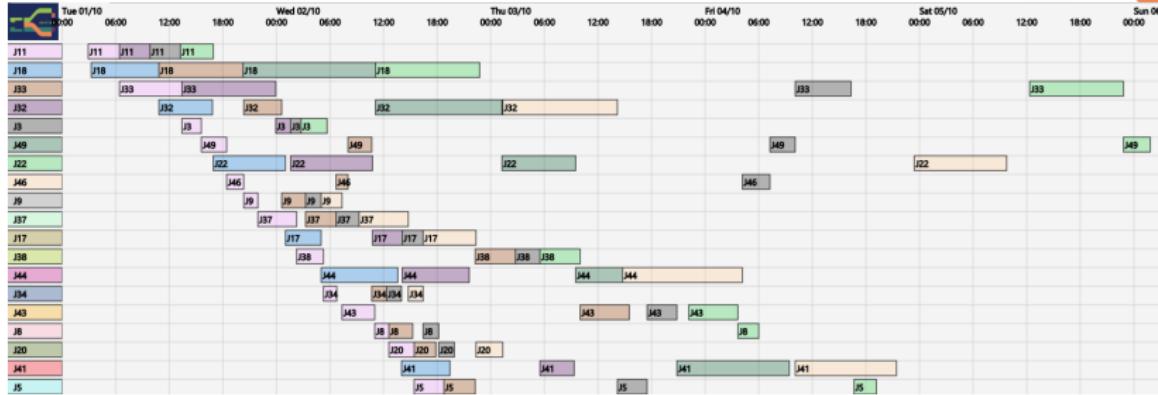
- Visualization is key to present and to understand results
- Many different ways to give an overview of schedule, and highlight problems
- Some diagrams types are used a lot, and are provided in our generic scheduling tool
- Customization is key

# Machine Gantt Chart ✓



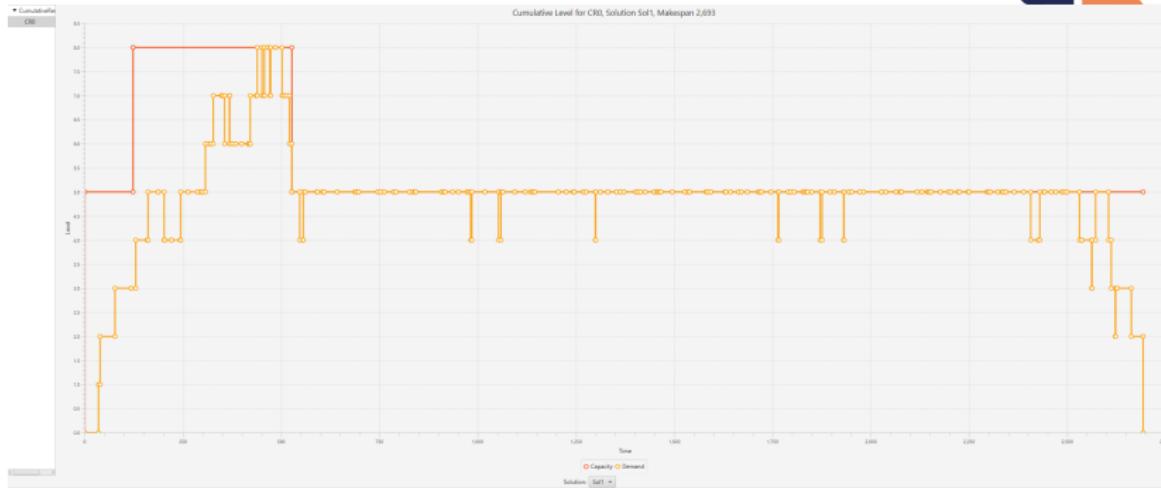
- Shows all tasks that are assigned to each machine
- Tasks should not overlap
- Also shows work in progress (WiP), down-times
- Optional display of setup and idle times

# Job Gantt Chart ✓



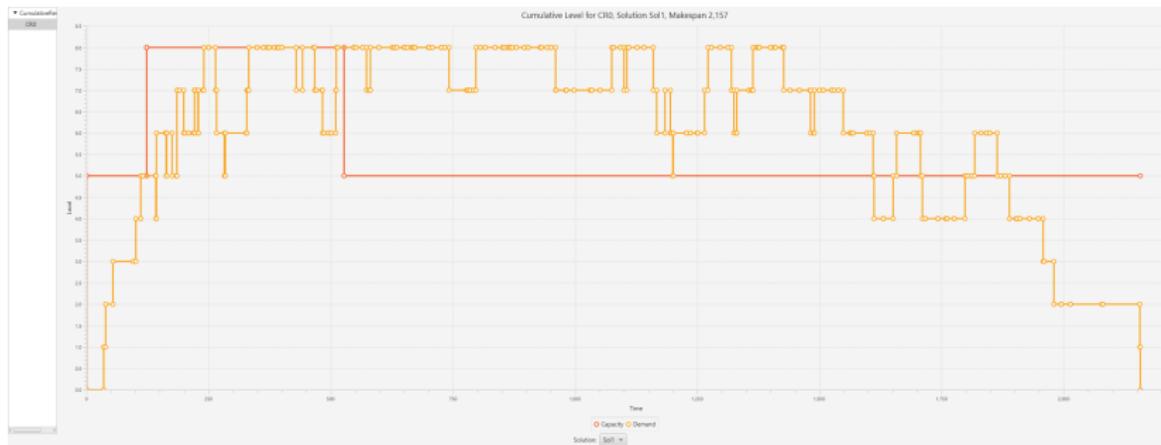
- Shows all tasks of a job in one line
- Only works for single chain of process steps
- Possible display of earliness, lateness
- Optional display of waiting and transport times

# Cumulative Resource Chart ✓

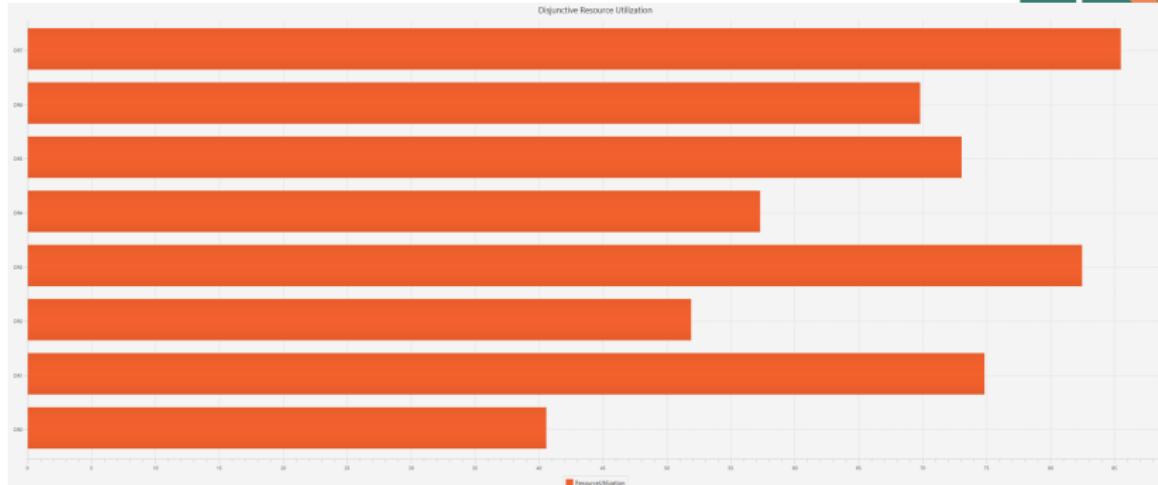


- Shows resource utilization of cumulative resource over time
- Utilization should be below capacity profile
- Unless we relax the cumulative resource constraint

# Cumulative Resource Constraint Relaxed

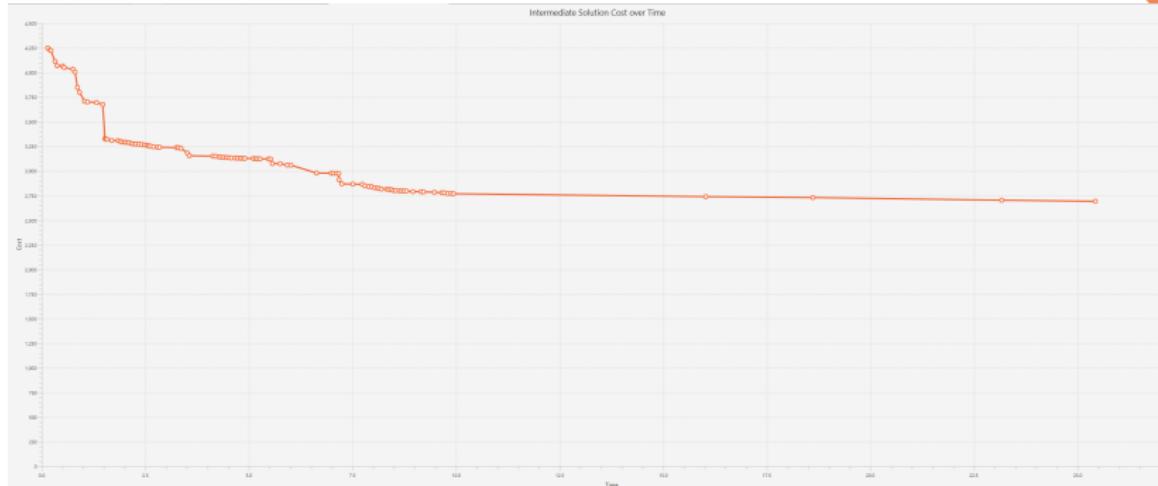


# Resource Utilization ✓



- Shows utilization of machines as percentage of active time
- Helpful to identify bottleneck machines
- Information also shown in Machine Gantt

# Intermediate Solutions ✓

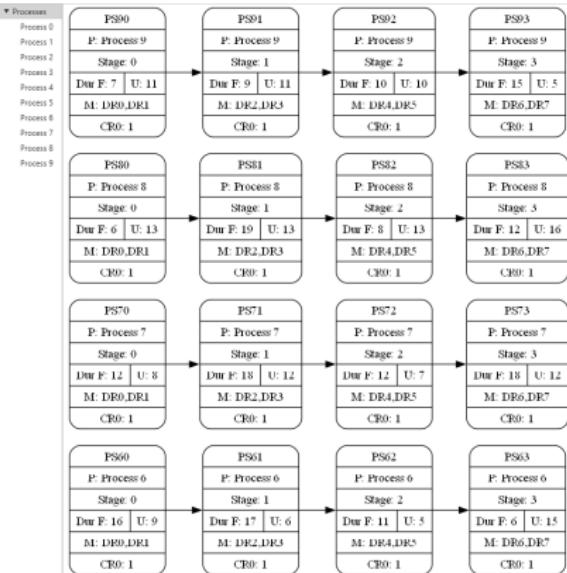


- Shows intermediate solutions found over time
- Useful to see if enough/too much time is allocated

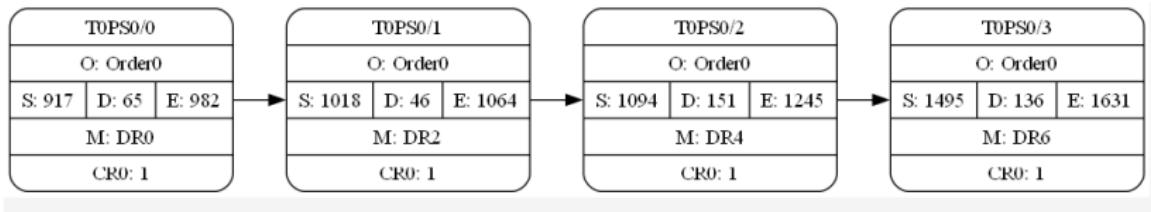
# Process Diagram ✓



- See all details of one process in one image
- Can also look at all processes in one diagram
- Options to show/hide different fields

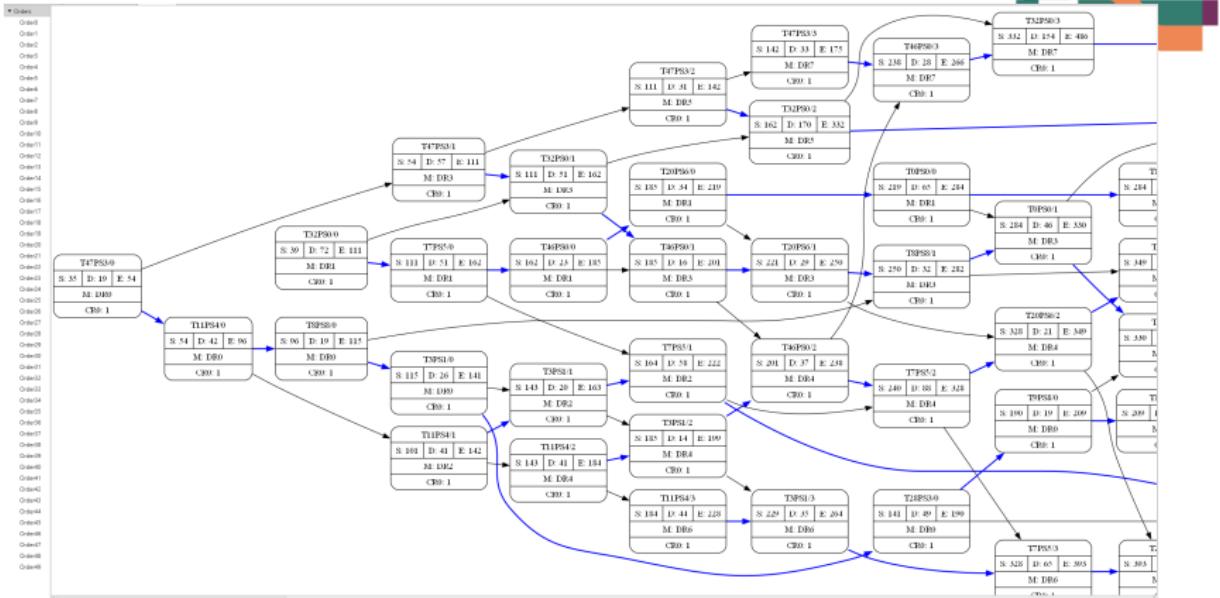


# PERT Chart(Program Evaluation Review Technique) ✓



- Show details of job as a graph
- Useful if task graph is not a chain
- Often used in project management

## PERT Charts become Confusing Quite Quickly



- Especially if all resource dependencies are included (in blue)

# Calendars X



	Jan 22	Feb 22	Mar 22	Apr 22	May 22	Jun 22	Jul 22	Aug 22	Sep 22	Oct 22	Nov 22	Dec 22
Mon												
Tue												
Wed												
Thu												
Fri												
Sat												
Sun												

- Shows weekly structure for one or more years
- Indicates public holidays, shut-downs, etc
- Indicating working days, KPI for each day

# Outline



Core Concepts

Temporal Relations

Alternative Processes, Bill of Materials

Problem Classification

Key Visualization Methods

Summary

# Summary



- We introduced the key concepts for scheduling problems
- Orders, products, processes
- Jobs and tasks
- Existing problem classifications
  - Academic
  - Limited practical usefulness
  - Used for benchmarking
- Key visualization ideas