A Decomposition Model for the NoWaithHFS Problem

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Abstract

1 Model

We describe a decomposition model for a NoWaitHFS with n jobs J and m stages S, scheduled on a set of disjunctive machines M. Each job consists of s tasks with start time s_{ij} and duration d_{ij} for stage i of job j, assigned to machine m_{ij}. Each stage has the same number of machines m_s.

We define the schedule by giving an assigned machine m_{ij} for each task, these values range from 1 to m_s. We also define the position p_{ij} of a task on its assigned machine, these values range from 1 to n. Two tasks of the same stage i only can have the same values p_{ij} if they are scheduled on different machines.

The following invariant must hold:

$$\forall_{j_1,j_2 \in J \ s.t. \ j_1 \neq j_2} \forall_{i \in S} : \quad (m_{ij_1} \neq m_{ij_2} \vee p_{ij_1} \neq p_{ij_2}) \tag{1}$$

The positions of two jobs in two consecutive stages must be compatible

$$\forall_{j_1,j_2 \in J \ s.t. \ j_1 \neq j_2} \forall_{1 \leq i < m} : \quad (m_{ij_1} = m_{ij_2} \wedge p_{i_1} < p_{ij_2}) \Rightarrow (m_{i+1,j_1} \neq m_{i+1,j_2} \vee p_{i+1,j_1} < p_{i+1,j_2}) \tag{2}$$

If two jobs are assigned on the same machine in stage i, then they must be eight assigned to different machines in stage $i+1$, or they must be in the same order in both stages.

The start times within a job are linked by equality constraints, as soon as one stage finished, the next stage must start, there is no wait between the tasks of the same job.

$$\forall_{j \in J} \forall_{1 \leq i < m} : \quad s_{i+1,j} = s_{ij} + d_{ij} \tag{3}$$

The total completion time $C_{\max}$ of the schedule is defined as

$$C_{\max} = \max_{j \in J} s_{mj} + d_{mj} \tag{4}$$

The overall end of the schedule is the last of the ends of any job. The objective is to minimize the overall completion time.

If two jobs j_1 and j_2 in stage i are scheduled on the same machine, then their start times must be arranged in such a way that they do not overlap:

$$\forall_{j_1,j_2 \in J \ s.t. \ j_1 \neq j_2} \forall_{i \in S} : \quad p_{ij_1} < p_{ij_2} \Rightarrow s_{ij_2} \geq s_{ij_1} + d_{ij_1} \tag{5}$$