

Concepts

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Constraint Based Production Scheduling

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Key Points

- We introduce the core concepts used in scheduling
- Different layers of description
 - What we are doing (jobs, tasks, resources)
 - Why we are scheduling (orders, products, processes)
- Temporal Relations
- Process description
- Problem classification
- Visualization

1 Core Concepts

1.1 Jobs, Tasks and Resources

Most basic description of scheduling problem

- *Job*
 - Collection of activities required to manufacture one object/lot/order
 - Overall start/end determined by starts and ends of its tasks
- *Task*
 - Individual activities required for manufacture
 - Have defined start, end (typical: variables) and duration (sometimes fixed)
 - Often performed on one specific resource (more on that later)
- *Resources*
 - Resources are needed to perform the tasks
- Very compact representation of scheduling problem
- But, where does that information come from?

1.2 Orders, Products, Processes

Scheduling orders

- An *order* specifies a need for a certain *product* at a given time in a specific quantity
- There may be multiple ways of making the *product* (multiple *processes*)
- We assume that the process to use is decided when placing the order
- Each order corresponds to a job, with its constituent tasks
- There may be limited visibility of future orders

Process Description

- Each *process* consists of one or more *process steps*
- A process step contains a duration formula to describe how long it lasts
- The order of *process steps* is defined by *process sequences*
- The resources needed are defined by *resource needs* (described later on)
- Tasks are created for each process step, their duration is based on the duration formula and order quantity

Where do the orders come from?

- Made to order
 - Each order is caused by a customer request
 - Defines due date, release date often implied
- Made to stock
 - Orders are satisfied from stock
 - Inventory control strategy decides when to make product
 - Often called stock orders
 - More complex variant integrates production planning and detailed scheduling
 - Example later in course

2 Temporal Relations

Temporal Relations

- Temporal constraints between tasks and/or jobs
- Defined by the manufacturing process
- In simple cases
 - A single sequence of process steps performed in that order
 - Each task must finish before the next one can start



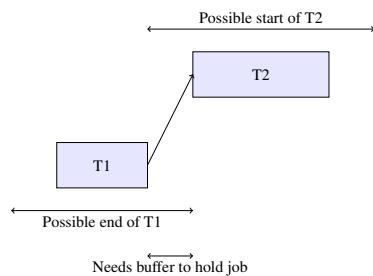
Annotations on Features

- ✓Currently available in scheduling tool
- (✓)Will be available shortly
- ✗Currently not available, may be added in future version

2.1 Relations between Tasks

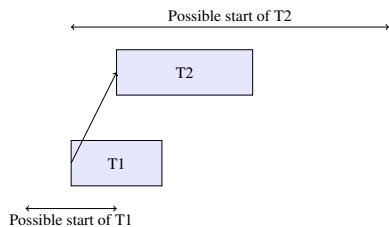
The Most Common Relation: EndBeforeStart ✓

- States that one task (T1) must end before the next one (T2) can start
- Typical for manufacturing process based on the same item
- Addition: offset
 - Wait at least offset units between end and start
 - For example cooling, drying time outside a machine



Less Common: StartBeforeStart ✓

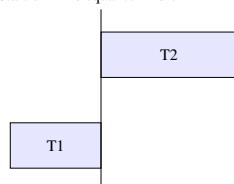
- States that one task (T2) can start any time after the start of another task (T1)
- Uncommon in manufacturing, occurs in project management
- Example later on on assembly line balancing



NoWait ✓

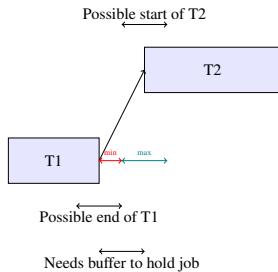
- Sometimes, two steps must follow each other immediately
- The item made would spoil
 - Product specific
- There is no space to hold item
 - Machine specific, buffers
- End of one task (T1) must be equal to start of next task (T2)
- May mean delay of start of task T1

Start of T2 is equal to End of T1



MaxWait (✓)

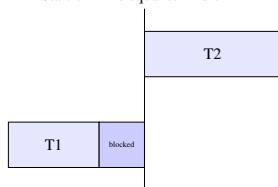
- Limit how long we can wait between tasks
 - Cooling enough, but not too much
 - Baking: rise time
- Impose both lower and upper waiting time limit
- Makes it more difficult to find solutions



Blocking ✓

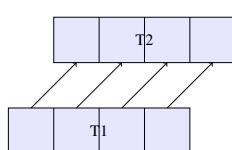
- Sometimes, two steps must follow each other immediately
- There is no space to store item between machines
- Keep item on previous machine until needed
- That machine is now *blocked*
- Duration of task T1 is extended until start of T2
- *Use with caution! Easy to deadlock*

Start of T2 is equal to End of T1



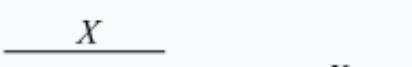
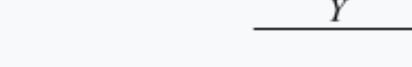
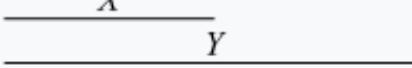
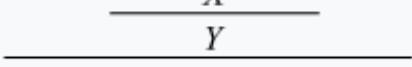
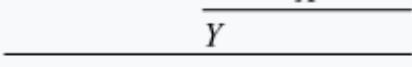
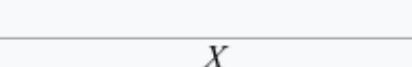
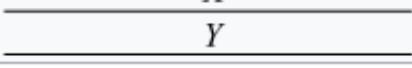
Special Case: Pipelining ✗

- Sometimes, we can start on the next task while the first is still running
- Possible if one job produces multiple items (lots,...)
- As soon as the first item is finished, take it to the next machine to process it there
- Overlaps T1 and T2 as much as possible
- Details can get complex



More General: Relations between Intervals

- First introduced by Allen (1983)
- 13 relations between intervals
- Allows composition of relations
- Constraint reasoning on sets of relations

Relation	Illustration	Interpretation
$X < Y$	 A horizontal line with two tick marks. The first tick mark is labeled X above the line. The second tick mark is labeled Y below the line.	X precedes Y
$Y > X$	 A horizontal line with two tick marks. The second tick mark is labeled Y above the line. The first tick mark is labeled X below the line.	Y is preceded by X
$X \mathbf{m} Y$	 A horizontal line with two tick marks. The first tick mark is labeled X above the line. The second tick mark is labeled Y below the line.	X meets Y
$Y \mathbf{mi} X$	 A horizontal line with two tick marks. The second tick mark is labeled Y above the line. The first tick mark is labeled X below the line.	Y is met by X (i stands for <i>inverse</i>)
$X \mathbf{o} Y$	 A horizontal line with two tick marks. The first tick mark is labeled X above the line. The second tick mark is labeled Y below the line.	X overlaps with Y
$Y \mathbf{oi} X$	 A horizontal line with two tick marks. The second tick mark is labeled Y above the line. The first tick mark is labeled X below the line.	Y is overlapped by X
$X \mathbf{s} Y$	 A horizontal line with two tick marks. The first tick mark is labeled X above the line. The second tick mark is labeled Y below the line.	X starts Y
$Y \mathbf{si} X$	 A horizontal line with two tick marks. The second tick mark is labeled Y above the line. The first tick mark is labeled X below the line.	Y is started by X
$X \mathbf{d} Y$	 A horizontal line with two tick marks. The first tick mark is labeled X above the line. The second tick mark is labeled Y below the line.	X during Y
$Y \mathbf{di} X$	 A horizontal line with two tick marks. The second tick mark is labeled Y above the line. The first tick mark is labeled X below the line.	Y contains X
$X \mathbf{f} Y$	 A horizontal line with two tick marks. The second tick mark is labeled X above the line. The first tick mark is labeled Y below the line.	X finishes Y
$Y \mathbf{fi} X$	 A horizontal line with two tick marks. The first tick mark is labeled Y above the line. The second tick mark is labeled X below the line.	Y is finished by X
$X = Y$	 A horizontal line with two tick marks. The first tick mark is labeled X above the line. The second tick mark is labeled Y below the line.	X is equal to Y

from Wikipedia: https://en.wikipedia.org/wiki/Allen%27s_interval_algebra

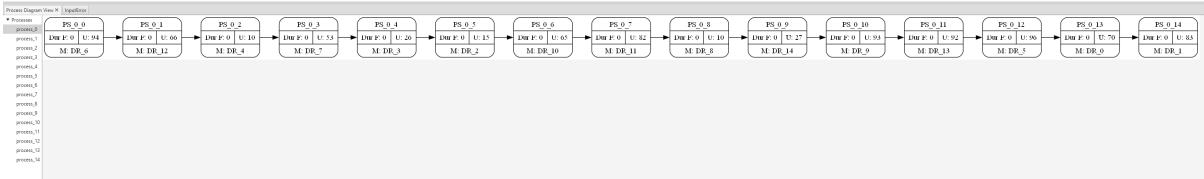
2.2 Relation between Tasks and Jobs

Start and End of Jobs

- The start of a job is equal to the start of the earliest task of the job

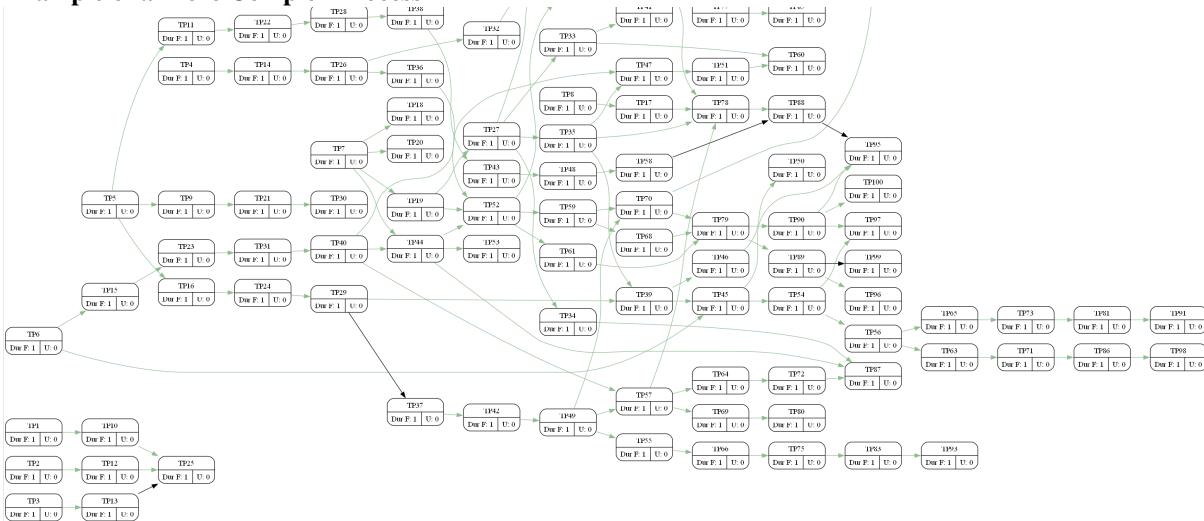
- The end of a job is equal to the latest end of any of its tasks
- Also called: the job *spans* its tasks
- Sometimes very simple
 - Start of job is start of first process step
 - End of job is end of last process step
 - But, do we know which steps will be first or last?

An Example of a Simple Process



- The steps form a precedence chain
- Easy to identify first and last step

An Example of a More Complex Process



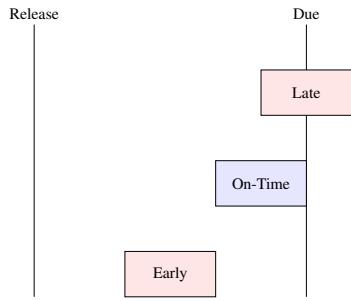
- There is no clear first or last process step

2.3 Jobs: Release and Due Date

Jobs: Release and Due Dates ✓

- The execution of a job may be constrained in time
- Release dates* states earliest time a job can start
- Due dates* states latest time a job can end
- These may or may not be hard constraints!
- A job will be *late* if it ends after the due date
- A job will be *early* if it ends before the due date

- A job will be *on-time* if it ends at the due date



2.4 Relations between jobs

Relations between Jobs X

- There may be relations between jobs as well
- For example, jobs for the same product may be arranged by due date
- Do not allow to run job for a later due date before any job with an earlier due date
- Orders for the same customer, but different products, may be constrained
- Most common:
 - Jobs for intermediate products must finish in time for their use later on

3 Alternative Processes, Bill of Materials

More Complexity

- We have ignored a lot of potential complications
 - Alternative processes
 - Alternative process paths
 - Alternative resources
- Intermediate products
- Impact of raw material availability

Intermediate products X

- Some production operations are assembly steps
- Combine multiple intermediate products together
- These intermediate products need to be made as well
- There are processes for those products

Raw materials ✗

- Sometimes, a process step needs certain raw materials
- These are not made within the scheduled part of the plant
- They come from stock, inventory control problem
- Do we schedule production and then order raw materials?
- Do we schedule based on the available raw materials?

Bill of Materials (BoM), Bill of Processes ✗

- Enterprise systems will describe which items are needed to make a product
- Tree like structure, indicates the intermediate product/raw material needed and its quantity
- *BoM explosion* derive all required input materials for a given set of orders
- We may want to know at which step of process we need which materials (Bill of processes)
- This is where you use SAP, big database, trivial calculation
- Becomes hard if processes not fixed

4 Problem Classification

Problem Classification

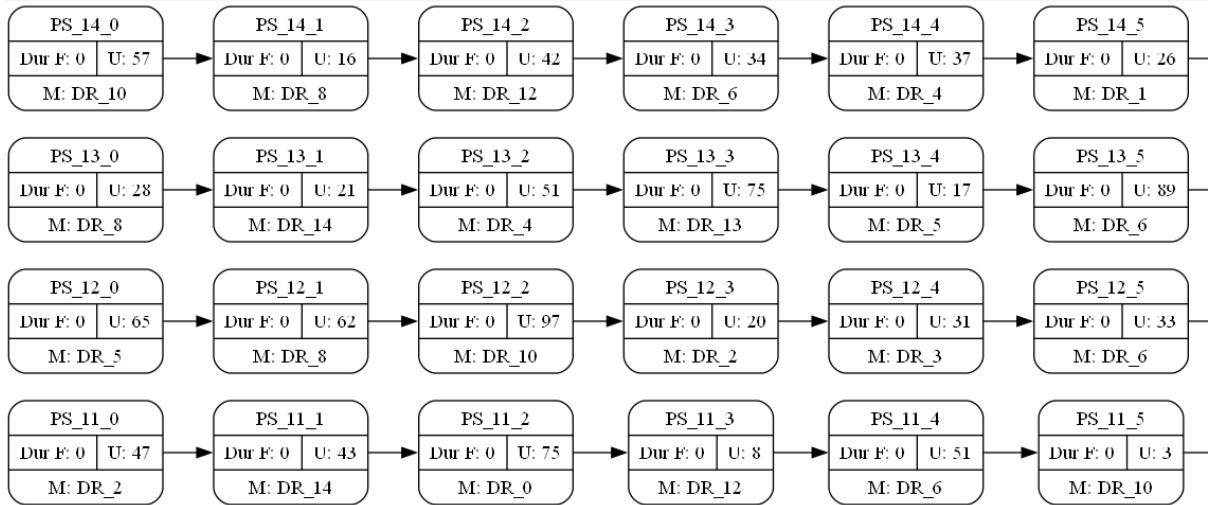
- Most real-world problems are messy, with many special conditions and exceptions
- Academic research prefers well-structured problems
- Scheduling research often focuses on well-structured problem types
 - Easier to understand
 - Possible to exploit structure
 - Easier to compare results
- A small number of problem types are very common in research

4.1 Job-Shop

Job-Shop ✓

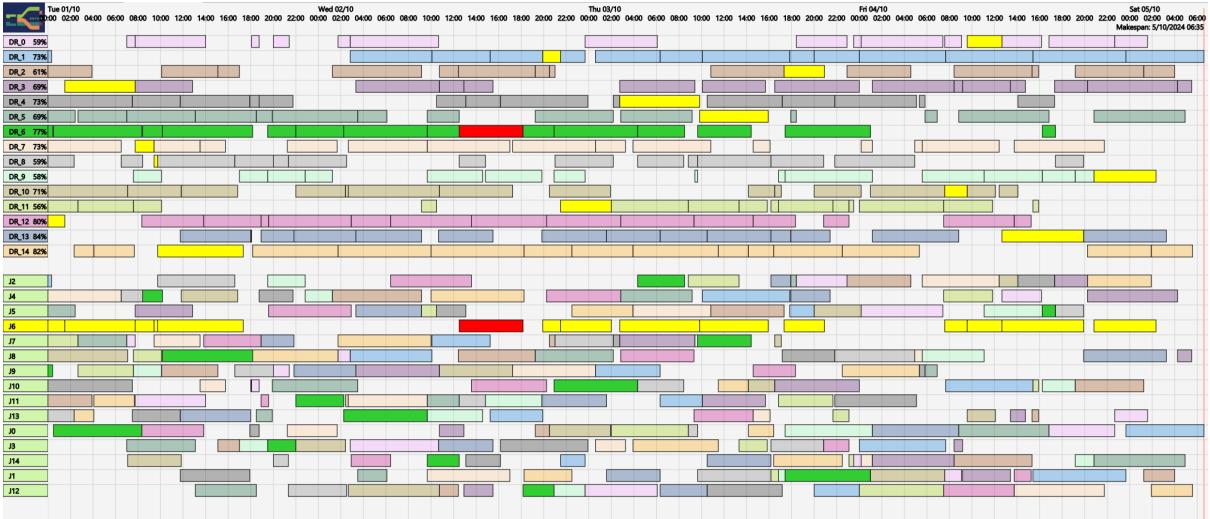
- Consists of a number of jobs and a number of machines
- Each job visits each machine, but possibly in a different order, depending on process
- Tasks of a job are linked as a precedence chain
- Objective is to minimize overall end, the *makespan*

Example Job-Shop Process



- Note that the order of machines visited is different for each process

Example Job-Shop Solution

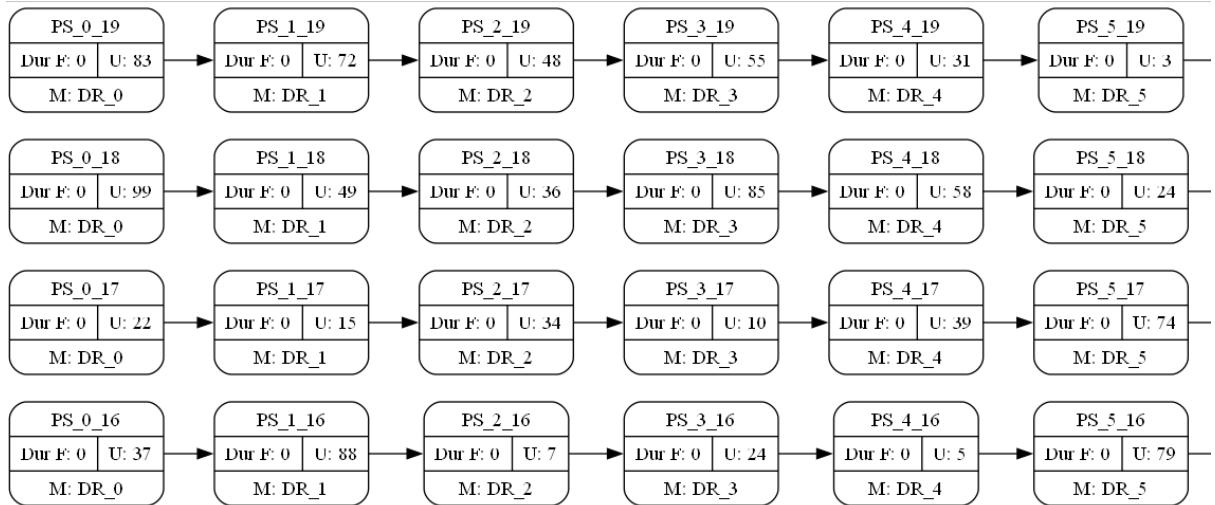


- One task is selected (in red), in both Machine and Job Gantt Chart
- Tasks are colored by machine, note coloring in jobs is different for each job

4.2 Flow-Shop

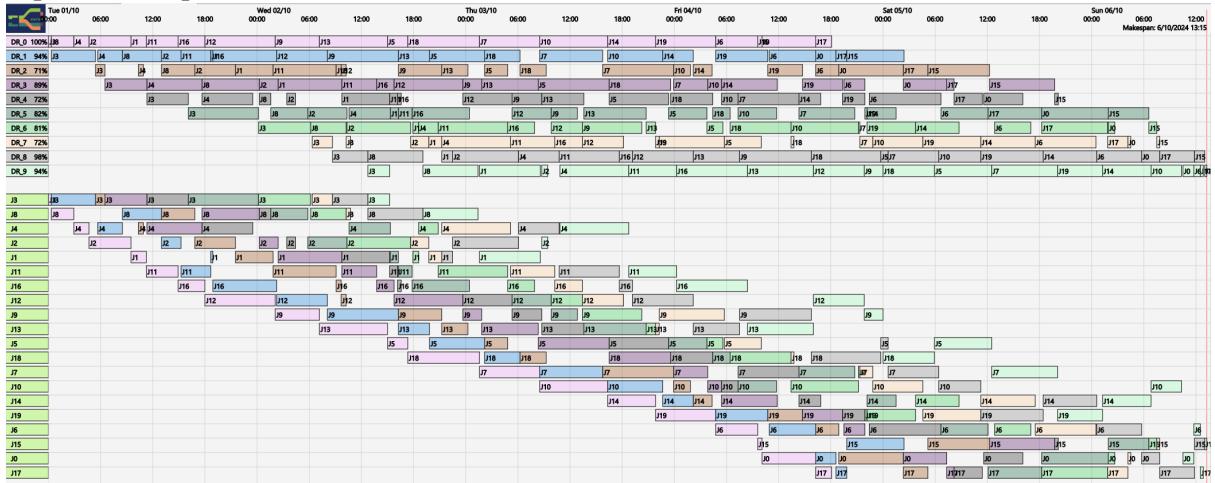
Flow-Shop ✓

- Consists of a number of jobs and a number of machines
- Each job visits each machine, all jobs in the same order
- Tasks of a job linked in a precedence chain
- Objective is to minimize overall end, the *makespan*



- Note that each process visits the machines in order DR_0, DR_1, ...

Example Flow-Shop Solution



- Tasks are colored by machine, note the regular pattern in the Job Gantt Chart

4.3 Open-Shop

Open-Shop ✓

- Consists of a number of jobs and a number of machines
- Each job visits each machine, we have to choose the sequence individually for each order
- There are no temporal constraints between tasks, but tasks of the same job cannot overlap
- Objective is to minimize overall end, the *makespan*

Open Shop Example Process

- Only showing details of one process
- No prescribed sequence between process steps

- Easier to find a task to run next
- Much larger search space

▼ Processes

process_0

process_1

process_2

process_3

process_4

process_5

process_6

PS_0_6	
Dur F: 0	U: 56
M: DR_4	

PS_0_5	
Dur F: 0	U: 92
M: DR_5	

PS_0_4	
Dur F: 0	U: 71
M: DR_0	

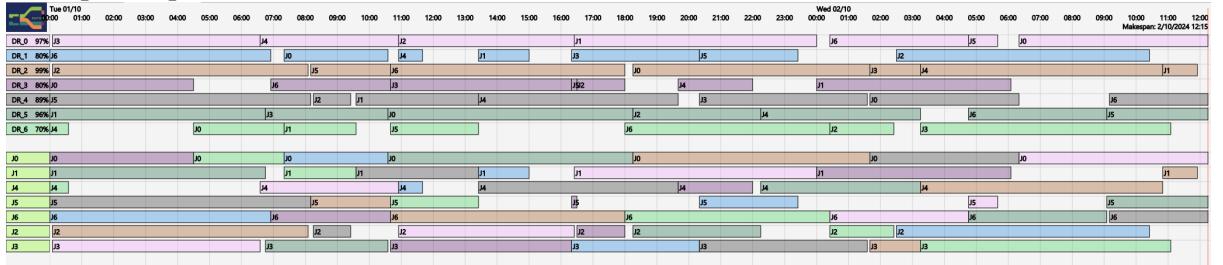
PS_0_3	
Dur F: 0	U: 34
M: DR_6	

PS_0_2	
Dur F: 0	U: 54
M: DR_3	

PS_0_1	
Dur F: 0	U: 39
M: DR_1	

PS_0_0	
Dur F: 0	U: 89
M: DR_2	

Open-Shop Example Solution



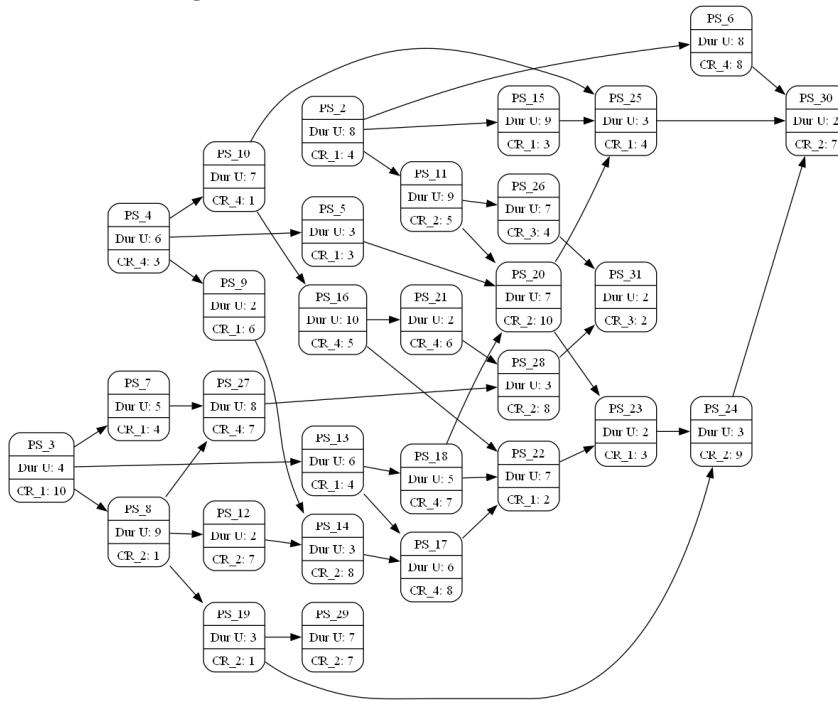
- Example solution for 7x7 open shop example
- Order of tasks within jobs not constrained
- Note that machines are still idle in optimal solution

4.4 RCPSP

Resource Constrained Project Scheduling Problem (RCPSP) ✓

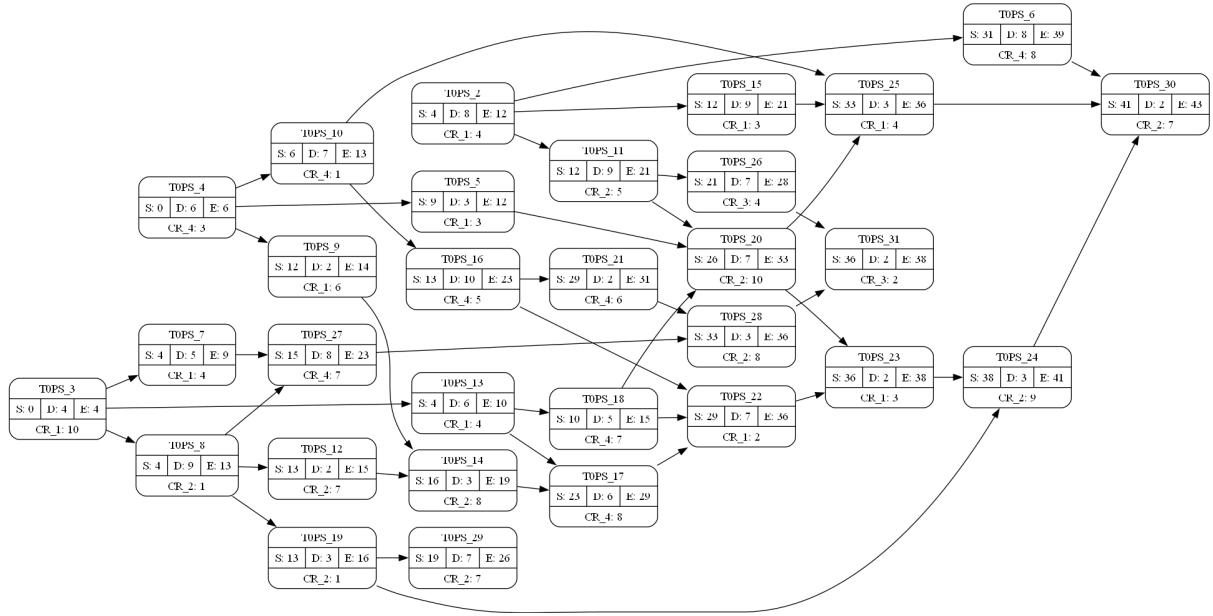
- Problem class from project management
- One project (one job), many tasks
- Precedence graph is arbitrary DAG
- Cumulative resources
- Variants with alternative modes, process alternatives ✗

RCPSP Process Diagram



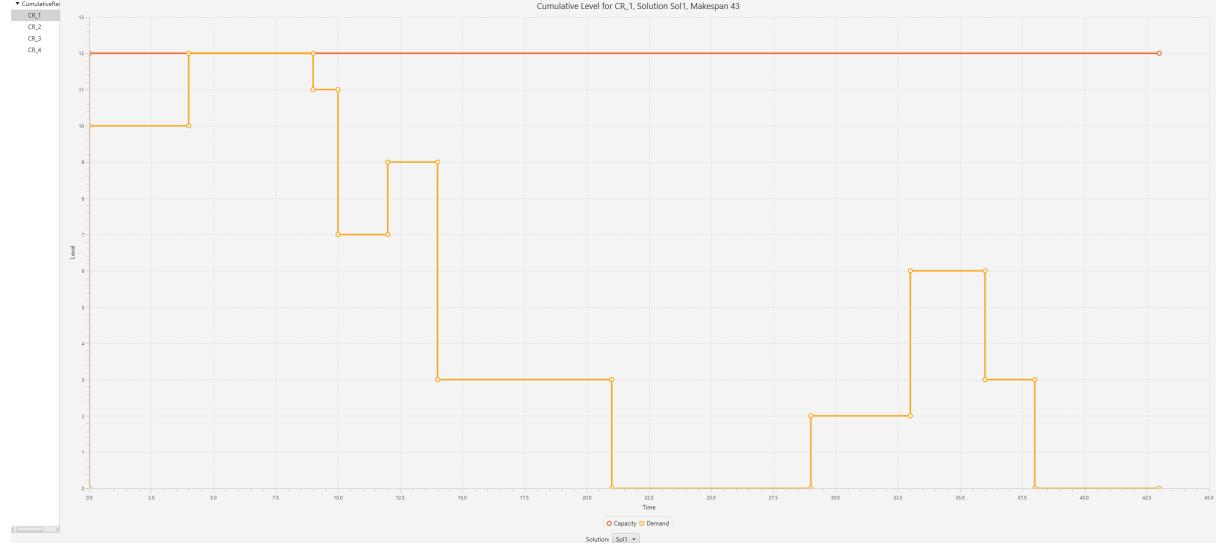
- Multiple sources, sinks
- Multiple cumulative resources

RCPSP PERT Diagram



- Resource constraints influence schedule, no single critical path

RCPSP Cumulative Chart



- Multiple cumulative resource, not busy all the time
- Not all instances are very hard to solve

4.5 $\alpha/\beta/\gamma$ Notation

$\alpha/\beta/\gamma$ Notation

- The previous classes are good for algorithm research, but not very practical
 - General scheme to describe problem type introduced in 1979
 - Based on three parameters
- α resource structure, stages

β temporal relations

γ objective

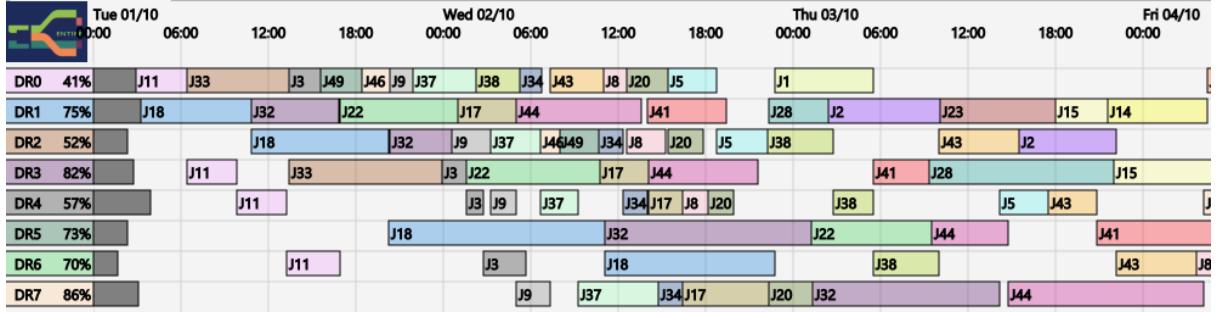
- $P2/r_j, \bar{d}_j/C_{\max}$: One stage, two identical parallel machines, hard release and due dates, objective makespan
- More detailed description at <https://encyclopedia.pub/entry/30497>

5 Key Visualization Methods

Visualization

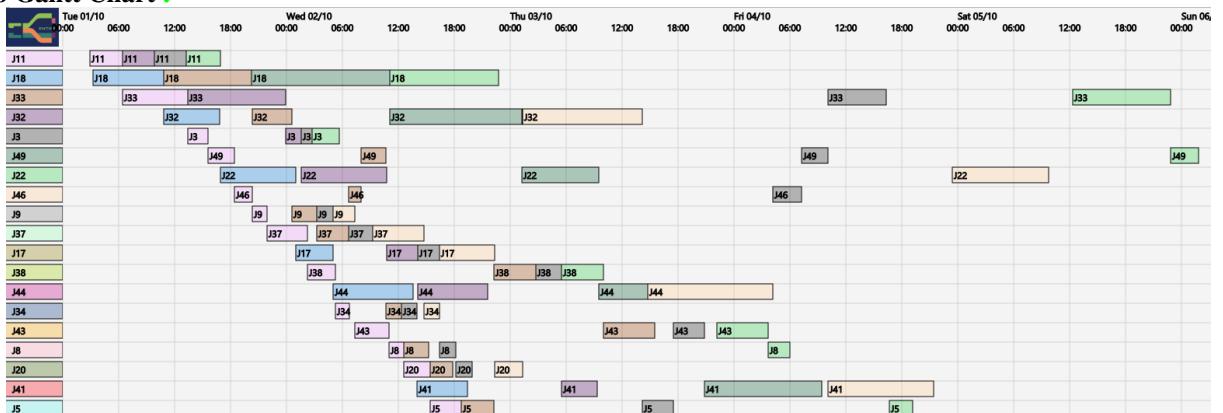
- Visualization is key to present and to understand results
- Many different ways to give an overview of schedule, and highlight problems
- Some diagrams types are used a lot, and are provided in our generic scheduling tool
- Customization is key

Machine Gantt Chart ✓



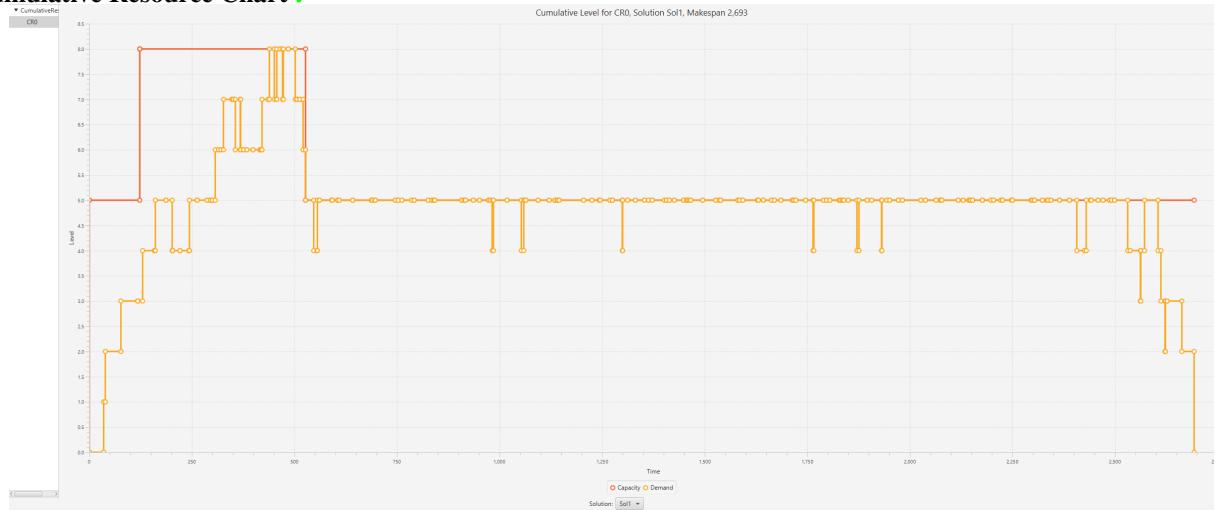
- Shows all tasks that are assigned to each machine
- Tasks should not overlap
- Also shows work in progress (WiP), down-times
- Optional display of setup and idle times

Job Gantt Chart ✓



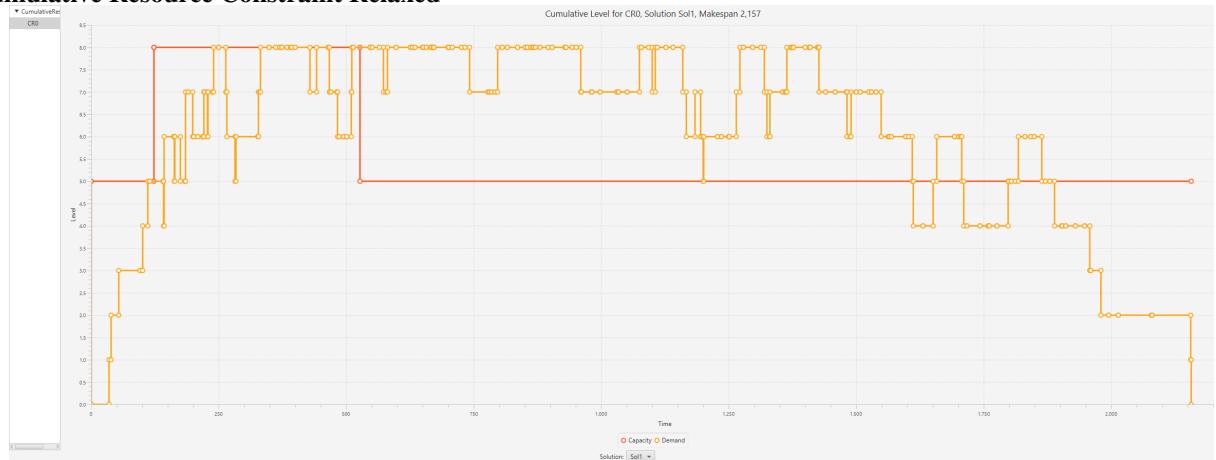
- Shows all tasks of a job in one line
- Only works for single chain of process steps
- Possible display of earliness, lateness
- Optional display of waiting and transport times

Cumulative Resource Chart ✓

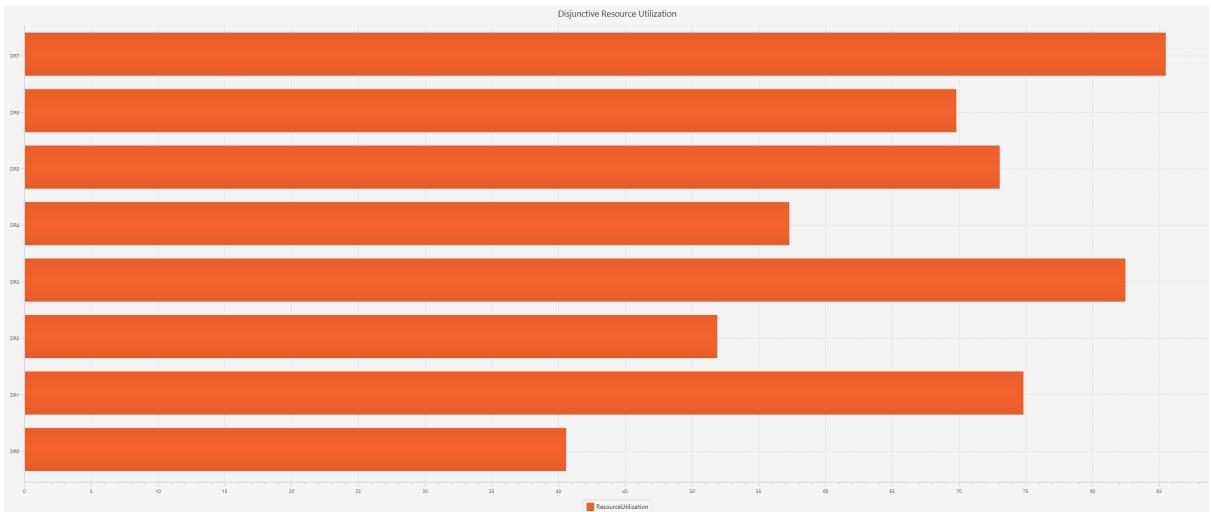


- Shows resource utilization of cumulative resource over time
- Utilization should be below capacity profile
- Unless we relax the cumulative resource constraint

Cumulative Resource Constraint Relaxed

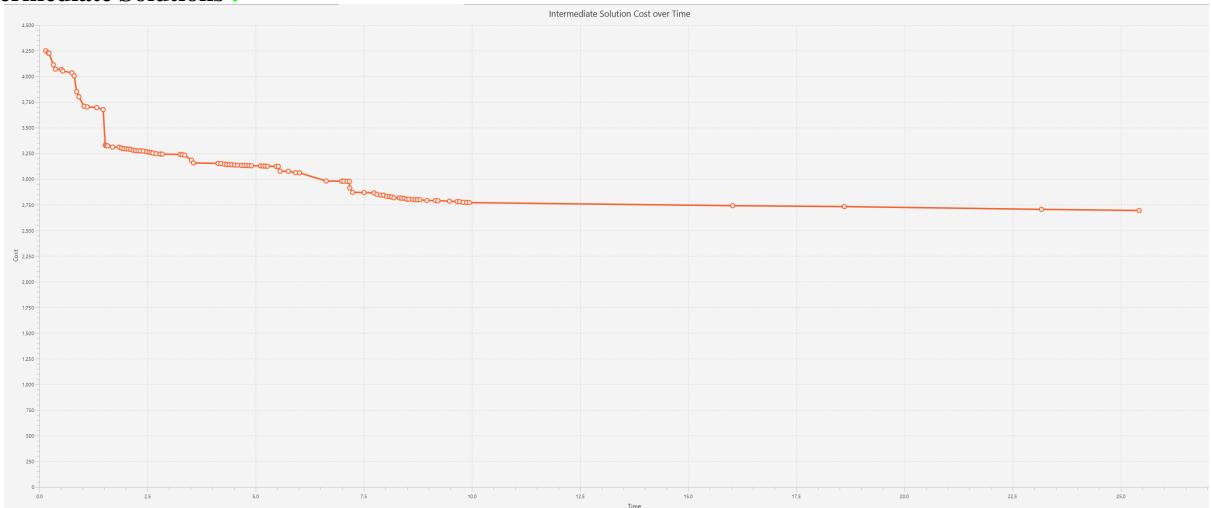


Resource Utilization ✓



- Shows utilization of machines as percentage of active time
- Helpful to identify bottleneck machines
- Information also shown in Machine Gantt

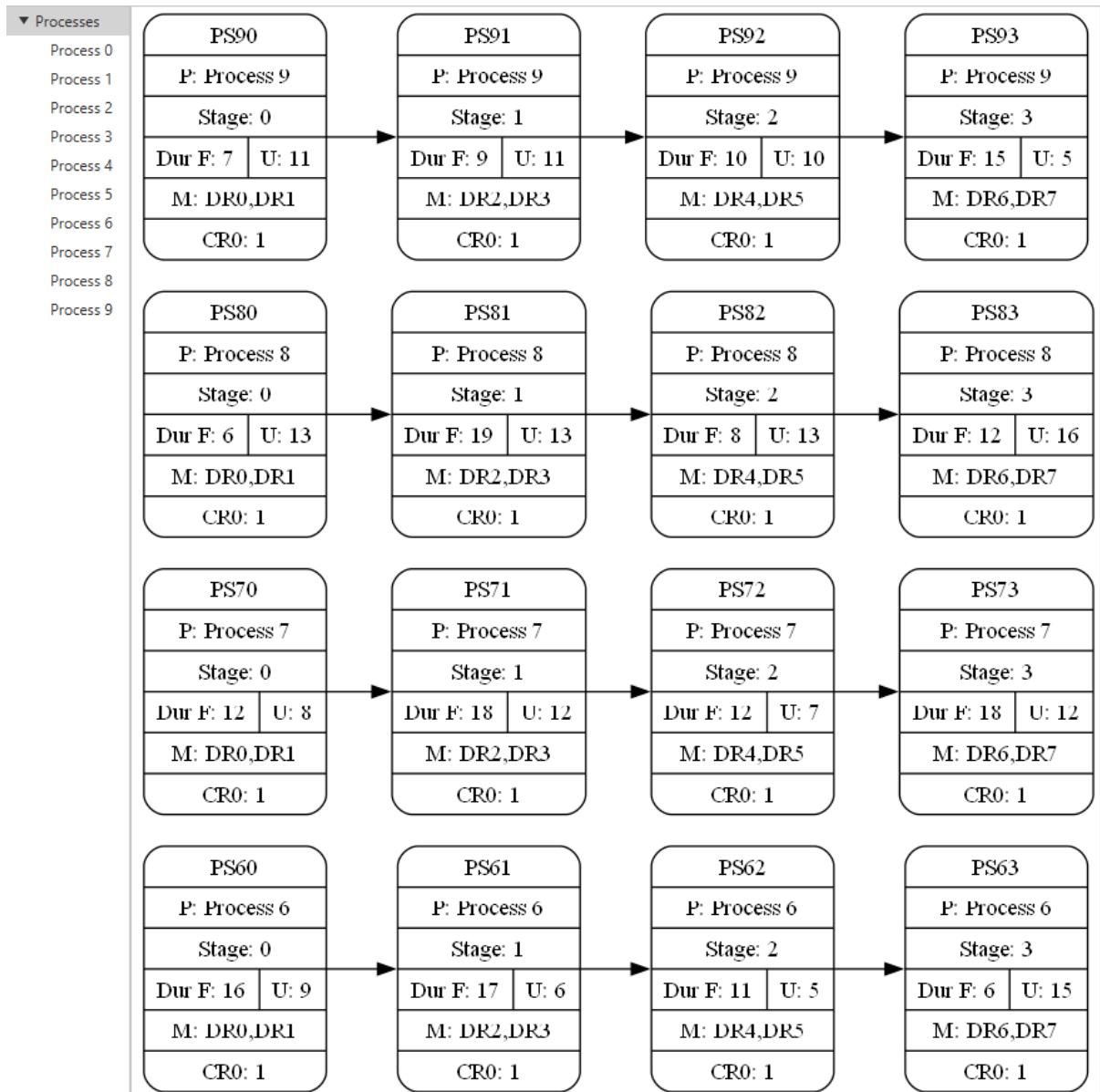
Intermediate Solutions ✓



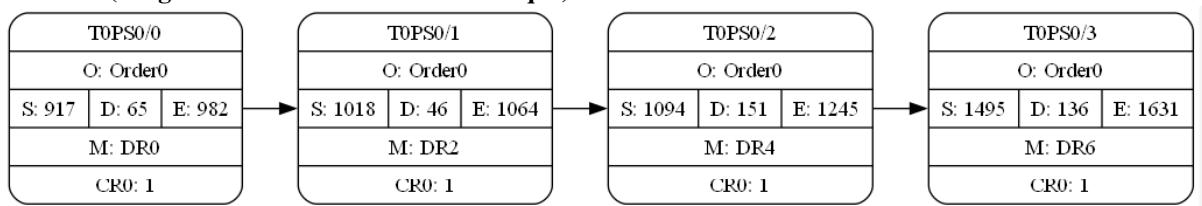
- Shows intermediate solutions found over time
- Useful to see if enough/too much time is allocated

Process Diagram ✓

- See all details of one process in one image
- Can also look at all processes in one diagram
- Options to show/hide different fields

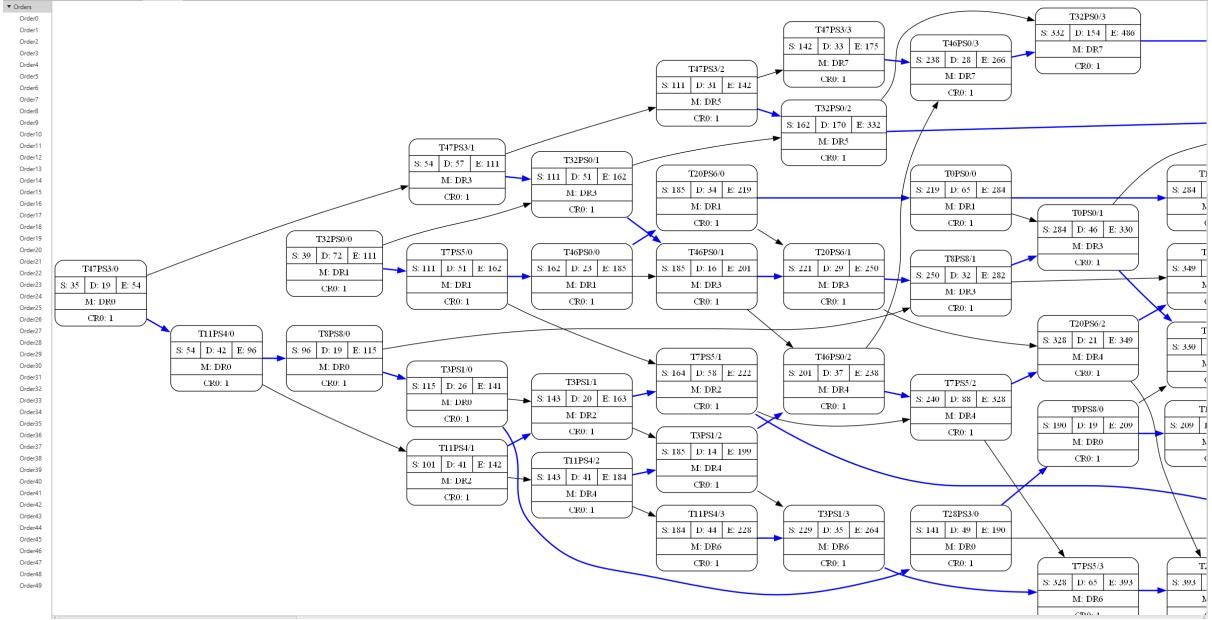


PERT Chart(Program Evaluation Review Technique) ✓



- Show details of job as a graph
- Useful if task graph is not a chain
- Often used in project management

PERT Charts become Confusing Quite Quickly



- Especially if all resource dependencies are included (in blue)

Calendars X

	Jan 22	Feb 22	Mar 22	Apr 22	May 22	Jun 22	Jul 22	Aug 22	Sep 22	Oct 22	Nov 22	Dec 22
Mon	3	3	3	3	3	3	3	3	3	3	3	3
Tue	3	3	3	3	3	3	3	3	3	3	3	3
Wed	3	3	3	3	3	3	3	3	3	3	3	3
Thu	3	3	3	3	3	3	3	3	3	3	3	3
Fri	3	3	3	3	3	3	3	3	3	3	3	3
Sat												
Sun												

- Shows weekly structure for one or more years
- Indicates public holidays, shut-downs, etc
- Indicating working days, KPI for each day

6 Summary

Summary

- We introduced the key concepts for scheduling problems
- Orders, products, processes
- Jobs and tasks
- Existing problem classifications
 - Academic
 - Limited practical usefulness
 - Used for benchmarking
- Key visualization ideas