Final

July 30, 2017

0.1 Regularized linear regression

In [27]:
```
import pandas as pd
import numpy as np


def linear_sol_constrain(x_in, y_in, lamb):
        sudo_inv_x_reg = np.linalg.inv(np.transpose(x_in).dot(x_in) + lamb * np.identit
        weight_reg = sudo_inv_x_reg.dot(y_in)
        return weight_reg

def one_vs_all(tar, train, lamb):
        Y = np.ones(train.shape[0])*-1 # set all to -1
        Y[train.dig == tar] = 1 # Y is the label
        X = train.iloc[0:, 1:] # X is the input
        ones = pd.DataFrame({'const': np.ones(train.shape[0])})
        X = pd.DataFrame.as_matrix(pd.concat([ones, X], axis=1))
        w = linear_sol_constrain(X, Y, lamb)
        return [w, X, Y]

def one_vs_one(tar1, tar2, train, lamb):
        ones = pd.DataFrame({'const': np.ones(train.shape[0])})
        X = train.copy()
        X.insert(1,'const', ones)
        X = X[X['dig'].isin([tar1, tar2])]
        Y = -1*np.ones(X.shape[0])
        Y[X.dig == tar1] = 1 # set up Y
        X = X.iloc[0:, 1:]
        w = linear_sol_constrain(X, Y, lamb)
        return[w, X, Y]

def error_est(x_in, y_in, weight):
        y_in_pred = np.sign(x_in.dot(weight))
        return np.mean(y_in != y_in_pred)

def n_l_transform(data_in):
        data_in_trans = data_in.assign(x3 = np.power(data_in['inten'],2))
        data_in_trans = data_in_trans.assign(x4 = np.power(data_in['symm'],2))
```

```python
        data_in_trans = data_in_trans.assign(x5 = data_in['inten']*data_in['symm'])
        return data_in_trans

def error_est_out(tar, test_nl, weight):
        X_out = test_nl.iloc[0:,1:]
        ones = ones = pd.DataFrame({'const': np.ones(test_nl.shape[0])})
        X_out.reset_index(drop = True, inplace = True)
        X_out = pd.concat([ones,X_out], axis = 1)
        Y_out = np.ones(test_nl.shape[0])*-1 # set all to -1
        Y_out[test_nl.dig == tar] = 1
        return error_est(X_out, Y_out, weight)

train = pd.read_excel('train.xlsx')
test = pd.read_excel('test.xlsx')
train.columns = ['dig','inten','symm']
test.columns = ['dig','inten','symm']

err = []
for i in range(5,10):
        sol = one_vs_all(i, train, 1)
        w = sol[0]
        X = sol[1]
        Y = sol[2]
        err.append(error_est(X,Y,w))
print(err)
```

```
[0.076258400768070222, 0.091071183651076665, 0.088465231106844053, 0.074338225209161987, 0.08832
```

Q7, 8 vs all has the lowest in sample error

In [24]:
```python
train_nl = n_l_transform(train)
test_nl = n_l_transform(test)
err = []
print(train_nl.shape)
for i in range(0,5):
        sol = one_vs_all(i, train_nl, 1)
        w = sol[0]
        X_in = sol[1]
        Y_in = sol[2]
        err.append(error_est_out(i, test_nl, w))
print(err)
```

```
(7291, 6)
[0.10662680617837568, 0.021923268560039861, 0.098654708520179366, 0.082710513203786751, 0.099651
```

Q8, as shown above, 1 has the lowest out of sample error

In [25]:
```
err = []
for i in range(0,10):
        sol = one_vs_all(i, train_nl, 1)
        w = sol[0]
        X_in = sol[1]
        Y_in = sol[2]
        err.append(error_est_out(i, test_nl, w))
res = pd.DataFrame({'with trans': np.array(err)})

err = []
for i in range(0,10):
        sol = one_vs_all(i, train, 1)
        w = sol[0]
        X = sol[1]
        Y = sol[2]
        err.append(error_est_out(i, test, w))
res = res.assign(without = np.array(err))
res = res.assign(without_95 = np.array(err)*0.95)

print(res)
```

```
   with trans   without  without_95
0    0.106627  0.115097    0.109342
1    0.021923  0.022422    0.021300
2    0.098655  0.098655    0.093722
3    0.082711  0.082711    0.078575
4    0.099651  0.099651    0.094669
5    0.079223  0.079721    0.075735
6    0.084704  0.084704    0.080468
7    0.073244  0.073244    0.069581
8    0.082711  0.082711    0.078575
9    0.088191  0.088191    0.083782
```

Q9, as shown above, for 5 vs all, transformation improves the performance but less than 5%

In [28]:
```
sol_transform0 = one_vs_one(1, 5, train_nl, 0.01)
sol_transform1 = one_vs_one(1, 5, train_nl, 1)
err_1 = error_est(sol_transform0[1], sol_transform0[2], sol_transform0[0])
err_2 = error_est(sol_transform1[1], sol_transform1[2], sol_transform1[0])
test_use = test_nl[test_nl['dig'].isin([1,5])]
err_1_out = error_est_out(1, test_use, sol_transform0[0])
err_2_out = error_est_out(1, test_use, sol_transform1[0])
print('in sample error, lambda = 0.01 vs lambda = 1')
print(err_1, err_2)
print('out of sample error, lambda = 0.01 vs lambda = 1')
print(err_1_out, err_2_out)
```

```
in sample error, lambda = 0.01 vs lambda = 1
0.00448430493274 0.00512491992313
```

```
out of sample error, lambda = 0.01 vs lambda = 1
0.0283018867925 0.0259433962264
```

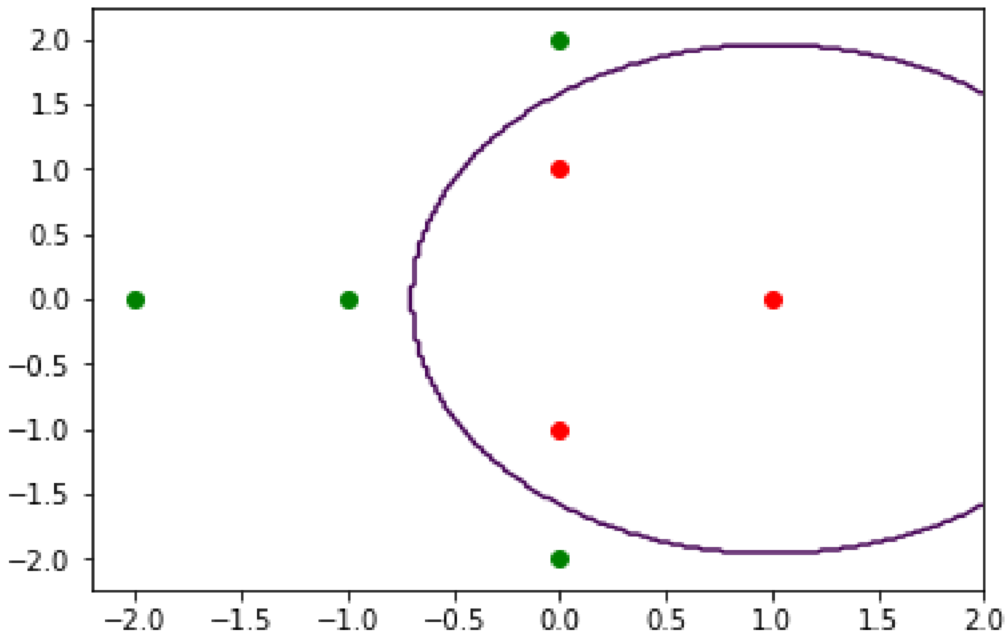
Q10, it can be seen that with lambda goes up, E_{in} goes up but E_{out} goes down, meaning that there is an overfitting happening.

0.2 SVM

```
In [2]: import pandas as pd
        import numpy as np
        from sklearn import svm
        import matplotlib.pyplot as plt

        indata = pd.DataFrame({'x1': [1,0,0,-1,0,0,-2], 'x2': [0,1,-1,0,2,-2,0],
        'y':[-1,-1,-1,1,1,1,1]})
        def mykernel(X,Y):
                return (1+np.dot(X,Y.T))**2
        clf = svm.SVC(kernel = mykernel, C = np.inf)
        clf.fit(indata.loc[:,['x1','x2']], indata.loc[:,'y'])
        print(clf.n_support_)
        s = plt.figure(1)
        plt.scatter(indata[indata.y == -1].loc[:,'x1'],
                indata[indata.y == -1].loc[:,'x2'], color='red')
        plt.scatter(indata[indata.y == 1].loc[:,'x1'],
                indata[indata.y == 1].loc[:,'x2'], color='green')
        x1 = np.linspace(-2,2,200)
        x2 = np.linspace(-2,2,200)
        X1,X2 = np.meshgrid(x1,x2)
        FX1 = np.ndarray.flatten(X1)
        FX2 = np.ndarray.flatten(X2)
        M = np.array((FX1,FX2))
        F = np.reshape(clf.predict(np.transpose(M)), np.shape(X1))
        plt.contour(X1,X2,F,[0])
        plt.show()
```

```
[2 3]
```



Q12, as shown above. n_support is 5.

0.3 RBF

```
In [34]: import numpy as np
         import pandas as pd
         from sklearn import svm
         from sklearn import metrics
         import matplotlib.pyplot as plt
         import sklearn.cluster as clt

         def targ_func(X): # X is numpy array organized as [x1,x2]
                 y = np.sign(X[:,1] - X[:,0] + 0.25*np.sin(np.pi*X[:,0]))
                 return y

         def training_set_generate(n):
                 training_set = np.random.uniform(-1.0,1.0,[n,2])
                 return training_set

         c = 0
         for i in range(0,10000):
                 train = training_set_generate(100)
                 y = targ_func(train)
                 clf = svm.SVC(C=np.inf, gamma = 1.5, kernel = 'rbf')
                 clf.fit(train, y)
                 E_in = 1 - clf.score(train, y)
```

```
            if(E_in > 0):
                c += 1
    print(c/10000)
```

```
0.0
```

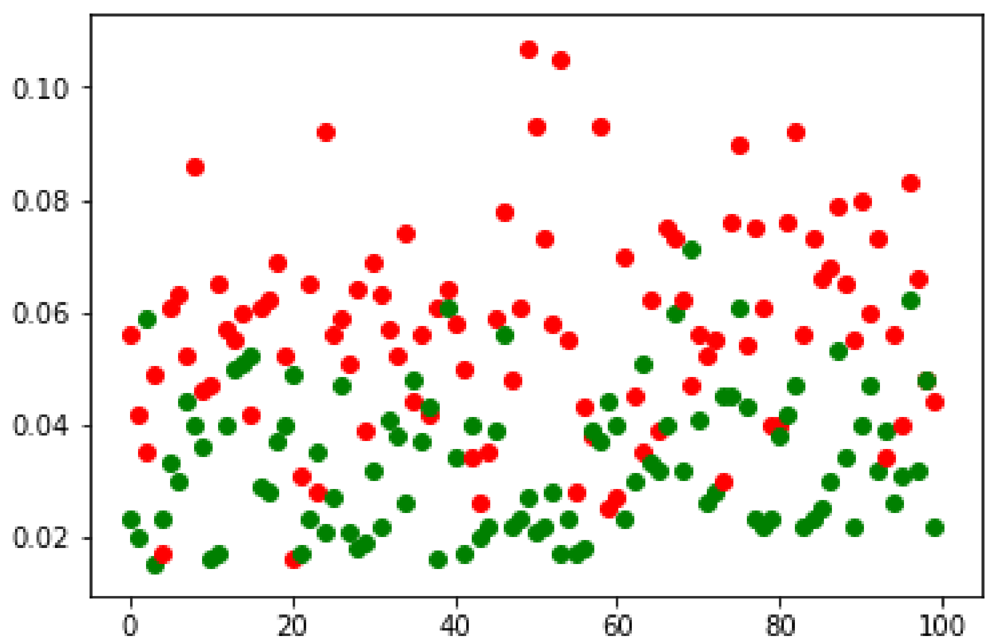
Q13, as shown above, svm with RBF kernel can always separate the sample.

```
In [35]: class RBF_regular:
             def __init__(self, K, gamma):
                 self.K = K
                 self.gamma = gamma
                 self.w = None
             def fit(self, train, y):
                 # step 1 find the centers
                 K = self.K
                 gamma = self.gamma
                 clust = clt.KMeans(n_clusters = K)
                 clust.fit(train)
                 centers = clust.cluster_centers_
                 # step2 build phi matrix
                 phi = np.empty([np.shape(train)[0], K])
                 for i in range(0, np.shape(phi)[0]):
                     for j in range(0,K):
                         phi[i,j] = np.exp(-gamma*np.sum((train[i,]-centers[j,])
                 # step3 calculate pseudo inverse
                 pseudo = np.linalg.pinv(phi)
                 w = np.dot(pseudo, y)
                 self.weight = w
                 self.centers = centers
             def predict(self, x):
                 w = self.weight
                 centers = self.centers
                 gamma = self.gamma
                 K = self.K
                 h = 0
                 for i in range(0,K):
                     h += w[i] * np.exp(-gamma*np.sum((x-centers[i,])**2, axis = 1))
                 return np.sign(h)
             def score(self, x, y):
                 y_pred = self.predict(x)
                 return metrics.accuracy_score(y_pred, y)

         c = 0
         s = plt.figure(1)
         for i in range(0, 100):
             # generate data sets
```

```
        train = training_set_generate(100)
        test = training_set_generate(1000)
        y_in = targ_func(train)
        y_out = targ_func(test)
        # svm
        clf = svm.SVC(C=np.inf, gamma = 1.5, kernel = 'rbf')
        clf.fit(train, y_in)
        E_out_svm = 1 - clf.score(test, y_out)
        # rbf regular
        rbf = RBF_regular(9,1.5)
        rbf.fit(train, y_in)
        E_out_rbf = 1 - rbf.score(test, y_out)
        # count the outperform times
        if(E_out_rbf > E_out_svm):
                c += 1
        # plot
        plt.scatter(i,E_out_rbf,c='red')
        plt.scatter(i,E_out_svm,c='green')
print(c/100)
plt.show()
```

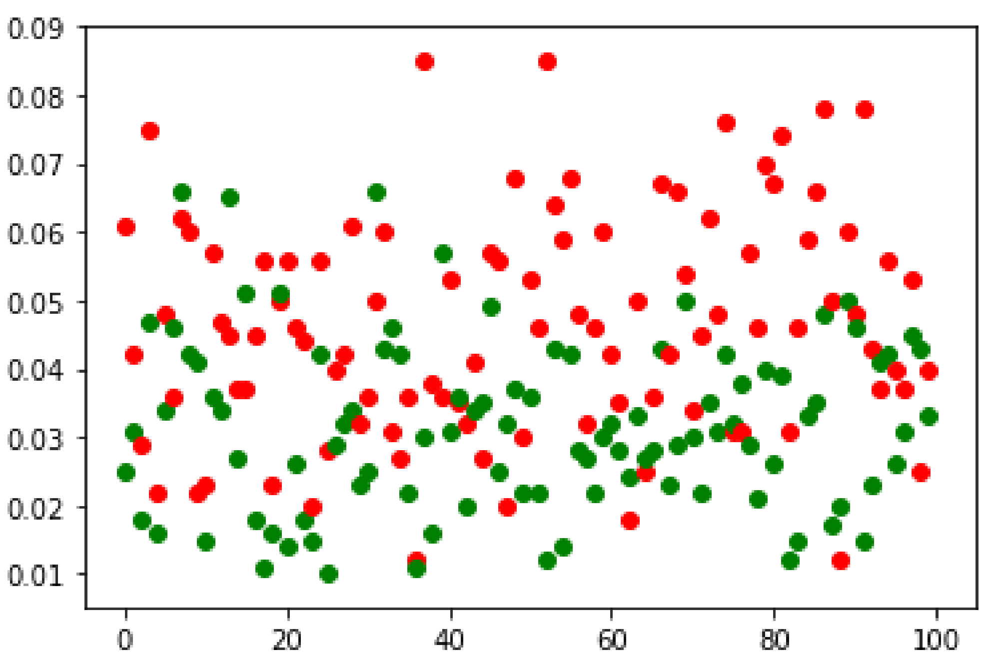
```
0.84
```



Q14, as shown above, in 100 experiments, 84% of the cases svm performs better than regular rbf method

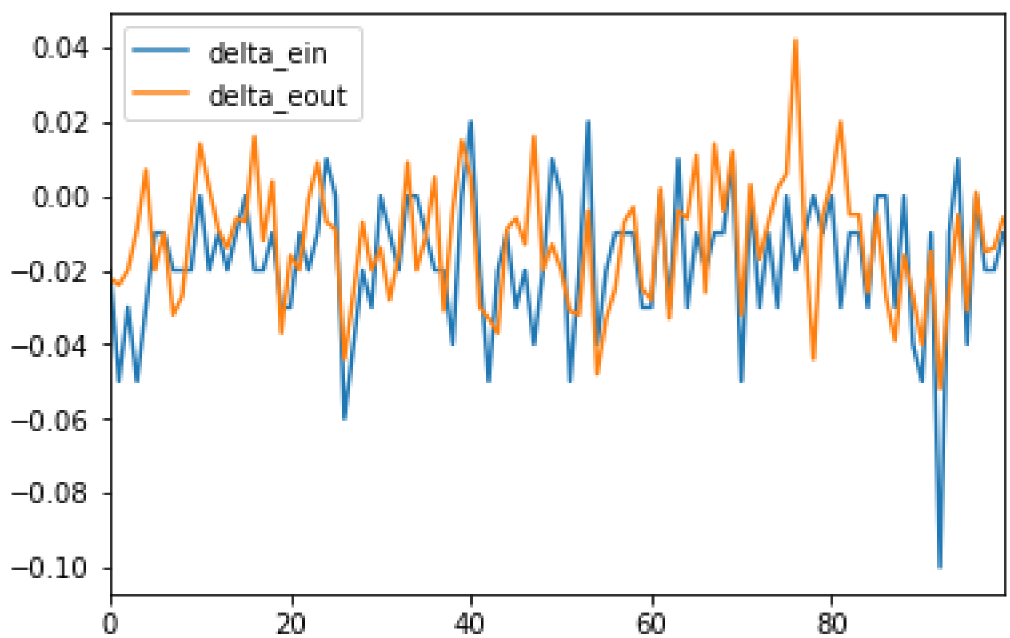
In [36]:
```python
c = 0
s = plt.figure(1)
for i in range(0, 100):
        # generate data sets
        train = training_set_generate(100)
        test = training_set_generate(1000)
        y_in = targ_func(train)
        y_out = targ_func(test)
        # svm
        clf = svm.SVC(C=np.inf, gamma = 1.5, kernel = 'rbf')
        clf.fit(train, y_in)
        E_out_svm = 1 - clf.score(test, y_out)
        # rbf regular
        rbf = RBF_regular(12, 1.5)
        rbf.fit(train, y_in)
        E_out_rbf = 1 - rbf.score(test, y_out)
        # count the outperform times
        if(E_out_rbf > E_out_svm):
                c += 1
        # plot
        plt.scatter(i,E_out_rbf,c='red')
        plt.scatter(i,E_out_svm,c='green')
print(c/100)
plt.show()
```

```
0.8
```



Q15, as shown above, increasing the centers will improve the performance of regular rbf cluster. 80% of the experiments svm performs better

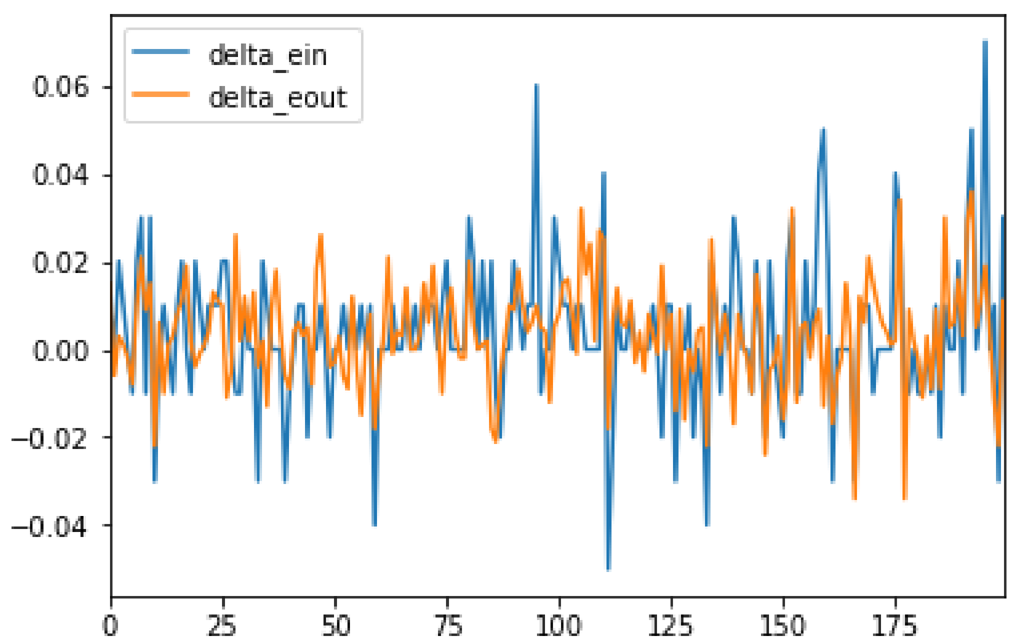
```
In [37]: err = pd.DataFrame({'delta_ein': np.empty(100), 'delta_eout': np.empty(100)})
         for i in range(0,100):
                 train = training_set_generate(100)
                 test = training_set_generate(1000)
                 y_in = targ_func(train)
                 y_out = targ_func(test)
                 ## cluster
                 rbf9 = RBF_regular(9,1.5)
                 rbf12 = RBF_regular(12,1.5)
                 rbf9.fit(train, y_in)
                 rbf12.fit(train, y_in)
                 e_in9 = 1-rbf9.score(train, y_in)
                 e_in12 = 1-rbf12.score(train, y_in)
                 e_out9 = 1-rbf9.score(test, y_out)
                 e_out12 = 1-rbf12.score(test, y_out)
                 err.iloc[i,0] = e_in12 - e_in9
                 err.iloc[i,1] = e_out12 - e_out9
         err.plot()
         plt.show()
```



Q16, as shown above, the curves are error_12 - error_9 for in sample and out-of sample errors. Most of the experiments, both delta values are negative meaning that from 9 to 12, both errors will go down.

In [38]:
```
err = pd.DataFrame({'delta_ein': np.empty(200), 'delta_eout': np.empty(200)})
for i in range(0,200):
        train = training_set_generate(100)
        test = training_set_generate(1000)
        y_in = targ_func(train)
        y_out = targ_func(test)
        ## cluster
        rbf15 = RBF_regular(9,1.5)
        rbf2 = RBF_regular(9,2)
        rbf15.fit(train, y_in)
        rbf2.fit(train, y_in)
        e_in15 = 1-rbf15.score(train, y_in)
        e_in2 = 1-rbf2.score(train, y_in)
        e_out15 = 1-rbf15.score(test, y_out)
        e_out2 = 1-rbf2.score(test, y_out)
        err.iloc[i,0] = e_in2 - e_in15
        err.iloc[i,1] = e_out2 - e_out15
print(err[err.delta_ein <= 0].shape[0]/200)
print(err[err.delta_eout <= 0].shape[0]/200)
err.plot()
plt.show()
```

```
0.58
0.335
```



Q17, as shown above. most of the cases, from gamma = 1.5 to gamma =2, E_{in} will go down while E_{out} will go up, suggesting an overfitting tendency

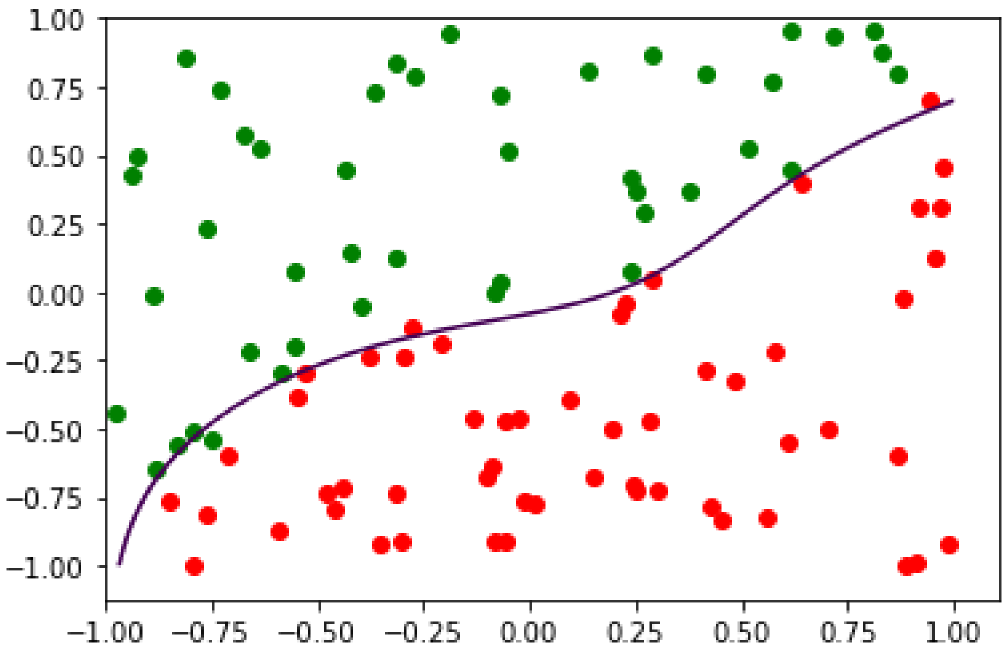
In [39]:
```
c = 0
for i in range(0, 1000):
        train = training_set_generate(100)
        y_in = targ_func(train)
        rbf = RBF_regular(9, 1.5)
        rbf.fit(train, y_in)
        score = rbf.score(train, y_in)
        if(score == 1):
                c += 1
print(c/1000)
```

```
0.023
```

Q18, as shown above, only 2% of the time the cluster will reach 0 E_{in}

In [43]:
```
## An example for RBF regular solver
f = plt.figure(0)
train = training_set_generate(100)
y_in = targ_func(train)
plt.scatter(train[y_in == 1,0],train[y_in == 1,1], c = 'green')
plt.scatter(train[y_in == -1,0], train[y_in == -1,1], c= 'red')
rbf = RBF_regular(9, 1.5)
rbf.fit(train, y_in)

x1 = np.linspace(-1,1,1000)
x2 = np.linspace(-1,1,1000)
X1,X2 = np.meshgrid(x1,x2)
FX1 = np.ndarray.flatten(X1)
FX2 = np.ndarray.flatten(X2)
M = np.array((FX1,FX2))
F = np.reshape(rbf.predict(np.transpose(M)), np.shape(X1))
plt.contour(X1,X2,F,[0])
plt.show()
```



The above code shows an example clustering process by RBF regular method.