Course Project for Statistical Inference by Hsin-Hua Lai

Overview

In this Course Project we will investigate the distribution of average 40 exponentials generated by a thousnad simulations and compare it with the Central Limit Theorem.

Simulation Logic

We follow the logic of the example code shown on the assignment website to first create a NULL vector called mns. We then generate 40 random exponentials with lambda = 0.2 and calculate its mean and append it to the NULL vector mns. We keep doing this for 10000 times and plot the histogram of the resulting data to check the Central Limit Theorem.

Simulation and Results

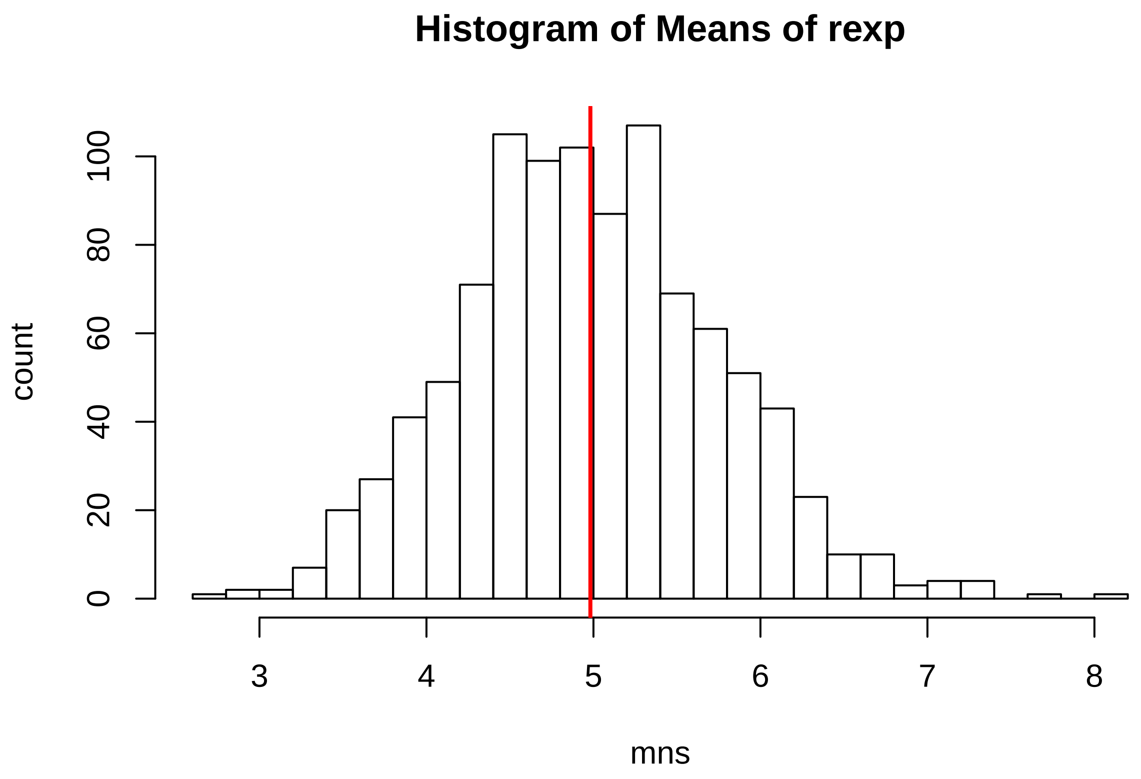
```r
## We first fix the value of lambda
lambda <- 0.2

## We create a Null vector called mns
mns <- c()

## For each i, we generate 40 data using exponential distribution with
## given lambda and calculate the mean of these 40 data and append it
## to the vector mns. We iterate it for 1000 times to generate a distribution.
for (i in 1:1000) mns <- c(mns, mean(rexp(40,lambda)))

## Let's first show the distribution and calculate the sample mean
hist(mns, xlab = "mns", ylab = "count", main = "Histogram of Means of rexp", breaks = 20)

## We add a verticle line to illustrate the sample mean
abline( v = mean(mns), col=2, lwd =2)
```



```
## The mean of the distribution gives the sample mean
samplemean <- mean(mns)
```

Therefore we can see that the Sample Mean is 4.9816477 pointed by the red vertical line, which is roughly equal to the Theoretical Mean 0.5.

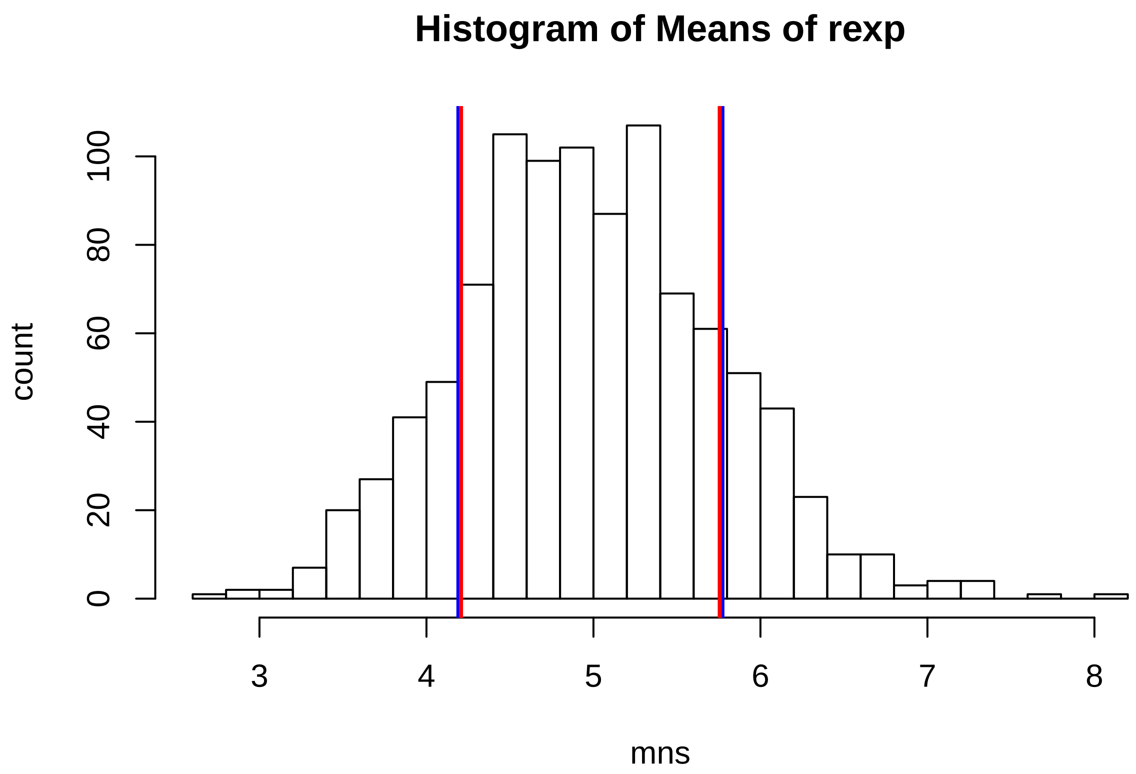
Sample Variance versus Theoretical Variance

We first use R to numerically extract the theoretical variance and the sample variance of the distribution

```
## The theoretical var can be obtained from the exponetial distribution
Tvar <- 1/(lambda^2)/40

## The sample var is
Svar <- var(mns)

## In order to compare Tvar and Svar using some plots, we can look at the
## standard deviation of the sample mean distribution which is
## basically the square root of the sample variance.
hist(mns, xlab = "mns", ylab = "count", main = "Histogram of Means of rexp", breaks = 20)
## illustrate the theoretical sample sd
abline( v = samplemean + sqrt(Tvar), col=4, lwd = 2)
## the sample sd
abline( v = samplemean + sqrt(Svar), col=2, lwd = 2)
## illustrate the theoretical sample sd
abline( v = samplemean - sqrt(Tvar), col=4, lwd = 2)
## the sample sd
abline( v = samplemean - sqrt(Svar), col=2, lwd = 2)
```



Again we can see that the Sample Variance 0.5998725 is nearly equal to the Theoretical Variance 0.625. The above histogram also include red lines and blue lines giving the locations of the sample mean standard deviation and the theoretical mean standard deviation, which almost overlap with each other.

Is the distribution approximately normal?

As discussed previously, we have obtained the sample mean and the sample variance (which gives sample standard deviation(Ssd)). We, therefore, first use the sample mean and the Ssd to plot the normal distribtion on top of the histogram and compare it with the normal distribution obtained using theoretical mean and theoretical sd (Tsd).

```
## Let's first replot the histogram
ht <- hist(mns, xlab = "mns", ylab = "count", main = "Histogram of Means of rexp", breaks = 20)

## The sample sd is as follows
Ssd <- sqrt(Svar)

## The theoretical sd is
Tsd <- sqrt(Tvar)

## Nowe we generate the normal distribution data using sample mean and Ssd
## We first generate sequence of data between samplemean +- 4 Ssd which
## should cover >99% data
xsample<- seq(samplemean - 4*Ssd, samplemean + 4*Ssd, length = 1000)

## generate the normal distribution
ysample <- dnorm(xsample, mean = samplemean, sd = Ssd)

## Rescale the y value
```

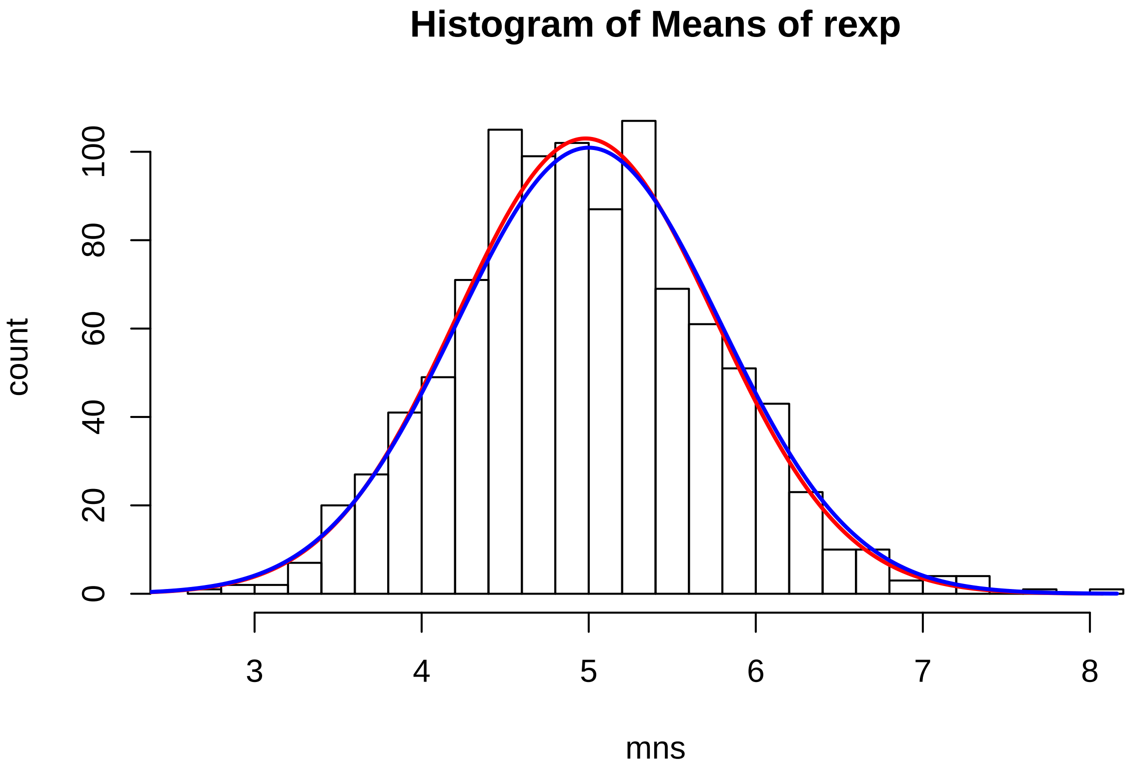
```r
## Below ht$mids gives the mid points of the bars and diff(ht$mids[1:2])
## gives the difference between the first two mid points which should be
## universal distance between nearby mid points
## We then multiply it by length(xsample) which tells that how many points
## are contained withing each difference between mid points
ysample <- ysample*diff(ht$mids[1:2])*length(xsample)

## Now we generate the normal distribution data using theoretical mean and Tsd
## Note that theoretical mean is 1/lambda = 5
xtheo <- seq(1/lambda - 4 *Tsd, 1/lambda + 4 * Tsd, length = 1000)

## general the theoretical normal distribution
ytheo <- dnorm(xtheo, mean = 1/lambda, sd = Tsd)

## Rescale
ytheo <- ytheo*diff(ht$mids[1:2])*length(xtheo)

## Now we plot both normal distributions based on sample mean data and theoretical analysis
lines(xsample,ysample, col =2, lwd = 2) ## sample normal
lines(xtheo, ytheo, col = 4, lwd = 2) ## theoretical normal
```



The red curve is obtained based on the sample mean data, including sample mean and sample variance or sample standard deviation. The blue curve is obtained purely based on the theory with theoretical mean and theoretical variance. We can see that for 1000 times of simulations, they match each other quite well, which gives confirms the central limit theorem.