

Poincaré conjecture

- ▶ Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.
- ▶ Proven by Grigori Perelman in 2003.
- ▶ Uses Richard S. Hamilton's Ricci flow.

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- ▶ Iwasawa theory with Euler systems (Kolyvagin–Rubin) is currently the most promising approach.
- ▶ Profound impact on number theory and cryptography; would clarify ranks of elliptic curves and reshape algorithms and security assumptions.

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- ▶ Motivic cohomology and derived algebraic geometry (Voevodsky–Deligne program) offer potential pathways.
- ▶ Would unify topology and algebraic geometry; enable algorithmic methods for classifying algebraic varieties and deepen understanding of the structure of complex manifolds.

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- ▶ The existence and smoothness of solutions in three dimensions is unknown.
- ▶ Most promising method: convex integration for constructing wild weak solutions (De Lellis–Székelyhidi), and scale-critical regularity via harmonic analysis (Koch–Tataru).

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- ▶ Key lines of attack: circuit lower bounds, barriers from natural proofs (Razborov–Rudich), and algebraic/geometric complexity (VP vs VNP).
- ▶ Consequences: if $P = NP$, many cryptosystems collapse; if $P \neq NP$, formal hardness underpins secure cryptography and explains intractability.