Poincaré conjecture

- Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.
- Proven by Grigori Perelman in 2003.
- Uses Richard S. Hamilton's Ricci flow.

Birch and Swinnerton-Dyer conjecture

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- Profound impact on number theory and cryptography; would clarify ranks of elliptic curves and reshape algorithms and security assumptions.

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- Motivic cohomology and derived algebraic geometry (Voevodsky–Deligne program) offer potential pathways.
- Would unify topology and algebraic geometry; enable algorithmic methods for classifying algebraic varieties and deepen understanding of the structure of complex manifolds.

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- Most promising method: convex integration for constructing wild weak solutions (De Lellis–Székelyhidi), and scale-critical regularity via harmonic analysis (Koch–Tataru).

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- Key lines of attack: circuit lower bounds, barriers from natural proofs (Razborov–Rudich), and algebraic/geometric complexity (VP vs VNP).
- Consequences: if P $=$ NP, many cryptosystems collapse; if P $\neq$ NP, formal hardness underpins secure cryptography and explains intractability.